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Concept of Probability in Sanskrit Texts on Classical Music

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Introduction

A classical music concert by a maestro, whether vocal or instrumental, either in the South Indian (Karnatic) style or in the northern (Hindustani) style turns out to be enjoyable and enchanting with the pleasant feeling remaining with a discerning listener for a long time. One wonders how transient groups of sounds without linguistic articulation, are made to convey some profound meaning, leading to satisfactory internal experience to a listener, even if it be called entertainment, so that this art form that literally builds castles in air has evolved as distinct from folk music and has sustained itself in this country over a period of two thousand years or more. It is usually pointed out that Indian classical music, known also as Rāga music in current parlance, has had an unbroken tradition of practitioners who have maintained the science of their art true to its original axioms but with many innovations and modifications over the centuries. Starting from Bharata's Nāṭyaśāstra (BNS) dated variously, but perhaps originating around 100 BCE, Sanskrit texts numbering nearly a hundred exist on the theory of music[1,2]. While some texts may repeat what a previous author had already said, there are authors questioning and differing from their predecessors to arrive at new findings. The stated aim of many texts has been to reconcile theory and practice, where practice refers to the performance of professionals who by their theoretical knowledge, training and innate intelligence explore now vistas within the boundaries of the tradition.

Almost all known treatises trace the origin of classical music to the chanting of the Sāmaveda. They also postulate one-to-one relation between the human body and the stringed instrument Vīṇā in sound production, a tenet adapted from the Aitareya Āraṇyaka (III.2.5)

[1] V. Raghavan, Some Names in Early Sangita Literature, (in 3 parts), *J of the Music Academy Madras*, 1932.
[2] M.S.Ramaswami Aiyar, Bibliography of Indian Music, *J. Royal Asiatic Soc. of Great Britain & Ireland*, No.3, pp.233-246, 1941.

belonging to the tradition of the Ṛgveda. These two are the foundational axioms or postulates accepted by all authors on Hindu music. The first tenet gradually led to the delineation of the seven *svara* (notes) and twenty-two *śruti* (ordered sound intervals) in an octave with special names and symbols. The second principle of similarity between the Vīṇā and the human body encouraged experiments with strings to clarify a variety of questions regarding human voice and vocal music, including theoretical understanding of *svara* and *śruti.* This tradition of experimenting with stringed instruments eventually culminated with the southern Sarasvatī Vīṇā and the northern Sitār that are too well known worldwide.

The texts generally narrate esoteric concepts about sound and state the theory of *svara*s separated by certain *śruti* intervals. Attempts at quantification or fixing of *svara* positions are made, but the authors largely depend on verbal descriptions and similes to bring out ideas that are abstruse but essential to the music. For example discussion on the difference between *svara* and *śruti* occupies considerable space and in some texts the explanations are quite confusing. Interestingly enough, while many authors propound that the *śruti*s in an octave are finite in number, Kohala's school holds that *śruti*s are infinite as noted in the text *Bṛhaddeśī* by Mataṅga (*c* 8th cent CE)[3]. It is easy to recognize that such diverse opinions are due to the fundamental question whether a line of finite length is to be described as a series of distinct points or is it adequate to take it as a sequence of sub-divisions. Several authors provide illustrative examples of their theory by providing *rāga* samples made out of *svara* symbols. These are actually short time series samples simulated by the authors with the full understanding that when sung the sound pattern will be continuous in time. How the internal composition of the various *svara*s relative to each other and to the whole, is to be handled by training, practice, intelligence and intuition to produce numerous pleasing *rāga* patterns with songs and accompaniments or extempore without any articulated song is the main subject matter of the texts.

A large technical vocabulary is developed in the texts to delineate enchanting creation of melodies in human voice and on the Vīṇā. These technical terms are like parameters that one introduces in making a mathematical model for a physical process, except that they are too many, vague, neither fixed nor quantifiable but expected to be internalized by the musician to be able to visualize beauty in such sound patterns and be capable of conveying the same to the audience. In the midst of this palpable uncertainty there is one precise word *dviguṇa* to

[3] आनन्त्यं तु श्रुतीनां तु दर्शयन्ति विपश्चितः। यथा ध्वनिविशेषाणामानन्त्यं गगनोदरे ॥ (*Bṛhaddeśī.* v. 29)

describe the doubling property of any *svara* from the lower to the upper register. In the present paper, we first discuss this *dviguṇa* property of the notes in successive registers. This is followed by a discussion on how uncertainty or unexpectedness is built into *rāga* representation through the often ignored probabilistic concept *alpatva, bahutva* in our ancient Sanskrit texts. The present study is an attempt at exploring scientific thought processes in the texts that helped our ancient musicologists to arrive at a theory for *rāga* music.

Sapta-svara **and** ***Dviguṇa***

The seven *svara*s with their well-known names and vocalization symbols are: Ṣadja (Sa), Ṛṣabha (Ri), Gāndhāra (Ga), Madhyama (Ma), Pañcama (Pa), Dhaivata (Da), Niṣāda (Ni); in the increasing order of pitch. The number count of seven is likely to have continued from the oral tradition of the Sāmaveda that uses largely five but occasionally six and seven *svara*s in some chants. The Vedic names for the *svara* are not only different but also arranged in the descending order as *Prathama, Dvitīya, Tṛtīya, Caturtha, Mandra*, corresponding to Ma, Ga, Ri, Sa, Da with the infrequent *Kruṣṭa* and *Atisvāra* equated with Ni and Pa in some places. It is noted that the starting *svara* in the sacred music is Ma in contrast with the *laukika* (worldly) music of BNS and other texts starting with Sa. A simple description of *sāmagāna* practice including variant traditions and opinions is available in *The Ragas of Karnatik Music* by Ramachandran[4]. The monograph by Prajñānānanda[5] and a recent article by Subhadrā Desai[6] also provide some preliminary information on the relation between *sāmagāna* and classical music.

Hymns of the Ṛgveda are chanted in three pitch levels, namely *anudātta* (low), *svarita* (middle) and *udātta* (high). This might have led to the five and later seven *svara*s of *sāmagāna* and also provided a model for the three registers of classical music. All traditional authors state that the same seven notes exist in the lower, middle and upper registers of human voice emanating from three levels corresponding with the chest, the throat and the head. The intriguing concept here is that of *dviguṇa* (double/twice) used to describe the relation between the *svara*s in the consecutive registers. The first clear statement of this

[4] N.S.Ramachandran *The Rāgas of Karnatik Music,* University of Madras, 1938
[5] Swami Prajnanananda, *A History of Indian Music Vol.1* Ramakrishna Vedanta Math, Calcutta, 1963.
[6] S. Desai, Vedic Chanting and its Relation to Indian Music, (published in 3 parts.) *The Vedanta Kesari*, July, August, September, 2014. Kolkata.

occurs in the *Saṅgīta Ratnākara* (SR) of Śārṅgadeva (12-13th cent)[7] esteemed to this day as an authority on classical music.

व्यवहारे त्वसौ त्रेधा हृदिमन्द्रोऽभिधीयते। कण्ठे मध्यो मूर्ध्नि तारो **द्विगुण**श्चोत्तरोत्तरः॥ SR (1.7)

The text has two famous commentaries namely, the *Sudhākara* of Simha-bhūpāla (14th cent CE)[8] and the *Kalānidhi* of Kallinātha (15th cent CE)[9]. The first commentator explains the above verse as

हृदि य उत्पद्यते नादः स मन्द्र इति कथ्यते; यस्तु कण्ठे उत्पद्यते स मध्यः; यस्तु मूर्ध्नि तारः । एषां मानं कथयति– द्विगुण इति । यावान्मन्द्रः ततो द्विगुणो मध्यः यावान्मध्यः ततो द्विगुणस्तारः ॥

He says sound produced in the chest, throat and head is known as *mandra*, *madhya* and *tāra* respectively. Their measure (*māna*) is stated to be *dviguṇa*. It is double that of the *mandra* in the *madhya*; and double that of the *madhya* in the *tāra sthāna*. However Kallinātha interprets this as the relative effort needed by a person to produce the sounds in the three registers. According to him,

.....द्विगुणः, गुणः उच्चारण प्रयत्नः.....अयमर्थः....द्विगुण–प्रयत्न–साध्यत्वाद् द्विगुण इति ।

There is no explanation on how the human effort required in a register could have been measured and found to be twice that needed in the lower register. Hence, we have to ignore this interpretation of *dviguṇa* as a passing opinion of the commentator. In any case he says that the *mandra-ṣaḍja* with twice the effort becomes the *madhya-ṣaḍja* and it is not a new note. He also further comments that when the octave is divided into 22-*śruti* intervals, the 23rd will be the *dviguṇa*[10]. Both the commentators and their predecessors have had the understanding that the same *svara* in the successive octave exhibits *the property of doubling*. Between the two commentators separated by nearly a century, the explanation of the former seems nearer to what the author of SR wants to convey. Somanātha in his *Rāgavibodha* (1609 CE) with auto commentary repeats previous authors on *dviguṇa* and emphasizes the sameness of the *svara* through an example that a person going from a lower level to an elevated place is

[7] Śārṅgadeva son of Soddhala was the Chief Accountant in the court of King Singhaṇa, who ruled at Devagiri in Maharashtra during 1210-1247 CE. His grandfather Bhāskara had moved out of Kashmir and migrated towards South India.

[8] Simha Bhūpāla a king belonging to the Racherla dynasty ruled at Rācakonda, in Andhra Pradesh *c* 1330 CE.

[9] Kallinātha was a scholar in the courts of King Vijaya I and of his successor King Devarāya II of the Vijayanagara Empire in the middle of 15th century.

[10] श्रुतिरूपनादविवक्षायान्तु त्रयोविंशो द्विगुणः ॥ (Kallinātha's commentary on SR I.7)

still recognized as the same individual[11]. The expression of frequency of vibration in terms of cycles per unit time is modern and it is known that the frequency of a given note (taken as a Sine wave) doubles in the immediate next octave. Hence the connotation *dviguṇa* to denote the interval measure of the octave, which is apt and precise, cannot be taken as a lucky coincidence. Beyond reasonable doubt, *dviguṇa* should have entered into the vocabulary of classical music through experiments with the stringed Vīṇā.

The Vīṇā

Almost all the texts have a section on the Vīṇā, representing stringed instruments. Starting from BNS it is taken as an accepted fact that the theoretical 22 *śruti*s in the three registers cannot be produced clearly by the human voice. Simha-bhūpāla in his commentary quotes a verse of Pārśvadeva (13th cent CE) to emphasize that a Vīṇā is necessary to demonstrate all the notes clearly[12]. Without going into all that is known and written about the evolution of string instruments from Vedic times we note that in the early period of BNS the single-string (*eka-tantrī*) and the bow shaped instrument with several strings must have been in wide use[13]. Bharata for his demonstration of the existence of 22-*śruti*s, invokes two identically tuned instruments one with fixed pitch positions and the other that can be changed further. How many strings these had? In the absence of direct evidence we have to infer that they must have had seven or more strings each. Abhinavagupta explaining the experiment of BNS says that the *pañcama-tantrī* has to be slackened[14]. This could mean either the fifth string or the one giving the Pa-*svara* had to be lowered by one *śruti*. Śārṅgadeva is quite clear that Bharata's experiment has to be carried out on two instruments each with 22 strings with their pitch in ascending order when played in the downward direction[15]. There is mention in BNS of the *matta-kokila vīṇā* that had 21 strings like a harp, seven for each of the three registers. Here, even if the tuning were to be done by the ear, since the width of the bow shaped instrument increases upwards, the length of the string associated with any *svara* from *tāra* to *madhya* and then to *mandra* should double sequentially. Or conversely, as in the human body the pitch is said to double upwards, in the case of the above Vīṇā the doubling of the pitch happens in the opposite direction. In the later stringed instruments with or without frets also

[11] यथा देवदत्तो नीचस्थलादुच्चस्थानगतोऽपि स एवेति प्रत्यभिज्ञायते तथेत्यभिप्रायः ॥

[12] ते तु द्वाविंशतिर्नादा न कण्ठेन परिस्फुटाः । शक्या दर्शयितुं तस्माद्वीणायां तन्निदर्शनम् ॥ (*Saṅgīta-samaya-sāra*)

[13] A.K.Coomaraswamy, The Parts of a Vīṇā, *J. of the American Oriental Society*, Vol. 50. (1930), pp. 244-253.

[14] वीणायामपरस्यां पञ्चमतन्त्री श्रुतिमात्रं शिथिलीकार्या तदा मध्यमग्रामो जायते॥ (*Abhinavabhāratī* Commentary on BNS)

[15] द्वे वीणे सदृशौ कार्ये यथा नादः समो भवेत् । तयोर्द्वाविंशतिस्तन्त्र्यः प्रत्येकं तासु चादिमा ॥
अधराधरतीव्राः तास्तज्ज्ञो नादः श्रुतिर्मताः । (SR I. 3. 11 & 13a)

the same condition applies. This is mentioned clearly by Kallinātha[16], Mahārāṇa Kumbha (1433-1468 CE)[17] and Dāmodara (1625 CE)[18] among several others. Authors starting from Bharata onwards knew that as the length of the freely vibrating part of the string reduces the *śruti* increases. This *viparyaya* or reverse order must have led to the word *dviguṇa* as reciprocal of half the length of the open string for hearing the *tāra*-Sa. That the sound of the open string can be considered as *Ṣaḍja,* is an ancient concept traced to Dattila (*c* 3rd cent CE) by Simha-bhūpāla[19]. Śārṅgadeva famed to have designed a new instrument *Niśśaṅka-vīṇā* known by his honorific *Niśśaṅka* meaning *Doubt-less*, must have taken the middle octave to be equivalent to the interval [1, 2]. He says as much in the third chapter of SR while describing the process of *ālāpana*, where he introduces the technical term *dvyardha* which literally means *one-and-half*. Kallinātha comments that since, relative to the *dviguṇa-svara* this note is at the middle it is called *dvyardha*. The very form of the word indicates this to be based on the Vīṇā referring to Ma if the physical length of the octave on the string is taken. If the *śruti* value were to be meant then *dvyardha* would refer to Pa.

Saṅgīta Pārijāta of Ahobala Paṇḍita (1665 CE) is the first known text to specifically state that at half the length of the open string the *dviguṇa ṣaḍja* arises. This work also provides the distances of the seven so called *śuddha-svara*s (pure notes) on the open string. Since theorization and experimental effort by Hindu musicologists is largely ignored by main stream historians of science in India, the relevant text is quoted here.

स्वरश्च हेतुभूताया वीणायाश्चाक्षुषत्वतः । तत्र स्वरविबोधार्थं स्थानलक्षणमुच्यते॥
ध्वन्यवच्छिन्नवीणायां मध्ये तारकसंस्थितः। उभयोः षड्जयोर्मध्ये मध्यमं स्वरमाचरेत्॥
त्रिभागात्मकवीणायां पञ्चमः स्यातदग्रिमे। षड्जपञ्चमयोर्मध्ये गान्धारस्यस्थितिर्भवेत्॥
सपयोः पूर्वभागे च स्थापनीयो रिस्वरः। सपयोर्मध्यदेशे तु धैवतं स्वरमाचरेत् ॥
तत्रांशद्वयसंत्यागात् निषादस्य स्थितिर्भवेत्॥ (*Sangīta Pārijāta* v. 314-318)

Ahobala remarks that on the Vīṅā the *svara* positions can be visually observed for better appreciation of their nature. The positions on the open string are given with the understanding that the free string produces the *madhya*-Sa. The positions are as follows: *tāraka* (upper-Sa)

[16] यथा शरीरे श्रुतयः उत्तरोत्तरोच्चा उत्पद्यन्ते तथा वीणायां अधराधरोच्चा उत्पद्यन्ते इति बोद्धव्यम् || Kallinātha on SR (3.12-14).

[17] Mahārāṇa Kumbha the Rājput King of Mewar is a great name in the political history of India. Besides waging wars, building forts and monuments he remains famous for his exhaustive encyclopaedic Sanskrit text of nearly 16000 verses on music, dance and *Rasa* titled *Saṅgīta-Rāja* and a commentary on the *Gīta-govinda* of the renowned poet Jayadeva.

[18] द्विगुणः पूर्वपूर्वस्माद् अयंस्यादुत्तरोत्तरः। एवं शरीरवीणायां दारव्यान्तु विपर्ययः॥ (*Saṅgīta-darpaṇa* 1.47)

[19] दत्तिलो हि स्वेच्छया यस्यां कस्यामपि श्रुतौ षड्जं स्थापयेत्तदपेक्षया च श्रुतिनियमेन अन्यस्वरान्स्थापयेदित्युक्तवान् । यदाह– षड्जत्वेन गृहीतो यः षड्जग्रामे ध्वनिर्भवेत् । तत ऊर्ध्वं तृतीयः स्याद् ऋषभो नात्र संशयः॥ (Comm. on SR I.4.15-16)

is at the centre of the string; Ma is at the midpoint between the middle and upper-Sa. If the string divided into three parts, Pa will be at the first one-third point (from the left). Gandhara (Ga) is located at the midpoint between Sa and Pa. By dividing the distance between Sa and Pa into three parts, Ri is placed at the first point from the left. Dhaivata (Da) is taken as the midpoint between Pa and the upper-Sa. By dividing the distance between Pa and the upper-Sa into three parts, Ni is placed after leaving two parts from Pa. He further gives the 22-*śruti* positions of unequal pitch intervals along with their traditional names, details of which can be read in the original[20]. The taut string of the Vīṇā is held at the two ends and plucked near the right support. The above statement of Ahobala translates into placing the frets for the middle register at (1, 8/9, 5/6, 3/4, 2/3, 7/12, 5/9, 1/2) of the full length of the string measured from the right end support to produce the seven *svaras* in their ascending order and then the *dviguṇa.* Since the fundamental natural frequency of a string is inversely proportional to its length the above values indicate the *śuddha svara* scale of Ahobala to be (1, 9/8, 6/5, 4/3, 3/2, 12/7, 9/5, 2). Naturally there are other ways of dividing the length to produce more number of differing notes which may or may not lead to a pleasant *rāga* scale. It is also evident that several ways of partitioning the string, holding the *dviguṇa* property as invariant was known to Bharata and his successors. But for some special reason they did not like to precisely discretize the octave into finite number of unconnected points, although the seven or the twenty one strings of the ancient Vīṇā were separated physically.

All the texts abound in discussions on how to combine, compare, discriminate, organize and accept or reject the *svara* pattern to generate innumerable melodies. What is glaringly evident in all the texts on music, starting from the Vedic period, is the symbiotic relation nurtured between the Vīṇā and the human voice. This experimental approach seems to have peaked during 1200-1700 CE. A philosophical question that has occupied the Hindu mind since ancient times has been whether the *svara* gamut is contrived artificially or is it something already existing in the cosmic *nāda* space only to be manifested through human voice? The genesis of this question is traceable to the metaphysical distinction made between *dhwani* and *nāda.* The former is the primordial all pervasive sound that is the cause of all creation[21]. A form of this *dhwani* known as *nāda* leads to music but it is again of five types. The *ati-sūkṣma-nāda* (most subtle sound) is in the Heart; the *sūkṣma-nāda* (subtle sound) is in the cave (of the Heart). The *avyakta-nāda* (non-manifest sound) is at the level of the jaw, while it

[20] *Saṅgīta Pārijāta* Printed and Published by R.S.Gondhalekar, Jagaddhitecchu Press. Pune, 1897

[21] ध्वनिर्योनिः परा ज़ेया ध्वनिः सर्वस्य कारणम्। आक्रान्तं ध्वनिना सर्वं जगत् स्थावरजङ्गमम् ॥ (*Bṛhaddeśī* v. 11)

is *vyakta* (manifest) at the throat centre and it is *kṛtrima* or artificial at the mouth[22]. The heart referred here is not the biological organ on the left side of the chest, but the Heart-chakra of the subtle body that is extolled in the Vedas[23]. The 22-*śruti*s of music are also conventionally stated to originate at this location due to 22-nodes present in the Heart-chakra that is perceptible in yogic meditation[24].

The question of artificiality of musical notes might have arisen due to academic interest also, since not all persons are capable of producing the musical *svara* sequence satisfactorily, but inarticulate sounds of some birds appear to follow a natural pattern if heard intently. *Nāradīyaśikṣā* traditionally revered as the Vedic ancillary text representing the interface between Vedic and *laukika* music, states that the peacock calls in *ṣaḍja* (Sa), the *krauñca* (heron) calls in *madhyama* (Ma) and the koel calls in *pañcama* (Pa)[25]. We need not dwell on this concept other than noting that Bharata, Mataṅga, and Śārṅgadeva cite this model whereas later authors particularly Rāmāmātya[26], Somanātha, Ahobala do not quote this, even though they were equally interested with the question of natural existence or otherwise of the musical notes. These three musicologists excelled in their experimentation, construction and classification of a variety of Vīṇā instruments. Rāmāmātya followed by Somanātha reports that in the four stringed instruments tuned as Sa-Pa-Sa-Ma in two registers some *svara*s are spontaneously excited even when the assigned frets are not pressed on the neighbouring string. The presence of the second and third harmonic corresponding to the *dviguṇa*-Sa and the Pa could be recognized aurally by plucking the open Sa-string and theorized to be *svayambhū* or self-generated. By a series of such arguments, Somanātha concludes all the notes in the octave to be natural and not of human creation or imagination. Besides recognizing the presence of higher harmonics in a *svara* sound, he also reports sympathetic vibration on neighbouring strings tuned to nearly the same *śruti.* The concept of *saṁvādi svara*s (consonant notes) known from Bharata's time, get a new interpretation nearer to the modern theory of vibration of strings by the experiments of Somanātha.

Śruti and *Svara*

[22] सूक्ष्मो नादो गुहावासी हृदये चातिसूक्ष्मकः। कण्ठमध्ये स्थितो व्यक्तः अव्यक्तस्तालुदेशके॥
कृत्रिमो मुखदेशे तु ज्ञेयः पञ्चविधो बुधैः । इति तावन्मया प्रोक्ता नादोत्पत्तिर्मनोहरा॥ (*Bṛhaddeśī* v.24-25)

[23] *Nārāyaṇa Sūkta* in the Taittirīya Āraṇyaka (10.13) of the Kṛṣṇa Yajurveda.

[24] तस्य द्वाविंशतिर्भेदाः श्रवणाच्छ्रुतयो मताः। हृद्यूर्ध्वनाडीसंलग्ना नाड्यो द्वाविंशतिर्मताः ॥ (*Saṅgīta Ratnākara* I.3.8)

[25] *Nāradīyaśikṣā*, Published by Srī Pītāmbarāpīṭha Samskṛta Pariṣad, Datiya,1964.

[26] Rāmāmātya grandson of Kallinātha is famous for his treatise on music *Svara-mela-kalānidhi* (*c* 1550 CE).

A point that has been vigorously discussed, in all the texts, is the difference between *śruti* and *svara.* Is *śruti* the cause of *svara* or the other way round? *Svara* are seven, but is there only one *śruti* or are there precisely 22-*śruti*s? Simha-bhūpāla says that there are 22 different types of *nāda* denoted as *śruti*, but also points out the confusion existing about the meaning of the word used[27]. Even though *śruti* and *svara* were held to be different, as Mataṅga says *svara*s get depicted always by *śruti*[28], theorists looked for physical demonstrations to discriminate the above two words and to clarify their meanings. Abhinavagupta (10th cent CE) the great philosopher from Kashmir in his illuminating commentary *Abhinavabhāratī* on BNS explains the above technical terms clearly referring to the stringed Vīṇā. According to him *svara* is that pleasant audible sound with *anuraṇana* emanating from excitation at the corresponding *śruti* position. *Śruti* is just the audible peculiarity arising immediately with sound and it is not any divisible part of sound[29]. Śārṅgadeva also refers to *anuraṇana* and defines *śruti* and *svara* (SR I.3.24) in the same way as Abhinavagupta.

Somanātha through his experiments with strings points out that *svaras* are not only self-emanating but have the property of *anuraṇana.* That is, with any *svara* there exists 'follower sound' which refers to presence of overtones and higher harmonics. On the other hand *śruti* is just any sound in the interval of the octave without *anuraṇana.* In the absence of a tuning fork to demonstrate *śruti* as the flat sound value due to a pure Sine wave, the above are to be considered as fairly clear explanations. The subtlety in this argument is due to the fact that the sound values of both are relative with respect to *madhya*-Sa taken as unity. Whereas *svaras* as dependent variables have nearly specified locations, *śruti* as the independent variable is just any point in the interval [½, 4] spanning the three registers and beyond. Ahobala points out that as per experts *śruti*s are abundant, placed at hair-tip interval, in the octave. It is recognized, he says, that there are 22 *śruti*s in the human voice and in the Vīṇā based on the Sa-Pa interval relation[30]. As we go through the texts, we find that essentially the numbers 7 and 22 are taken to be conventional to illustrate how *svara*s can be produced to a first approximation by dividing the octave, but the *nāda* of *svara* is not same as *śruti-nāda.* The *Nāradīyaśikṣā* figuratively explains that variation of *svara* over *śruti* cannot be known,

[27] तस्य नादस्य द्वाविंशतिसंख्यका भेदा भवन्ति । ते च श्रुतिसंज्ञया उच्यन्ते ।.....तत्र श्रुतेः एकत्वानेकत्व विषये महती विप्रतिपत्तिः॥

[28] षड्जादयः स्वराः सप्तव्यज्यन्ते श्रुतिभिः सदा । अन्धकारस्थिता यद्वत् प्रदीपेन घटादयः ॥ (*Bṛhaddeśī* v. 36)

[29] वयं तु श्रुतिस्थानाभिघातप्रभवशब्दप्रभावितोऽनुरणनात्मा स्निग्धमधुरः शब्द एव स्वर इति वक्ष्यामः ।... ... श्रुतिश्च नाम श्रोत्रगम्यं वैलक्षण्यं यावता शब्देनोत्पद्यते । ... ... शब्दावयवो न श्रुतिः इत्युक्तमेव ॥ (*Abhinavabhāratī* Commentary on BNS)

[30] केशाग्रव्यवधानेन बहव्योऽपि श्रुतयः स्मृताः। वीणायांच तथा गात्रे संगीतज्ञानिनां मते॥
मध्ये पूर्वोत्तराबद्धवीणायां गात्र एव वा । षड्जपंचमभावेन श्रुतिर्द्वाविंशतिं जगुः ॥ (Saṅgīta-pārijāta v. 42, 43)

but the text prescribes that the transit from *svara* to *svara* should be smooth like the movement of shadow and sunlight[31]. This is as good as a mathematical condition that a *svara* must vary as a continuous function of *śruti*. The principle of similarity between the human body and the stringed Vīṇā instrument goes deep back into the Vedic period as pointed out in the beginning of this article. Musicologists have explored this doctrine further since this is all about the tonal quality of the voiced *svara* that is the basic ingredient of *rāga*. What was suspected to exist in voiced *svara* was confirmed by them on the string so that through a feedback the two modes of music could get mutually enriched. Modern science helps us to visualize the presence of overtones and higher harmonics through frequency domain analysis. As an aid to follow the previous discussion, the frequency spectrum of three *svara*s as sung by a musician is shown in Fig. 1[32].

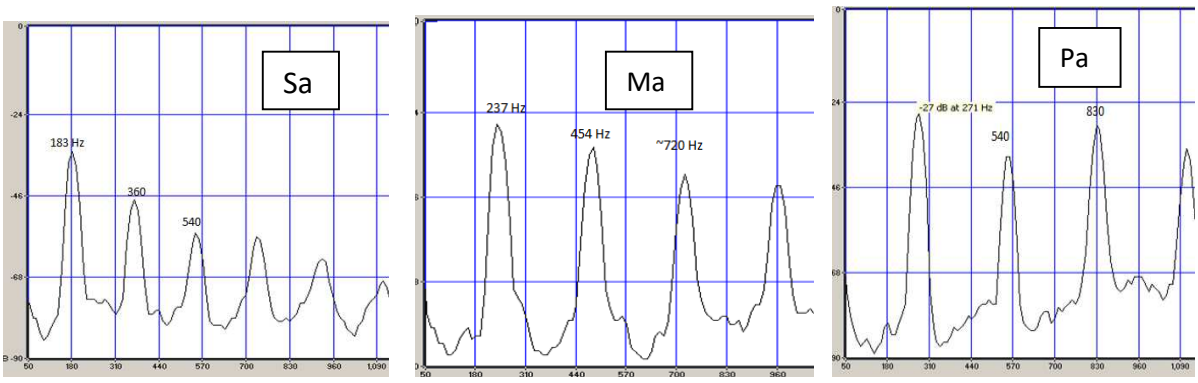

Fig1. Spectrum of voiced *svara* Sa, Ma and Pa. The *svara* are very nearly centred on Sa = 180 Hz; Ma=240 Hz; Pa=270 Hz. Observe that in the first graph of Sa higher harmonics namely, *dviguṇa*-Sa and *dviguṇa*-Pa are present. The peaks are not spiked and pointed like arrow heads. Sound due to a narrow band of frequencies around the corresponding primary *śruti* and also due to *dviguṇa* harmonics would be present in any good quality *svara*. The term *śruti* can be taken as non-dimensional proxy for the modern frequency.

Rāga Characterization

Indian music from the time of Bharata or earlier has had syllabic symbols for the seven notes as *Sa, Ri, Ga, Ma, Pa, Da, Ni* that can be vocalized with or without the pronunciation of the vowels and also produced on the string. A select number of *svaras* out of the seven pure and twelve *vikṛta svaras* are arranged and combined in myriad ways leading to thousands of *tāna, mūrchana, alaṅkāra* and still other patterns for voice training and practice on the Vīṇā. Exposition of this part is invariably based on sophisticated mathematical methods of

[31] स्वरात्स्वरसंक्रमस्तु स्वरसन्धिमनुल्बणम्। अविच्छिन्नं समं कुर्यात् सूक्ष्मं छायाऽऽतपोपमम्॥ *Nāradīyaśikṣā* (I. 6.18)

[32] This is taken from the M.Sc Dissertation, *Spectral Analysis of Gamaka Svaras* by Karuna Nagarajan, SVYASA University, 2006. Figures personally communicated by Dr.Karuna Nagarajan.

permutation and combination for forming chains of *svara*s of varied length and hence cannot be precisely described. But, any definition or description of *rāga* will be as incomplete as one can possibly describe in words the flowing river, the blowing wind or the thunder storm. A very general statement due to Mataṅga is that *rāga* is a special imaginative sound sequence that is aesthetically pleasing and melodious to captivate the minds of the listeners[33]. However, musicologists with keen hearing and long observation have found several special normative features as essential for depicting any *rāga* out of a selection of *svara*s.

Bharata lists ten general characters for what he denotes as *Jāti* (genus) the forerunner of the later *rāga* of Mataṅga and Śārṅgadeva. The ten characters considered fundamental to describe a particular *rāga* and also to discriminate one from another (*i.e.* with a given name) are *graha, amśa, nyāsa, apanyāsa, mandra, tāra, ṣāḍavita, auḍuvita,* and *alpatva, bahutva*. All the ten points with further divisions and subdivisions are described in SR and the two commentaries in great detail. The first four characters are about the starting, major, intermediate and ending notes. The next four refer to the lower and upper registers and six- and five-*svara* possibilities. The last two are the most intriguing importance of which is missed in literal translations.

Alpatva-Bahutva

Rāga depiction can happen either with a song already composed or extempore without any words. The former is the *nibaddha* (*kalpita*) music while the latter is the *ālāpana*. For any *Rāga* with a defined *āroha-avaroha* (scale), its individuality is infused through the concept of *alpatva* and *bahutva* of the *svara*s. Except for the *alpatva-bahutva* characteristic, the other eight listed above can be classified as in the case of *tāna, mūrcchana, alṅkāra* that are deterministic *svara* groups[34]. The first *jāti-rāga* named *Ṣāḍjī* exemplified through a song with *svara-prastāra* appears in the seventh section of the first chapter of SR. The notes of the song of twelve lines to be sung row by row are shown here in Table 1. Kallinātha, in his commentary explains that this example is useful in understanding the property *alpatva-bahutva*.

अत्र स्वरसंख्या अल्पत्वबहुत्व परिज्ञानाय लिख्यते । षड्जाः षट्त्रिंशत्; ऋषभा द्वादश; गान्धारा विंशतिः; मध्यमा अष्टौ; पंचमा अष्टौ; धैवताः षोडश; निषादा द्वादशेति मिलिता द्वादशोत्तरं शतम्॥

[33] योऽसौ ध्वनिविशेषस्तु स्वरवर्ण विभूषितः। रञ्जको जनचित्तानां स रागः कथितो बुधैः ॥

[34] A table is available for the 18-Jāti group with name, *aṁśa, nyāsa, apanyāsa, mūrcchana, ṣāḍava, auduva* in the book *Sangeeta Ratnakaram: A Study* by R.R.Ayyangar, Wilco Publishing House, 1978, Bombay.

He shows the relative frequency of the *svara*s by counting their occurrence in the table as Sa=36; Ri=12; Ga=20; Ma=8; Pa=8; Da=16 and Ni=12; the sum total is 112. He does not differentiate between long and short notes and the registers. He also points out that the *aṁśa* (important) *svara* Sa is the most frequent. This art of mixing *svara* in different proportions to derive melodies goes back to Bharata. Śārṅgadeva in the first chapter of SR presents illustrative examples, like the above, for all the 18 *jāti-rāgas* of BNS demonstrating the *alpatva-bahutva* property as explained by Kallinātha. It is easy to see from the above example the ratio of the counts to the sample size 112 is, in modern data analysis, an estimate for the probability of occurrence of the seven *svara*s. Here, the *svara* is seen to be treated as a discrete random variable with seven possible outcomes with different probabilities. Starting from such elementary discrete structures, more complex compositions are described and transition to continuous variation of *alpatva-bahutva* property is achieved in *rāgālāpana*.

Table 1. Swara Prastāra of song in the Rāga Ṣāḍjī

[Lower dot: *Mandra*; Upper dot: *Tāra*; others: *Madhya*]

Sā	Sā	Sā	Sā	Pā	NiDa	Pā	DaNi
Rī	GaMa	Gā	Gā	Sā	RiGa	DaSa	Dā
RiGa	Sā	Rī	Gā	Sā	Sā	Sā	Sā
Dā	Dā	Nī	NiSa˙	NiDa	Pā	Sā˙	Sā˙
Nī	Dā	Pā	DaNi	Rī	Gā	Sā	Gā
Sā	Dā.	Da.Ni.	Pā.	Sā	Sā	Sā	Sā
Sā	Sā	Gā	Sā	Mā	Pā	Mā	Mā
Sā	Gā	Mā	DaNi	NiDa	Pā	Gā	RiGa
Gā	Gā	Gā	Gā	Sā	Sā	Sā	Sā
Dā	Sā	Rī	GaRi	Sā	Mā	Mā	Mā
Dā	Nī	Pā	DaNi	Rī	Gā	Rī	Sā
RiGa	Sā	Rī	Gā	Sā	Sā	Sā	Sā

In the second chapter of SR more than two hundred *Rāga*s are described some of them with elaborate examples. A major contribution of Śārṅgadeva is the illustration of the structure of several *rāgas* through songs and direct *ālāpana,* as a succession of sounds arising out of different groupings of *svara*s with the property of *alpatva-bahutva* in a pleasant manner. These examples are actually long sample data showing the *svara*s to be used in a particular *rāga* and their proportion and mix. SR does not report any quantified proportions, but that

can be easily estimated from the samples as explained above by Kallinātha. There are further divisions to the ten principal characters of *rāga* and many more delicate features are described in SR to introduce continuous variability in a nuanced fashion in *rāga* presentation. Only a few such artifices can be noted here. *Alpatva* is of two kinds known as *langhana* and *anabhyāsa*[35]. The first is to leap over a *svara* whereas the latter is infrequent use. Popularly *langhana* is interpreted as skipping a *svara.* But even in the pentatonic and hexatonic scales, transitions in the *rāga* are continuous. Hence, Śārṅgadeva is careful to define *laṅghana* as *īṣat-sparśa* that is infinitesimally vanishing touch most probably observed with deleted *svara*[36]. Kallinātha explains this further as the absence of (*svara*) form produced by effort[37]. The second type of *alpatva* is *anabhyāsa* that is infrequent use. Hence *alpatva* is better translated as *low probability*. The opposite of the above is *bahutva* with *alanghana* and *abhyāsa* that refer to emphasis and frequent use of particular *svara*s. Thus, this pair of property is both qualitative and quantitative and also to be effected relative to each other. For every *rāga*, *svara*s having low probability are specified in SR. This automatically implies the remaining *svara*s to be relatively more frequent. It is not that always the probabilities of the *svara*s have to be unequal. For example in the Rāga Mālavaśrī the seven *svara*s have equal weight[38] and this is confirmed by Kallinātha also in his commentary[39]. This example illustrates that among all the theoretical characters it is the probabilistic property of *alpatva-bahutva* that is central to Rāga individuality. The other features such as *sañcāra* and *antaramārga*, *ṛju* and *vakra* and variation in speed introduce more variability into the sound structure so much so the pitch position of a particular *svara* in any *rāga* will not be single valued. Already we have seen in Fig.1, the basic *svara* is not a strict Sine wave, but it is only nearly so, such that its energy is spread in a narrow band carrying overtones contributed by closely spaced *śruti*s or frequencies. Additionally ornamentation due to *gamaka* which is defined as a delicate modulation around a *svara* is a must to please the listeners[40]. This modulation is further classified into fifteen types of refined movements such as quivering, throbbing, wavering, shaking, oscillating, swinging, sliding, and rotating with further combinations thereof. There are too many possible technical variations that cannot be discussed here except to remark that all of these add variability to the *rāga* on different time

[35] अल्पत्वं च द्विधा प्रोक्तमनभ्यासाच्च लङ्घनात् । अनभ्यासस्त्वनंशेषु प्रायो लोप्येष्वपीष्यते॥ (SR I.7.50)
[36] ईषत्स्पर्शो लङ्घनं स्यात्प्रायः तल्लोप्यगोचरम् । उशन्ति तदनंशेऽपि क्वचिद्गीतविशारदाः॥ (SR I.7.51)
[37] लङ्घनमीषत् स्पर्शः स्वरस्य स्थान–प्रयत्न–कृत–स्वरूप–न्यूनता ॥ (Kallinātha on SR above)
[38] दिनस्य केशवप्रीत्यै मालवश्रीस्तदुद्भवा । समस्वरा तारमन्द्रषड्जांशन्यासषड्जभाक् ॥ (SR II. 2.53)
[39] मालवश्रीलक्षणे – समा स्वरा यस्याः सा समस्वरा | स्वराणां समत्वं अत्र अल्पत्वबहुत्वकृत वैषम्यरहितत्वं विवक्षितम्॥
[40] स्वरस्य कम्पो गमकः श्रोतृ चित्तसुखावहः । (SR III.87a)

scales. Finally it is the imagination of the singer that decides the detailing of *ālāpana* in a session, so much so musicologists neither visualize nor explain *rāga* as a deterministic process in time.

If musicologists are expected to analyze the best of the music heard by them to delineate the theory behind Rāga depiction, we can see that the texts starting from BNS, with the available vocabulary at their disposal, foreshadow concepts that are basic to the Theory of Probability. Rāga as a random process may sound unusual and even jarring for those who connect with the aesthetics of *rāga* music only through emotion. But Śārṅgadeva following his predecessors not only analyzes classical music in terms of a hierarchically complex set of sound structures but also describes through similes the tantalizingly unpredictable nature of Rāga while describing the procedure of *ālāpana* in the third chapter of SR. His approach is analytical as one can realize from hundreds of technical words used to delineate not only the grammar of Rāga but also the sound quality of voice and classification of good and bad singers. To this already heavy vocabulary Somanātha adds a further set of technical terms and a unique system of notations in his text for rendering *rāga* music on the Vīṇa.

Ālāpana or *ālapti* is defined in SR as exhibition (*prakaṭīkaraṇam*) of the Rāga. Some broad guide lines are stated for this aural depiction that can be vocal or presentation on the string. Four modes of transit are mentioned as dwelling states. From the *sthāyisvara* up to but excluding the fourth *svara* (*dvyardha*) and return is the first transit. The second is the one including the fourth *svara.* The third transit is dwelling in between the fourth and the eighth *svara* to return to the *sthāyi*. The fourth would include the eighth note (*dviguṇa*) and occasionally above to return back to the end note that is like a stationary point. In this process conditions previously stated such as *alpatva-bahutva*, *langhana*, *nigraha*, *praveśa* and varieties of ornamental *gamaka* as found pleasing, have to be maintained. The exposition is done in calm, composed and leisurely pace, little by little, bringing out the temporal pattern of Rāga as a whole that is sometimes hidden, sometimes prominent but always modulated within the region of attraction prescribed by the ascent and descent of the *svara*s. Śārṅgadeva cryptically but effectively states

स्तोकस्तोकैस्ततः स्थायैः प्रसन्नैर्बहुभङ्गिभिः। जीवस्वरव्याप्तिमुख्यै रागस्य स्थापना भवेत् ॥ (SR 3.196)

The word *stoka-stokaiḥ* has the meaning of *little-by-little*, and the word *stoka* in Indian mathematics stands for a small unit of time equal to about 5.35 seconds[41]. The phrase *bahubhaṅgi* is equally meaningful. This refers to another important point that with all general characters of the *rāga* remaining same a refreshingly different realization is possible in each session. Each musician can also render the same *rāga* in individual style with several little-by-little embellishments. Thus the theoretical characterization of *rāga* music as developed in the Sanskrit texts is nothing but stochastic.

Discussion

In modern times, Ramachandran[42] was perhaps the first person to point out somewhat hesitantly "Hitherto single figures have been given by various writers as the value of the śrutis in each raga. But it may be clearly seen that in each raga a note assumes different shapes……It is more or less a general tendency for the śruti of a particular note to appear sharp when that note is relinquished for a higher note and to appear flat when a descent is made for that note…..And gamakas afford a wide choice of śrutis in the treatment of a note." He found with the help of a sound analyzer that in the Rāga Kanakāṅgi the svara Ri appears to take at least two values 10/9 and 256/243. He experimented with singers and found in some cases three different *śruti* values were used for the same *svara*. He seems to take *svara* to be a Sine function which has two or three distinct frequency values that are rational fractions. But as per the ancient texts the variation over the *śruti* is generally continuous and hence careful analysis should show any *svara* to occupy a narrow band of *śruti* values in songs set to *rāga* by learned composers and also in appealing *ālāpana* performances. This fact has been experimentally verified by Komaragiri[43] by analyzing time series of several *rāga* performances by three famous contemporary musicians. In Fig. 1 three isolated *svara* were depicted using modern recording and analysis techniques to find that their peak *śruti* values, although nearly equal to (1, 1.33, 1.5) still show considerable spread. In the presentation of any *rāga* by an inspired maestro, in either the southern or the northern style, the *svara* flow will be continuously weaving patterns in time. In such a case to find the *śruti* value of a *svara* would be, as aptly said by Nārada, like finding the path of fish in water and flight of birds in air[44]. In recent years there is increased interest in computational methods to

[41] *The Gaṇita-sāra-sangraha of Mahāvīrācārya,* (Ed.) M. Rangācārya. Madras Govt.Press, Madras 1912.
[42] N.S.Ramachandran *(ibid)*
[43] M.M.Komaragiri. Departures from the Acoustical Parameters in the Intonation of South Indian Musical Intervals. *Canadian Acoustics* Vol.40 No.4, 2012, pp.3-11.
[44] यथाप्सुचरतां मार्गो मीनानां नोपलभ्यते। आकाशे वा विहङ्गानां तद्वत् स्वरगताश्रुतिः ॥ *Nāradīyaśikṣā* (I. 6.16)

identify *rāga*s of classical music based on actual recordings of performances[45]. As an off shoot of such investigations one can construct pitch histogram of a *rāga* which gives the number of times finely divided *śruti* bins in an octave get occupied in a sample *rāga* performance over a short time period. In Fig.2, two such plots are shown for the number of occurrences of different *svaras* over the *śruti* interval in the performance of two popular *rāga*s. The X-axis is in Cents defined as

$$\text{Cent} = 1200 \log_2(\textit{śruti})$$

Thus, the middle register covering the *śruti* interval [1, 2] corresponds to the interval [0, 1200] Cents. The figure shows that the histograms do not peak at every one of the theoretically prescribed *svara*s even though their contribution is present in the distribution. Also interesting to note is the very small contribution from *svara* that are popularly said not to belong to that particular Rāga. This seems to happen since in *gamaka* phrases, neighbouring *śruti*s and even slight movements of another *rāga* (*svara-kāku, rāga-kāku*) are permitted but with very low probability. Description of such niceties is available in the 3rd Chapter (*prakīrṇādhyāya)* of SR and the commentaries. The relative heights and spreads in the figures are visual representations of the *alpatva-bahutva* character

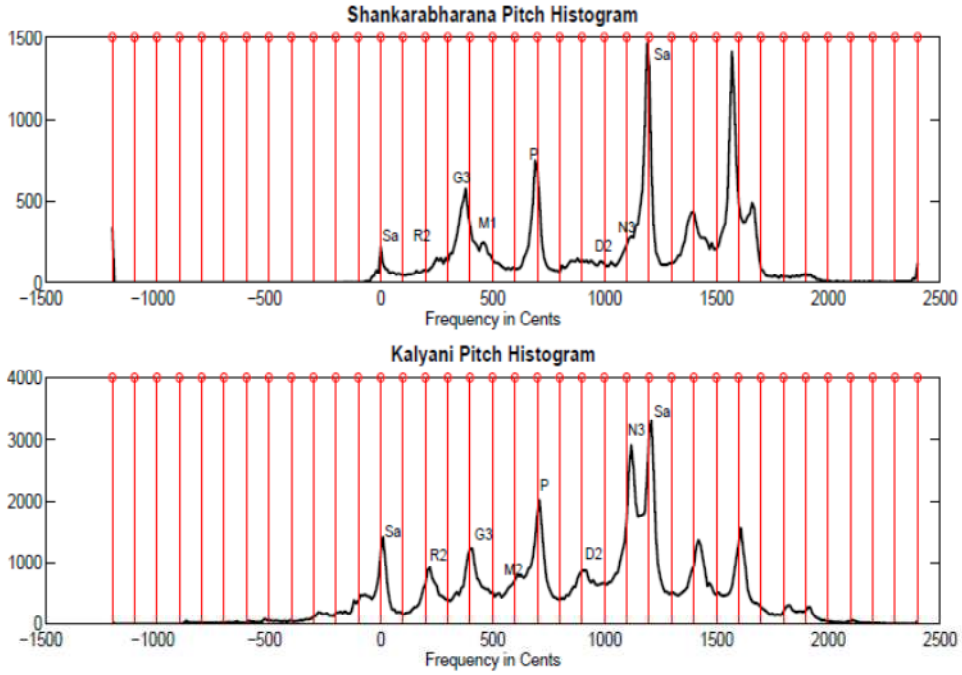

Fig.2. Pitch Histograms showing the property of *alpatva-bahutva* or relative probability of occurrence of the seven prescribed *svara*s for two popular Rāgas of Karnatik Music in current practice.
[Figure after Prof. Hema Murthy; IITM, Chennai Accessed on 21-11-2017;
https://www.google.co.in/Hema-Murthy-slides-3rd-CompMusic-workshop.pdf]

[45] G.K.Koduri, J.Serra and X.Serra. Characterization of Intonation in Carnatic Music by Parametrizing Pitch Histograms. *Proc. 13th Intnl. Soc. For Music Information Retrieval Conference (ISMIR 2012)* pp.199-204.

of the particular *rāga* within the time interval of its exposition and also as discriminated from another *rāga*. More importantly, from a historical view point, this is a modern generalization of the *svara* counting method first demonstrated by Kallinātha six centuries ago. The variation of the pitch in Fig. 2 is continuous and hence for proper comparison between two ragas the figures have to be normalized such that the area under the curve is unity. This will be in fact an estimate of the first order probability density function of the continuous random variable ***Svara*** of that particular *Rāga*. Each *Rāga* will be characterized by one such multimodal probability density function with several peaks. It must be noted that this is only a first order mathematical description or model with no claims for completeness in explaining *rāga* music in all its delicate intricacies. Like with any stochastic process joint density functions of higher order and time wise variation of statistical properties greatly matter.

Among the structural characters of a *rāga*, apart from the ten properties stated by Bharata, *antaramārg*a (internal path) is considered important by all authors. This is a very general phrase and only some vague information of this is available in SR and the commentaries. Roughly speaking this allows use of *svara*s displaced from their assigned positions to be used in between other *svara*s as though they have *alpatva* (low probability) character. Seen from the perspective of a stochastic process *antaramārga* may be referring to internal relation among *svara*s in time. The simplest measure of this second order property is the autocorrelation function. How is this *antaramārga* concept brought into the education of music students? This is where Śārṅgadeva highlights the importance of *vāggeyakāra*s who are accomplished scholars having the ability to compose songs, set them to *rāga* music and also sing, for the sustenance of the classical tradition. Thus the *nibaddha-saṅgīta* or composed music for which the *svara-prastāra* is made available by tradition is indispensable not only for training purposes but also for the theory of *rāga* music. We have seen the example of Rāga Ṣāḍjī already. SR provides several such examples with samples of *ālāpana* also as a sequence of *svara*s. One can analyse these to some extent by assigning *śruti* values to the *svara*s. Out of curiosity in Fig. 3(a) the time series of the *gīti* in the Rāga Ṣāḍjī given in SR as per Table 1 is plotted by taking the short syllable as a reference (*mātrā*) time unit. The autocorrelation of the *rāga* as it evolves in time is shown in Fig. 3(b).

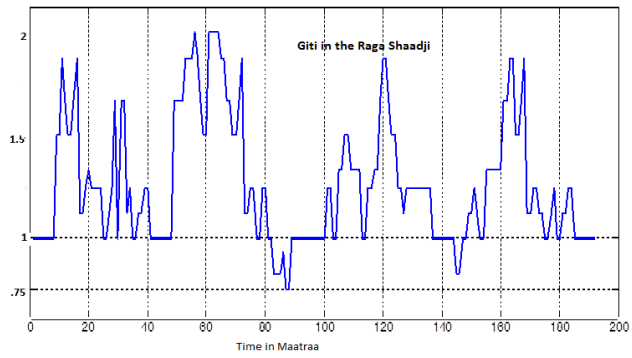

Fig. 3 (a) Time series of the song in Rāga Ṣāḍjī

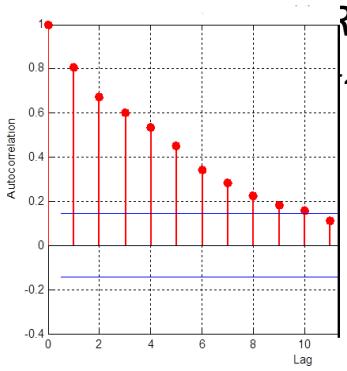

Fig. 3(b) Autocorrelation

It is observed that the *prastāra* is strongly correlated with the autocorrelation decaying very slowly. It would be interesting to compare the above with simple songs currently in use. Saint Purandaradāsa (*c* 1550 CE) considered the grandsire of Karnatic music systematized teaching at the elementary level to begin with *sarale-varase*, *janti-varase*, and *alamkāra* series. He also composed hundreds of songs in different *rāga*s, many of which are popular to this day. His *gīti* compositions *śrī-gaṇanātha* and *kereya nīranu* in the Rāga Malahari are the very first songs that students learn even now. These are shown in Fig 4 (a) and Fig 5 (a) as time series plotted with the prescribed *svara*s. The points are joined by lines to get a visual impression of the *svara* pattern. Fig 4(b) and Fig 5(b) show the autocorrelation function which for up to two steps appears to be decreasing geometrically but later shows slow decay, but faster than the song of Śarṅgadeva above.

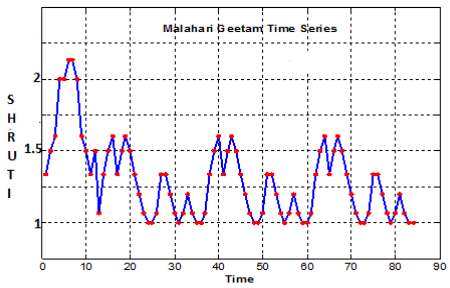

Fig. 4(a) *Śrī Gaṇanātha*…time series plot

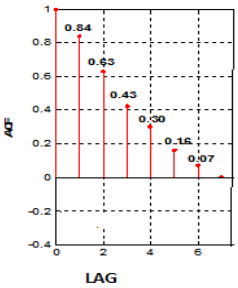

Fig. 4 (b) Autocorrelation

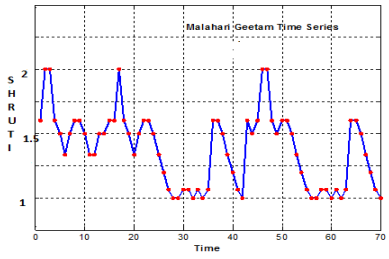

Fig. 5(a) *Kereya Nīranu*…time series plot

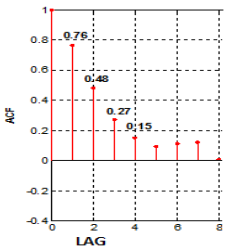

Fig 5(b) Auocorrelation

These types of songs are simple, without *gamaka* embellishment and speed variations. These can be faithfully presented even on keyboard instruments using the nominal *svara* positions. These are introduced in music classes after training in the strictly deterministic and periodic *alaṅkāra* strings. The point to be noted is that the autocorrelation decreases in two different ways caused by two different internal arrangements among the same *svara* groups indicating the two songs to be sample presentations of the same *rāga* among many other possibilities.

Next, in the graded order of complexity students are trained in *swarajate* and *varṇa* compositions. In Figure 5 (a,b) the time series and autocorrelation for a *varṇa* composition in the Rāga Hamsadhwani are shown. These results are by their very nature approximate since the time series is artificially constructed with discrete *svara* positions as given in printed text books and class notes. Nevertheless, one can get an idea of possible second order property of a *rāga.* Here also after two or three steps, the autocorrelation decreases too slowly, indicating long time memory in the composition as far as the song structure is concerned.

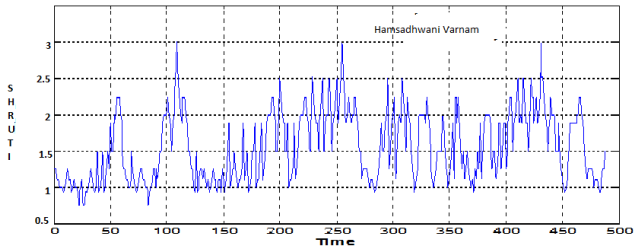

Fig. 5(a) Hamsadhwani Varṇa Time series.

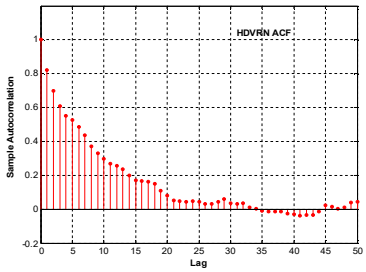

Fig. 5(b) Autocorrelation

Note: Mandra-Sa =0.5, Madhya-Sa=1, Ri=1.125, Ga=1.25, Pa=1.5, Ni=1.875; Tāra-Sa=2

Composed songs mark time with the help of external cyclic *tāla* measure to produce a feeling of rhythm in the background of the *rāga*. This may introduce hidden trends leading to slow decay of the autocorrelation as seen above. In *ālāpana* without *tāla* the external time keeping is removed so that the time series of *rāga* can evolve in different time scales, but staying within the boundary defined by the *āroha-avaroha* and the ever present *gamaka*. In this perspective with or without a vocalized song the theoretical description in the texts of any ***Rāga*** corresponds to a sample time series of the continuous random variable ***Svara*** evolving over its sample space namely, the *śruti* interval [½, 4] as per a pre-defined and culturally imbibed or intuitively innovated probability measure.

Summary and Conclusion

In this paper we have briefly reviewed Sanskrit texts by Bharata, Dattila, Matanga, and a few later musicologists on classical music, to bring out that the theory of melody or *rāga* music is based on the principle of uncertainty of *svara* positions in the octave. The *dviguṇa* limit or interval of the octave is precise, but the positions of the seven primary and twelve modified *svaras* are actually treated as variables except for their sequential order within the octave. This is not to say that the Vīṇā string positions as stated by Ahobala are invalid. As Ahobala and before him Somanātha points out, the visual fret positions are of great help for learners to get a feeling to the tonal variability of a *svara,* around its central or dominant *śruti* value. It is generally accepted that *rāgālāpana* or *melody-depiction*, for want of a better word, is the real test of a musician. The textual theory can be seen as a pedagogical support to train the future musician to carry out *ālāpana*, which will be an extempore performance without accompaniments except for the drone or *ādhāra-śruti.* It is with reference to this basic pitch level that the *ālāpana* will be gauged and perceived as an aesthetically appealing production of a particular Rāga. To educate and train a learner for this purpose a heavy hierarchy of increasingly complex sound structures, ranging from the discrete to the continuous, periodic to the stochastic, have been developed by the classical tradition layer by layer. The preliminary teaching is with deterministic discrete *svara* groups. These patterns are without exception pre-determined and hence repeat periodically after a prescribed interval. Such structures are musical and may even exhibit some elements of *rāga* but they are not yet *rāga* music. Rāga enters for the first time through the probabilistic *alpatva-bahutva* property in the increasingly complex musical patterns of songs sung to external time keeping with the help of *tāla*. *Rāgālāpana* is a further generalization without external time support where the artist has the freedom to bring out the features of the *rāga* as a sample time series with its mathematically structured aural pattern that is emotionally enjoyable and satisfying to the performer and the listener. Apart from a variety of other factors, Bharata and his successors highlight unexpectedness and particular types of *svara* mix as the key to bring out *rasa-bhāva* or emotional appeal in melodies. Further investigations are necessary to understand higher order organization of *rāga* music, beyond the ancient *alpatva-bahutva* and the modern first order pitch histogram. This is a rich area for further exploration that will help analytical understanding of how emotions (*rasa*) of listeners get evoked in particular *rāga*s by their special aural patterns.

This knowledge tradition combines experimentation on the Vīṇā with vocal music in a mutually beneficial feedback relationship. While rigorous training, aptitude and intuition play

obviously decisive roles in shaping a musician, knowledge of the grammar expounded in the texts adds sophistication and refinement to the art without obstructing the originality of the performer. The historically well evidenced culture of classical *rāga* music as a dialogue between the artist and the audience, independent of any human language, is well preserved in the texts by expounding the theory repeatedly with minor additions and alterations to keep up with contemporary practices. An amazing edifice of Sanskrit texts has been built by our ancient musicologists to explain the theory behind classical music. A formidable intellectual foundation that is partly speculative and narrative but systematic and analytical using mathematical and experimental methods is available for present day students, scholars and practitioners of music. This has been possible only because the art and the science of *rāga* music were encouraged to develop together as *lakṣya* and *lakṣaṇa* with mutual dependence in the traditional academic ecosystem. While the scholarly growth of musicology is an achievement of the Indic knowledge system with its roots going as far back as the Vedas and even though classical music has large number of votaries to this day all over the country, there is visible stagnation in the theoretical tradition with no important Sanskrit text on music appearing after mid-19th century. It is hoped that the relevance of the inquisitive mind set and scientific ideas left behind by the textual corpus will be recognized by our educational institutions including Sanskrit Universities, to bridge the above gap, so that Indian classical music as a fine art can rejuvenate itself side by side with its traditional partners namely, mathematics and experiments with musical instruments.

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