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Dynamic response of a beam on elastic foundation of finite depth under a moving force

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Summary. In this paper, dynamic response of an infinitely long beam resting on a foundation of finite depth, under a moving force is studied. The effect of foundation inertia is included in the analysis by modelling the foundation as a series of closely spaced axially vibrating rods of finite depth, fixed at the bottom and connected to the beam at the top. Viscous damping in the beam and foundation is included in the analysis. Steady state response of the beam-foundation system is obtained. Detailed numerical results are presented to study the effect of various parameters such as foundation mass, velocity of the moving load, damping and axial force on the beam. It is shown that foundation inertia can considerably reduce the critical velocity and can also amplify the beam response.

List of symbols

b	width of the beam
C_b	coefficient of viscous damping for the beam
C_f	coefficient of viscous damping for the foundation
E	Young's modulus
f	frequency
H	foundation depth
I	moment of inertia
i	$=(-1)^{0.5}$
K, k	indexing variables
k_f	foundation modulus
m	mass per unit length of the beam
N	total number of frequency points in Eqs. (25) and (26)
n	indexing variable
P	moving force
Q	axial force on the beam
$q(x, t)$	foundation pressure per unit length of the beam
$q(\xi)$	foundation pressure in the moving co-ordinate system
t	time variable in sec.
$U_j(\xi)$	generalized coordinate in Eq. (4)
$U_j^*(f)$	Fourier transform of U_j
$u(y, t; x)$	axial displacement in the foundation at a particular x value
$u(y, \xi)$	foundation displacement in the moving coordinate system
$\bar{u}(\bar{y}, \xi)$	$= u(\bar{y}, \xi)/L$, nondimensionalized foundation deflection
v	velocity in meters/sec.
v_{cr}	critical velocity corresponding to massless foundation
$w(x, t)$	beam deflection
$w(\xi)$	beam deflection in the moving coordinate system
$\bar{w}(\xi)$	$= w(\xi)/L$ nondimensionalized beam deflection
$\bar{w}^*(f)$	Fourier transform of $\bar{w}(\xi)$

x, y	coordinate axis
α	velocity parameter
α_{cr}	critical velocity parameter
β	mass parameter
ξ	moving coordinate
η_b	beam damping parameter
η_f	foundation damping parameter
$\sigma(y, t, x)$	vertical stress in the foundation
$\delta(\ )$	Dirac delta function
ρ	foundation mass per unit depth per unit length of the beam

1 Introduction

The study of the dynamics of a beam on an elastic foundation (BEF) is relevant in several fields of engineering. In particular, this class of problems finds application in the analysis of railway tracks. A comprehensive study on the response of BEF under a moving load has been presented by Fryba [1]. One of the earliest study of a beam on an elastic foundation was performed by Timoshenko [2]. His work was concerned with the response of a rail subjected to a moving force with constant velocity. He obtained the steady state solution and showed that there exists a critical velocity at which the beam deflection becomes unbounded. Kenney [3] included into the BEF model the effect of linear damping in the beam. He showed that in the steady state the wavelength behind the load increases with the damping. He also found the critical damping value at which the wavelength behind the load becomes infinite. Mathews [4], [5] extended Kenney's solution for an oscillating moving force. He studied the variation of the critical velocity with the forcing frequency. Achenbach and Sun [6] obtained the steady state response of a Timoshenko beam resting on an elastic foundation under a moving force. They also included the effect of linear damping in the analysis. Recently Bogacz, Kezyzynski and Popp [7] have extended this analysis to a moving oscillating force. The effect of an axial force acting along the beam has been studied by Kerr [8]. This has been extended to the more general case of a Timoshenko beam by Chonan [9]. Bogacz, Nowakowski and Popp [10] studied the stability of a Timoshenko beam on an elastic foundation under a moving spring mass system. They assumed a steady state solution in the form of travelling waves and showed that the critical velocity may become less than the velocity of shear waves or the velocity of longitudinal waves.

The usual BEF model as applied to the railway track was initially modified by Kerr [11] to include the rotational resistance of the cross-ties (sleepers). This was further refined by Kerr and Zarembski [12] by treating the tack as a long repetitive structure consisting of identical units. This model included the effect of cross-tie stiffness to describe the lateral and vertical static response. Kerr and Accorsi [13] generalized this model to include dynamic response.

The classical Winkler foundation model used in the study of BEF takes into account only the stiffness or spring effect of the foundation. It does not include the effect of the foundation mass or inertia which may be sometimes more predominant than the stiffness effect. Attempts have been made in the past to generalize the Winkler foundation model to take into account the foundation inertia. An approximate approach for including the foundation inertia is by adding an equivalent mass to the beam. Rades [14] has used this approach to account for the foundation inertia. Saito and Murakami [15] proposed a natural generalization of the Winkler foundation model to account for the foundation inertia. They studied the wave propagation in an infinitely long Timoshenko beam resting on an elastic foundation with inertia. They represented the foundation as a series of closely spaced axial bars of finite length, fixed at the bottom and connected to the beam at the top. They found that for this inertial foundation, nodal planes appear in the foundation at certain frequencies. They concluded that a massless foundation is a good approximation for low frequency ranges only. This foundation model was used by Holder and Michalopoulos [16] to obtain the steady state response of an infinitely long Euler-Bernoulli beam under a moving force. They showed that the critical velocity decreases with the foundation inertia. They also studied the effect of damping in the beam. Using the same foundation model, Iyengar and Pranesh [17] showed that the natural frequency of a free-free beam decreases with the foundation mass.

A fresh study of the BEF with foundation inertia, under a moving force is undertaken in the present paper. The solution approach is different from that adopted by Holder and Michalopoulos [16]. Also, it may be noted here that the above authors did not consider the effects of foundation damping and axial force. These are included in the present work.

2 Equations of motion

With reference to Fig. 1, the equation of motion for a beam on which a concentrated force is moving at a uniform velocity v is

$$EI(\partial^4 w/\partial x^4) + Q(\partial^2 w/\partial x^2) + C_b(\partial w/\partial t) + m(\partial^2 w/\partial t^2) = P\delta(x - vt) - q(x, t). \tag{1}$$

Here, $w(x, t)$ is the transverse deflection of the beam, P and Q are the moving force and the axial force respectively. C_b is the viscous damping coefficient of the beam. (See the list of symbols for complete nomenclature). The foundation pressure $q(x, t)$ has to be computed from the foundation equation of motion given by

$$\rho(\partial^2 u/\partial t^2) + C_f(\partial u_r/\partial t) = k_f H(\partial^2 u/\partial y^2). \tag{2}$$

Here the damping force is taken proportional to the relative velocity $(\partial u_r/\partial t)$ between the beam and the foundation.

The boundary conditions on $u(y, t; x)$ are

$$\begin{aligned} &u(o, t; x) = w(x, t) \\ &u(H, t; x) = 0. \end{aligned} \tag{3}$$

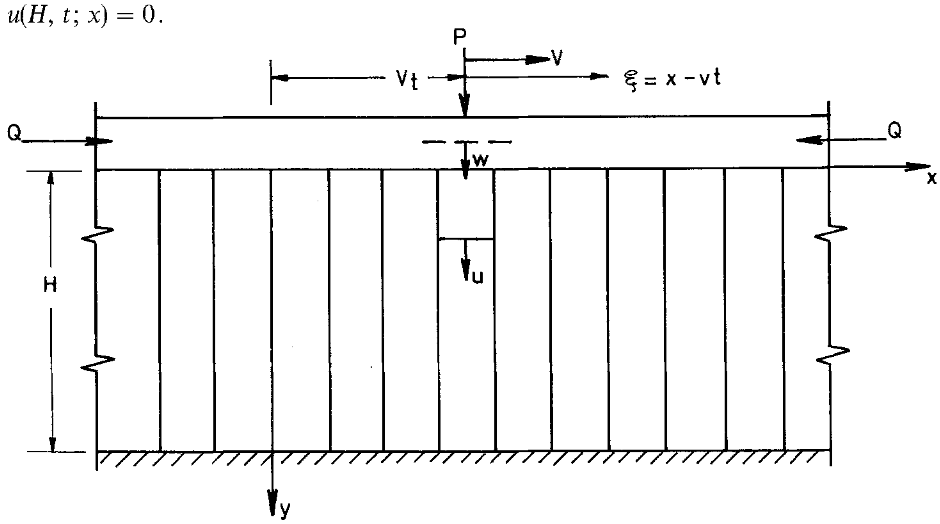

Fig. 1. Beam-foundation system

Since the foundation is connected only at the top by the beam here x appears only as a parameter. The pressure exerted by the foundation on the beam is given by

$$q(x, t) = -k_f H(\partial u/\partial y)|_{y=0}. \tag{4}$$

For further work, it would be convenient to nondimensionalize the variables in terms of the static characteristic length $L = (3\pi/4)(4EI/k_f)^{0.25}$ and the natural frequency parameter $\Omega = (k_f/m)^{0.5}$. The new variables are

$$\bar{w} = w/L, \quad \bar{u} = u/L, \quad \bar{u}_r = u_r/L, \quad \bar{x} = x/L, \quad \bar{y} = y/L, \quad \bar{H} = H/L$$

$$\tau = \Omega t \quad \text{and} \quad \bar{v} = v/(\Omega L). \tag{5}$$

Since the beam is infinitely long, following Kerr [8], it is convenient to introduce the moving coordinate system

$$\xi = \bar{x} - \bar{v}\tau, \quad \bar{y} = \bar{y}. \tag{6}$$

In terms of these coordinates, the deflection profile of the beam becomes time invariant, in the steady state. Now, Eqs. (1) and (2) are transformed as

$$d^4\bar{w}/d\xi^4 + (3\pi/2)^2(\gamma + \alpha^2)(d^2\bar{w}/d\xi^2) - (3\pi/2)^3 \alpha \cdot \eta_b(d\bar{w}/d\xi) = P_0\delta(\xi) - qL^3/(EI) \tag{7}$$

$$\partial^2\bar{u}/\partial\xi^2 - (3\pi C_f/4\rho\Omega\alpha)(\partial\bar{u}_r/\partial\xi) = (3\pi\bar{H}/4\alpha\beta)^2(\partial^2\bar{u}/\partial\bar{y}^2). \tag{8}$$

The boundary conditions are

$$\bar{u}(o, \xi) = \bar{w}(\xi), \quad \bar{u}(\bar{H}, \xi) = 0. \tag{9}$$

The foundation pressure is given by

$$q(\xi) = -k_f H(\partial\bar{u}/\partial\bar{y})|_{\bar{y}=0}. \tag{10}$$

The following nondimensional parameters can be easily identified.

Mass parameter: $\beta^2 = \rho H/m$
Velocity parameter: $\alpha = v/v_{cr}$
Axial force parameter: $\gamma = Q/Q_{cr}$
Moving force parameter: $P_0 = (3\pi/2)^2 P/\{2(k_f EI)^{0.5}\}$
Beam damping parameter: $\eta_b = C_b/\{2(k_f m)^{0.5}\}$.

Here, $v_{cr} = (4EIk_f/m^2)^{0.25}$ is the critical velocity and $Q_{cr} = 2(k_f EI)^{0.5}$ is the static buckling load of a BEF on massless Winkler's foundation.

3 Analysis

It is convenient to solve the problem, starting with the foundation equation. It is seen that the boundary conditions of Eq. (9) for the foundation are inhomogeneous. With the transformation

$$\bar{u}(\bar{y}, \xi) = \bar{u}_r(\bar{y}, \xi) + \{(\bar{H} - \bar{y})/\bar{H}\}\,\bar{w}(\xi) \tag{11}$$

the boundary conditions are

$$\bar{u}_r(o, \xi) = 0, \quad \bar{u}_r(\bar{H}, \xi) = 0. \tag{12}$$

Now Eq. (8) becomes

$$\partial^2\bar{u}_r/\partial\xi^2 - \{3\pi C_f/(4\rho\Omega\alpha)\}(\partial\bar{u}_r/\partial\xi) = \{(3\pi\bar{H})/(4\alpha\beta)\}^2(\partial^2\bar{u}_r/\partial\bar{y}^2) - \{(\bar{H}-\bar{y})/\bar{H}\}(d^2\bar{w}(\xi)/d\xi^2). \quad (13)$$

The solution for $\bar{u}_r(\bar{y}, \xi)$ can be taken as

$$\bar{u}_r(\bar{y}, \xi) = \sum_{j=1}^{\infty} U_j(\xi)\sin(j\pi\bar{y}/\bar{H}). \quad (14)$$

After substituting Eq. (14) in (13), multiplying by $\sin(j\pi\bar{y}/\bar{H})$ and integrating over y from 0 to $\bar{H}$ one gets

$$(d^2U_j/d\xi^2) - 2\eta_f\{3\pi^2 j/(4\alpha\beta)\}(dU_j/d\xi) + \{3\pi^2 j/(4\alpha\beta)\}^2 U_j = -(2/j\pi)(d^2\bar{w}/d\xi^2) \quad (15)$$

where $\eta_f = C_f/\{(2j\pi/H)(k_f H\rho)^{0.5}\}$ is the foundation damping ratio. The Fourier transform of $U_j(\xi)$, namely

$$U_j^*(f) = \int_{-\infty}^{\infty} U_j(\xi)\exp(-i2\pi f\xi)\,d\xi, \quad (16)$$

can be obtained from Eq. (15) as

$$U_j^*(f) = [(2/j\pi)(2\alpha\beta f)^2\,\bar{w}^*(f)]/\{-(2\alpha\beta f)^2 - i(3\pi\eta_f j\alpha\beta f) + (3\pi j/4)^2\}. \quad (17)$$

It is seen that $U_j^*(f)$ is expressed in terms of $\bar{W}^*(f)$ which is the Fourier transform of the beam displacement $\bar{W}(\xi)$. This can be obtained from Eq. (7), where now

$$q(\xi) = -k_f H\left\{\sum_{j=1}^{\infty}(j\pi/\bar{H})\,U_j(\xi) - \bar{W}(\xi)/\bar{H}\right\}. \quad (18)$$

It follows that

$$\bar{W}^*(f) = P_0/\{(2\pi f)^4 - (3\pi/2)^2(\gamma+\alpha^2)(2\pi f)^2 - (3\pi/2)^3\eta_b\alpha(i2\pi f) + 4(3\pi/4)^4 + S_e\} \quad (19)$$

where

$$S_e = 8(3\pi/4)^4\sum_{K=1}^{\infty}(2\alpha\beta f)^2/\{(2\alpha\beta f)^2 + i3\pi\eta_f K\alpha\beta f - (3\pi K/4)^2\}.$$

Further, substitution of Eq. (19) in Eq. (17) leads to

$$U_j^*(f) = (2/j\pi)(2\alpha\beta f)^2 P_0/[\{-(2\alpha\beta f)^2 - i3\pi\eta_f j\alpha\beta f + (3\pi j/4)^2\} \times\{(2\pi f)^4 - (3\pi/2)^2(\gamma+\alpha^2)(2\pi f)^2 - (3\pi/2)^3\eta_b\alpha(i2\pi f) + 4(3\pi/4)^4 + S_e\}]. \quad (20)$$

Now $\bar{w}(\xi)$ and $U_j(\xi)$ are the inverse Fourier transforms given by

$$\bar{w}(\xi) = \int_{-\infty}^{\infty}\bar{w}^*(f)\exp(i2\pi f\xi)\,df \quad (21)$$

$$U_j(\xi) = \int_{-\infty}^{\infty} U_j^*(f)\exp(i2\pi f\xi)\,df. \quad (22)$$

The bending moment in the beam can be found as

$$M(\xi) = (EI/L)\,(d^2\bar{w}/d\xi^2)$$
$$= -(2\pi f)^2\,(EI/L) \int_{-\infty}^{\infty} \bar{w}^*(f)\,\exp\,(i2\pi f\xi)\,df. \tag{23}$$

Similarly, the stress in the vertical direction in the foundation is

$$\sigma(\bar{y}, \xi) = -(k_f H/b) \left\{ \sum_{j=1}^{\infty} (j\pi/\bar{H})\,U_j(\xi)\,\cos\,(j\pi\bar{y}/\bar{H}) - \bar{w}(\xi)/\bar{H} \right\} \tag{24}$$

where b is the width of the beam.

Thus, once the integrals in Eqs. (21) and (22) are evaluated the various responses can be obtained.

4 Numerical approach

A closed form expression for the integrals in Eqs. (21) and (22) seems difficult, if not impossible. Hence, here the discrete Fourier transform approach is used to evaluate these integrals. On discretization one gets

$$\bar{w}(n) = \Delta f \sum_{k=0}^{N-1} \bar{w}^*(k\Delta f)\,e^{i2\pi nk/N} \qquad (n = 0, 1, 2, \ldots, N-1)$$

$$U_j(n) = \Delta f \sum_{k=0}^{N-1} U_j^*(k\Delta f)\,e^{i2\pi nk/N} \qquad (n = 0, 1, 2, \ldots, N-1) \tag{26}$$

where Δf is the frequency step size and N is total number of points. The above summations are evaluated using the fast Fourier transform (FFT) algorithm as given in the book by Brigham [18]. It may be mentioned here, that, for the success of the FFT algorithm, Δf should be such that the effect of aliasing is minimised. Also the value of N should be sufficiently large to avoid spurious rippling effects in the results. Based on several trials runs, the values of Δf and N are chosen as 0.025 and 2^{10} respectively, for further numerical work. It is to be noted that Eqs. (19)

Table 1. Effect of N_t on convergence of peak beam response. $\alpha = 0.155$, $\beta = 8.0$, $\gamma = 0.3$, $\eta_b = 0.02$, $\eta_f = 0.05$, $\Delta f = 0.025$, $N = 2^{10}$

N_t	w_p/w_s	M_p/M_s	N_t	w_p/w_s	M_p/M_s
1	2.04402	1.85101	13	3.71997	3.63976
2	2.53957	2.23343	14	3.74260	3.66511
3	2.86969	2.56891	15	3.76167	3.68594
4	3.09316	2.86079	16	3.77790	3.70322
5	3.25876	3.07421	17	3.79183	3.71769
6	3.37329	3.22029	18	3.80388	3.72990
7	3.46434	3.33130	19	3.81439	3.74028
8	3.53562	3.41985	20	3.82361	3.74916
9	3.58962	3.48459	21	3.83175	3.75681
10	3.63130	3.53271	22	3.83898	3.76343
11	3.66401	3.56976	23	3.84543	3.76919
12	3.69281	3.60858			

and (20) for the Fourier transforms contain an infinite series. The convergence of this series is studied by finding the peak beam response for a different number of terms N_t in the series. The results of the convergence study are shown in Table 1. The peak beam deflection w_p and bending moment M_p shown are normalized with respect to the peak static deflection w_s and bending moment M_s respectively. For the static case the peak beam deflection and bending moment are given by

$$W_s = PL^3/\{(3\pi/2)^3\, EI\} \tag{27}$$

$$M_s = PL/(3\pi). \tag{28}$$

It is observed that about twenty terms are required in the series for good convergence.

5 Range of parameters

Among the five parameters, the velocity parameter α and the mass parameter β are the most important. The parameter α depends on the critical velocity v_{cr} of the massless foundation. Thus, the critical velocity ratio α_{cr} defined as the value of v/v_{cr} at which the beam deflection is a maximum will be unity when $\beta = 0$, which corresponds to the case when the foundation inertia is zero. However, when the foundation mass effect is included the corresponding α_{cr} will be less than unity. Here, the numerical results are obtained up to a value of α slightly exceeding α_{cr}. The mass parameter β essentially depends on the depth of the foundation. The minimum value of β would be zero corresponding to the massless foundation case. At the other end, the maximum value of β is taken as $\beta = 8$. This value of β corresponds to a foundation depth of nearly eight meters for railway tracks. The axial force parameter γ is varied in the range $0-0.3$. The damping coefficients η_b and η_f are varied in the ranges $0-0.2$ and $0.05-0.2$ respectively.

6 Numerical results

For the above range of parameters, numerical results have been obtained to study the effect of various parameters on the beam response and critical velocity.

6.1 Effect of β

In Fig. 2 the effect of foundation mass on the critical velocity is shown for two different values of γ. The critical velocity α_{cr} is the velocity parameter at which the peak beam deflection is the highest for fixed values of other parameters. The inset sketch in this figure shows the procedure for finding α_{cr}.

In Fig. 3 the effect of foundation mass on beam deflection and bending moment in the ξ coordinate system is shown. This figure shows the normalized beam deflection and bending moment for four different values of β, at a specified value of α, γ, η_b and η_f. This figure brings out the fact that β influences the peak response considerably. For the case of $\beta = 8.0$, the value of $\alpha = 0.175$ selected is very near to the critical velocity. Hence, the response shows considerable amplification for this case. Since in practical applications the peak values are more important than the actual profiles, the effect of the other parameters is further studied with reference to the peak response.

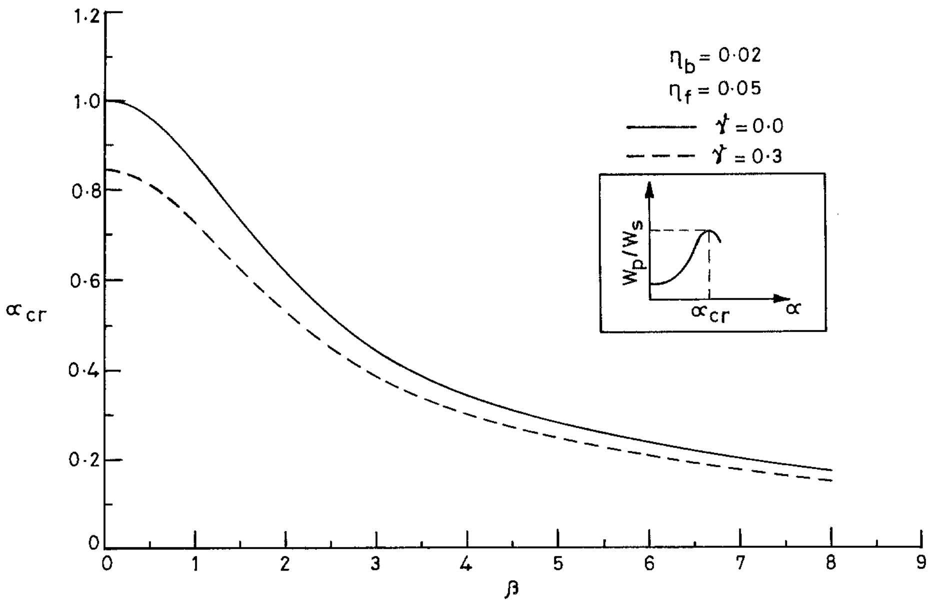

Fig. 2. Effect of foundation mass on the critical velocity

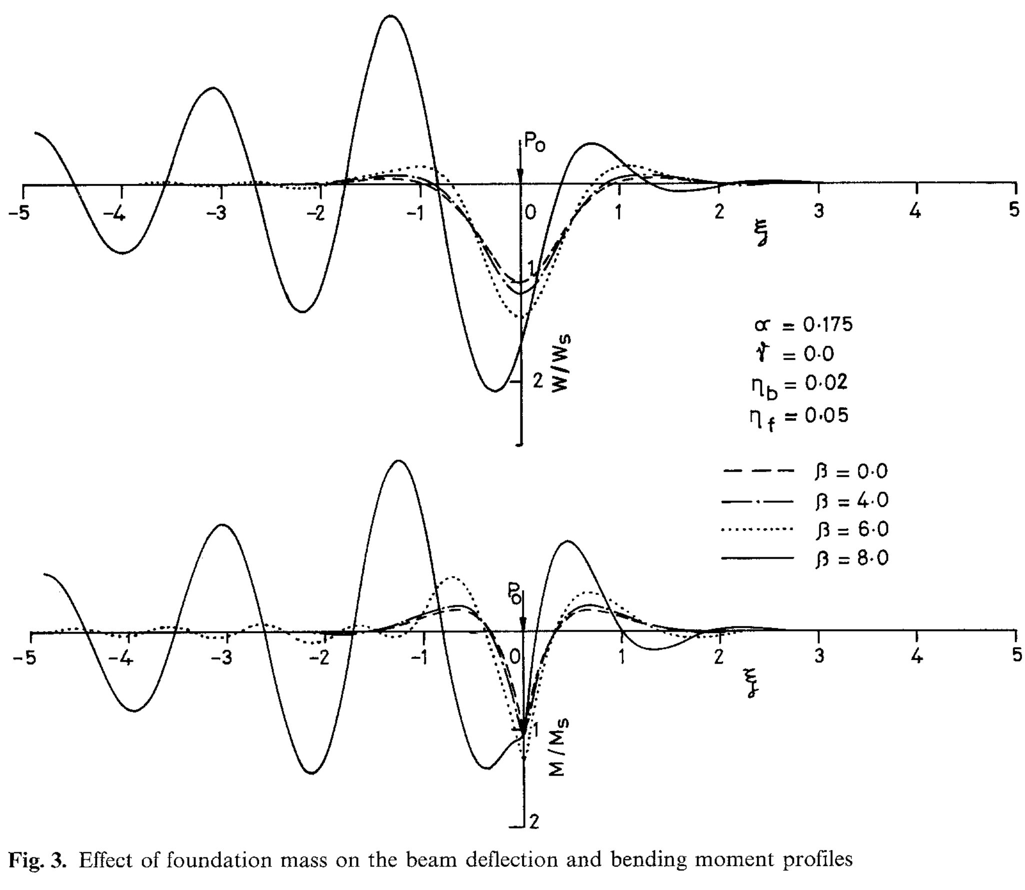

Fig. 3. Effect of foundation mass on the beam deflection and bending moment profiles

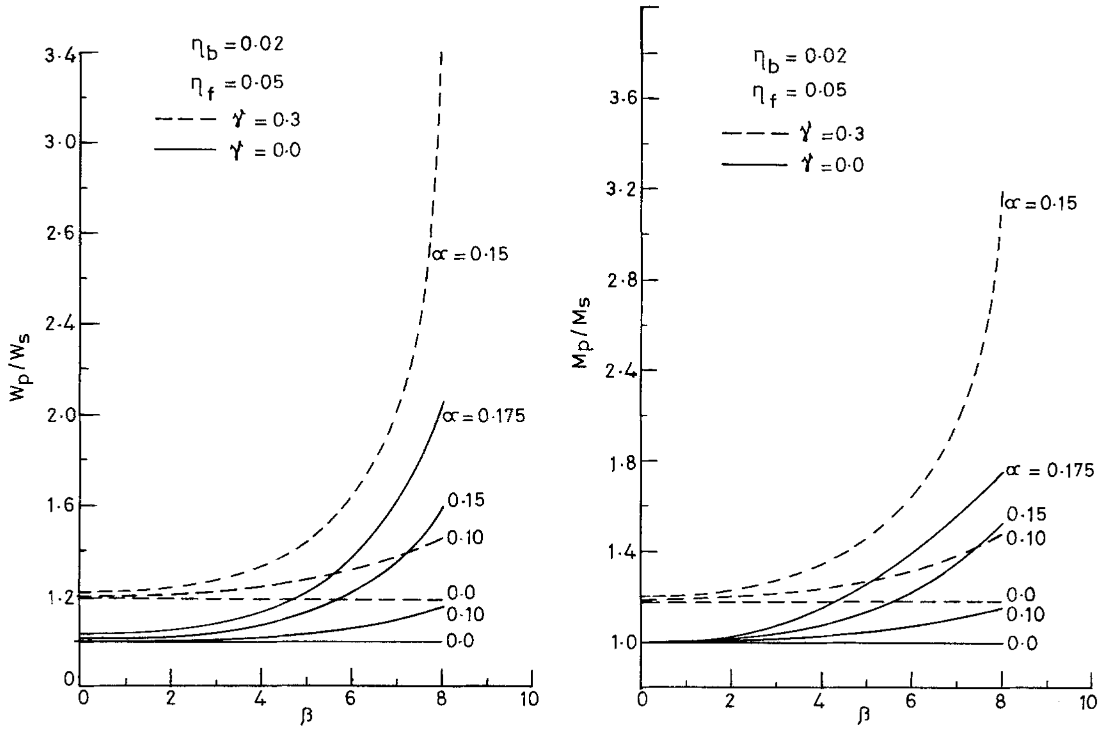

Fig. 4. Effect of foundation mass on peak beam response

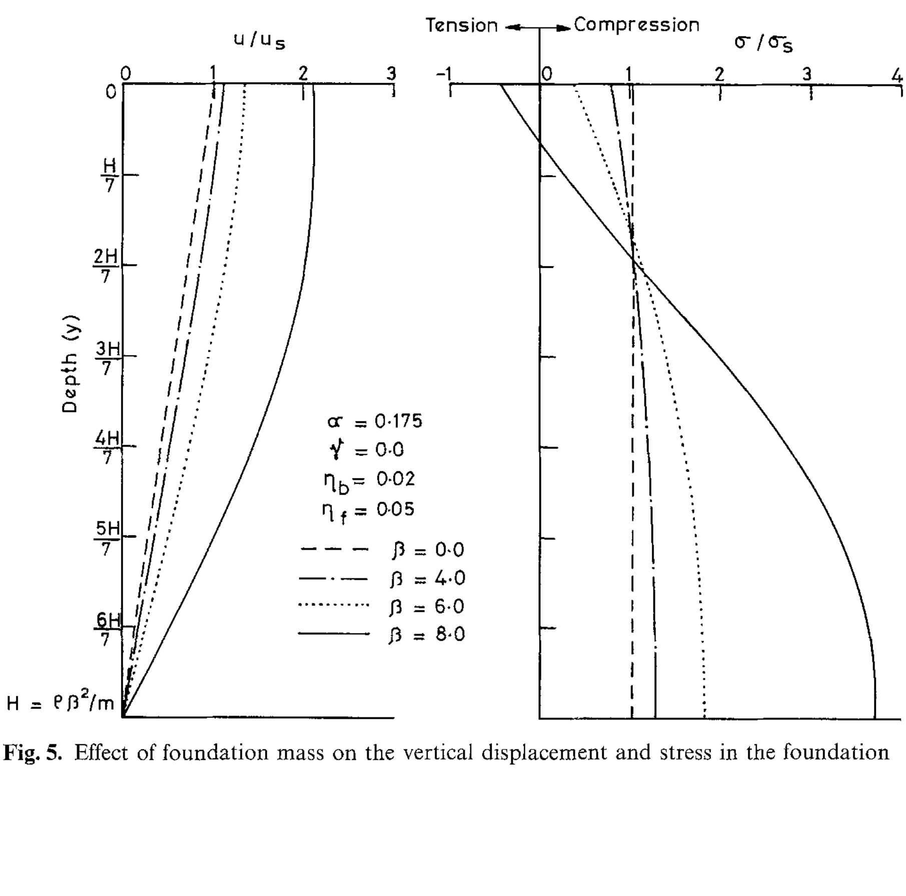

Fig. 5. Effect of foundation mass on the vertical displacement and stress in the foundation

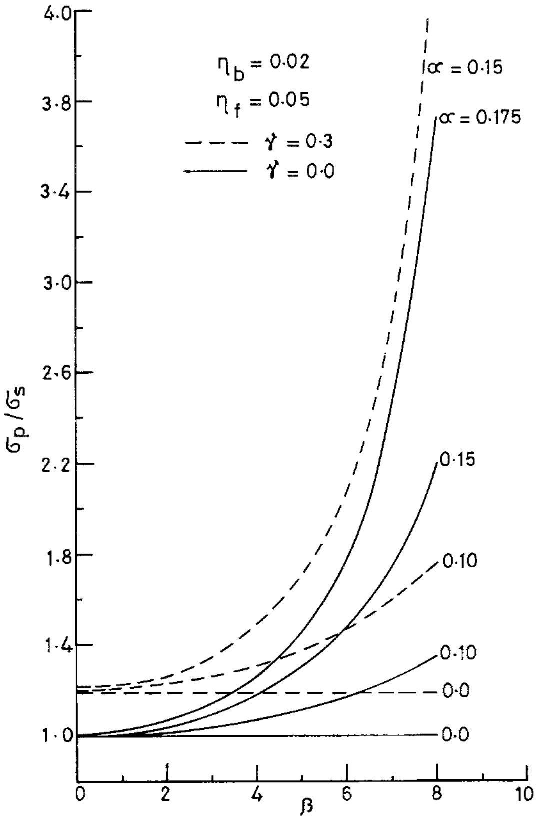

Fig. 6. Effect of foundation mass on peak stress in the foundation

In Fig. 4a the variation of peak beam deflection is shown as a function of β for several values of α. The effect of γ is also presented in this figure. In Fig. 4b corresponding results for the peak bending moment are shown.

In Fig. 5 the effect of a foundation mass on the axial displacement (u/u_s) and stress (σ/σ_s) in the foundation is shown. The axial displacement has been normalized with respect to the static displacement $u_s = W_s$. Similarly, the stress is normalized with respect to the static stress $\sigma_s = (k_f/b)\, w_s$. This figure shows the variation of foundation displacement and stress along the depth for four different values of β, at a specified value of α, η_b, η_f and γ. The foundation depth is expressed in terms of β, ρ and m as $H = m\beta^2/\rho$. The foundation response shown in this figure corresponds to the ξ value, where the beam deflection is a maximum. The variation of the peak stress (σ_p/σ_s) in the foundation with respect to β is shown in Fig. 6.

6.2 Effect of α

In Fig. 7 the effect of the velocity of the moving force on the beam deflection and bending moment profiles is shown. Again as in the case of Fig. 3, as α approaches α_{cr}, the waviness in the profile gets amplified dramatically. Previously in Fig. 4a the effect of the velocity was studied only for a few selected values of $\alpha < \alpha_{cr}$. In Fig. 8 the effect of α is studied in a more detailed fashion by covering the range of α slightly exceeding α_{cr}. In Figs. 9 and 10 some limited results on the effect of beam damping and foundation damping on the peak beam deflection are also shown.

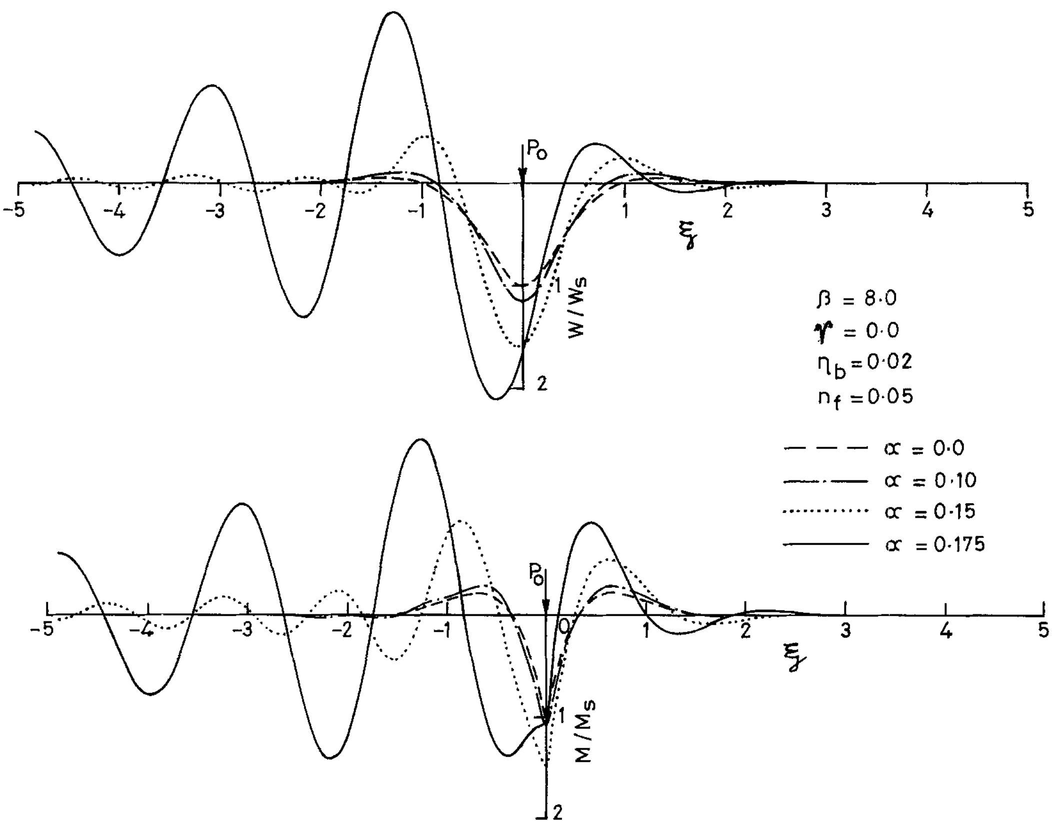

Fig. 7. Effect of velocity on beam deflection and bending moment profiles

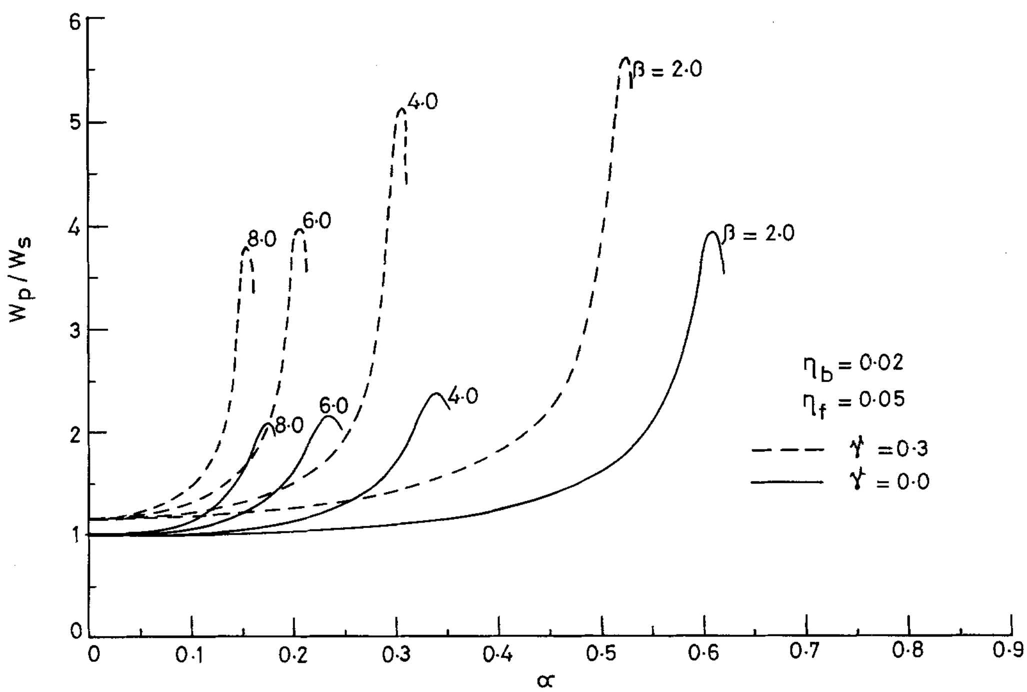

Fig. 8. Effect of velocity on peak beam deflection

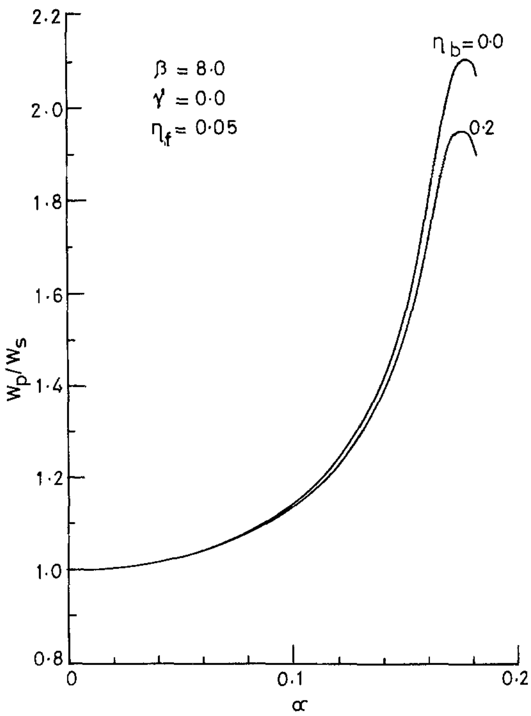

Fig. 9. Effect of beam damping on peak beam deflection

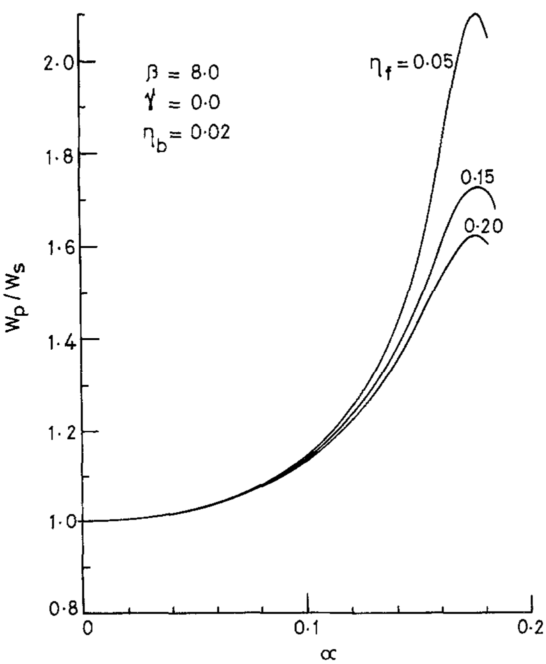

Fig. 10. Effect of foundation damping on peak beam deflection

In Fig. 11 the effect of the velocity on the foundation response is presented. This figure shows the variation of foundation vertical displacement and stress along the depth for different values of α. These results correspond to the ξ value at which the beam deflection is a maximum. It is interesting to note that as α approaches α_{cr}, tensile stresses get developed in the top portion of the foundation.

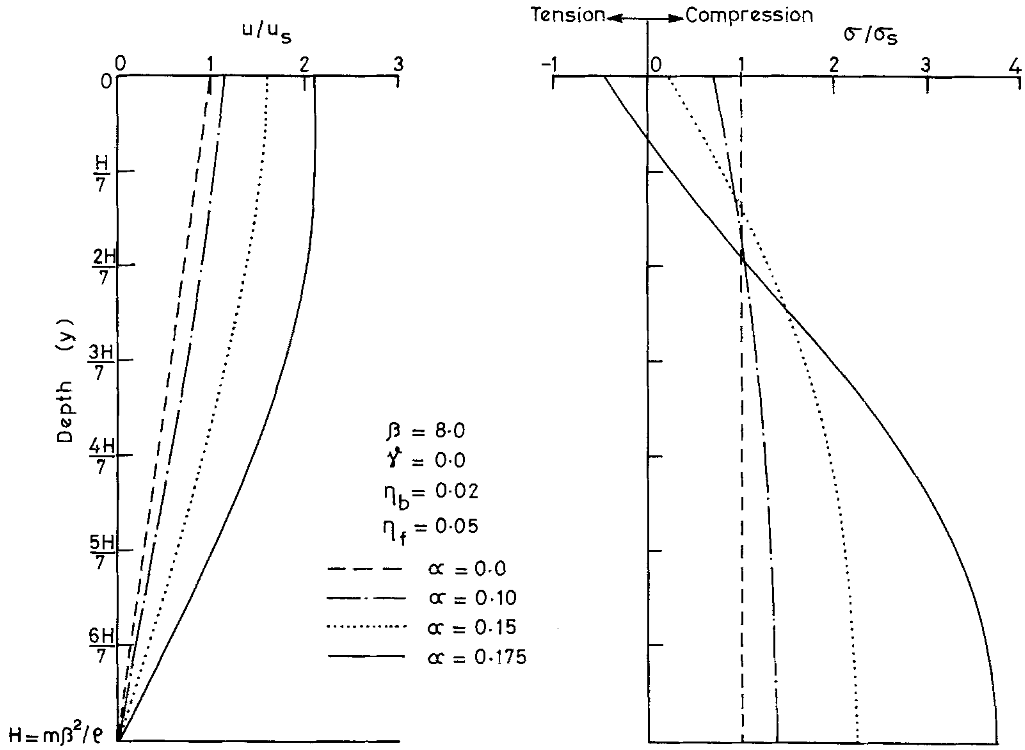

Fig. 11. Effect of velocity on vertical displacement and stress in the foundation

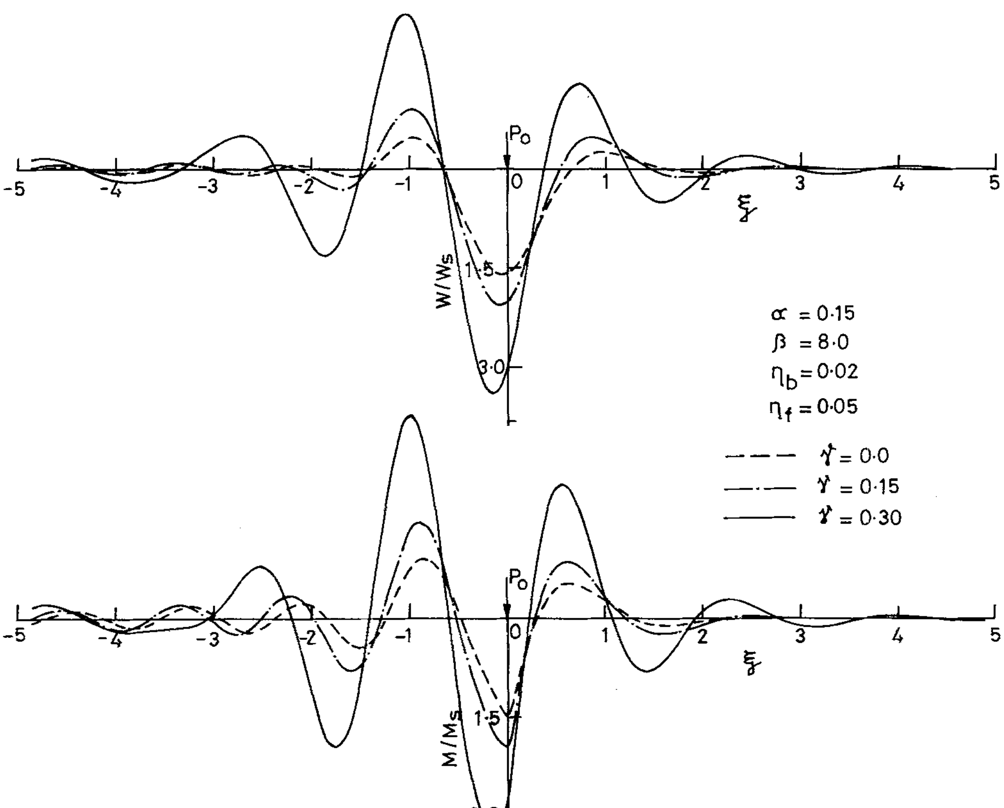

Fig. 12. Effect of axial force on the beam deflection and bending moment profiles

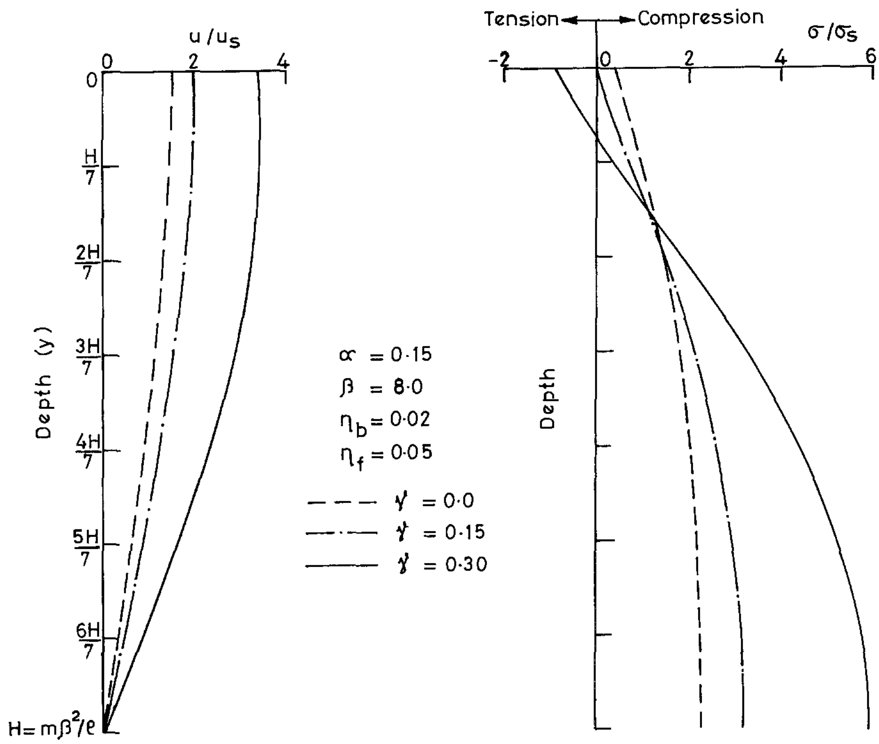

Fig. 13. Effect of axial force on the vertical displacement and stress in the foundation

6.3 Effect of γ

In Fig. 12 the effect of axial force on the beam deflection and bending moment profiles is shown. It is seen from this figure that the amplitudes in the profiles increase with γ. This is due to the loss of stiffness in the presence of an axial force on the beam. In Fig. 13 the effect of axial force on the foundation vertical displacement and stress is shown. This figure shows the variation of foundation displacement and stress along its depth for three different values of γ. Again it is seen that the presence of axial force can include tensile stresses in the top layers of the foundation.

6.4 Effect of η_b and η_f

In Figs. 14 and 15 the effect of beam damping and foundation damping on the foundation response is presented.

7 Discussion

The inclusion of foundation mass influences the combined beam-foundation response quantitatively as well as qualitatively. The most interesting and important feature is the effect on the critical velocity. Knowledge of the critical velocity is of considerable importance in moving load problems. It is shown in Fig. 2 that the foundation inertia can dramatically reduce the critical velocity. For example, when $\beta = 8.0$, which corresponds to a foundation depth of about 8 meters for a railway track, the critical velocity is only about 20% of the value which one gets if the

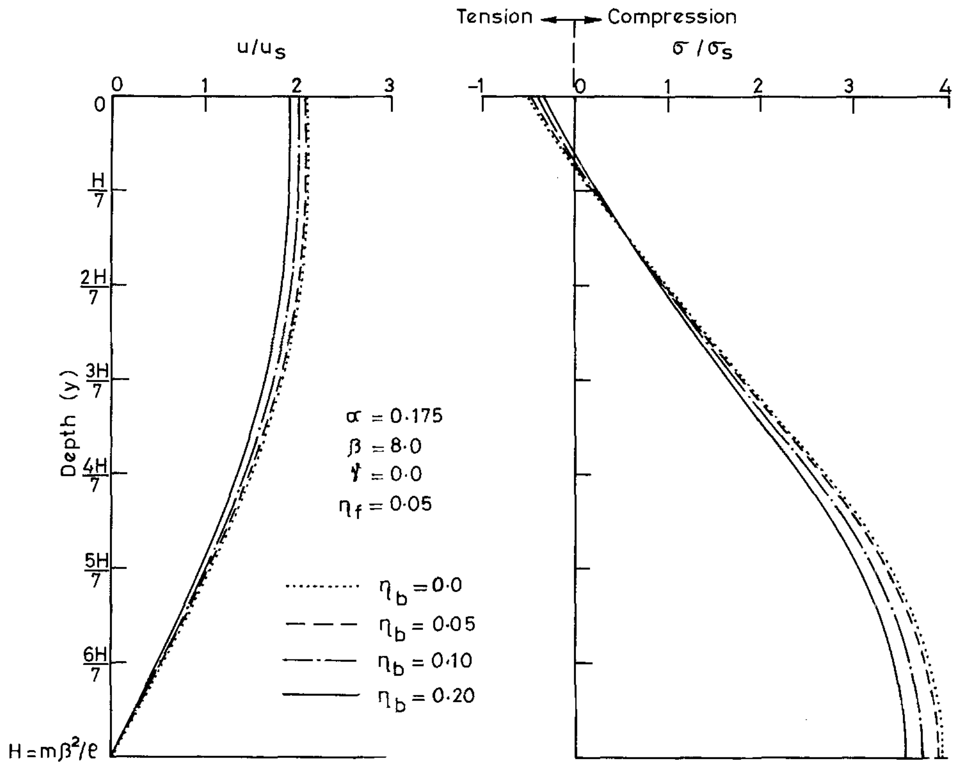

Fig. 14. Effect of beam damping on the vertical displacement and stress in the foundation

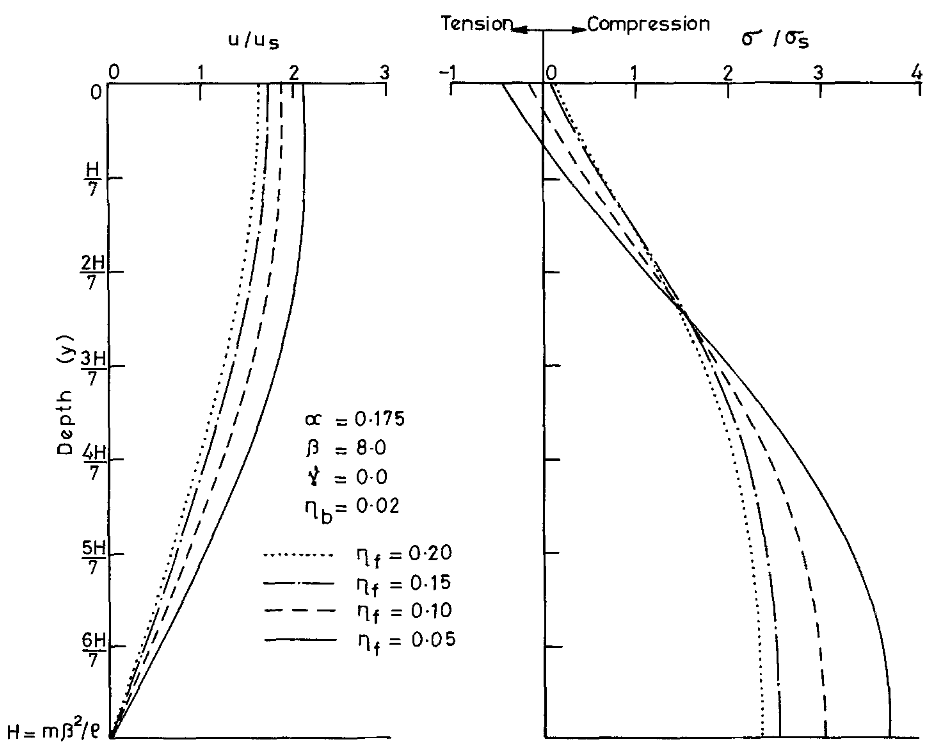

Fig. 15. Effect of foundation damping on the vertical displacement and stress in the foundation

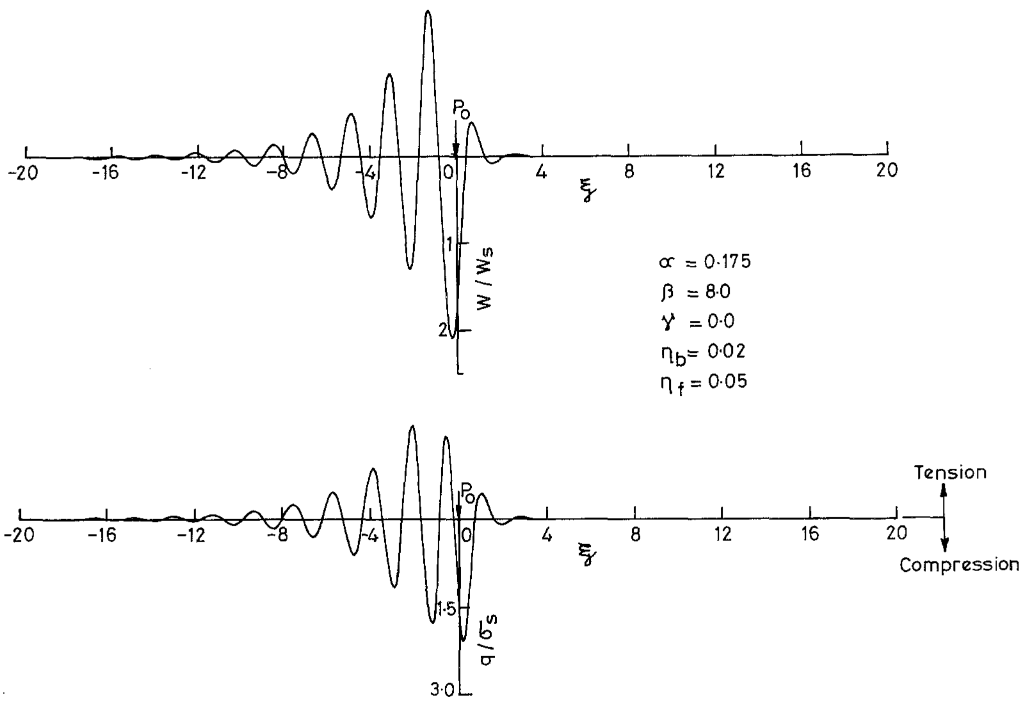

Fig. 16. Profiles of beam deflection and foundation pressure

foundation inertia were to be neglected. At any velocity less than the critical velocity, the inclusion of foundation mass effect increases the beam deflection as shown in Fig. 3. Moreover, near the critical velocity the spatial waviness in the response profiles increases considerably. Also, near the critical velocity the peak beam response increases significantly. The effect of foundation mass on the peak beam deflection is seen more clearly in Fig. 4a. It is observed from this figure that the variation of peak dynamic deflection with respect to β is dependent on the load velocity. For very low velocities, the foundation mass is not of much importance but at higher velocities the parameter β considerably increases the response. For example at $\alpha = 0.1$ an increase in β value from 2 to 7 increases the peak dynamic deflection by only 10% over the peak static deflection. However, at $\alpha = 0.175$ the corresponding increase is nearly 65%. A similar dependence is observed in the peak bending moment variation with respect to β as shown in Fig. 4b. The mass parameter β influences the foundation response also. This is clearly seen in Fig. 5. An important effect in the foundation is the development of tension at the top portion, as the velocity approaches the critical velocity. Also there can be a spatial phase lag between the beam deflection $w(\xi)$ and the foundation pressure $q(\xi)$ profiles as shown in Fig. 16. The effect of damping either in the beam or foundation is to reduce the peak response in the vicinity of $\alpha = \alpha_{cr}$, as in Figs. 9 and 10. However, it is generally seen that the foundation damping is more effective than the beam damping in reducing the response amplitudes. This in turn, again indicates the importance of including the foundation in the dynamic analysis of beams on elastic foundation. The effect of axial force, as expected, is to increase the response and to decrease the critical velocity. Interestingly, this may also cause tensile stresses in top layers of the foundation, as in Fig. 13.

It may be noted here that numerical results have been obtained for values of α only upto and slightly exceeding α_{cr}. If one desires to extend the results much beyond α_{cr}, suitable new values of Δf and N will have to be selected to avoid aliasing and rippling effects.

8 Conclusion

In this paper a comprehensive study on the effect of foundation inertia on the dynamics of a beam on an elastic foundation under a moving force has been presented. The foundation is modelled as a series of closely spaced, independent axial rods, fixed at the bottom and connected to the beam at the top. This foundation model is a simple generalization of Winkler's model to include foundation inertia. The effects of axial forces, beam and foundation damping have been included in the analysis. The foundation effect has been shown to be very important inasmuch as it can reduce the critical velocity and also can induce tensile stresses leading to separation between the beam and foundation. These results can be used for finding the critical velocity, and peak stresses in a beam-foundation system which would be directly useful in the study of railway tracks.

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