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EFFECT OF SELF-WEIGHT AND VERTICAL ACCELERATION ON THE BEHAVIOUR OF TALL STRUCTURES DURING EARTHQUAKE*

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SUMMARY

The effect of self-weight and vertical ground acceleration during earthquakes on vertical cantilevers has been studied. The input is taken to be a bivariate normal random process, digitally simulated on a computer. The tip deflection, base moment and shear force have been obtained numerically for three structures of different natural frequencies. It is found that the presence of self-weight and vertical ground excitation could alter these three quantities considerably. This leads to the conclusion that with tall structures a refined analysis, similar to the one presented here, is advisable.

INTRODUCTION

In recent years there has been a steady growth of understanding of earthquakes and their effects on structures. In the analysis of tall buildings, chimneys and towers it is customary to replace the structure by a suitable model such as a shear beam or a cantilever.[1] One then proceeds to obtain the deflection, bending moment and shear force under the horizontal ground motion. Though the effect of the horizontal component seems to be more important, the effects of vertical acceleration and distributed mass need to be understood. It may be pointed out here that in buckling analyses of columns free at the top, self-weight is also usually included.[2] In the present paper an effort is made to study these two effects on tall structures which could be idealized as uniform cantilevers. The effect of the vertical acceleration is considered only to change the weight of the structure. The extensional motion and the effect of the second horizontal component are not included in the analysis.

ANALYSIS

Referring to Figure 1, the equation of motion of the cantilever can be written as

$$\frac{\partial^2}{\partial y^2}\left\{EI(y)\frac{\partial^2 U}{\partial y^2}+(g+\ddot{y}_g)\int_y^h m(\xi)\,[U(\xi)-U(y)]\,\mathrm{d}\xi\right\} = m(y)\frac{\partial^2 U}{\partial t^2}-c\frac{\partial(U-x_g)}{\partial t} \tag{1}$$

where

$U(y,t)$ = absolute lateral deflection

$EI(y)$ = flexural rigidity

$m(y)$ = mass density

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c = viscous damping coefficient

$\ddot{x}_g, \ddot{y}_g$ = horizontal and vertical ground acceleration

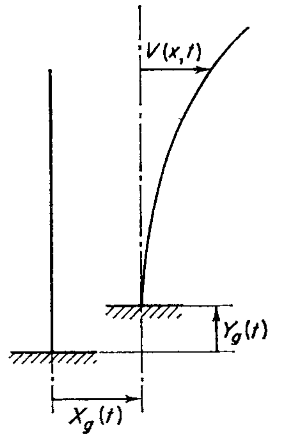


Figure 1. Cantilever model

The second term on the right-hand side of equation (1) represents viscous damping assumed to be proportional to the relative velocity. If the flexural rigidity and mass distribution remain uniform equation (1) can be simplified as

$$\frac{\partial^4 V}{\partial y^4}+\frac{m}{EI}(g+\ddot{y}_g)\left[(y-h)\frac{\partial^2 V}{\partial y^2}+\frac{\partial V}{\partial y}\right]=\frac{m}{EI}\left(\frac{\partial^2 V}{\partial t^2}+\ddot{x}_g\right)-\frac{c}{EI}\frac{\partial V}{\partial t} \tag{2}$$

where V is the relative motion with respect to the ground,

$$V(y,t)=U(y,t)-x_g(t) \tag{3}$$

An approximation in a Galerkin sense (boundary conditions are satisfied but not the differential equation) is used to solve equation (2) using the eigenfunctions of a cantilever beam.[3] Accordingly, the solution of equation (2) is assumed in the form

$$V(y,t)=\sum_{n=1}^{N} A_n(t)\,\Phi_n(y) \tag{4}$$

where N is a positive integer, $A_n(t)$ are the generalized co-ordinates and $\Phi_n(y)$ are the eigenfunctions;[3]

$$\Phi_n(y)=\left(\cosh\frac{\lambda_n y}{h}-\cos\frac{\lambda_n y}{h}\right)-\frac{\cosh\lambda_n+\cos\lambda_n}{\sinh\lambda_n+\sin\lambda_n}\left(\sinh\frac{\lambda_n y}{h}-\sin\frac{\lambda_n y}{h}\right) \tag{5}$$

where

$$\lambda_n^4 = m\omega_n^2 h^4/EI$$

$$\lambda_1 = 1{\cdot}8751,\quad \lambda_2 = 4{\cdot}6941,\quad \lambda_3 = 7{\cdot}8548$$

$$\lambda_4 = 10{\cdot}9955,\quad \lambda_5 = 14{\cdot}1371,\quad \lambda_6 = 17{\cdot}2787,\ldots$$

Substituting equation (4) into equation (2) and minimizing the mean square error over h one gets

$$\ddot{A}_i+\frac{c}{m}\dot{A}_i+\omega_j^2 A_i+\frac{g+\ddot{y}_g}{C_i}\left[\sum_{j=1}^{N} A_j(\mathrm{d}_{ij}-hf_{ij}+C_{ij})\right]=-\frac{\ddot{x}_g}{C_i}g_i \quad (i=1,2,\ldots,N) \tag{6}$$

in which

$$C_i=\int_0^h \Phi_i^2\,\mathrm{d}y$$

$$d_{ij} = \int_0^h y\Phi_i \Phi_j'' \, dy \qquad C_{ij} = \int_0^h \Phi_i \Phi_j' \, dy$$

$$f_{ij} = \int_0^h \Phi_i \Phi_j'' \, dy \qquad g_i = \int_0^h \Phi_i \, dy$$

where the prime indicates differentiation with respect to y. For given inputs $\ddot{x}_g$ and $\ddot{y}_g$ the number N of equations in the above system has to be determined weighing the accuracy of the result against the computational work involved. It is to be noted that the existence of $\ddot{y}_g$ gives rise to a system of equations [equation (6)] with time-varying coefficients.

The ground excitations are random functions. In the literature[4–8] many models have been proposed for the horizontal component. At present, however, it appears that not much is known about the vertical component and its correlation with the horizontal component. It may be pointed out here that there is no reason to believe the two components to be uncorrelated or the statistical structure of $\ddot{y}_g$ to have insignificant effects on the solution. In the face of lack of definite information, it is decided in this study to choose a bivariate stationary Gaussian process for $\ddot{x}_g$ and $\ddot{y}_g$ with the following spectral properties:

Power spectrum of $\ddot{x}_g$:

$$S_{11}(\omega) = \frac{S_1 \omega_g^4}{(\omega_g^2 - \omega^2)^2 + 4(\rho\omega\omega_g)^2} \tag{7}$$

Power spectrum of $\ddot{y}_g$:

$$S_{22}(\omega) = S_2 \frac{\omega_g^2(\omega_g^2 + 4\rho^2 \omega^2)}{(\omega_g^2 - \omega^2)^2 + 4(\rho\omega\omega_g)^2} \tag{8}$$

Cross spectrum between $\ddot{x}_g$ and $\ddot{y}_g$:

$$S_{12}(\omega) = S_3 \exp(-\omega^2/\omega_g^2) \tag{9}$$

The two components are generated using the expressions

$$\ddot{x}_g(t) = \sqrt{(2\Delta\omega)} \sum_{i=1}^{n} H_{11}(\omega_i) \cos(\omega_i' t + \Phi_{1i}) \tag{10}$$

$$\ddot{y}_g(t) = \sqrt{(2\Delta\omega)} \sum_{i=1}^{n} \{H_{22}(\omega_i) \cos(\omega_i' t + \Phi_{2i}) + |H_{21}(\omega_i)| \cos[\omega_i' t + \theta(\omega_i) + \Phi_{1i}]\} \tag{11}$$

where

$$H_{11} = (S_{11})^{\frac{1}{2}}$$

$$H_{22} = \left(\frac{S_{11} S_{22} - |S_{12}|^2}{S_{11}}\right)^{\frac{1}{2}}$$

$$H_{21} = \bar{S}_{12}/H_{11}$$

$$\Delta\omega = \frac{(\omega_u - \omega_l)}{n}$$

$$\omega_i = \omega_l + (i - \tfrac{1}{2})\Delta\omega, \quad \omega_i' = \omega_i + \delta\omega$$

Φ_{1i}, Φ_{2i} = uniformly distributed random number in $(0, 2\pi)$

$$\theta = \tan^{-1} \frac{\operatorname{Im} H_{21}(\omega)}{\operatorname{Re} H_{21}(\omega)}$$

ω_l, ω_u = lower and upper cut-off frequencies in the spectra

$\delta\omega$ = very small random frequency uniformly distributed in $-\Delta\omega/2$ and $\Delta\omega/2$

The theory behind the above procedure and other details have been presented elsewhere.[9,10] It is expected that the structure subjected to such an excitation will produce a typical motion from which a good understanding of the structural response under real seismic excitation can be obtained.

RESULTS AND DISCUSSION

The most significant responses of the structure under study would be the relative deflection at the tip, bending moment and shear force at the base. In the present case these are given by

$$V(h,t) = \sum_{i=1}^{N} A_i(t)\,\Phi_i(h) \tag{12}$$

$$M(0,t) = \sum_{i=1}^{N} A_i(t)\,[EI\Phi_i''(0) + m(g+\ddot{y}_g)\,g_i] \tag{13}$$

$$S(0,t) = \sum_{i=1}^{N} A_i(t)\,EI\Phi_i'''(0) \tag{14}$$

These three responses have been studied in some detail for three cantilevers of different natural frequencies and heights. These are designated as structures I, II and III and their properties are as in Table I. For

Table I

Structure	h (ft)	m (lb ft^{-2} sec^2)	EI (psf)	f_1* (cps)
I	50	10	$3{\cdot}6\times10^8$	1·343
II	100	10	$3{\cdot}6\times10^8$	0·336
III	200	10	$25{\cdot}2\times10^8$	0·222

* Fundamental frequency.

purposes of comparison, the responses have been computed also by neglecting the self-weight and vertical acceleration. In each case, the system of equations given by equation (6) has been solved numerically by a Runge–Kutta–Gill procedure on an IBM 360 computer for various sample inputs. The damping coefficient c has been taken as

$$c = 2\eta\omega_1 m \tag{15}$$

where $\eta = 0{\cdot}01$ and ω_1 is the fundamental natural frequency of the structure in radians per second. For the simulated ground motion the various constants in equations (7–9) have been selected as

$$\left.\begin{aligned} &\omega_g = 6\pi, \quad \rho = 0{\cdot}5 \\ &S_1 = 0{\cdot}02\rho/\pi\omega_g \\ &S_2 = 2{\cdot}25S_1/(1+4\rho^2) \\ &S_3 = 0{\cdot}05S_1/\rho \end{aligned}\right\} \tag{16}$$

A suitable choice of the number N of equations to be retained in equation (6) is essential for numerical accuracy as well as economy in computer cost. In view of this, equation (6) has also been studied in the frequency domain considering

$$\left.\begin{aligned} &\ddot{x}_g = e^{i\lambda t} \\ &\ddot{y}_g = 0 \\ &A_j = H_j e^{i\lambda t} \end{aligned}\right\} \tag{17}$$

Substitution of this into equation (6) yields a system of algebraic equations for H_j, which could be solved easily. Using these values of H_j, the frequency response functions of $V(h,t)$, $M(0,t)$ and $S(0,t)$ can be obtained as

$$H_V(\lambda) = \sum_{j=1}^{N} H_j(\lambda)\,\Phi_j(h) \tag{18}$$

$$H_M(\lambda) = \sum_{j=1}^{N} H_j(\lambda)\,(EI\Phi_j''(0)+mgg_j) \tag{19}$$

$$H_S(\lambda) = \sum_{j=1}^{N} H_j(\lambda)\,EI\Phi_j'''(0) \tag{20}$$

Figure 2 shows the result of such analysis performed on structure II plotting the amplitude of the frequency response function. The input spectra are also shown in the same figure. The input spectra decay very fast

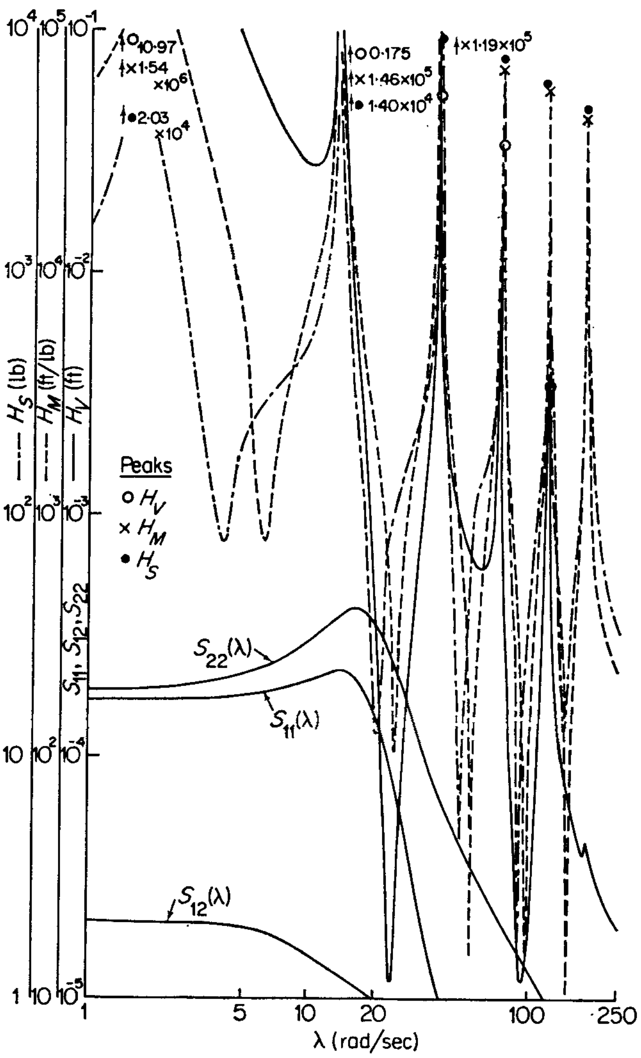


Figure 2. Frequency response function of structure II and input spectra

and it is seen from the figure that only the first five modes significantly contribute to the response. Accordingly in equation (6) five equations ($N = 5$) have been retained. Since the responses are random, one needs to know their statistical properties in as much detail as possible. Herein estimates have been obtained for the r.m.s. responses of all the three structures considered. Nine samples for structure I and nineteen samples for the other two have been used in arriving at the estimates by ensemble-averaging. Three typical sample responses are shown in Figures 3, 4 and 5. Only nine sample responses are computed for structure I since its natural frequencies are much higher than those corresponding to other structures and hence it is much more time-consuming to perform a time-domain analysis including up to the fifth mode. Figures 6, 7 and 8 show the

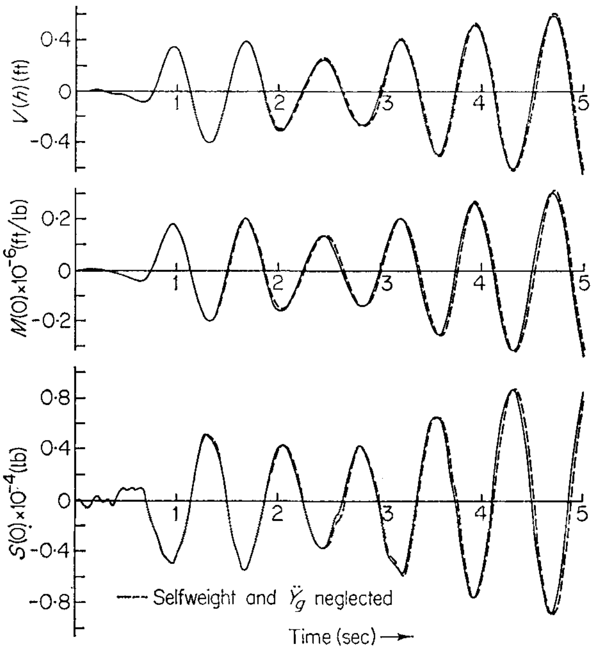


Figure 3. Response history of structure I

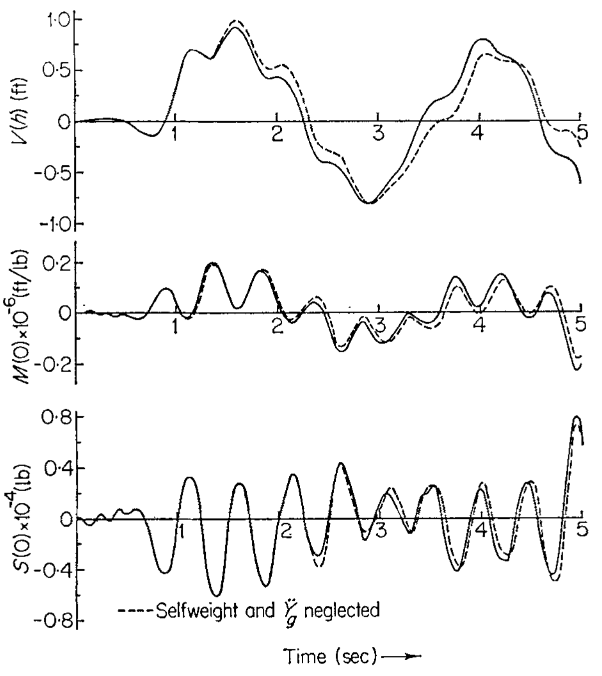


Figure 4. Response history of structure II

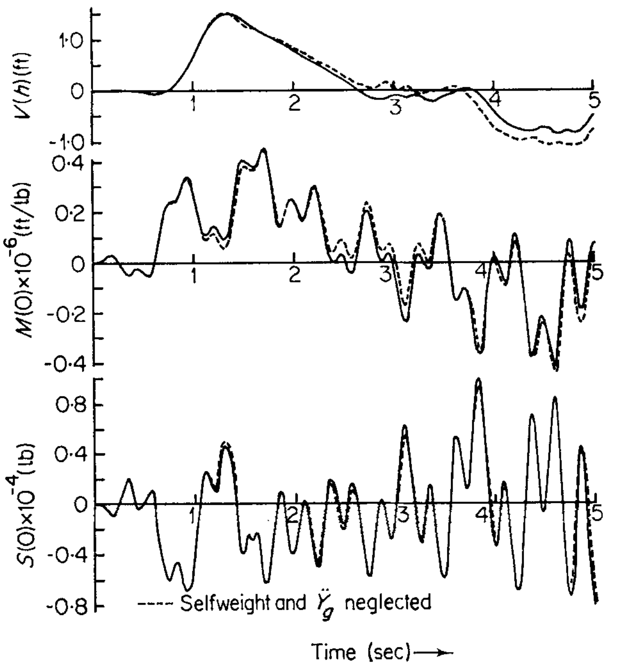


Figure 5. Response history of structure III

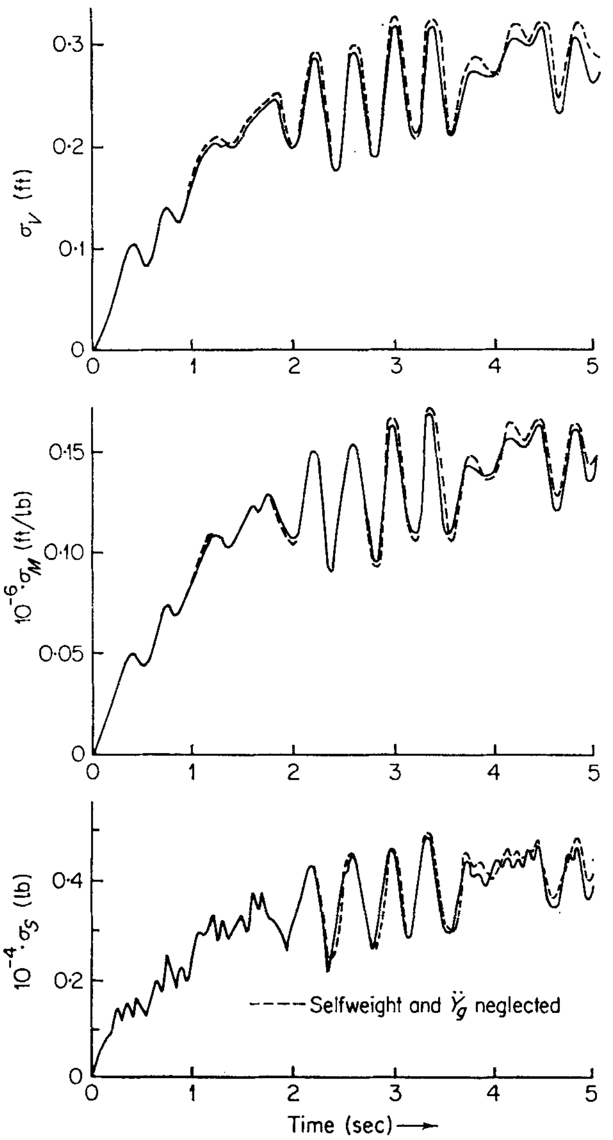


Figure 6. R.m.s. response of structure I

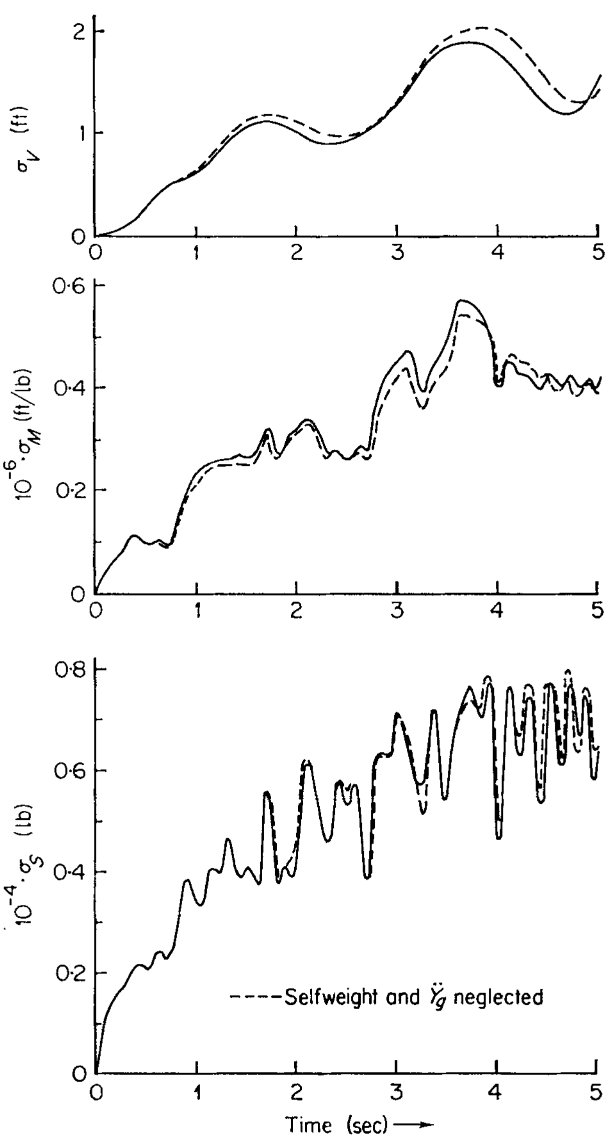


Figure 7. R.m.s. response of structure II

estimates of the r.m.s. responses as functions of time. These figures clearly indicate the effect of the self-weight and the vertical acceleration. As one could expect, the difference is more considerable with the taller structures (II, III) than with the shorter one. For structural design the highest absolute peak responses are more significant and hence these are also obtained in all the samples during the first five seconds of earthquake. Since the number of samples is small, it is not possible to obtain a reliable estimate of the probability density function. Instead, the (absolute) maximum responses of all the samples have been presented in Figures 9–11. It is seen that the consideration of self-weight and $\ddot{y}_g$ might either increase or decrease the peak responses. However, the difference either way seems to be considerable in most cases.

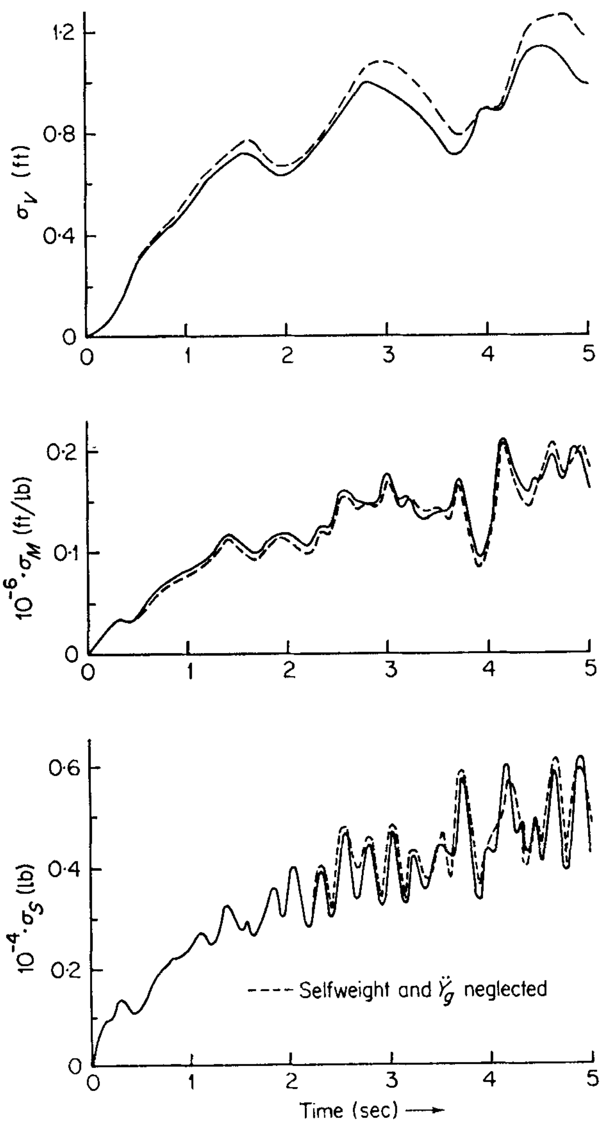


Figure 8. R.m.s. response of structure III

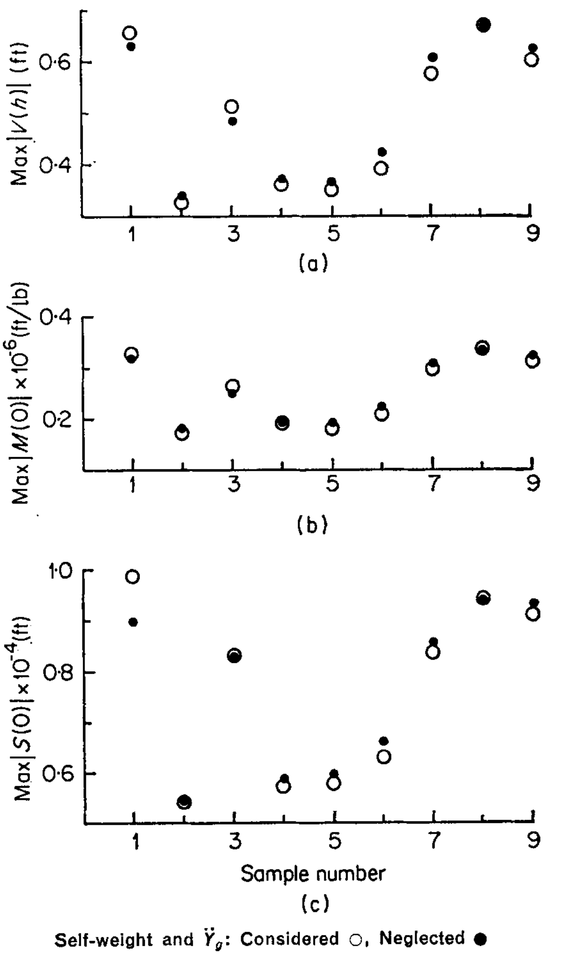
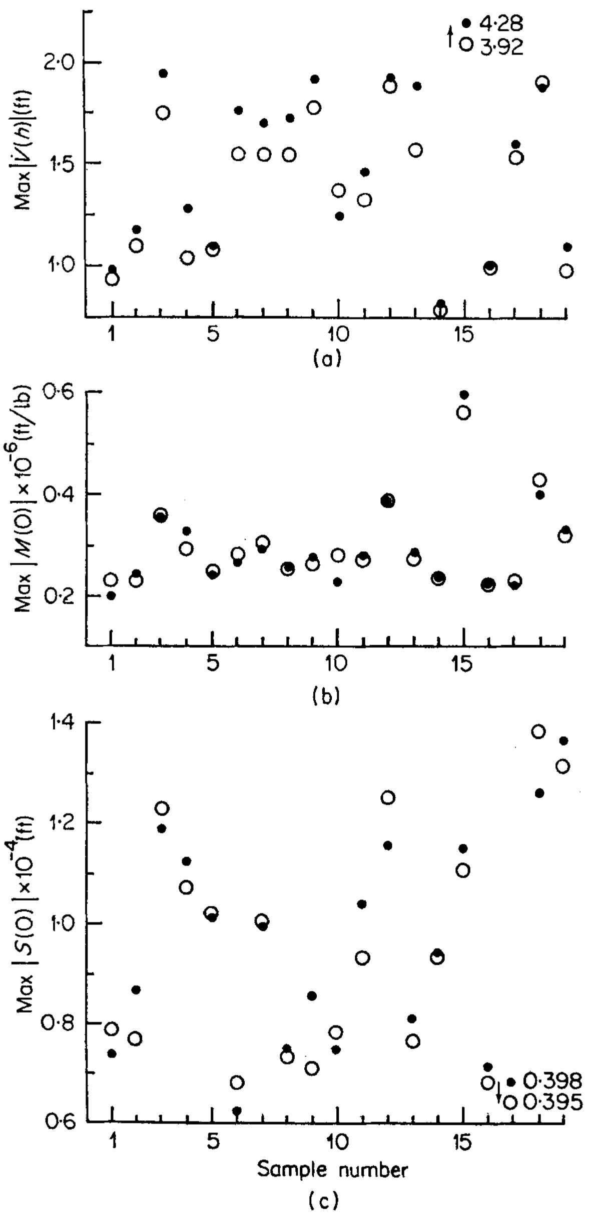
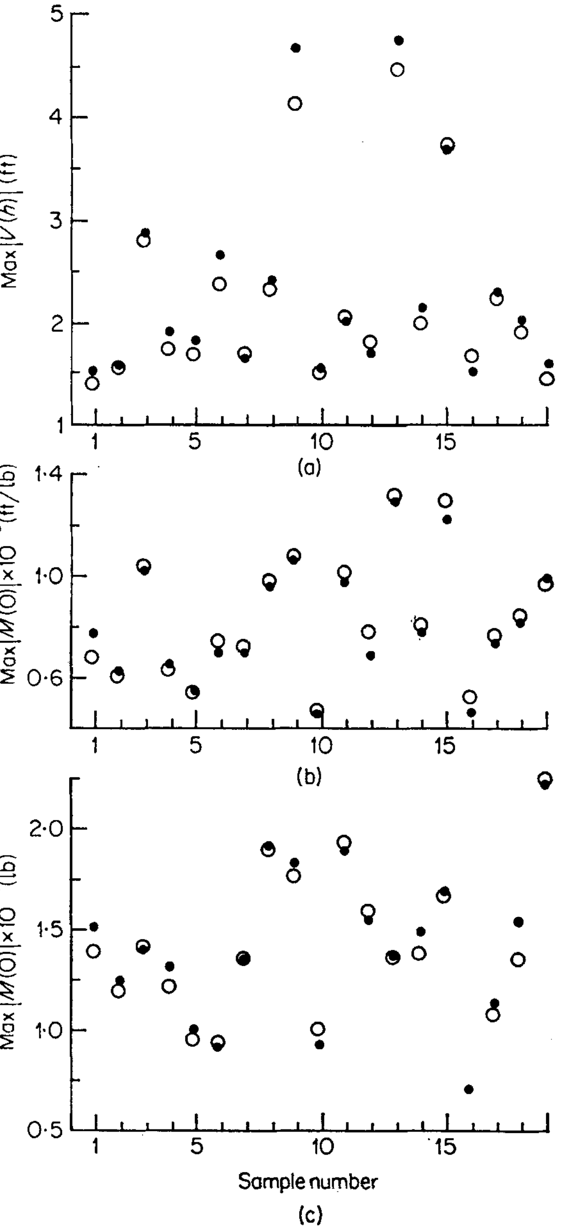


Figure 9. Effect of self-weight and vertical acceleration for structure I in terms of absolute maximum response





Self-weight and $\ddot{Y}_g$: Considered ○, Neglected ●

Figure 10. Effect of self-weight and vertical acceleration for structure II in terms of absolute maximum response

Figure 11. Effect of self-weight and vertical acceleration for structure III in terms of absolute maximum response

The above result implies that the effect of vertical acceleration could be much more pronounced, particularly in beam response, when a frame structure is considered without recourse to the shear building assumption. A general method of dynamic response analysis in which the distributed mass of the frame structure can be taken into consideration was developed in Reference 11. It will be an interesting future study to assess the effect of vertical acceleration on the response of the frame structure by combining the method in Reference 11 with the simulation technique described here.

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