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ARTICLE

# Planetary nodes and apses in the *Sūrya-Siddhānta*

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**Abstract**
There are a few aspects of Indian astronomy that border on the implausible, and none more so than the nodal and apsidal movements of planetary orbits as specified in the *Sūrya-Siddhānta*. In this paper we examine some aspects of this data in the light of modern astronomy. Given the very-slow rates of motion specified for these two parameters, we speculate that this data might be heliocentric (as opposed to geocentric), and proceed with that assumption in this paper. It is determined that while this assumption proves true for the outer planets (Mars, Jupiter and Saturn), it does not conform well with the inner planets (Mercury and Venus). It is also determined that while the directions specified in the *Sūrya-Siddhānta* for these subtle motions are correct, the rates of motion given are slower by a couple orders of magnitude compared to actuality. Further analysis also appears to confirm that the data is genuine, and the epoch of the data is likely many thousands of years old.

**Keywords** Nodes · Apsis · *Pāta* · *Manda* · Heliocentric · *Sūrya-Siddhānta*

## 1 Introduction

There are, in Indian astronomy, several intriguing and wondrous items which have exercised the sagacity of the best minds of the past and continues to do so.

A case in point is the earth's precessional constant. It is well known that this quantity is exceedingly difficult to determine without sophisticated instrumentation, requiring a very long period of sustained observation. Most civilizations of the past have not even been aware of this subtle movement of the earth, much less measuring it with any degree of accuracy. The Greeks, for example, only suspected that such a movement exists, and estimated it at 36 arc-seconds per year. The ancient Indian value of 54 arc-seconds per year has been the subject of wonder and speculation among the scientific community due to its proximity to the actual value today of 50.4 arc-seconds, that is, 1° in 71 years.

Another such wonderment in Indian astronomy, which sits at a level even greater than the knowledge of precession, and perhaps borders on the incredible, are the movements of the nodes and apses of planetary orbits as given in the *Sūrya-Siddhānta* (Burgess, 1858). These movements are many orders finer than that of precession. For example, the apsidal motion given for Saturn is 1° in 300,000 years!

We will not, in this paper, delve deep into how the ancient Indians may have obtained these numbers, but focus our attention on other aspects of the topic, including the curious presence of heliocentricity that is implied in the data.

We begin with an overview of planetary Nodes and Apses.

## 2 Planetary Nodes and Apses

It is common knowledge that the earth revolves around the sun. Due to relative motion, to an observer on the earth, it appears that the sun revolves around the earth. Fig. 1 shows the plane of the sun's apparent orbit around the earth (e), known as the ecliptic-plane.

The figure also shows the apparent orbit of a planet (say Mars) around the earth, which too, like the apparent sun, travels in an eastwards direction. It can be seen that Mars's orbit is inclined with respect to the ecliptic-plane, meeting it at two points—the nodes. The node at which the planet crosses from south of the ecliptic to north is known as the ascending-node ($n_a$), while that at which the planet moves

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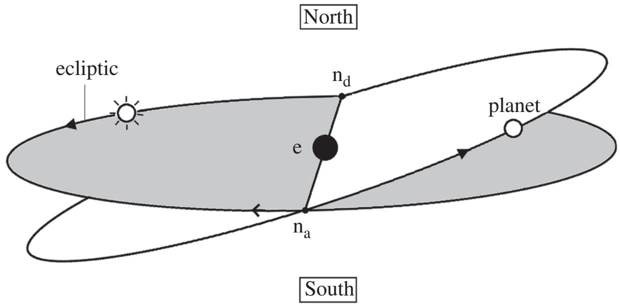


**Fig. 1** Planetary Nodes

from north of the ecliptic to south is called the descending-node ($n_d$).

Due to gravitational influences among the planets, these nodal points, called *pāta* in Indian astronomy, are not stationary in space, but have a slow movement westwards, that is, in a direction opposite to the movement of the planet itself, as shown in the figure. This nodal movement is extremely slow, taking many hundreds of years to show a noticeable change.

Another variation in planetary orbits is the apsidal motion. As shown in Fig. 2, the apparent path of the planets around the earth is ovular in shape. The closest point of the orbit to the earth is called perigee (P) while the farthest point is called the apogee (A).

The apogee is called *manda* in Indian astronomy. The line joining the apogee and perigee is known as the apsidal-line. As shown on the right side of Fig. 2, the apsidal line has a slow rotation in time, just like the nodes. Unlike the nodal movement, however, this apsidal motion is in the same direction as that of the planet itself, that is, eastwards.

With that overview, let us now look at the data for the ascending-node (*pāta*) and apogee (*manda*) motion of planetary orbits, as given in the *Sūrya-Siddhānta*.

## 3 The data

The *Sūrya-Siddhānta* presents the number of orbits (revolutions) completed by the ascending-nodes of the five visible planets in a *Kalpa* of 4,320,000,000 years, as shown in Table 1.

Similarly, the revolutions of the apses of various planets in a *kalpa* are given as shown in Table 2.

Note that the Sun's apsidal motion given in Table 2 is actually that of the earth, due to relative motion

The first three columns of these tables comprise the *Sūrya-Siddhānta* data; the latter two columns are calculated by us. The fourth column results from dividing 4.32 billion years (a *Kalpa*) by the second column, which gives us the number of years required for one revolution. The

**Table 1** Planetary Nodal Movement in the *Sūrya-Siddhānta*

| Planet | Revolutions in a *Kalpa* | Direction | Years per revolution | Years per arc-minute |
|---|---|---|---|---|
| Mercury | 488 | Westwards | 8,852,459 | 410 |
| Venus | 903 | Westwards | 4,784,053 | 221 |
| Mars | 214 | Westwards | 20,186,916 | 935 |
| Jupiter | 174 | Westwards | 24,827,586 | 1149 |
| Saturn | 662 | Westwards | 6,525,680 | 302 |

**Table 2** Planetary Apsidal Movement in the *Sūrya-Siddhānta*

| Planet | Revolutions in a *Kalpa* | Direction | Years per revolution | Years per arc-minute |
|---|---|---|---|---|
| Sun (Earth) | 387 | Eastwards | 11,162,791 | 517 |
| Mercury | 368 | Eastwards | 11,739,130 | 543 |
| Venus | 535 | Eastwards | 8,074,766 | 374 |
| Mars | 204 | Eastwards | 21,176,471 | 980 |
| Jupiter | 900 | Eastwards | 4,800,000 | 222 |
| Saturn | 39 | Eastwards | 110,769,231 | 5128 |

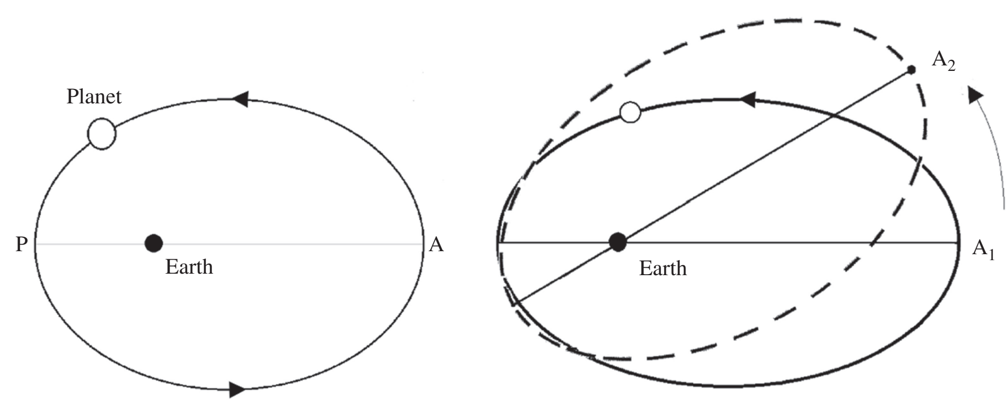


**Fig. 2** Planetary Apsidal Motion

last column is obtained by dividing the fourth column, first by 360, and then again by 60, which gives us the number of years required to move by 1 arc-minute. The significance of 1 arc-minute is that it is the smallest angular distance that can be detected by the human eye, that is, by naked-eye observation.

The last column of these tables is quite conclusive. It is seen that it would take many centuries, and even millennia (in the case of Saturn), for the node/apsis locations to shift by 1 arc-minute, which is the smallest change that can be detected by the human eye.

It is quite inconceivable that such sustained and very accurate observations could have been conducted by the ancients, uninterrupted for many centuries, and even millennia, to have obtained these numbers in the *Sūrya-Siddhānta*. Therefore, it is almost certain that these numbers were produced as a result of tweaking the Indian planetary model to match observational data, that is, the observed motion of the planet itself (not its Nodes/Apses).

To bolster this conclusion, we present below another aspect of the data.

Fig. 3 shows the orbits of Earth (E) and Mars (M) around the Sun (S). It presents three cases comprising successive passages of Mars through its ascending-node in the years 2003, 2005, and 2007, respectively, and the relative positions of the Earth, Mars and Sun. Note that the orbital period of Mars is 687 days, and V represents the direction of the reference point, namely, the vernal-equinox, from which all longitudes are measured.

It can be seen that in the year 2003, towards the end of the year, when Mars was at its ascending-node, its ecliptic longitude (shaded), as observed from the earth, was around 8°. In 2005 this quantity was 42°, and in 2007 it was 92°.

Thus, while the sun-based heliocentric longitude of Mars's ascending-node (angle VSM) is more or less

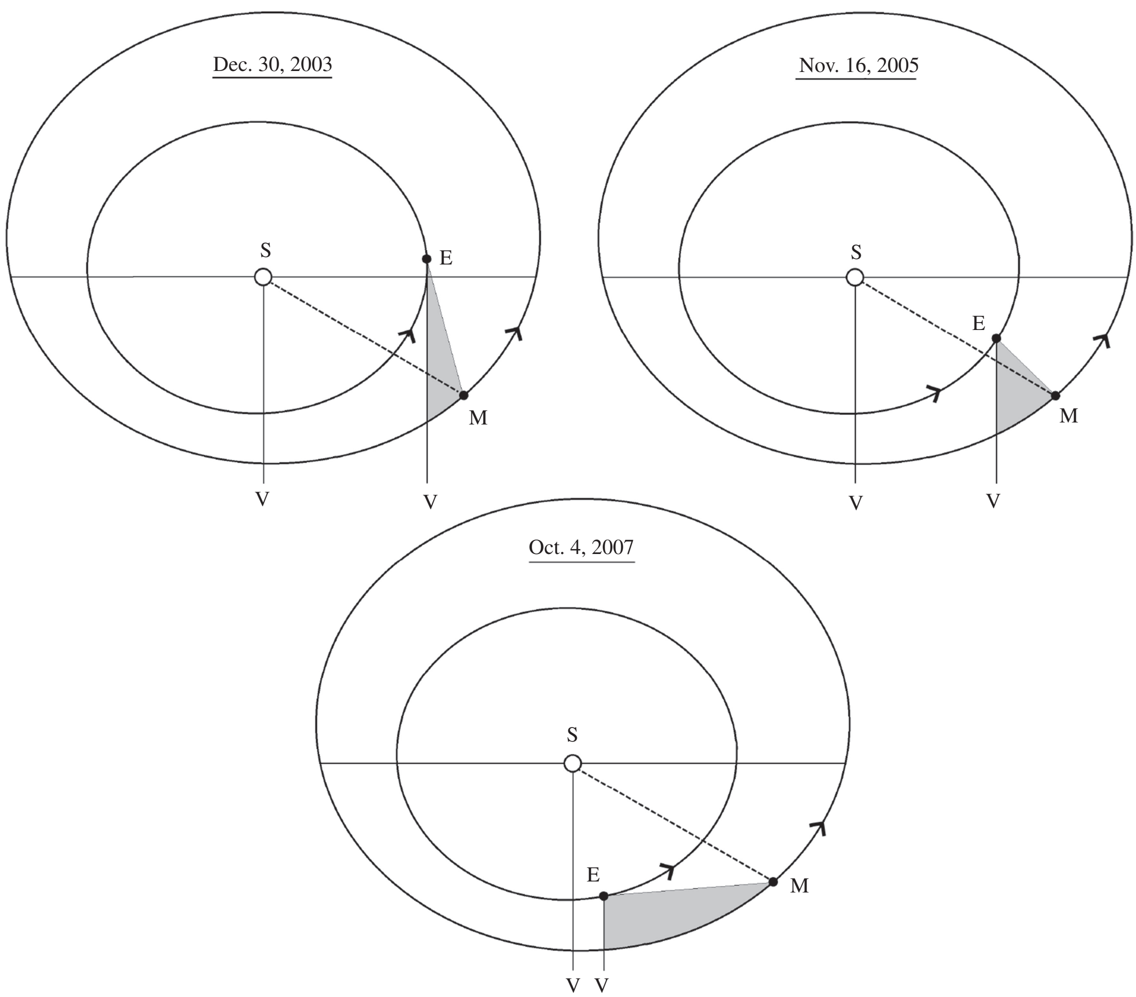


**Fig. 3** Successive Passages of Mars through its Ascending Node

constant, observations of the ascending-node made from the earth are all very different. Therefore, the very-slowly changing node and apses locations given in the *Sūrya-Siddhānta* are not the result of earth-based observation of these parameters.

## 4 Computation and discussion

With the preliminaries out of the way, let us now make some computations of the nodes and apses of various planets as given in the *Sūrya-Siddhānta*, and compare them with results from modern formulae (Meeus, 2000).

### 4.1 Age of the Universe

Note that the *Sūrya-Siddhānta* gives us only the rate-of-motion of the nodes and apses. To calculate their locations at any given time, we must first compute the time passed (age of the universe until that time), and multiply that by the rate-of-motion.

According to Indian mythology, the universe is expected to last for a *kalpa* of 4.32 billion years. Within this period are found sub-periods called *Mānavantaras* of about 300 million years each. A *Mānavantara* comprises 71 odd *Mahāyugas* of 4.32 million years each, and finally, within each *Mahāyuga* are four ages (*Kṛta, Tretā, Dvāpara*, and *Kali*).

Per the *Sūrya-Siddhānta*, six *Mānavantaras* have passed in the present *Kalpa*. Of this, the seventh, 27 *Mahāyugas* have passed. And finally, of this, the 28th *Mahāyuga*, the *Kṛta*, *Tretā*, and *Dvāpara* ages are past, and we have recently entered into the *Kali* age. Note that each of these Indian periods also has a 'dawn' and a 'dusk' sub-period, which must be factored into the calculation. Finally, note also that creation was not instantaneous, but took a little over 17 million years according to the *Sūrya-Siddhānta*. For further details refer to the Indian Time Cycles (Narayanan, 2022). Thus, we have:

**Table 3** Age of the Universe at start of *Kali*

| Cycle | Time (years) |
|---|---|
| Dawn of *Kalpa* | 1,728,000 |
| 6 *Mānavantaras* | 1,850,688,000 |
| 27 *Mahāyugas* | 116,640,000 |
| *Kṛta* | 1,728,000 |
| *Tretā* | 1,296,000 |
| *Dvāpara* | 864,000 |
| Subtract time for creation | − 17,064,000 |
| Sum Total = Age of Universe at start of *Kali* | 1,955,880,000 |

**Table 4** Test dates

| Test date (year) | Age of Universe (years) |
|---|---|
| 3102 BCE (*Kali* Start) | 1,955,880,000 |
| 4700 BCE | 1,955,878,402 |
| 4000 BCE | 1,955,879,102 |
| 3000 BCE | 1,955,880,102 |
| 2000 BCE | 1,955,881,102 |
| 1000 BCE | 1,955,882,102 |
| 0 CE | 1,955,883,102 |
| 1000 CE | 1,955,884,102 |
| 2000 CE | 1,955,885,102 |

As shown in Table 3, summing up the periods that have passed, and subtracting the time for creation, we obtain the age of the universe at the start of *Kali* (universally accepted to be 3102 BCE) as 1.96 billion years. With that as reference, we calculate in Table 4 the total time passed for various test dates which we will employ.

### 4.2 Location of Nodes and Apses, per the *Sūrya-Siddhānta*

The *Sūrya-Siddhānta* assumes that at creation everything in the heavens was situated at a fixed origin, including the planets, their nodes, and their apses. From that moment, the planets and their apses move eastward, while their nodes move westward. Having determined above the total time passed since creation, we can now compute the positions of the planetary nodes and apses at various test dates, simply by multiplying the rate-of-motion of the nodes/apses (from Tables 1, 2) by the time passed. Note that nodal motion is westwards, whereas longitudes are measured eastwards; therefore the raw result for nodes must be subtracted from 360°. The results are given in Tables 5 and 6.

Observe, for each planet, the decreasing longitude in time, which shows that the nodal movement of all planets is westwards, according to the *Sūrya-Siddhānta*.

As a matter of interest, we quote below from Aryabhata's concise booklet, the *Āryabhaṭīyam* (Clark, 1930), written around 500 CE, which presents the same numbers in Table 5 in rounded form:*verse 9a:the ascending nodes of Mercury, Venus, Mars, Jupiter and Saturn,having moved, are situated at 20, 60, 40, 80 and 100 degrees from the origin*

In contrast to the nodes, the increasing longitude with time in Table 6 shows that the apsis movement of all planets is eastwards, according to the *Sūrya-Siddhānta*. Once again, referring to the *Āryabhaṭīyam* on this:

*Verse 9b:the apsides of the Sun and the above-mentioned planetsare at 78, 210, 90, 118, 180 and 236*

We note that there appears to be a discord of about 10 degrees each for the apses of Mercury, Venus, Mars and

**Table 5** Nodal locations for test dates

| Location of ascending Nodes, per *Sūrya-Siddhānta* (in deg) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Planet | 4700 BCE | 4000 BCE | 3000 BCE | 2000 BCE | 1000 BCE | 0 CE | 1000 CE | 2000 CE |
| Mercury | 20.94 | 20.92 | 20.88 | 20.84 | 20.79 | 20.75 | 20.71 | 20.67 |
| Venus | 60.15 | 60.1 | 60.02 | 59.95 | 59.87 | 59.8 | 59.72 | 59.65 |
| Mars | 40.17 | 40.16 | 40.14 | 40.12 | 40.1 | 40.08 | 40.07 | 40.05 |
| Jupiter | 79.76 | 79.75 | 79.74 | 79.72 | 79.71 | 79.7 | 79.68 | 79.67 |
| Saturn | 100.71 | 100.67 | 100.61 | 100.56 | 100.5 | 100.45 | 100.39 | 100.34 |

**Table 6** Apsidal locations for test dates

| Location of Apogee, per *Sūrya-Siddhānta* (in deg) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Planet | 4700 BCE | 4000 BCE | 3000 BCE | 2000 BCE | 1000 BCE | 0 CE | 1000 CE | 2000 CE |
| Sun (Earth) | 77.08 | 77.10 | 77.13 | 77.16 | 77.19 | 77.23 | 77.26 | 77.29 |
| Mercury | 220.27 | 220.29 | 220.32 | 220.35 | 220.38 | 220.41 | 220.44 | 220.47 |
| Venus | 79.57 | 79.61 | 79.65 | 79.69 | 79.74 | 79.78 | 79.83 | 79.87 |
| Mars | 129.93 | 129.94 | 129.96 | 129.98 | 129.99 | 130.01 | 130.03 | 130.05 |
| Jupiter | 170.87 | 170.93 | 171.00 | 171.08 | 171.15 | 171.22 | 171.30 | 171.38 |
| Saturn | 236.60 | 236.61 | 236.61 | 236.61 | 236.62 | 236.62 | 236.62 | 236.63 |

Jupiter between the *Āryabhaṭīyam* and *Sūrya-Siddhānta*. We will not, however, probe into this matter in this paper.

### 4.3 Location of Nodes and Apses, per modern formulae

Before attempting to apply modern formulae to calculate the positions of the nodes and apses for the test dates, we must first be clear on what the *Pāta* (node) and *Manda* (apogee) imply in modern technical parlance.

As mentioned earlier, the very-slowly changing planetary nodes/apses specified by the *Sūrya-Siddhānta* cannot possibly be earth-based (geocentric) data, since earth-based measurements, as we saw, will be quite different from year to year. Thus, we will assume that the *Sūrya-Siddhānta* data is perhaps the very-slowly changing heliocentric longitude of these two quantities, and see what we get.

Table 7 shows the computed ecliptical longitudes of heliocentric planetary nodes for the test dates, using modern formulae.

In a similar fashion, we compute the heliocentric aphelion locations in Table 8.

There are two points to keep in mind while comparing Tables 5 and 6 with Tables 7 and 8: (a) While the *Sūrya-Siddhānta* employs a fixed *Sidereal* origin, modern formulae use the non-fixed *Tropical* system, whose origin moves in time. (b) The change of the Tropical system's origin is due to precession, which has an average rate-of-motion of 50.4 arc-seconds per year, or 14° in 1000 years. Thus, to compare the two sets of tables, we will need to compensate for the changing origin in Tables 7 and 8.

**Table 7** Modern heliocentric nodal positions for test dates

| Location of heliocentric ascending Node (in deg) (Modern) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Planet | 4700 BCE | 4000 BCE | 3000 BCE | 2000 BCE | 1000 BCE | 0 CE | 1000 CE | 2000 CE |
| Mercury | 325.7 | 333.63 | 345.24 | 358.07 | 9.79 | 21.6 | 33.5 | 45.49 |
| Venus | 17.11 | 23.13 | 32.1 | 41.26 | 48.74 | 57.42 | 66.66 | 75.49 |
| Mars | 354.76 | 0.96 | 9.38 | 17.54 | 25.5 | 33.75 | 41.66 | 49.32 |
| Jupiter | 33.79 | 40.57 | 50.56 | 60.1 | 70.09 | 80.1 | 90.26 | 100.45 |
| Saturn | 54.86 | 61.3 | 69.93 | 78.52 | 87.32 | 96.14 | 104.96 | 113.66 |

**Table 8** Modern aphelion positions for test dates

| Location of heliocentric aphelion (in deg) (Modern) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Planet | 4700 BCE | 4000 BCE | 3000 BCE | 2000 BCE | 1000 BCE | 0 CE | 1000 CE | 2000 CE |
| Sun (Earth) | 171.20 | 182.11 | 199.60 | 213.74 | 232.10 | 249.90 | 264.71 | 283.70 |
| Mercury | 154.56 | 165.49 | 180.80 | 195.60 | 211.00 | 226.46 | 241.70 | 257.28 |
| Venus | 214.18 | 224.90 | 239.70 | 254.05 | 268.95 | 283.11 | 297.32 | 311.77 |
| Mars | 33.28 | 46.37 | 64.63 | 82.40 | 100.74 | 119.16 | 137.62 | 156.16 |
| Jupiter | 92.58 | 102.97 | 116.32 | 130.99 | 148.10 | 162.60 | 178.17 | 194.83 |
| Saturn | 144.73 | 157.44 | 178.76 | 196.53 | 216.58 | 235.30 | 253.50 | 273.73 |

### 4.4 Direction of motion

The first thing we will verify is whether the direction of motion of the node/apses given in the *Sūrya-Siddhānta* is in line with modern data.

To obtain the nodal change per 1000 years, we will subtract the values of adjacent columns in Table 7. Note that the difference of the first two columns (4700–4000 BCE) will need to be multiplied by 10/7, since the change is only for 700 years, and not 1000. The results are shown in Table 9.

To find the movement with respect to a fixed origin (like the *Sūrya-Siddhānta* data) we must subtract the precessional movement in 1000 years, namely, of 50.4 arc-seconds × 1000/3600 = 14°, from each cell. Doing that we obtain the values shown in Table 10.

Thus, we see that all actual (modern) planetary nodal movement is indeed westwards (negative), as stated in the *Sūrya-Siddhānta*.

Repeating the above steps for apsidal motion, we obtain the values shown in Table 11.

After subtracting the precessional movement of 14°, we obtain the figures in Table 12.

With one exception (Jupiter in the time range 4700–4000 BCE), we see that all actual (modern) planetary apsidal movement is eastwards (positive), once again verifying what has been stated in the *Sūrya-Siddhānta*. We are unable to explain the one discrepancy with Jupiter at this time.

### 4.5 Rates of motion

The following Table 13 compares the rate-of-motion of the planetary nodes per the *Sūrya-Siddhānta* (Table 1) with modern data (Table 10).

The average rate-of-motion (degrees per 1000 years) given in Table 10 was multiplied by 60 and the product used as a divisor to 1000, the result giving us the number of years

**Table 9** Nodal movement per 1000 years

| Modern nodal movement in 1000 years (eastwards, in deg) | | | | | | | |
|---|---|---|---|---|---|---|---|
| Planet | 4700–4000 BCE | 4000–3000 BCE | 3000–2000 BCE | 2000–1000 BCE | 1000–0 BCE | 0–1000 CE | 1000–2000 CE |
| Mercury | 11.33 | 11.61 | 12.83 | 11.72 | 11.81 | 11.9 | 11.99 |
| Venus | 8.6 | 8.97 | 9.16 | 7.48 | 8.68 | 9.24 | 8.83 |
| Mars | 8.86 | 8.42 | 8.16 | 7.96 | 8.25 | 7.91 | 7.66 |
| Jupiter | 9.69 | 9.99 | 9.54 | 9.99 | 10.01 | 10.16 | 10.19 |
| Saturn | 9.2 | 8.63 | 8.59 | 8.8 | 8.82 | 8.82 | 8.7 |

**Table 10** Actual nodal movement per 1000 years

| Actual modern nodal movement in 1000 years (in deg) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Planet | 4700–4000 BCE | 4000–3000 BCE | 3000–2000 BCE | 2000–1000 BCE | 1000–0 BCE | 0–1000 CE | 1000–2000 CE | AVG |
| Mercury | − 2.67 | − 2.39 | − 1.17 | − 2.28 | − 2.19 | − 2.1 | − 2.01 | − 2.12 |
| Venus | − 5.4 | − 5.03 | − 4.84 | − 6.52 | − 5.32 | − 4.76 | − 5.17 | − 5.29 |
| Mars | − 5.14 | − 5.58 | − 5.84 | − 6.04 | − 5.75 | − 6.09 | − 6.34 | − 5.83 |
| Jupiter | − 4.31 | − 4.01 | − 4.46 | − 4.01 | − 3.99 | − 3.84 | − 3.81 | − 4.06 |
| Saturn | − 4.8 | − 5.37 | − 5.41 | − 5.2 | − 5.18 | − 5.18 | − 5.3 | − 5.21 |

**Table 11** Apsidal movement per 1000 years

| Modern apsidal movement in 1000 years (eastwards, in deg) | | | | | | | |
|---|---|---|---|---|---|---|---|
| Planet | 4700–4000 BCE | 4000–3000 BCE | 3000–2000 BCE | 2000–1000 BCE | 1000–0 BCE | 0–1000 CE | 1000–2000 CE |
| Sun | 15.4 | 17.6 | 16.7 | 16.1 | 17.8 | 14.6 | 17.8 |
| Mercury | 15.3 | 15.1 | 15.5 | 15.2 | 15.4 | 15.6 | 15.5 |
| Venus | 14.6 | 14.9 | 14.9 | 14.7 | 14.3 | 14.3 | 14.5 |
| Mars | 18.4 | 18.4 | 18.3 | 18.4 | 18.3 | 18.4 | 18.3 |
| Jupiter | 11.4 | 15.3 | 14.0 | 17.3 | 14.1 | 17.3 | 15.4 |
| Saturn | 18.7 | 21.4 | 17.7 | 20.1 | 18.7 | 18.2 | 20.2 |

**Table 12** Actual apsidal movement per 1000 years

| Actual modern apsidal movement in 1000 years (in deg) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Planet | 4700–4000 BCE | 4000–3000 BCE | 3000–2000 BCE | 2000–1000 BCE | 1000–0 BCE | 0–1000 CE | 1000–2000 CE | AVG |
| Sun | 1.4 | 3.6 | 2.7 | 2.1 | 3.8 | 0.6 | 3.8 | 2.6 |
| Mercury | 1.3 | 1.1 | 1.5 | 1.2 | 1.4 | 1.6 | 1.5 | 1.4 |
| Venus | 0.6 | 0.9 | 0.9 | 0.7 | 0.3 | 0.3 | 0.5 | 0.6 |
| Mars | 4.4 | 4.4 | 4.3 | 4.4 | 4.3 | 4.4 | 4.3 | 4.4 |
| Jupiter | − 2.6 | 1.3 | 0 | 3.3 | 0.1 | 3.3 | 1.4 | 1.6 |
| Saturn | 4.7 | 7.4 | 3.7 | 6.1 | 4.7 | 4.2 | 6.2 | 5.3 |

**Table 13** Comparing nodal rate-of-motion

| Nodal movement (years per arc-minute) | | |
|---|---|---|
| Planet | SS (westwards) | Modern (westwards) |
| Mercury | 410 | 7.9 |
| Venus | 221 | 3.2 |
| Mars | 935 | 2.9 |
| Jupiter | 1149 | 4.1 |
| Saturn | 302 | 3.2 |

**Table 14** Comparing apsidal rate-of-motion

| Apsidal movement (years per arc-minute) | | |
|---|---|---|
| Planet | SS (eastwards) | Modern (eastwards) |
| Sun | 543 | 6.4 |
| Mercury | 374 | 11.9 |
| Venus | 517 | 27.8 |
| Mars | 980 | 3.8 |
| Jupiter | 222 | 10.4 |
| Saturn | 5128 | 3.1 |

required to move by 1 arc-minute (as presented in the last column below). For e.g.: $1000 / (2.12 \times 60) = 7.9$.

And similarly for Apsidal Data in Table 14 (from Tables 2, 12).

From these tables we observe that the rates of motion given in the *Sūrya-Siddhānta* are quite different, about two orders of magnitude slower, than what modern formulae give us for heliocentric nodal and apsidal motion of the planets.

As mentioned, if our conjecture regarding these numbers in the *Sūrya-Siddhānta* is correct, they arise, not from direct observation, but from the complex Indian model for planetary motion, with its dual pulsating epicycle scheme. Clearly, nothing further can be stated here until the Indian planetary model is analyzed and understood thoroughly.

## 4.6 Epoch of the data

In an earlier paper (Narayanan, 2012), we had presented the result that the *manda* specified in the *Sūrya-Siddhānta* for the outer planets (Mars, Jupiter and Saturn) coincided with their aphelion (heliocentric apsis). For the inner planets (Mercury and Venus), however, this was not the case, and their specified *mandas* were quite distant from their aphelions. Thus, it would appear that the epicycle-based Indian planetary model treats the outer and inner planets differently, and, of course, it would be technically correct to do so. The gist of it is that we do not know what the *pāta* and *manda* of the inner planets mean physically, so we have no basis to compare them using modern formulae.

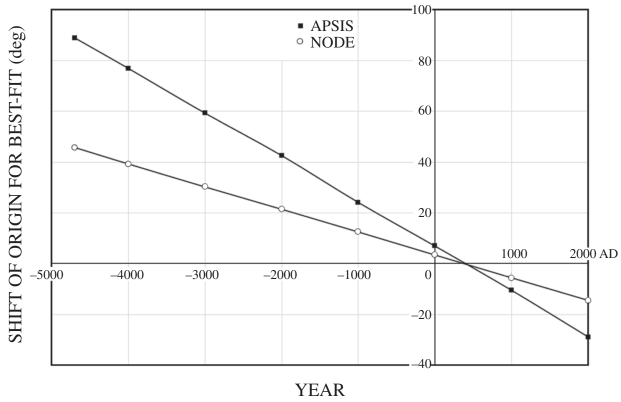


**Fig. 4** Variation of Best-fit Shifts in Time

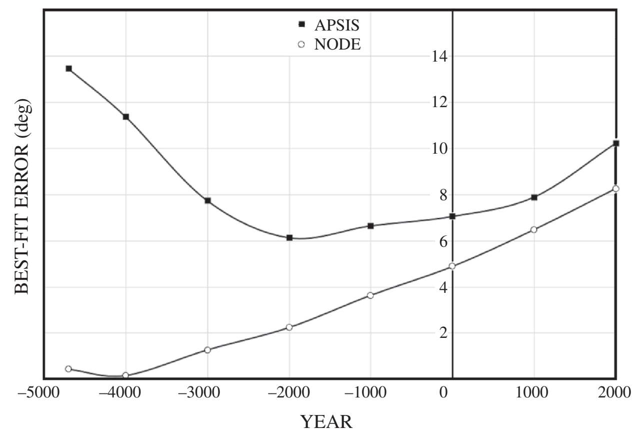


**Fig. 5** Variation of Nodal and Apsis Error using Best-Fit Schemes

Table 5 gives us the locations of the outer-planet nodes (e.g.: 40.17°, 79.76°, 100.71°) per the *Sūrya-Siddhānta*, while Table 6 gives us the locations of their aphelions (e.g.: 129.93°, 170.87°, 236.60°).

Recall that the Indian system uses a fixed (sidereal) origin, whereas modern formulae employ a movable (tropical) origin. Thus, for any heavenly body, the longitudes in both systems will differ, although the difference will be a constant, for that epoch.

We will, therefore, take the modern nodal/apsidal data of the outer planets for all test dates, and shift their origins step-by-step till we obtain a best fit for the *Sūrya-Siddhānta* data for that date, using the Least-Squares (LS) method. This shift will be the sum of the precessional movement and the movement of the node/apsis itself. The error for each best-fit is also mentioned. The results are shown in Tables 15 and 16.

Fig. 4 shows a plot of the best-fit shifts for the node and apsis for various test dates. As expected, the shifts are seen to vary more or less linearly with time. The magnitudes for apsidal data are seen to be greater than that for nodal data, due to the fact that precessional and apsidal movements are in the same direction (eastwards), and therefore add up, whereas the nodal movement, being westwards, is subtracted from the precessional movement, resulting in a smaller shift.

There are two remarkable things in the plot. Firstly, the best-fit origin shift is seen to be zero in the timeframe 400–500 CE, and secondly, both nodal and apsidal lines reach zero in the same timeframe.

The former observation comes from a well-known fact. As stated earlier, while the sidereal Indian origin is a fixed point on the ecliptic, the modern tropical system employs a movable origin. Most scholars agree that both these systems synced up around 450–500 CE, that is, the movable (tropical) origin circumnavigated the ecliptic and reached back to the fixed Indian sidereal origin around 500 CE. Fig. 4 appears to agree very well with that conclusion.

The second observation, showing the agreement of both nodal and apsidal data, with both lines meeting up at the same point in time where the sidereal Indian origin meets the tropical modern origin, reflects the fact that the Indian nodal/apsidal data is genuine, and not made-up as some colonial scholars have insinuated.

The plot is also a strong testament in favor of the hypothesis that the data given in Indian texts regarding *pāta* (node) and *manda* (apsis) for the outer planets are their heliocentric equivalents.

Fig. 5 shows a plot of the variation of error in the best-fit scheme (Least Squares method) for node and apsis locations with time, from data presented in Tables 15 and 16.

**Table 15** Best-fit for nodal data of the outer planets

| Nodal locations, *Sūrya-Siddhānta* and best-fit modern data (in deg) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Planet | SS Ref | 4700 BCE | 4000 BCE | 3000 BCE | 2000 BCE | 1000 BCE | 0 CE | 1000 CE | 2000 CE |
| Mars | 40.17 | 40.5 | 40.21 | 39.59 | 38.95 | 37.97 | 37.16 | 36.08 | 34.86 |
| Jupiter | 79.76 | 79.53 | 79.82 | 80.77 | 81.51 | 82.56 | 83.51 | 84.68 | 85.99 |
| Saturn | 100.71 | 100.6 | 100.55 | 100.14 | 99.93 | 99.79 | 99.55 | 99.38 | 99.2 |
| Shift | | +45.74 | +39.25 | +30.21 | +21.41 | +12.47 | +3.41 | − 5.58 | − 14.5 |
| Error | | 0.42 | 0.15 | 1.26 | 2.23 | 3.63 | 4.88 | 6.48 | 8.26 |

**Table 16** Best-fit for Apsidal Data of the Outer Planets

| Aphelion locations, *Sūrya-Siddhānta* and best-fit modern data (in deg) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Planet | SS Ref | 4700 BCE | 4000 BCE | 3000 BCE | 2000 BCE | 1000 BCE | 0 CE | 1000 CE | 2000 CE |
| Mars | 129.93 | 122.22 | 123.27 | 123.92 | 124.98 | 124.85 | 126.09 | 127.12 | 127.27 |
| Jupiter | 170.87 | 181.52 | 179.87 | 175.61 | 173.57 | 172.21 | 169.53 | 167.67 | 165.94 |
| Saturn | 236.60 | 233.67 | 234.34 | 238.05 | 239.11 | 240.69 | 242.23 | 243.00 | 244.84 |
| Shift | | +88.94 | +76.9 | +59.29 | +42.58 | +24.11 | +6.93 | − 10.5 | − 28.89 |
| Error | | 13.47 | 11.38 | 7.73 | 6.12 | 6.64 | 7.05 | 7.89 | 10.23 |

We observe that for nodal data the error is almost down to zero around the 4000 BCE timeframe, whereas the minimum error for apsidal data occurs in the 2000 BCE timeframe. While not conclusive, these curves are indicative of the great antiquity of the nodal/apsidal data given in the *Sūrya-Siddhānta*.

## 5 Conclusions

Some conclusions that may be drawn from this brief study on the nodes and apses of planets as given in the *Sūrya-Siddhānta* are as follows:

(1) The very-minute nodal/apsidal movement for planetary orbits specified in ancient Indian astronomical works like the *Sūrya-Siddhānta* have long been a matter of speculation and conjecture as regards its genuineness. It appears from the present study that there are good reasons to consider this data genuine.
(2) The direction of these fine movements as specified in the *Sūrya-Siddhānta*, that is, eastwards for all apsidal motion and westwards for all nodal, are found to be true per modern data.
(3) The apses (*manda*) and nodes (*pāta*) of the outer planets, as given in the *Sūrya-Siddhānta*, correlate well with their heliocentric equivalents in modern astronomy. Their given rates of motion, however, were found quite distant from the actual rates of motion of the heliocentric equivalents.
(4) It is very likely that the nodal/apsidal rates of motion were not derived from direct observation, but are a result of the ancients tinkering with the Indian planetary model to conform with observational data of the planet itself.
(5) This nodal/apsidal analysis confirms what is generally accepted among scholars, namely, that the Indian sidereal origin synced up with the tropical origin (Vernal Equinox) in the period 400–500 CE.
(6) An error analysis using the least squares method found the minimum error for nodal data in the 4000 BCE timeframe, while that for the apsidal data was seen to occur around 2000 BCE.

More research is clearly needed before anything concrete can be established. Some areas to explore further on this subject are: (1) The physical meaning of *Pāta*/*Manda* of the inner planets; (2) Discrepancy between the nodal/apsidal rates of motion given in the *Sūrya-Siddhānta* and their heliocentric equivalents; (3) Discrepancy between the *Āryabhaṭīyam* and the *Sūrya-Siddhānta* regarding the planetary apsis locations.

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