THE *ĀRDHARĀTRIKA* SYSTEM OF ĀRYABHAṬA I

R. N. Rai
Indian National Science Academy, Bahadur Shah Zafar Marg, New Delhi 1

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In this paper it has been shown that the *Ārdharātrika* system of Āryabhaṭa I is described not only in the *Puliśa Siddhānta* quoted by Bhaṭṭotpala (tenth century A.D.) and Al-Bīrūnī (eleventh century A.D.) and the *Sūrya-Siddhānta* of the *Pañcasiddhāntikā* (sixth century A.D.) but also in *Karaṇatilaka* by Vijayananda (tenth century A.D.) of Vārāṇasi and in another Karaṇa type book written by an unknown author of Vārāṇasi in Śaka 560.

It is now well known that Āryabhaṭa I (fifth century A.D.) had propounded a midnight system of reckoning in addition to the sunrise system of reckoning.[1] The important points in which the first differs from the second have been given by Bhāskara I[2] (seventh century A.D.). The *Puliśa Siddhānta* quoted by Bhaṭṭotpala (tenth century A.D.) and Al-Bīrūnī (eleventh century A.D.), the *Sūrya-siddhānta* of the *Pañcasiddhāntikā* (sixth century A.D.) and the *Khaṇḍakhādyaka* of Brahmagupta (seventh century A.D.) describe systems based on the midnight reckoning of Āryabhaṭa I. It will be shown in this paper that there must have been other works describing the midnight system of reckoning.

There is no difference between the two systems as regards the number of revolutions of the moon and of its apogee and its node in a *caturyuga*. However, the two systems differ as regards the number of days in a *caturyuga* and therefore their daily motions are not the same in the two systems. Stanzas 1–6 of chapter IX of the *Pañcasiddhāntikā* give methods of calculating the longitudes of the sun, moon, moon's apogee and its node, and their *kṣepa* quantities at the epoch of the *Pañcasiddhāntikā*.

In calculating the *kṣepa* quantities, Pundit Sudhakara Dvivedi first calculates the *ahargaṇa* since the beginning of creation according to the constants of the modern *Sūrya Siddhānta* and then uses the rate of motion of the above quantities according to the midnight system of reckoning to calculate the *kṣepa*.[3] Thibaut says: 'from tentative calculations it appears that the *kṣepas* exhibited by Varāha Mihira are calculated, not from the beginning of *Kaliyuga* or the *Mahāyuga*, but from the beginning of the *Kalpa*'.[4] Actually, as will be shown presently, Varāha Mihira made his calculations from the beginning of

Kaliyuga and it is just fortuitous that Pundit Dvivedi got the correct result. The reason is as follows:

The value of the *ahargaṇa* calculated by Pundit Dvivedi up to Śaka 427 is 714,403,601,073. The value of the *ahargaṇa* from the beginning of *Kaliyuga* is 1,317,123. Subtracting it from the first *ahargaṇa*, the value of the *ahargaṇa* from the beginning of creation up to the beginning of *Kaliyuga* is 714,402,283, 950 which is $452\frac{3}{4} \times 1{,}577{,}917{,}800$. Hence according to the midnight system of Āryabhaṭa the number of revolutions made by the sun, moon, moon's apogee and the node will be $452\frac{3}{4} \times$ number of revolutions of these quantities in a *caturyuga*. The revolution numbers of the sun and moon in a *caturyuga* are divisible by 4 and their longitudes are zero at the beginning of *Kaliyuga*. The revolution number of moon's apogee gives a remainder 3 when divided by 4 and its longitude is 90° in the beginning of *Kaliyuga*, while the revolution number of its node leaves a remainder 2 on dividing by 4 and the position of the node is 180° at the beginning of *Kaliyuga*. Although the revolution numbers of moon's apogee and its node are not the same according to the *Sūrya Siddhānta* as those given by Āryabhaṭa, the remainders, when divided by 4, are the same and the positions of the apogee and node at the beginning of *Kaliyuga* are the same according to the *Sūrya Siddhānta* and the *Āryabhaṭīyaṃ* because the number of years according to the *Sūrya Siddhānta* from creation till the beginning of *Kaliyuga* is $452\frac{3}{4} \times 4{,}320{,}000$.[5] These positions have been clearly stated by Parameśwara in his commentary of stanza 24 of the first chapter of the *Sūrya Siddhānta*.[6]

If we now use the *ahargaṇa* since the beginning of *Kaliyuga* and calculate the *kṣepas*, keeping in mind the value of the *kṣepas* in the beginning of *Kaliyuga*, we obtain the same values for sun, moon and moon's apogee as have been obtained by Dvivedi. He did not calculate the value of the *kṣepa* for the node of the moon. If we calculate it, we can easily emend the text of the *Pañcasiddhāntikā* which is certainly very corrupt. In 1,317,123 days, the motion of moon's node is given by

$$Y = \frac{232{,}226 \times 1{,}317{,}123}{1{,}577{,}917{,}800} = 194 - \frac{13{,}658{,}189}{87{,}662{,}100}.$$

If we add to this $\frac{1}{2}$, the *kṣepa* at the beginning of *Kaliyuga*, the position of the node at the end of Śaka 427 is $\frac{30{,}172{,}861}{87{,}662{,}100}$ which is very nearly equal to $\frac{631{,}454}{1{,}834{,}582}$. If we subtract from 631,454 half of 270, the *kṣepa* at midday is 631,319.

The text of the *Pañcasiddhāntikā* at present reads[7]:

> '*Trighanadaśaghne navakaikpakṣarāmendudahaśabāḥ* |
> *Sahite yamavasubhutārṇavaguṇadhṛtibhiḥ Kramādrāhoḥ*.' ||

The emended reading should be

'Trighanadasaghne svake navakaikarāmendudahaṣaṭkāḥ sahite yamavasubhutārṇavaguṇadhṛtibhiḥ kramādrāhoḥ.

The *kṣepa* now reads 631,319 which is the value obtained by calculation.

Bhaṭṭotpala in his commentary on the *Bṛhatsaṃhitā* has given a large number of quotations from the *Puliśa Siddhānta.*[8] Similarly, Al-Bīrūnī quotes from the *Puliśa Siddhānta* at many places in his book on India. The constants given by him are the same as those given in the midnight reckoning of Āryabhaṭa.[9] According to both Al-Bīrūnī and Bhaṭṭotpala, *Puliśa Siddhānta* takes a *Kalpa* to be composed of 1,008 *caturyugas.*[10] However, the number of revolutions of the *Śīghrocca* of Venus, given by Bhaṭṭotpala, is not the correct one.[11] This is due to the faulty reading of the manuscript used by Dvivedi. According to Bhaṭṭotpala the distance travelled by each planet in a *caturyuga* is the same and equal to 18,712,080,864,000 *yojanas.*[12] This must be equal to the product of the length of the orbit of a planet and its revolution number. The length of the orbit of Venus, according to the *Puliśa Siddhānta,* is 2,664,632 yojanas.[13] Dividing by this the previous number, the quotient is 7,022,388·08. The reading in the *Bṛhatsaṃhitā* should, therefore, be:

'*aṣṭavasuhutāvahāśviyamakhanagairbhārgavaścāpi*' and not '*aṣṭavasuhutāvahānalayamakhanagairbhārgavaścāpi*'

Another book, mentioned by Al-Bīrūnī and following the midnight system of reckoning, is the *Karaṇatilaka,* composed in the year Śaka 888 by Vijayananda of Vārāṇasi.[14] For calculating the *ahargaṇa,* the *Karaṇatilaka* says: 'Take the years of the *Śakakāla,* subtract therefrom 888, multiply the remainder by 12, and add to the product the complete months of the current year which have elapsed. Write down the sum in two different places. Multiply the one number by 900, and add 661 to the product, and divide the sum by 29,282. The quotient represents *adhimāsa* months. Add it to the number in the second place, multiply the sum by 30, and add to the product the days which have elapsed of the current month. The sum represents the lunar days. Write down this number in two different places. Multiply the one number by 3,300, add to the product 64,106 and divide the sum by 210,902. The quotient represents *ūnarātra* days, and the remainder the *avamas.* Subtract the *ūnarātra* days from the lunar days. The remainder is the *ahargaṇa,* being reckoned from *midnight as beginning*'.

There is no difference between the two systems of Āryabhaṭa as regards the number of *adhimāsa* in a *caturyuga.* But the method of calculating *adhimāsa* and *ūnarātra* days in the *Karaṇatilaka* is slightly different from that given in other books. We will therefore prove the method as well as deduce the values of the *kṣepa* quantities. If the total number of *saura* months at any instant is X, the number of *adhimāsas* Y will be given by

$$Y = \frac{1{,}593{,}336 \times X}{51{,}840{,}000} = \frac{X \times 900}{\frac{51{,}840{,}000 \times 900}{1{,}593{,}336}},$$

$$= \frac{900X}{29{,}282 - \frac{2{,}468}{66{,}389}} \simeq \frac{900X}{29{,}282}.$$

Again if the number of lunar days is Z, the number of *ūnarātra* days U will be given by

$$U = \frac{25{,}082{,}280 \times Z}{1{,}603{,}000{,}080} = \frac{3{,}300 \times Z}{\frac{1{,}603{,}000{,}080 \times 3{,}300}{25{,}082{,}280}}$$

$$= \frac{3{,}300 \times Z}{210{,}902 - \frac{7{,}646}{69{,}673}} \simeq \frac{3{,}300Z}{210{,}092}.$$

The number of *adhimāsas* since the beginning of *Kaliyuga* up to Śaka 888 is

$$\frac{4{,}067 \times 1{,}593{,}336}{4{,}320{,}000} = 1{,}500 + \frac{4{,}063}{180{,}000}.$$

Now

$$\frac{4{,}063}{180{,}000} = \frac{4{,}063 \times 29{,}282}{29{,}282 \times 180{,}000} = \frac{660 + \frac{71{,}683}{90{,}000}}{29{,}282}.$$

The *adhimāsa kṣepa* is therefore 661. However, if we calculate the *kṣepa* for calculating the *ūnarātra*, we are unable to get the figure given in the *Karaṇatilaka*. The number of lunar months from the beginning of *Kaliyuga* to Śaka 888 is
$(4{,}067 \times 12 + 1{,}500) = 50{,}304$, so that the number of lunar tithis is $50{,}304 \times 30 = 1{,}509{,}120$. The value of *ūnarātra* days is

$$\frac{1{,}509{,}120 \times 25{,}082{,}280}{1{,}603{,}000{,}080} = 23{,}613 + \frac{735{,}423}{2{,}226{,}389},$$

and

$$\frac{735{,}423}{2{,}226{,}389} \simeq \frac{69{,}665}{210{,}092}.$$

The *kṣepa* therefore comes out to be 69,665 and not 64,106 as stated by Al-Bīrūnī.[15]

Al-Bīrūnī further refers to another astronomer, Auliatta (?), the son of Sahāwī (?), who uses the epoch of *Śakakāla* 918 and follows the methods of the *Puliśa Siddhānta* for making the calculations.[16]

We will now refer to some astronomical tables and rules extracted from a Siamese manuscript and brought to France by M. La Loubere in 1687. The rules enable one to calculate the places of the sun and moon.[17] These rules were explained by Giovanni Domenico Cassini, an Italian, who was working in Paris and had been appointed by the French King as the first director of the Paris observatory. Cassini came to the conclusion that the epoch of the tables corresponded 'to the 21st of March, in the year 638 of our era, at 3 in the morning, on the meridian of Siam'.[18] From the epoch, 'the mean place of the sun for any other time is deduced, on the supposition that in 800 years, there are contained 292,207 days'.[19] From Cunningham's tables we find that in 638 the initial day of the solar year was 20th of March.[20] It is, therefore, evident that the epoch of the tables is 560 Śaka elapsed. The number of days in a year shows that the motion of the sun is in accordance with the midnight reckoning of Āryabhaṭa.[21] The greatest inequality of the sun according to the tables is 2° 12′ and they are given at intervals of 15°. The apogee of the sun is supposed to be fixed amongst the stars and at 80° from the beginning of the Zodiac.[22] The value of the greatest inequality of the moon is given to be 4° 56′.[23] These constants are the same as those given in *Khaṇḍakhādyaka* except that the inequality of the sun differs from the value given in *Khaṇḍakhādyaka* by 2′.

M. Cassini observes, 'that they (the tables) are not originally constructed for the meridian of Siam, because the rules direct to take away 3′ for the sun, and 40′ for the moon (being the motion of each for 1^h, 13′), from their longitudes calculated as above. The meridian of the tables is therefore 1^h, 13′ or 18° 15′ west of Siam; and it is remarkable that this brings us very near to the meridian of Benares, the ancient seat of Indian learning'.[24] This shows that the tables are based on a Karaṇa type of book composed at Benares and based on the midnight system of reckoning of Āryabhaṭa.

Playfair's paper refers to other tables and rules obtained by the French from South India in the eighteenth century and published in the Memoirs of the Academy of Sciences for 1772 and in the volumes of M. Bailly's History of Astronomy. Much could be known about the state of the knowledge of Indian Astronomy in the eighteenth century if these are studied.

References

1 Shukla, K. S., *Gaṇita*, XVII, 83 (1967).

2 *Mahābhāskarīya*, Chap VII, stanzas 22–35.

3 Thibaut, G., and Dvivedi, S., *Pañcasiddhāntikā*, Sanskrit commentary, p. 44.

4 *Ibid.*, English Commentary, p. 56.

5 *Sūrya-Siddhānta*—Edited with Parameśwara's commentary by K. S. Shukla, 1–46. The figure given here is up to the end of *Kṛtayuga*. This has been stated to be equal to 452¼ *caturyugas* by Parameśwara in his commentary.

6 *Ibid.*, p. 6.

7 *Pañcasiddhāntikā*, text, p. 26 stanza 5.

[8] *Bṛhatsaṃhitā* with the commentary of Bhaṭṭotpala, edited by M. M. Pundit Sudhakara Dvivedi, Vol. I, pp. 24–29; pp. 36–38; pp. 46–53.

[9] Al-Bīrūnī's India, translated by Sachau, Vol. II, p. 18.

[10] *Ibid.*, Vol. I, p. 370; *Bṛhatsaṃhitā*, Vol. I, p. 48.

[11] *Bṛhatsaṃhitā*, Vol. I, p. 49.

[12] *Ibid.*, Vol. 1, p. 49.

[13] *Ibid.*, Vol. I, p. 49.

[14] Al-Bīrūnī's India, Vol. II, p. 50.

[15] Making the calculations from the beginning of the present *caturyuga* and using the method of *Khaṇḍakhādyaka* for calculating the *ūnarātra* days, Schram gets 69,601 for the value of the *kṣepa*. It may be remarked that the method of calculation given by Vijayananda is less exact than that given in *Khaṇḍakhādyaka.*

[16] Al-Bīrūnī's India, Vol. II, p. 190.

[17] This information has been taken from an article entitled 'Remarks on the Astronomy of the Brahmins', by Prof. John Playfair, published first in the Transactions of the Royal Society of Edinburgh and reproduced by Shri Dharampal in his book 'Indian Science and Technology in the eighteenth century', p. 10.

[18] 'Indian Science and Technology in the eighteenth century' by Dharampal, p. 16.

[19] *Ibid.*, p. 17.

[20] Cunningham, A., Indian Eras, p. 158.

[21] *Pañcasiddhāntikā*, chap. IX, stanza I.

[22] 'Indian Science and Technology in the eighteenth century' by Dharampal, p. 18.

[23] *Ibid.*, p. 19.

[24] *Ibid.*, p. 20.