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THE GNOMON IN EARLY INDIAN ASTRONOMY

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The data in the Arthaśāstra for the variation of the gnomon shadow length with time is found to fit, with one exception, a formula given in the *Yavanajātaka* and *Pañcasiddhāntikā*. A comparison of this formula is made with the corresponding trigonometric formula.

I. The fraction of day-time, t/d, which has elapsed from sunrise, corresponding to specified lengths s of the shadow of the gnomon of length g, is given in the second book of Kauṭilya's *Arthaśāstra*, chapter 20, called *Adhyakṣapracāra*. The values, as given in Ganapati Sastri's edition[1], can be arranged in the following table.

TABLE I

s/g	8	6	4	2	1	2/3	1/3	0
t/d	1/18	1/14	1/8	1/6	1/4	3/10	3/8	1/2
$d/2t$	9	7	4	3	2	5/3	4/3	1

In the more recent edition[2] of Harihara Sastri, the commentator Bhikṣuprabhāmati takes 1/10 instead of 1/8 for t/d, when $s=4g$, without specifying any reason. In this paper, we point out that, with this alteration, the entries in Table I follow the formula.

$$\frac{d}{2t}=\frac{s}{g}+1\cdots\cdots \tag{1}$$

This is the special case of the fomula found in the *Yavanajātaka*[3] (*YJ*) chapter 79, verse 32, and *Pañcasiddāntikā*[4] (*PS*), IV 48-49, which can be expressed in the form

$$\frac{d}{2t}=\frac{s-s_0}{g}+1\cdots\cdots \tag{2}$$

by putting $g=12$ in equation (4) p. 46, part II, of reference 4. The formula (1) neglects the noon-shadow, s_0.

The same section of the *Arthaśāstra* gives the rule for the uniform variation of the noon-shadow s_0/g from 0 at the summer solstice to 1 at the winter solstice, which is a reasonable approximation, in early astronomy, for an observer on or near one of the Tropics, for example at Ujjain. But the rule for the uniform variation of the length of daylight from 12 to 18 *muhūrtas*, also found here and in the *YJ* and *PS*, implies a latitude of about 35°, and so seems to have been uncritically borrowed from Babylonia.

The derivation of formulae (1) and (2) may be based on the following rough considerations :

(a) s/g and d/t are taken to be linearly related, i.e. we assume

$$\frac{d}{t} = a\frac{s}{g} + b.$$

(b) When the sun is at the zenith, there is no shadow, i.e. we have $s=s_0$ at $t=d/2$. On this day, the sun's altitude at mid-morning is roughly 45°, i.e. we have $s=g$ at $t=d/4$. This gives formula (1).

(c) On other days when $s \neq s_0$ the length of the shadow is increased by s_0, i.e. we replace s in(1) by $s-s_0$.

The above argument is the same as that given by Kuppanna Sastri[5] for justifying the similar relation between the *lagna* and the gnomon shadow found in *PS* II, 11-13.

II. In this section, we indicate some similar relations for the gnomon shadow in Babylonian and Greek astromony.

The second tablet of the series *MUL APIN* records a table of shadow lengths which follow the formula

$$t = \frac{c}{s} \quad . \quad . \quad . \qquad (3)$$

with entries only for $s=1, 2, \ldots\ldots 10$, measured in cubits, c having the values 4 hours, 5 hours, 6 hours at Winter Solstice, Equinox and Summer Solstice. The tablet does not give any value for the noon-shadow. The formula (2) is therefore more complete than the earlier formula (3) besides being more relevant to India.

Neugebauer[6] mentions a Greek type of table of shadow lengths, in which the noon-shadow changes uniformly (i.e. the linear zig-zag pattern) as in the Indian works we have mentioned above, but between different limits. Another common feature is that the variation of $(s-s_0)$ with day-time is the same at any time of the year, but the form of variation does not have the $\frac{1}{t}$ dependence as in (1), (2), (3)

III. We have seen in section I that the simple formula (2) seems to have designed to roughly fit observation at latitude φ nearly equal to the obliquity $23\frac{1}{2}°$. To test this in some special cases, we have calculated the variation of shadow-length by the trigonometric formulas given for example in *PS* IV, 41-44 (reference 4, Part II, p. 43-44) taking $\varphi = 23\frac{1}{2}°$ and declination equal to $23\frac{1}{2}°$ (Table 2), 0° (Table 3), $-23\frac{1}{2}°$ (Table 4). The second row in these tables gives the trigonometric result and the third row that of formula (2).

TABLE II

s/g	8	6	4	2	1	2/3	1/3	0
$(2t/d)_1$	.08	.11	.16	.31	.51	.63	.80	1
$(2t/d)_2$	.11	.14	.20	.33	.50	.60	.75	1

TABLE III

s/g	8	6	4	2	1	2/3	1/2	.43
$(2t/d)_1$	.09	.11	.17	.32	.55	.72	.86	1
$(2t/d)_2$	.12	.15	.22	.40	.67	.86	1	—

TABLE IV

s/g	8	6	4	2	4/3	1.07	1
$(2t/d)_1$	.11	.15	.22	.45	.68	1	—
$(2t/d)_2$	.125	.17	.25	.50	.75	.93	1

There are no entries in the last column, third row of Table 3, and in the last column, second row of Table 4, because the specified shadow lengths, in all the Tables, are only in the interval before noon, i.e. the maximum value of t is $d/2$. After noon the shadow lengths follow the same law except that t is now the time that has to elapse till sunset.

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