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PLUTO AND A TRANSPLUTONIAN PLANET AS PREDICTED BY VENKATESHA KETAKARA*

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In the last quarter of nineteenth and first quarter of twentieth century there were many attempts for predictions of trans-neptunian planets. Among the first researchers in this field were Forbes[1] and Todd[2] (1880), Gaillot[3] (1909), Pickering[4] (1909 and 1928). The successful prediction of Pluto is credited to Perceival Lowell[5] who first calculated in 1915 and observed the same on photographic plates in 1930, but an Indian astronomer Shri Venkatesha Ketakara successfully predicted the existence of Pluto and also of another trans-plutonian planet (still un-discovered) by extending the theory of satellites of Jupiter and applying law of equilibrium of gravitational moments[6,7,8] and also studying the motions of comets. The independent and earlier prediction by Ketakara deserves recognition after complete analysis. It may be pointed out that earlier predictions of two trans-neptunian bodies were based on perturbation theory while Ketakara's prediction was based on a different type of law governing the solar system. It may be pointed out that after successful search of one trans-neptunian planet, Pluto, in 1930[9], attempts were made to find the positions of another member of the solar system beyond Pluto. Sevin[10,11] (1946 and 1949), Strubell[12] (1953), Krizinger[13] (1958 and 1963) continued the work in this field. Tombaugh[14] continued searching for trans-plutonian planet for 16 years (up to 1945) but failed to find the predicted planet. Again the topic was restarted by Joseph L. Brady of California Institute of Technology in 1967[15]. His attempts are based on studies of periods of Halley's comet using old European and Chinese records[16]. He could correct the time period of this planet and after recursions using lot of old data he has been able to predict the time period of the trans-plutonian planet to be 464 years. The residuals of apparitions of Halley's comet were analysed[17] and the conclusions led to the belief that

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there is a trans-plutonian planet (X planet as he calls it) with highly inclined orbit ($i=120°$, $e=\pm 0.07$). Forces acting on the Halley's comet according to Brady, are gravitational[18,19]. Also T. Kiang has worked in this direction using old records of perihelion passage timings of Halley's comet and believes in the existence of a trans-plutonian planet[20]. The recent works of Brady are the results of sophisticated computer-computations using perturbations in the solar family members, but experimental observations have shown negative results in Licks and Greenwich observatories[21,22,23]. In these investigations a solid angle about 6.3 degree square was scanned (around Brady's predicted position in the constellation of Cassiopia, but no moving object brighter than 17th magnitude is found in these investigations[24]. Also the theoretical arguments against the X-planet are advocated by Goldreich and Ward[25]. Using perturbations due to this planet in the solar system, it is proved that this Brady's object will affect, resulting into large variations of the angle between solar axis and the normal to the ecliptic with a period of 10^7 years and also the presence of such a body will disrupt the co-planar configuration of the solar system on a time scale of 10^6 years. These arguments now have resulted into negligence of further investigations in this field[26] but still the researchers believe that these are gravitational forces (and not non-gravitational) which disrupt the Halley's Comet's motion and considering the apogee of this comet, it is believable that the affecting bodies must be Pluto or another trans-plutonian celestial body. If the X-planet does not have unusual constants like the 120 degrees inclination then the arguments of Goldreich and Ward do not stand against these beliefs. Brady believes that the planet may not be existing as a single body[27]. Now let us analyse the work done by Shri Venkatesha Ketakara in 1911 on theoretical predictions of two trans-neptunian planets.

It may be pointed out that, Lowell's dynamical evidence of existence of a trans-neptunian planet (Pluto) was first given in 1915[28] and the search to discover it telescopically was done after first world war and the approximate calculations of its position were done in 1920 by the Western scholars. The extract of Venkatesha Ketakara's findings was reported in Bulletin of Astronomical Society of France[29] (May 1911 issue). This is reported below from one of the works of Ketakara[30,31] where he reported this part in preface to his Gujrati text of Astronomy.

*"M. V. B. KETAKAR, á Dharwar (Indes), a recherché, non d'après les pérturbations planétaires, mais d'aprés les considerations et conditions qui semblent réagir le system solaire, la position de deux planétes transneptuniennes

* Extract from Page 277, of May, 1911, *J. of Societie Astronomique France*

et leur assigne, au 1er Janvier 1911, les positions suivantes :

1er Janvier 1911	1er planéte	2°planéte
Longitude moyenne...	289°,4	109°,0
Distance moyenne...	38, 95	59, 60
Revolution...	242ans.28	458ans.27

Le resumé de la note de M. le lieutenant-colonel DELAUNEY sur les distances des satellites (Bulletin de mars 1911, page 138), doit être ainsi rectifié et complete :

Les distances des satellites sont donnés par la formule $A+Bn$, dans laquelle A et B sont des constantes et n un entier, On a :

	A	B
Systeme de Jupiter...	0.27	1.14
de Saturne...	0.27	0.46
d'Uranus...	0.249	0.151
du Soleil...	0.04	0.12

Dans les systémes planetaires, il semble que A soit proportionnel au diamétre de la planéte et B proportionnel á la masse."

Note : In this report the longitudes are interchanged. An explanation to this is given later in this very paper.*

Recently the author has found some of the original documents of Ketakara. Following is the extract from his own daily work diary[8 293]. This gives some of the ideas of actual theory used by him.

The date 1st November 1910 was astronomically a very important one on account of *Saptagrahi* (seven planets, celestial bodies and one lunar node being near each other in the same sign). According to Ketakara it afforded many important hints as regards the probable place in the heavens of planets beyond Neptune (Ketakara has shown calculations and data showing this astronomical event). The first consideration that helped him to guess the distance of a new planet was that there was always a planet at a distance double that of the preceeding one and sometimes at a distance and a half. There ought to be a planet at a mean distance double that of Uranus $19.18 \times 2 \simeq 38.36 A.\ U.$

Ketakara thought that a peculiar law of daily mean motions similar to the singular relations between mean motions of Jupiter's satellites appear to

*Recently Ketakara's handwritten manuscripts have been procured. These show that the longitudes actually calculated were interchanged in the printed version. In the actual computations he got the right longitude for Pluto.

hold good in case of Uranus, Neptune and the hypothetical trans-neptunian planet 'Brahma' as he named it. The law is as follows :

Let a, b, c be the mean motions of Uranus, 'Brahma' and Neptune respectively, then these hold a relation :

$$a+3b-4c=0. \tag{1}$$

Also let A, B, C represent the mean longitudes of them, then

$$A+3B-4C=180°. \tag{2}$$

This may be pointed out that similar laws are found to hold in case of satellites of Jupiter.

In 1910 about the month of August, Uranus and Neptune were in opposition. From this it was concluded that the new planet must have been in conjunction with Neptune, i.e. the longitude of the new planet must have been equal to that of the Neptune on the day of opposition of Uranus and Neptune. Ketakara calculated longitudes of these planets using French annare for 1911 and found that on 23 July 1911 the longitude would be 109°51′ 57″ and used equation (1) to calculate the mean daily motion and found that $b=14''.6364$. Using this mean motion he computed the mean longitude on 1 January 1911. It was found to be 109°2′26″. In order to find out the periodic time he used the relation :

Period of Neptune × daily angular motion of Neptune=Period of 'Brahma' × daily angular motion of 'Brahma.'

Using this relation, he found the period=242.28 years. The distance was calculated using Kepler's third law and was found to be equal to 38.95 *A.U.* The law of motion in this case is :

Uranus−4 Neptune+3 Brahma=180°.

In order to find the elements of another planet, say 'Vishnu' (as he named it), of longitude V, revolving around the Sun beyond the orbit of 'Brahma' (longitude B), he used the equation :

$$N-2B+V=180°$$

so that when Neptune and 'Brahma' are in conjunction

$$V=180°+B.$$

On July 23 of 1910 the longitude of 'Vishnu'=289°51′52″.

If n, b and r are mean daily motions of Neptune, 'Brahma' and 'Vishnu' respectively then :

$$n-2b+r=0 \quad \therefore r=7''.7318 \text{ per day.}$$

Motion for 203 days from 23rd July 1911 to 1st January 1911=26′10″.
Longitude of 'Vishnu' on 1st Jan. 1911=289°25′42″
To find periodic time he used the relation :
Period of Uranus × daily motion of Uranus
=Period of 'Vishnu' × daily motion of 'Vishnu'
∴ Period = 458.27 years.

Using Kepler's third law he found mean distance=59.573 *A.U.* The law of motion for this being :

$$N - 2B + V = 180°.$$

After all these treatments, Ketakara in his notes tries to verify if the rule of motions observed among the Jupiter's satellites holds good also in case of some consecutive planets. He wrote a letter to M. C. Flammarian, Secretary, Astronomical Society of France informing his results on 14.4. 910, (the date as it appears in his notes) but his notss on verification of the rule of motions are dated 14th April 1911 (Good Friday). He tries to verify the rules of motions also for the triads (Mercury, Venus and Earth), (Jupiter, Uranus and Neptune) .. and some more discussions are there which deserve careful analysis. He also tries to find equations of motion among four planets and verifies for (Jupiter, Saturn, Uranus and Neptune).

In the original documents of Ketakara on next pages appears a letter to M.C. Flammarian dated 14.4.1912. Here is the copy of the letter sent to M.C. Flammarian. It may be remarked that this letter is another approach based on capture of comets which occured to Ketakara late in April, 1912.[34]

Letter of M.C. Flammarian, Secretary General, Society Ast. de France.

The planets and Comets, dated 15. 4. 1912

The dynamical laws clearly show that the Sun, single handed, is unable to change the parabolic orbits of comets into elliptic ones unless assisted by the planets and the resisting medium. Thus, the former agent lessens the velocity and increases the curvature of the orbits and so compels them to motion in elliptical orbits. The latter shortens the periodic time and the mean distance.

The largest planet Jupiter has captured all the comets, the aphelion distances of which hang between 5 to 6 astronomical units. I give below the comets that are suspected to be periodic, with the names of the planets that have been probably captured by them.

Comet	*Aphelion*		*Capturing planet*
	Dist. A. U.	Lat.	
Temple 1866 I	10.325	2°.1 +	Saturn
Goggia 1867 I	11.714	0.7 +	-do-
De Vico 1846 IV	18.000	12.8 −	Uranus
Brorsen 1847 V	18.681	14.6 −	-do-
Pong Borks. 1884	33.690	18.1 +	Neptune
Halley 1910	35.503	11.5 +	-do-
Peters 1857 IV	38.050	0.5 +	Janus (Brahma)
Gia Cobini 1905 III	44.530	1.2 −	-do-

The number of comets thus captured in the past ages must have been enormous and our solar system would have been over-crowded had it not been for the fact that they are often split or slowly disintegrated by the thick medium of the ashes of their predecessors.

Thus the comets have become a new source of information about the unknown planets. I strongly believe in the existence of an unknown trans-neptunian planet revolving around the Sun at a distance of 39 to 40 units (See *Bull. French Ast. Soc.*, May 1911 page 277). I am sanguine that the hidden planet will be easily found out and will perpetuate the name of another Galle if he carries on the work of searching for this about the position indicated there by me, patiently and methodically. The unknown planet Janus, as he might be named may not be much inferior to Neptune."

It may be noted that Ketakara has here named the new planet Janus instead of 'Brahma' in order to avoid any lack of understanding on the part of a Christian respondent. It is clear from his notes that the date of despatch of an earlier letter quoting the results (published in Bulletin of the Soc. of May 1911) was 14 April 1911 and later on, in the above letter dated 15.4.1912 Ketakara has supported the existence of the new planet on the basis of his independent thinking about capture of comets, Peters 1857 IV and Gia Cobini 1905, III by Janus, like Halley's and other comets captured by other members of the solar system. It may be remarked that although these notions were already prevalent and in response to Ketakara's letters his thinkings were supported by others in the West (as is clear from his letters in his file not being reported here). There is no doubt that Ketakara's work was based on radically different approach.

The validity of the theory of Jovian Satellite's motions and its extension to planetary dynamics deserves full analysis. One has to go through all

relevant historical backgrounds in order to understand fully the Ketakara's approach based on rules of motions of Jupiter's satellites. The work in this direction is in progress.

It may be pointed out that H. P. Bhatta had a meeting with Ketakara in 1926 in which the latter explained that his theory[35] was based on the study of motion of comets and generalized sense of equilibrium and in this formulation he assumed that perturbations are small and it is only the gravitational forces between Sun and other constituents and the centrifugal force which govern the system. He used law of moments to predict the two planets. While reporting his results he assumed the mass of distant (trans-plutonian) planet smaller than that of nearer (Pluto) one. He stated categorically that his figures for the longitudes of these two planets will have to be interchanged in case this assumption for relative magnitudes of their masses does not hold. It may be noted that now on comparison of photographic records and prediction of Pluto by Lowell and Pickering, one feels that the results were quite far off the actual value and more-over the gravitational perturbations on Neptune are so small as is clear now that one cannot predict the existence or position of Pluto with any certainty[36,37] on the basis of perturbation theory. This fact supports Ketakara's approach. It may be emphasized that Ernest W. Brown a celestial mechanics-scholar in Yale University in 1930 agreed that Predictions could not be made from Perturbative effects on Uranus or Neptune. We must regard the fact that it was found near the predicted place as purely accidentally[38,39]. It may be remarked that on calculations we find that Ketakara's predicted longitude of Pluto on 1st January 1911 was in error by+0°.6 only, while Lowell's and Pickering's results were too off.

Now it is found that pluto has orbital constants as given by Ketakara for the first planet but its longitude is that of second planet. Here we are reminded of the statement by Ketakara (Note : *X*-planet has greater mass as advocated by Brady[39] in his meeting with H. P. Bhatta. Thus we can justify by interchanging the longitude of the two. So,

longitude of 1st planet=109°.0
distance=38.95 A.U.
period =242.28 years.

It can be verified that the prediction of first planet comes out to be true for Pluto and thus for second planet we have heliocentric longitude on January 1911 as 289°.4, distance=59.60 A.U. and time period=458.27. As the prediction for Pluto came out to be true and we believe that prediction for the second planet too may be reliable. Bhatta[40] assuming

latitude of this planet to be zero and almost of circular orbit, calculated the geocentric position of this planet in 1954, Jan. 1. He found R.A.=$21\frac{3}{4}$ hours but these days Brady of California has worked on *X*-planet and found inclination of its orbit=120° on the basis of some arguments and eccentricity $e=0.07$ and $T=464$ years. The time period with lesser number of Halley's periods was much greater than this figure but iterating with inclusion of more and more periods reported by old European and Chinese astronomers he found the results converging to this figure (a value very near to the one predicted by Ketakara). If the planet is always in general retrograde motion (as is evident from 120° inclination) then its sidereal period will be $T=458$ years, a value just near Ketakara's for second planet, but the longitude on being converted to a geocentric right ascension and declination using coplaner hypothesis yields values quite different than the ones given by Brady. It is suggested that the arguments for such a high inclination as 120° may be checked and results corrected accordingly. The position so found may be searched out again. It may be pointed out that such high inclinations were estimated earlier in case of Pluto too.

As Ketakara's approach was entirely different it was based on a theory similar to that of satellites of Jupiter[41,42,43] and capture of comets and his prediction for Pluto came out to be true and all other findings for *X*-planet (except some difference in position) tally with Brady's. Ketakara's work for prediction of Pluto must get due recognition and it is hoped that the transplutonian planet again deserves telescopic searches after useful theoretical analysis of Ketakara's and other's findings.

It may be pointed out that in fact Pluto got photographed in Mount Wilson Observatory in 1919 which was one of the photographs taken to check the planets beyond Neptune predicted by Pickering, but unfortunately the Pluto (The O' planet according to Pickering) was missed but Pluto was found in Lowell's photographs in 1930. Mt. Wilson plates were re-examined and Pluto was found on the photographs. The observed position was found differing by 1°.15 in longitude on comparison with Pickering's computed results. It may be pointed out that earlier computations of O' planet by Pickering in 1919,[44] were in error by 5°.6 while Lowell's prediction in 1914 was in error by 5°.9.[45] Thus Pickering's prediction of longitude of Pluto was more accurate than Lowell's. It may be pointed out that Ketakara's predicted longitude of Pluto had an error of 0°.6 although the details of basis of prediction in 1910 still need analysis. After observational discovery of Pluto in 1930 when some pointed out that pickering should have credit for discovery of Pluto rather than Lowell, Pickering replied that neither he nor Lowell was responsible for finding Pluto on the plates except in so far as Lowell

had left money to have the search made. He (Pickering) had made no such light and therefore, Lowell should receive full credit for the discovery of Pluto.[46] It may also be pointed out that Ketakara died in August 1930 but he suffered from paralysis since 1929 and his tounge had been affected for the last 1½ years of his life.[47] He did not make any such comment, being detatched from the scientific world due to paralysis during last years of his life. After discovery of Pluto, it was realized that in fact Pluto could not be reliably predicted on the basis of perturbations caused on it by Uranus or Neptune, because of their being very small and thus capable of being masked by experimental errors. It was a fortunate chance for Lowell that Pluto was found on the photographic plates at such a large distance from the position predicted on the basis of perturbation theory. Ketakara's approach on the contrary was radically new, better acceptable on physical grounds. It may be remarked that :

i) Ketakara did not use the perturbations caused by Uranus or Neptune, which is a justified approach ;

ii) He used the coupling of motions (singular solutions) in the three-body problem just like in case of satellites of Jupiter. In a telescope, Jovian satellites appear to oscillate backward and forward in almost a straight line across or behind the planet's disc[48] and repeat some configurations periodically. All the three cannot be in conjunction simultaneously when two of them are in conjunction with each other. The third one stays in opposition to both of them.[49,50]. Same situation seems to hold for distant Solar planet also.

iii) Ketakara's extension of theory of satellites of Jupiter to the dynamics of these distant planets in the solar system is justifiable. The following points are to be remarked.

a) In 1910, Pluto was actually in conjunction with Neptune, and Uranus was in opposition to them. The mean values of the longitudes can be computed easily from the standard formulae[51] given for the epoch beginning from January 1900.

b) In 1910 the longitude of node of the orbit of Pluto was almost 109°, a value very near to the longitude of Pluto. Hence the latitude of Pluto was almost zero. The latitudes of Uranus and Neptune too were not of much magnitude, so the coplanar hypothesis did hold approximately at least for the year 1910.

Ketakara's intuition on the basis of *Saptagrahi* deserves full analysis.

Also the extension of Jovian satellite's theory to the planetary dynamics needs complete mathematical analysis, but it is evident the ratio of velocities of these planets do conform to these possibilities of conjunction and opposition. The theory of periodic solution is to be checked in the case of Uranus, Neptune and Pluto. There is no doubt that the physical situations which were arrived at by Ketakara did happen in the dynamical history of this planetary triad. He could predict the position most accurately (and historically, earliest of all). If Lowell's prediction is speculated to be a fortunate chance, I believe Ketakara's was a better fortunate chance that he could predict so accurately on entirely different grounds appealing to the theoreticians of Astronomy. Perceival Lowell got the credit for this discovery, Pickering claimed that his predictions were more accurate on theoretical grounds, but finally he was satisfied by connotating that the symbol P_L for Pluto stands for Pickering Lowell—(P) at the top, (L) at the bottom, and not Perceival Lowell as understood by historians. He felt the credit for Lowell on account of the fact that Lowell left so much money for the experimental side of the project in the discovery of Pluto. Ketakara's results were better than even Pickering's, so chronologically Ketakara deserves due credit at least for the theoretical prediction of this planet. An exhaustive analysis is in progress by Mr. J. G. Chhabra under the guidance of the author.

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