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SPREAD AND TRIUMPH OF INDIAN NUMERALS*

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According to Menninger[1], it is quite probable that due to active commercial relations with India, the first Indian numerals became known in Alexandria sometime in the 5th century A.D. and from there they might have penetrated farther westward.

Menninger says that the Indian numerals did not arrive in Egypt as a scientific treasure, but rather like the numerals of alien peoples that become known in the harbours and ports. However, we may point out that some Brahmins (who are not supposed to be traders) who visited Alexandria in A.D. 470 were the guests of Consul Severus.[2]

Hindu numerals are found in several manuscripts of the Geometry of Boethius (*c.* 500) and if the relevant portions of the manuscripts be regarded as genuine, it will show that the Indian numerals had reached southern Europe about the close of the fifth century.[3]

The Mayan vigesimal abstract place-value notation contains the oldest zero in the New World. Although the Mayan system occurred in apparent isolation, Menninger (*CHN*, 405) suspects a possible borrowing from India, the Mayan culture (now extinct) being at its height during the period from the 6th to 11th centuries A.D.

According to Werner,[4] the Chinese adopted the Indian decimal system and notation introduced by the Buddhists and changed their custom of writing figures from top to bottom for the Indian custom from left to right.

Wei Chih's (died 643) *Sui-shu* (Records of the Sui Dynasty, 581-618 A.D.) mentions the Chinese translations of Indian works like[5]

(*i*) *Brahman Suan-fa* (Brahman Arithmetical Rules) in one book.

(*ii*) *Brahman Suan-ching* (Brahman Arithmetical Classic) in 3 books.

This shows that Indian calculation methods and numerals (?) were known in

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China already about the end of the 6th century. It is unfortunate that these Chinese translations are lost.

That the fame of the Indian numerals had already reached the banks of Euphrates in the 7th century, is shown by a passage in a work of the Syrian monk Severus Sebokht (A.D. 662) who lived in a monastery at Kenneshre (*HHM*, I, 95-96). He[6] refers to the Hindus and "their valuable methods of calculation; and their computing that surpasses description. I wish only to say that this computation is done by means of nine signs."

This clearly shows that the Syrian scholar understood the full significance of the Indian numerals although, like so many others, he mentions only the nine Indian number signs which, of course, without a zero symbol would not have been considered at all remarkable. Karpinski[7] is also of the opinion that the numerals referred by the Syrian Bishop are those "which we now use".

The Khmere inscription at Sambor (683 A.D.) and the Malay inscription (684 A.D.) at Palembang (Indonesia, Sumatra) give their dates (in *Śaka* years) by employing Indian decimal place system of numerals with zero.[8]

The Malay inscription at Kotakapur (716 A.D.) also gives its *Śaka* year 608 in the same system.[9]

In the Khai-yuan Period (713-741), Chhüthan Hsita (Gautama Siddhārtha) was appointed Royal Astronomer of China. Under imperial order he made the Chinese translation, called *Chiu-chih li* (*Navagraha* Calendar), of an Indian work. The Chinese version contained the Indian numerals following decimal place value notation and using a dot (instead of a circle) for zero, and further remarked[10] that "with these numerals, calculation is easy to the eyes". Of course, it is.

The Dinaya Sanskrit inscription (760 A.D.) at Java gives the *Śaka* year both in Indian place value notation and in Indian word numerals.[11]

During the reign of Caliph al-Manṣūr (755-775), works on Indian mathematics and astronomy (including those of Brahmagupta of 7th century) were translated into Arabic at Baghdad (*HHM*, I, 89). It is believed that it was at that time that the Hindu numerals were definitely introduced amongst the Arabs and the Baghdad scholars greatly appreciated the Indian system.[12]

An inscription (813 A.D.) at Po-nagar, Champa, gives the *Śaka* year in two slightly different forms employing the Indian system of positional numerals.[13]

The famous Abū Jafar Muḥammad al-Khwārizmī (*c.* 800-850) wrote about 820 a work on Indian numerals. The original in Arabic is not extant but we have its Latin

version, entitled *Liber Algorismi de numero Indorum* (The Book of al-Khwārizmī on Indian numerals), possibly by Adelard of Bath (*c.* 1120)[14] or by Robert of Chester (*CHN*. 411). Menninger thinks (*CHN*, 411) that al-Khwārizmī had probably learned the numerals himself from the Indian writings (several of which were available in Arabic translations).

According to Ibn al-Qifṭī's indication, the title of the Arabic original may have been like *Kitāb hisāb al-adad al-ḥindī* (Treatise on calculation with Hindu Numerals) but J. Ruska[15] conjectures its title equivalent to "Book of Addition and Subtraction by the Method of Calculation of the Hindus".

The treatise of al-Khwārizmī, as we have it, expounds the use of the Hindu (or as they are misnamed "Arabic") numerals 1 to 9 and 0 and the place value system, then explains various applications. It is an elementary arithmetic treatise using Indian numerals.

Arabs were already using alphabetic numeral system (as shown by an 8th century Arabic Papyrus from Egypt) similar to Greek. The Indian decimal place value system was also already known, but al-Khwārizmī's work was the first to expound it systematically. Unfortunately, even with this important introduction of useful symbols into more general use in Islamic lands, there was delay in adopting them quickly in all spheres of life. And the treatise achieved greatest success only when introduced to the West through Latin translations in early 12th century.[16]

As regards the forms of the number symbols, al-Khwārizmī stated that the new numerals, particularly 5, 6, 7 and 8 were written differently by different peoples but that this circumstance was no obstacle to their use as a place-value notation (*CHN*, 413).

Another example of the use of Indian numerals in S. E. Asia is provided by the inscription at Bakul (839 A.D.) in which the corresponding *Śaka* year is mentioned in decimal place value system.[17]

In his work, *Ancient Alphabets and Hieroglyphic Characters Explained etc.* the Syrian Ibn Wahshīya (*c.* 855) gives three forms of Hindu numerals as three species of Hindu alphabets which shows that the forms were well known in his time in Arabia (*HHM*, I, 96-97).

The Indian numerals are also mentioned by the Arab philosopher al-Jāḥiẓ (*d.* 868/869), who calls them 'figures of Hind' and observes that with these numerals large numbers can be represented with great facility (*HHM*, I, 97).

A concrete example of the use of the decimal place value system is provided by an Egyptian papyrus (written in the year 873) in which the year 260 is expressed in Indian numerals (*CHN*, 414).

Abū Yūṣuf al-Kindī (died *c.* 873) and al-Dīnawarī (died 895) each wrote a tract on Indian Computation (Hiṣābul Hindī).[18] Al-Dīnawarī was a lawyer and attempted to introduce Hindu methods in business.

Abū Sahl Ibn Tamim (d. 950), a native of Kairwān a village in Tunis in the north of Africa (*HHM*, I, 98), wrote in the *Sefer Yezirah* that he had used the Indian nine signs in his work on Hindu calculation, *Ḥisāb al-ghubār*.[19]

Abul-Ḥasan Masūdī (d. 956/957) visited India about 915 and later on mentioned the Indian numerals in his work (*c.* 943) with the remark that "a congress of sages at the command of the Creator Brahmā invented the nine figures" which shows that no inventor was known in India even at that time (*HHM*, I. 97).

Abul-Ḥasan al-Uqlīdisī wrote his *Kitāb al-Fuṣūl fī al-Ḥisāb al-Ḥindī* (Book on Principles of Hindu Computation) in Arabic at Damascus in 952/953 A.D.[20] It is said to be the earliest extant Arabic book that presents Indian system.

In the introduction al-Uqlīdisī states that he has travelled extensively and read all books on Indian arithmetic that he found. Hindu numerals and place value notation is discussed in the first part of the work.

He has made several interesting suggestions such as[21] (*i*) Modifications of Indian schemes whereby the (dust) abacus can be dispensed with, and ink and paper used instead. (This was the first step in discarding abacus slowly). (*ii*) Greek letters might replace the nine Indian numerals. (This was, in fact, done sometimes). (*iii*) The Indian numerals with superimposed dots might form a new Arabic alphabet.

Gerbert (*c.* 940-1003) visited Spanish border country around 967 and enriched his mathematical knowledge. It was probably here that he first became acquainted with the Indian numerals, for the Arabs had been in Spain since 713 (*CHN*, 322).

He carried the *ghubār* forms of Indian numerals (learned in Spain) back to his home place (Auvergne, France) and inscribed them on the counters of the monastic abacus in the form of *apices*. In this form the Indian numerals (without zero, as columns with no digits were simply left vacant) made their first definite excursion into the West towards the end of the tenth century.

But Europe was not ready for them; neither their nature nor their advantages were appreciated, and they soon retreated into the cells of learned monks as they failed to survive in ordinary use (*CHN*, 417).

However it must be pointed out that the full set of the *ghubār* forms of Indian numerals (including the zero symbol) is found in an Arabic manuscript dated 970 A.D.[22]

Although Indian numerals were known and appreciated by the Baghdad scholars as early as the 8th century, yet when Abūl Wafā (died 997/998 at Baghdad) wrote his Arabic text book on practical arithmetic, called *Book on What Is Necessary From the Science of Arithmetic for Scribes and Businessmen* (written between 961 and 976), he avoided the use of numerals by writing the numbers in words.[23]

Some historians (such as M. Cantor and H. Zeuthen) explain the lack of Indian numerals by presuming the existence of two opposing schools among the Arabic mathematicians one following Greek models and the other Indian models. However M. I. Medovy[24] shows that such a hypothesis is not supported by facts (as some writers/scholars are found to use both the systems).

It is more probable that the use of Indian numerals simply spread very slowly among the businessmen, scribes, and general public whose needs were heeded by the text-book writers. Whatever be the reason, the victory of the Indian numerals, though delayed, was unavoidable.

A good example of Indian numerals is found in an European manuscript written in Spain in 976 A.D.[25]

Alī Ibn Aḥmad al-Mujtabā, who lived in Baghdad and died in 987, wrote the *Kitāb al-takht al-kabīr fī al-ḥisāb al Hindī* (The Great Book of the Board on Hindu Arithmetic); and his contemporary, al-Kalwādānī, (living at Baghdad) also wrote a similar work, *The Book of the Board on the Hindu Arithmetic*.[26] Both the works employ the dust or *ghubār* form of Indian numerals and are among the several Arabic books written by the Eastern Muslim scholars on the subject in the tenth century.

The Indian numerals are mentioned by al-Nadīm (*d.* 995) in his *Kitāb al-Fihrist* (*c.* 987) and are called *hindisah* (*HHM*, I, 98). Ibn Nadīm reports of a custom of writing the zeros beneath the figures.[27]

The Indian numerals of the *ghubār* type (but without zero) are given, as an addition (992 A.D.) in a Spanish copy of the *Origines* by Isidorus of Seville (*d.* 636).[28]

The Indian numerals (with dot for zero) are found in the philosophical treatises of the brothers Ikhwān as-Ṣafā (*c.* 1000) along with their Arabic names and the old Arabic alphabetic numerals following the *abjād* system.[29]

A similar set of Indian numerals (but with different form of 5) is found in a treatise on Hindu arithmetic written about 1000 A.D. by the famous Arabic scientist al-Bīrūnī.[30]

In his *Āthār al-Baqiyah* (Vestiges of the Past), written in 1000 A.D., Bīrūnī calls the then modern numerals as 'al-arqam al-hind', i.e. 'the Indian Ciphers' distinguishing them from other systems (*HHM*, I, 99).

Kūshyār Ibn Labbān (*c.* 1010) wrote his *Kitāb fī Uṣul Ḥisāb al-Hind* (Book of Principles of Hindu Reckoning) in Arabic and is based on Indian system of numerals including zero which is represented by a circle.[31] His importance lies in his having written the work to introduce the Hindu methods into astronomical calculations.

Abu Bakr al-Karkhī (*d.* 1029) wrote (Probably between 1010 and 1016) his *Kāfī fī al-Ḥisāb* (Sufficient of the Computation) which was largely based on Hindu sources.[32] His other work using Indian numerals is the *Kitāb fī al-Ḥisāb al-Hindī* (Book of Indian Computation) which is cited in his Algebra.[33]

Al-Bīrūnī (whom we have already mentioned) visited India and studied Indian sciences between 1017 and 1030 and so he was more qualified than his predecessors to speak with authority about Indian numerals (*HHM*, I, 98). Two of his works, namely *Kitāb al-arqam* (Book of Ciphers) and another called *A Treatise on Arithmetic and the system of Counting with the Ciphers of Sindh and India* are quite relevant in the matter (*ibid*).

His knowledge and opinion about Indian numerals are expressed in the following words:[34]

> "As in different parts of India the letters have different shape, the numeral signs too, which are called *aṅka*, differ. The numeral signs which we use are derived from the finest forms of Hindu signs."

For some nice examples of the actual use of Indian numerals by al-Bīrunī, reference may be made to the facsimile of pages from his original Arabic work as reproduced in L. C. Karpinaski's *The History of Arithmetic* (New York, 1965), pp. 47 and 51 (*vide* ref. no. 7). These show the use of Indian form of several numbers up to 1000.

Abu'l-Ḥasan al-Nasawī, who lived in Baghdad (1029-1044), wrote his *Al-Muqni' fī al-Ḥisāb al-Hindī* (An Account of Indian Computation) which employs the numerical symbols obtained from the Indians. Introduction of the book shows that al-Nasawī wrote in Persian a book on Indian arithmetic for presentation to Magd al-Dawla, the Buwayhid ruler (who was dethroned in 1029/1030). Later on it was presented to Sharaf al-Mulūk, vizier of Jalā al-Dawla, ruler of Baghdad. But the vizier ordered al-Nasawī to write it in Arabic and the result was the above work *al-Muqni*.[35]

In the Islamic astronomical literature, sexagesimal digits were written from right to left in Arabic alphabetic numerals. But al-Nasawī placed successive digits in a vertical column and used Indian numerals only.

A nice detail about the transmission of Indian numerals to the Islamic world is accidentally preserved in the autobiography of Ibn Sīnā or Avicenna (*c.* 980-1037). When he was about 10 years old, missionaries of an Islamic sect, called Ismaelites, came to his native place, Bukhara (then under the Iranian Dynasty of the Samanids)

from Egypt. Through the teachings of these missionaries, Ibn Sīnā learned the Hindu method of computing. Without this explicit bit of information, no body would have dreamt that Indian influence (and numerals) reached southern Russia via Egypt![36]

It has been stated by Ali bin Abil-Regal Abul-Hasan, called Abenragel (1048), in the preface to his treatise on astronomy, that the invention of reckoning with nine ciphers is due to Hindu philosophers (*HHM*, I, 99).

Abu Jafar al-Ṭabarī, who lived in the town of Āmul (south-east of the Caspian) in the last half of the 11th century, wrote the *Shumār-nāme* (Reckoning Book). It is a text-book on Hindu computation and is said to be the earliest extant book on the subject in Persian.[37]

We have already mentioned that al-Khwārizmī's book on Indian computation was translated into Latin in the early 12th century by Adelard of Bath or by Robert of Chester. In fact the book quickly spawned a number of adaptations and off-shoots such as the *Liber alghoarismi* of John of Seville (*c.* 1135), the *Algorismus* of John of Sacrobosco (13th century) and the 12th century work *Ysagogarum Alchorizmi*.[38] Other 12th century epitomes exist in manuscript form in the Royal Library, Vienna and the University Library, Heidelberg (*CHN*, p. 411). (Also see *Math. Reviews* **44**, 481-482.)

Two more such Latin 'algorisms' are reported to exist in the British Museum, the one is the Royal MS. 15B. IX and the other is the Egerton MS. 2261. The Royal manuscript begins:[39]

> "The intention of al-Khwārizmī in this work is to present the teaching of numeration, addition, subtraction, duplication and mediation, multiplication, and division by the ten characters of the Hindus (per X karacteres indorum)."

In fact al-Kwārizmī's name became so closely associated with the 'new arithmetic' using the Hindu numerals that the Latin form of his name, algorismus, was given to any treatise on that topic. Hence by a devious path, is derived the modern word 'algorism' (corrupted by false etymology to 'algorithm').[40]

The oldest year-date to appear in Europe in the new Indian numerals occurs on a Sicilian coin of the Norman King Roger II (*CHN*, 439). The year marked is 533 A.H. (=1138 A.D.).

Out of several works on number written by Abraham Ibn Ezra (*d.* 1167), the most important is his *Sefer ha-Mispar* (Book of the Number). It is based on the Indian system of positional numerals but uses the first nine Hebrew letters for the figures 1 to 9 and the zero as in algorism.[41] The zero symbol is given as *galgal* ('wheel' or 'circle').

Saraf Eddin (*c.* 1172) of Mecca wrote a treatise entitled *Fi al-handasa wa al-arqam al-hindi* (On Geometry and the Indian Ciphers) (*HHM*, I, 99).

Al-Samaw'al (died *c*. 1180) was a native of Baghdad and studied the Hindu computational methods. In writing polynomials, he assigned to each power of x a place in a table in which the polynomial was represented by the sequence of its coefficients, written in Indian numerals. Only by employing the new numerals, he could easily handle large polynomials and large number of equations. His techniques helped development of symbolism necessary for the progress of algebra.[42]

Leonardo of Pisa or Fibonacci (*c*. 1170-1250) wrote his great work, the *Liber Abaci* (Book of Computation) in 1202 fully based on Indian numerals. The work prepared the ground for the widespread adoption of the Indian numerals in the West (*CHN*, 425).

As a young man he travelled about the Mediterranean visiting Egypt, Syria, Greece, Sicily and southern France, meeting the scholars and becoming acquainted with the various computational systems in use among the merchants of different lands. But he reports that all systems appeared to him in error as compared to the Indian mode ("quasi errorem computavi respectu modi indorum").[43]

He introduced the new numerals in the following words (*CHN*, 425):

> "The nine Indian numerals (figure *indorum*) are 9, 8, 7, 6, 5, 4, 3, 2, 1. With them and with the sign 0, which in Arabic is called *zephirum* (cipher), any desired number can be written."

Fibonacci's works did pioneering service in bringing Indian numerals into ordinary use. With him a new epoch in Western mathematics began. Although all his ideas were not taken up immediately, great influence was exerted by those portions of his works that served to introduce Indian numerals and methods.[44]

It is unfortunate that two of his works, namely *Dimmor Guisa* (a book on commercial arithmetic) and a tract on book X of *Euclid's Elements* (in which he promised a numerical treatment of irrationals instead of Euclid's geometrical presentation) are lost.

We have already mentioned the name of John of Sacrobosco who was educated at Oxford and later on taught mathematics in Paris where he died in 1244 or 1256. His *Algorismus* or *Tractatus de Arte Numerandi* was the first arithmetic, based on Indian numerals, written by an Englishman. His work was widely used all over western Europe for centuries and thus he did much to spread the Indian numerals and computation.[45]

But the most interesting among the computational works based on Indian numerals is the *Carmen de Algorismo* (Song of Algorismus) by the French monk Alexandre de Ville Dieu who taught in Paris about 1240. In his version, an Indian king named Algor figures as the inventor of the new art which itself is called algorismus (*CHN*, 412).

The opening lines from a 13th century manuscript (at Darmstadt) of his work may be translated thus (*CHN*, 412):

> "Here begins the alogorismus. This present art is called algorismus, in which we use twice the five figures of the Indians (*bis quinque figuris indorum*)."

These lines and the myth of king Algor again appear in the first English arithmetic (*c*. 1300), the anonymous *The Crafte of Nombryng* whose manuscript is in the British Museum (Egerton MS. 2622).[46]

We find that the nine Indian numerals were called *figurae* in the 13th century and the name was retained in English and French. Thus zero, the 'figure of nothing' was no numeral or no figure at all, '*nulla figura*' in Latin whence came the name 'null' for zero (*CHN*, 403).

In the 8th century, when Indian numerals were definitely introduced in China, a dot was used for zero (see above). A small circle as the symbol for zero is first found in print in the Chinese work *Su Shu Chiu Chang* (Mathematical Treatise in Nine Sections) of Chhin Chiu-Shao (1247 A.D.) but many believe in its use in the earlier period of the Sung Dynasty (950-1280) after its arrival from India.[47]

It is stated that *The Comprehensive Work on Computation with Board and Dust* (in Arabic) by Naṣir al-Dīn al-Tūṣī (1201-1274) marks an important stage in the development of the Indian numerals.[48]

A 13th century monastic manuscript (State Library, Munich) contains the Indian numerals along with Roman (*CHN*, 282).

Towards the end of the 13th century, an enemy suddenly appeared from an unexpected direction. As numbers began to be written in the new Indian numerals by some Italian trading houses, the City Council of Florence in 1299 issued an ordinance which forbade to enter the amounts of money in the accounts book in Indian numerals (*CHN*, 426).

The argument was that the new numerals were more easily forged or changed than Roman numerals. People were still too insecure about the new numerals. It was not only their forms that were unfamiliar but also the method of writing them. It is not therefore surprising that the local chambers of commerce in Italy resisted the adoption of Indian numerals.

Thus, although computations with Indian numerals were known to commercial and trading establishments in the 13th century, book-keeping continued in old manner. This, of course, was a serious obstacle to the spread of the Indian numerals. However, the teachers and students of universities at Paris, Oxford, Padua, and Naples, kept alive the knowledge of Indian numerals.

Gregory Chioniades, who studied astronomy in Tabriz (in Azerbaijan) around 1290, used the Eastern Arabic forms of Indian numerals while he was in Byzantium (from 1298 to 1302).[49] These forms of Indian numerals may have been learned from him by Planudes (see below) who also used them.

Like Abraham Ibn Ezra (12th century), Levi ben Gershon used, in his *Sefer Maasei Hoshev* (Book of the Calculator) (completed in 1321), the Indian place-value numeration but employs the first nine letters of the Hebrew alphabet for numerals 1 to 9 (and a circle for zero).[50]

About 1330 (?), the Byzantine scholar Maximus Planudes (a Greek monk and Constantinople ambassador to Venice in 1327?) wrote the *Psēphophoria kat' Indous e Legomenē Megalē* (Computation According to the Indians, Which is Great) based on Indian numerals. It sets forth the system of notation by the "nine figures received from the Hindus" together with the zero, and is the first of the Greek works to give any attention to Indian methods.[51]

T. L. Heath (*Hist. of Greek Math*, Oxford, 1965; II, 547) quotes Planudes more fully as follows:

> "(The symbols were) invented by certain distinguished astronomers for the most convenient and accurate expression of numbers. There are nine of these symbols (our 1, 2, 3, 4, 5, 6, 7, 8, 9), to which is added another called *zifra* (cypher), written 0 and denoting zero. The nine signs as well as this are Indian."

Heath mentions an earlier Greek work, with similar title, written in 1252 (extant as Paris MS. Suppl. Gr. 387) and believes that Planudes may have raided it. But the forms of numerals are stated to be different in the two works. Planudes is placed earlier by Heath and still earlier by Pingree (ref. 49).

Al-Umawī taught arithmetic in Damascus in 14th century. He wrote about 1373 the *Marāsīm al-intisāb fī 'ilm al-ḥisāb* which represents a trend of Arabic arithmetic in which the Indian dust board calculations had began to be modified to suit paper and ink.[52] In a table of sequences he used the Western Arabic forms of Indian numerals.

Indian numerals appear also on several manuscripts such as:[53]

(*i*) Latin manuscript (*c.* 1294) of Boethius arithmetic.
(*ii*) Latin manuscript (*c.* 1294) of Euclid.
(*iii*) Italian manuscript (*c.* 1339) of the *Trattato d' Abbaco, etc.* by Paolo Dagomari (*d.* 1373/1374).
(*iv*) French manuscript (14th century) of *Algorismus Proportionum* by Nicole Oresme (*c.* 1323-82).

In spite of wide-spread use of Indian numerals, a class of arithmetical works, called *The Computi* (which were treatises on Church Calendar), was mostly confined to

Roman numerals. Was it due to orthodoxy or prejudice? However, Indian numerals were known to the authors who occasionally used them—sometimes in a peculiar way. For instance a Latin manuscript (dated A.D. 1384) of an anonymous *computus* gives its date as (*Rara*, 443):

anno dnj 1000.300.80. $\mathring{4}$

As the Indian place-value notation penetrated deeper and deeper in the West, it gradually displaced computations with alphabetic numerals which, like Roman numerals, were deeply rooted. In the beginning there was a sort of "equilibrium" (or compromise) between the two systems. The Greek alphabetic numerals for the units (ϛ to θ including the now obsolete 'vau', 'stigma' or 'digamma' which stood for six) were used as "Indian numerals". Of course a zero symbol had to be adopted for there was no such thing in the alphabetic system. Thus a 15th century Greek manuscript of a text-book on arithmetic contains the following (*CHN*, 274):

ιϵ for 15
$\delta\gamma\bullet$ for 430
$\alpha\beta\theta\bullet$ for 1290

It may be recalled that such a compromise with Hebrew alphabetic numerals was already employed by Abraham Ibn Ezra (12th century) and Levi Ben Gershon (14th century).

The oldest German coin which gives the date of the year in Indian numerals is a silver medallion struck by the town of St. Gall in 1424 (*CHN*, 439).

The progress in the use of Indian numerals in the accounts books of the imperial free city of Augsburg (West Germany) may be summarized as follows (*CHN*, 289-293):

Sl.No.	Year	Use of numerals
(*i*)	1410	iiij^{M} lb vij^{c} lb lxxxx viij lb for 4798 lb.
(*ii*)	1430	iiij^{C} for amount 400; but year in Indian numerals.
(*iii*)	1470	amount as ij^{C} and lxiij which is repeated in Indian numerals as 363.
(*iv*)	1500	amounts in both systems but their total in Indian numerals only.
(*v*)	1533	all entries in Indian numerals only.

Before their regular appearance in printed arithmetical books, the Indian numerals were extensively used in several computational treatises of which the following 15th century manuscripts may be noted (*Rara*, pp. 443-465):

1. Italian Ms. (1422) of *Trattato di aritmetica* by Giovanni son of Luca da Firenze.

2. Latin Ms. (1424) of *Scientia de namero ac virtute numeri* by Rollandus (*c.* 1425).

3. Italian Ms. (*c.* 1430) of an anonymous work on Florentine commercial arithmetic.

4. Latin Ms. (*c.* 1442) of a work on *algorismus* by John of Sacrobosco (13th century).

5. Italian Ms. (*c.* 1456) of an anonymous work on business arithmetic.

6. Italian Ms. (*c.* 1460) of a work on mathematics possibly by Raffaele Canacci (of Florence).

7. Italian Ms. (*c.* 1460) of a work on mercantile arithmetic by Benedetto da Firenze.

The work on *Etymologies* (also called *Origines*) by Isidorus of Seville (*d.* 636 A.D.) was printed at Augsburg in 1472. The subject of arithmetic is treated in its book III (*Rara*, 8). It is not known whether this printed version contains the Indian numerals which were added to chapter one of book III in some 10th century copies of the work (*HHM*, I, 102).

Indian numerals are profusely used in Regiomontanus's *Calendar des Magister* which was printed in Nuremberg in 1473 A.D.[54]

The first truly dated computational work (using Indian numerals) to appear in print in the West is called *Treviso Arithmetic* (Treviso, 1478) from its place of printing in Italy, the author being unknown. The numeral 1 was printed as i generally (*Rara*, 3-7).

Wide penetration of Indian numerals and methods can be ascertained from the fact that in Italy the very first computation text-books has no traces of counting board (*CHN*, 441). In England, Indian numerals appeared on an illustration in an English work printed about 1480 (see below). In Germany, the first printed arithmetic text-book employing Indian numerals is the Bamberg arithmetic of 1483 by the Nuremberg rechenmeister Ulrich Wagner (*CHN*, 335 and 434).

Pietro Borghi's *Arithmetic* (Venice, 1484) is more elaborate than Treviso arithmetic and had far greater influence on education. Borghi first treats of the Indian place value notation, carrying his numbers as high as 'numero de million de million de million', and making no mention whatever of the Roman numerals (*Rara*, 16-19).

Abū'l-Ḥasan al-Qalaṣādī (1412-1486) is the last known Muslim mathematician

of Spain who wrote several books on arithmetic. His *Kashf al-asrār 'an waḍ 'ḥurūf al-ghubār* (Unfolding the Secrets of the Use of Dust Letters, i.e. Indian Numerals) was a text-book in schools of North Africa.[55]

Even at this juncture when Indian numerals were marching to victorius triumph, some set-backs did exist. For instance, the Frankfurt Mayor's Book of 1494 ordered the rechenmeister to abstain from calculating with Indian numerals (*CHN*, 427).

Great were the successes which the Indian numerals achieved. Greater was the revolution which they were creating. Opposition to them attracted more attention.

With the introduction of printing in the 15th century, the contest between the old counting board and the new Indian place-value numerals in Europe becomes visible in various ways. Thus the old and the new are symbolically represented in the *Margarita Philosophica* of Gregor Reisch (1503 A.D.). Next to Pythagoras with his sorrowful face working at a counting board sits a cheerful and serene Boethius contemplating his computations in Indian numerals. Arithmetic (personified as a female figure) hovers with her books between them, looking at the computer with digits and indicates her approval of him by two geometric series in Indian numerals on her garment (*CHN*, 350 and 431).

In another illustration, a woodcut by the Nuremberg artist Hans Sebald Beham (*d.* 1550), Winged Arithmetic is shown to turn her back on the counting board and point emphatically to the tablet with the new Indian numerals (*CHN*, 431).

Roman numerals were so deep rooted in Europe that it made exceedingly difficult for the Indian numerals to replace the old numerals even from those situations where the latter deserved no place. For some time they boiled in the same pot leading to a sort of confusion and multiplicity as illustrated by the following examples (*CHN*, 287).

M.CCCC.8II for 1482
15× 5 for 1515
I.O.VIII.IX for 1089
ICCOO or I.II. τ τ for 1200
(τ for zero from Greek word τζιφρα, i.e. *tzifra* or *cifra*).

As mentioned above, the Indian numerals appeared in England in an English work printed as early as 1480 (*Rara*, 10) or 1481 by the Caxton Press.[56] This was *the Mirrour of the World* in which arithmetic is briefly discussed but the author is not known and the numeral forms appear only on an illustration.

The *De Arte Supputandi* (The Art of Computation) by Cuthbert Tonstall (London, 1522) is the first book wholly on arithmetic (using Indian numerals) that was printed in England (*Rara*, 132-135). But the English treatise which was most influential in

popularising the Indian numerals was Robert Recorde's *The Grounde of Artes.* It appeared first about 1542 and in 27 further editions up to 1699.[57]

In Germany the work of Jakob Köbel (died 1533) played similar role. He wrote several books on arithmetic out of which his *Rechenbiechlin* first appeared at Augsburg in 1514. This has a table for learning the Indian numerals (*Rara,* 104) which were still considered difficult. (In this table, the number 89 is wrongly shown as LXXXXI).

Like Ibn Ezra (*d.* 1167), Elias Misrachi (*d.* 1526) used the Hebrew letters and O in his arithmetic which was based on former's work and bore same title (*Rara,* 521).

We have already mentioned the interesting illustration, contained in Reisch's *Margarita Philosophica* (1503 A.D.) in which Boethius is shown as the representative of Indian numerals. In fact the Medieval Europe (among some circles atleast) believed erroneously that Boethius (*c.* 500 A.D.) was the inventor of 'Indian' computation (*CHN,* 350). This belief may be partly due to the fact that Indian numerals are found in the manuscripts of his *Geometry* as early as tenth century (*HHM,* I, 92) or may be due to hero-worship.

Similarly some Latin writers, in their desire to exalt the classical (Greek) learning, assigned the Indian numerals to the Pythagoreans. For instance, Valentin Nabod did so in his *De Calculatoria Numerorum* (Cologne, 1556) which was written for the classical schools of Germany (*Rara,* 281). But could such writers succeed in misleading their readers and discredit the Indians ?

Any way, Indian numerals and computations continued to spread and hundreds of books appeared in print in the 16th century Europe on the subject.

The first arithmetical work (based on Indian numerals) printed in America appeared in Mexico in 1556. This was the *Sumario Compendioso* of Juan Diez Freyle.[58]

Towards the end of the 16th century, a book on arithmetic based on Indian numerals was submitted to a deacon of the cathedral at Antwerp for his approval. The decision was (*CHN,* 427):

> "These rules and procedures for computation and for finding the answers to problems are admittedly useful for merchants, and for their sake permission is granted for them to be printed; but they (the merchants) must see to it that they avoid usury and other illicit transactions and exchanges."

That is, the new numerals are not to be used for dealings that are not approved of. Prejudice and suspicion continued to exist about the Indian numerals and the long struggle between the 'abacists' and the 'algorithmicists' extended even beyond the 16th century. Orthodoxy was hard but cases of prosecution (like that of Galileo for his astronomical theories) have not come to light.

The duality of computation on the counting board and writing the numbers in Roman numerals were finally (in 17th cent.) replaced by the single-step procedure of written computations with Indian numerals.

This was, no doubt, the victory of Indian culture; but it was also a victory for the mind of man, who finally, in the long history of written numerals, arrived at a mature, abstract decimal place-value notation.

The Indian numerals are now used all over the civilized world. In vain Charles XII, King of Sweden (1682-1718), tried to abolish the Indian decimal system in favour of duo-decimal.[59] The decimal system is not the best but God favoured it by giving us 10 fingers.

References and Notes

[1] Menninger, Karl, *Number Words and Number Symbols (A Cultural History of Numbers)*. Translated from German, M.I.T. Press, Cambridge, Mass., 1970; p. 406. This important work will be referred to as *CHN* now.

[2] Majumdar, R. C. (General Editor), *The History and Culture of the Indian People*. Vol. II, Bharatiya Vidya Bhavan, Bombay, p. 625, 1968.

[3] Datta, B. and Singh, A. N., *History of Hindu Mathematics* (= *HHM*), Single volume edition, Asia Publishing House, Bombay, Part I, p. 92, 1962.

[4] Sarkar, B. K., *Hindu Achievements in Exact Sciences*. London, p. 11, 1918.

[5] Mikami, Y., *The Development of Mathematics in China and Japan*. Chelsea Publishing Co., New York, p. 58, 1961.

[6] Cajori, F., *A History of Mathematical Notations*. Vol. I, Open Court Publishing Co., La Salle, p. 48, 1974.

[7] Karpinski, L. C., *The History of Arithmetic*. Russell and Russell, New York, p. 48, 1965.
Due to newness and peculiarity of the zero symbol, it was generally mentioned separately. Hence, even when zero was fully known, writers spoke of the nine significant figures (*CHN*, 402 and 425). Also note the Indian word numeral *aṅka* which stands for 'nine' only.

[8] Bag, A. K., Symbol for Zero in Mathematical Notation in India. *Boletin Acad. Noc. Ciencias*, Vol. 48, pp. 250-251, 1970.

[9] Bag (see serial no. 8 above), p. 251.

[10] Yabuuti, K., Researches on the *Chiu-chih li* (Indian Astronomy under the Thang Dynasty, 618-906). *Acta Asiatica*, Vol. 36, p. 12, 1979.
The use of a (thick) dot for the zero symbol was not uncommon.

[11] Bag (serial no. 8), p. 251.

[12] Gillispie, C. C. (Chief Editor), *Dictionary of Scientific Biography* (= *DSB*). In 14 volumes, C. Scribners, New York, Vol. I, p. 40, 1970-1976.

[13] Bag (serial no. 8), p. 251.

[14] Smith, D. E., *History of Mathematics*. 2 volumes Dover, New York, Vol. I, p. 170 and Vol. II, p. 72, 1958.

[15] Rule, G. J., Article on al-Khwārizmī in *DSB*, Vol. 7, p. 360, 1973.
The conjecture of Ruska is not proper as the work (as extant) deals with multiplication and division (also see below).

[16] ———(serial no. 15), p. 362.

[17] Bag (serial no. 8), p. 251.

[18] Jairazbhoy, R. A., *Foreign Influence in Ancient India*. Asia Publ. House, Bombay, p. 173, 1963 and Smith (serial no. 14), I, 174.

[19] Levey, M. and Petruck, M. (ed. and trasl.), *Kūshyār ibn Labbān's Principles of Hindu Reckoning*. Univ. of Wisconsin Press, Madison, p. 6, 1965.

[20] ———A. S., Article on al-Uqlīdisī in *DSB*, Vol. 13, p. 544, 1976.

[21] Rule (serial no. 20), p. 545.

[22] Smith (serial no. 14), Vol. II, p. 74.

[23] ————————A. P., Article on Abū'l Wafā in *DSB*, Vol. I, p. 40, 1970.

[24] Rule (serial no. 23), p. 41.

[25] Smith (serial no. 14), Vol. II, p. 75.

[26] Smith, D. E. and Mourad, S., The Dust Numerals Among the Ancient Arabs. *American Math. Monthly*, Vol. 34, p. 258, 1927.
[27] See the *Islamic Culture* (Hyderabad), Vol. 42, No. 2 p. 124, April 1968.
[28] *HHM*, I, 102 and Cajori (serial no. 6), p. 50.
[29] Cajori (serial no. 6), p. 52.
[30] Karpinski (serial no. 7), pp. 47 and 51.
The treatise referred seems to be Bīrūnī's *Kitāb fī rāshikāt al-Hind* (The Book on Indian *raśikas*) devoted to composite proportions (see *Math. Reviews*, Vol. 41, p. 3).
[31] Levey and Petruck (serial no. 19), pp. 6, 86-92.
[32] Smith (serial no. 14), Vol. I, p. 283.
[33] Anbouba, A. (editor), *The Algebra al-Badī of al-Karajī* (= Karkhi). University of Lebanon, Beirut, p. 19, 1964.
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[35] Saidan, A. S., Article on al-Nasawī in *DSB*, Vol. 9, p. 614, 1974.
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[38] Toomer (serial no. 15), p. 362.
[39] Karpinski, L. C., Two Twelfth Century Algorisms. *Isis*, Vol. 3, p. 398, 1920-21.
[40] Toomer (serial no. 15) p. 363.
[41] Smith (serial no. 14), I, 207.
[42] Anbouba, A., Article on al-Samaw'al in *DSB*, Vol. 12, pp. 91-92, 1975.
[43] Smith (serial no. 14), I, 215.
[44] Vogel, K., Article on Fibonacci in *DSB*, Vol. 4, p. 612, 1971.
[45] Smith (serial no. 14), I, 221-222; and *CHN*, 402.
[46] ———(serial no. 14), II, 78-79. Karpinski (serial no. 7), pp. 56-57, however gives *c.* 1450 as the date of the same manuscript.
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[51] Smith (serial no. 14), I, 233-234; and Pingree (serial no. 49).
[52] Saidan, A. S., Article on al-Umawī in *DSB*, Vol. 13, p. 539, 1976.
[53] Smith, D. E., *Rara Arithmetica*. Chelsea Co., New York, pp. 434-438, 1970. *Rara* stands for this work.
[54] Cajori (serial no. 6), I, 96-97.
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[56] Karpinsk (serial no. 7), p. 58.
[57] ———(serial no. 7), p. 60.
[58] ———(serial no. 7), pp. 78 and 109.
[59] Kramer, E., *The Main Stream of Mathematics*. Oxford Univ. Press, New York, p. 17, 1951.
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