6 THE YUGA SYSTEM AND THE COMPUTATIONS OF MEAN AND TRUE PLANETARY LONGITUDES

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THE YUGA SYSTEM

INTRODUCTION

We find the mention of the *Yuga* system not only in the Indian astronomical works but also in the Paurāṇic literature. It is now difficult to trace whether the astronomical works of India borrowed the *Yuga*-concept from the *Purāṇās,* or whether the latter from the former. We shall first note the details of the *Yuga*-system, as is now in vogue, in the light of the information given in the most important astronomical works. (It may be mentioned that this word *yuga* appears even in many places in the *Ṛgveda* (Vide 1-158-6).

We are told that we are now in *Kaliyuga* which began at the mid-night of Laṅkā, between the 17th and 18th February 3102 B.C. The duration of this *Kaliyuga* is given to be 432,000 solar years. *Dvāpara yuga* which preceded this *Kaliyuga,* is reported to have the duration of two *Kaliyugas.* The *Tretā-yuga,* which preceded the *Dvāpara* was equal to three *Kaliyugas* in duration and the *Kṛta-yuga,* otherwise called *Satyayuga,* which preceded the *Tretā,* was equal to four *Kaliyugas* in duration. These four *yugas, Kṛta, Tretā, Dvāpara* and *Kali,* are often called *Yuga-pādas,* or the quarters of their sum, called *yuga,* otherwise called *Mahāyuga.* Thus a *Mahāyuga* is equal to $4+3+2+1=10$ *Kaliyugas* in duration$=$4320,000 solar years. *Sūrya-siddhānta* uses the word *yuga* for *Mahāyuga*[1]. The same *Sūrya-siddhānta* says in another breath, "*asmin kṛtayugasyānta sarve madhyagatāḥ grahāḥ*"[2] thus speaking *Kṛta* again as *yuga.* Let us call these *Kṛta* etc., as *yugas,* and the sum of the four namely *Kṛta, Tretā, Dvāpara* and *Kali* as a *Mahāyuga* to avoid equivocation. Then a *Mahāyuga* consists of 43,20,000 solar years. Seventyone *Mahāyugas* are reported to constitute a *Manvantara* or duration of a Manu's reign. Fourteen *Manvantaras* constitute what is called a Kalpa. *Sūrya-siddhānta* says: "A *Kalpa* is the day-time of a day of Brahmā, the Creator, and the night is also of the same duration."[3] The Cosmic manifestation is reported to last during the day-time, and to be withdrawn or annihilated during the night time.

Further, we are told that there will be what are called, *Sandhyās* in between two *Manvantaras* each equal to a *Kṛta Yuga* in duration, which is equal to four Kaliyugas or $\frac{4}{10}$, i.e. $\frac{2}{5}$ of a *Mahāyuga.* Since there will be fifteen such *Sandhyās* among the fourteen *Manvantaras,* the duration of these fifteen *Sandhyās* put together is equal to $15\times\frac{2}{5}=6$ *Mahāyugās.*

Thus a Kalpa is equal to $14 \times 71 + 6 = 1000$ *Mahāyugas*=4,320,000,000 solar years.

Before we go into the rationale of all this, it is better to know the other nomenclature used in this context. The *Sūrya-siddhānta* speaks of what is called a *Divyābda*, or an year of the gods of the heavens equal to 360 of our solar years, our one year equalling a day of theirs. It is interesting to note that Hindu mythology reports that gods (By 'gods' it should not be misinterpreted that the Hindus were that primitive as to postulate the existence of many gods. As per Hindu philosophy, there is the godhead, a supreme supernal Consciousness or Intelligence otherwise called Brahman, which is the architect of this universe. This Brahman, (note that it is in neuter gender), while creating is called Brahmā (masculine gender), while sustaining the universe Viṣṇu, and while annihilating the universe Rudrā. Though this Godhead, i.e. the Brahman is one, we are told that there is a hierarchy of Gods, inferior to this Brahman, besides such celestial species like Gandharvas, Kinnaras, Kimpuruṣas etc., all of whom may be deemed to correspond to the so-called angels spoken of in the English literature. These gods are reported to reside on the Meru mount, which we may identify as the north-pole. There, we have a perpetual day equal to six-months and a perpetual night equal to six months, so that one day there is equal to one year. The word *Surālaya* which is a synonym of the *Meru* means that it is the abode of such gods.

Thus, one year of ours, equalling one day of those gods, 360 years of men constitute an year of gods, which is the reason why they are called *Divyābdas*, or years of gods. We cannot help here going into the Hindu mythology, to expound the *Yuga*-system, though ultimately the *Yuga*-system is astronomical in concept. A mention is made of this *Divyābda* because *Sūrya-siddhānta* has used it. Incidentally, it may be mentioned here, that a day on the Moon is roughly equal to a lunation, otherwise called *Cāndramāsa*. We are told that for the Manes, who are reported to reside on the Moon, a lunation of men is equal to a day. This is all by the bye.

Here and now, it may be mentioned that as per Āryabhaṭa, Lalla, and Vaṭeśvara, who are reported to constitute the Keral school of astronomers, the four *Yuga-pādas* namely *Kṛta, Tretā, Dvāpara* and *Kali* are of equal duration. This is a reason why Brahmagupta criticized Āryabhaṭa for having said so, against the canons of *Smṛtis*. Indeed there seem to have been two schools of astronomy in ancient India, the first consisting of Brahmagupta, Śrīpati and Bhāskara etc., and the other Āryabhaṭa, Lalla, Vaṭeśvara and others. *Sūrya-siddhānta* belongs to the former school, the name of the author not being known as against the orthodox tradition, which claims that a deputy of the Sun taught Maya this siddhānta at the end of the *Kṛta Yuga*. There is another Āryabhaṭa, who may be termed Āryabhaṭa-II, who was the author of one *Mahā-siddhānta*. M. M. Sudhākara Dwivedi makes out that the author of the *Sūrya-siddhānta* lived between Āryabhaṭa-I and Āryabhaṭa-II. Incidentally, it may be mentioned that there was also Bhāskara-I who wrote a commentary on the work of Āryabhaṭa-I.

The rationale behind the statement that 71 *Mahāyugas* constitute a *Manvantara* is not traceable. The number 71 is indeed a peculiar number. At present, we are told that we are under what is called *Śveta Varāhakalpa.* Therein, six *Manvantaras* had elapsed and we are now under what is termed the duration of *Vaivasvata Manvantara.* In this *Manvantara,* twenty-seven *Mahāyugas* lapsed away and we are now under the 28th *Mahāyuga.* The three *Yugas—Kṛta, Tretā* and *Dvāpara* also lapsed away. Under the present *Kaliyuga,* 5083 years lapsed by 26-3-1982 as per the *Cāndra Māna* reckoning, and by 14-4-1982 as per the *Saura-Māna* reckoning.

Bhāskara (hereafter Bhāskara means Bhāskara-II) says that 1,97,29,47,179 years had elapsed by the beginning of the Śālivāhana Śaka, the Zero point of time as per the siddhānta calculations which occurred after the lapse of 3179 years from the beginning of this *Kaliyuga,* as per the details cited above. Here it must be pointed out that, whereas the *Sūrya-siddhānta* mentions that creation of the stars, planets began after the lapse of 47400 *Divyābdas* or 47400 × 360 years, after the beginning of the Kalpa, Brahmagupta, Śrīpati and Bhāskara mention that creation started from the beginning of the Kalpa.

Bhāskara quotes the *Sūrya-siddhānta* saying that the equinoctial points precede at the rate of 30,000 in a Kalpa. Inasmuch as this rate comes to 9″ per year, which is far less than 50″, the approximate rate of precession, and inasmuch as the *Sūrya-siddhānta* which was incorporated by Varāhamihira in his *Pañcasiddhāntikā* does not mention precession at all, it must be deemed that Bhāskara had the present *Sūrya-siddhānta* in his hands but read only the verse *Triṃśat-Kṛtvo yuge bhānām cakram prāk parilambate.* This indicates that the latter verses in this context, which make out that the rate of precession per year was 54″, postulating the libration theory (i.e. the theory that the equinoctial points do not precess secularly, but oscillate about the zero-point of the zodiac, i.e. the begining of Aśvinī to an amplitude of 27° on either side) were interpolated.

Brahmagupta was a great mathematician, whom therefore two great astronomers Śrīpati and Bhāskara followed. These three were mathematically rational astronomers. The very fact that, when the *Sūrya-siddhānta* was already extant in the times of Śrīpati and Bhāskara, they still chose to write fresh works of their own, signifies that the *Sūrya-siddhānta* did not enjoy the same popularity among the orthodox Hindus, as it enjoys now. This is really curious.

Rationale of the Yuga System

Before reading into the rationale of the *Yuga*-System we have got to look at the data given by the various astronomers with respect to the number of sidereal revolutions of the planets in a Kalpa etc. The following tables give those data, in addition to the modern data taken from Ball's *Spherical Astronomy.*

Table 6.1. *Number of sidereal revolutions of planets and planetary points in a Kalpa*

	Modern *Sūrya Siddhānta*	Bhāskara	Āryabhaṭa	*Khaṇḍa Khādyaka*	*Mahā-siddhānta*
Sun	4320000000	4320000000	4320000000	4320000000	4320000000
Moon	57753336000	57753300000	57753336000	57753336999	57753334000
Mars	2296832000	2296828522	2296824000	2296824000	2296831000
Mercury	17937060000	17936998984	1793702000	17937000000	17937054671
Jupiter	364220000	364226455	364224000	364220000	364219682
Venus	7022376000	7022389492	7022388000	7022388000	7022371432
Saturn	146568000	146567298	146564000	146564000	146569000
Moon's Apogee	488203000	488205858	488219000	488219000	488208674
Moon's Node	232238000	232311168	232116000	232226000	232313354

Table 6.2. *Number of mean solar days, lunar days, omitted lunar days, diurnal revolutions of stars etc. in a Kalpa*

	According to *Sūrya Siddhānta*	According to Bhāskara
(a) Number of mean solar days in a *Kalpa*	157791728000	1577916450000
(b) Number of *Adhikamāsas* in a *Kalpa*	1593336000	1593300000
(c) Number of *kṣayāhas* in a *Kalpa*	25082252000	25082550000
(d) Diurnal revolutions of stars	1582237828000	1582236450000
(e) *Tithis* in a *Kalpa*	1603000080000	1602999000000

Table 6.3. *Daily mean motions of planets*

	Brahmagupta Śrīpati and Bhāskara
Sun	0-59′-8″-10‴-21⁗
Moon	13°-10-34-53-0
Mars	0-31-26-28-7
Mercury	4°5′-32″-18-28
Jupiter	0-4-59-9-9
Venus	1-36-7-44-35
Saturn	0-2-0-22-51
Moon's Apogee	0-6-40-53-56
Moon's Node	0-3-10-48-20

Table 6.4. *Elements of the Solar System. Epoch* 1900 *(from Ball's Astronomy)*

Name of Planet	Semi. Major Axis of Orbit =Unity	*Sidereal Period* Mean Solar Day	Mean Daily Motion	Longitude of Perihelion	Longitude of Ascending Node	Inclination of Orbit	Eccentricity
Mercuty	0.3870986	87.96926	4°5'32".4	75°53'59"	47°8'45"	7°0'10"	0.205614
Venus	0.7233315	224.7008	1 36 7.7	130 9 50	75 46 47	3 23 37	0.006821
Earth	1.000000	365.2564	59 8.2	101 13 15	8 0 0	0 0 0	0.016751
Mars	1.523688	686.9797	31 26.5	334 13 7	48 47 9	1 51 1	0.093309
The Asteroids							
Jupiter	5.202803	4332.588	4 59.1	12 36 20	99 26 42	1 18 42	0.048254
Saturn	9.538844	10759.20	2 0.5	90 48 32	112 47 12	2 29 39	0.056061

We see a good divergence with respect to the sidereal revolutions and some other items, pertinent to our context. Why was there such difference, we have got to investigate. The argument may be given as follows.

The ancient Hindu astronomers could find the mean motions of the planets with sufficient accuracy though they did not have the modern instruments like the telescope etc. But one point must be clearly noted. *Knowing those mean daily motions, and the positions of those planets at their respective times, each of the great astronomers calculated back, as to when all the planets should have been at the Hindu zero point of the ecliptic.* It may be mentioned here, that the Hindu zero-point of the ecliptic is not the position of the equinoctial point, but the beginning point of the asterism Aświnī, which is now identified as ξ Piscium. Ranganatha (birth date about A.D. 1573) a commentator of the *Sūrya-siddhānta* said that the zero-point of the ecliptic was 10′ behind this ξ Piscium. Other astronomers did not specify where it was, probably because it was popularly known in their times. Unfortunately we are not in a position now to know which was exactly their zero-point.

The present-day Hindu astronomers are of two kinds : (1) those who are competent to teach the major texts like *Āryabhaṭīya*, *Sūrya-siddhānta* and *Siddhānta-śiromaṇi* of Bhāskara, with procfs; and (2) those who compute *pañcāṅgas* only basing on works called *karaṇa-granthas*. Those *karaṇa-granthas* are manuals which simplify the siddhāntas and give easier procedures to compute the planetary positions taking the starting point not from the beginning of the Kalpa nor even from the beginning of the *Kaliyuga*, as prescribed by the *Siddhāntas*, but from their recent epochs, which facilitated computation. It is unfortunate that neither of these categories of scholars namely those who teach the siddhānta texts, nor those who compute *pañcāṅgas* today are in the habit of observing the planetary positions. The second category of *pañcāṅga*-computers, whose number is thousand-fold the number of the former category, are ignorant of the rationale of the siddhāntas. They learn just the method of computation only. This is why Hindu astronomy fell long ago on evil days, creative activity having been already defunct. This is by the bye.

Śrīpati and Bhāskara took their data regarding the sidereal revolutions of planets etc. from Brahmagupta. Brahmagupta says—*Brahmoktam grahagaṇitam mahatā kālena khilībhūtam abhidhīyate shutam tat jiṣṇusuta brahmaguptena,* i.e. inasmuch as the ancient *Brahma Siddhānta* got obsolete; I am now rectifying it. Siddhāntas are prone to grow obsolete because of two important reasons: (1) the presumption that at the beginning of Kalpa all the planets with their aphelia and nodes were situated at the Hindu zero-point of the ecliptic; and (2) the mean daily motions as taken by the siddhāntas could not be expected to be as accurate as it should be. On these two counts it was but natural that the computed positions of the planets could not accord with their observed positions. Hence new siddhānta came to be written, which gave results which held good at their times. But again, these were bound to grow obsolete because of the same two reasons cited above.

In this context, one point must be stated. The Hindu astronomical texts construed that the Sun, Moon, Rāhu and Ketu, the latter two being the nodes of lunar orbit are also planets along with the five planets Mercury, Venus, Mars, Jupiter and Saturn. These latter five are of course planets as per modern astronomy, whereas calling the Sun, Moon, Rāhu and Ketu as planets may sound odd to modern astronomers. But what the Hindu astronomers meant by the word *grahas* (now talked of as planets) was that the Sun, Moon Rāhu and Ketu also had astrological influence on the fates of men as per the etymological significance of the word *graha*. The planets Uranus, Neptune and Pluto not observable by the naked eye came to be discovered during recent times and as such did not receive mention in the Hindu astronomical texts.

Now let us seek the rationale of the *yuga* system. It is easy to see that a Kalpa is the period in which all the planets, with their nodes and aphelia, make an integral number of revolutions with respect to the stars. This period was therefore computed, by the Hindu astronomers, noting their mean daily motions, and their positions at a particular time. It was therefore an extrapolation. We can take it that Brahmagupta who was indeed a great mathematician did this extrapolation. Śrīpati and Bhāskara indeed quote his name with reverence in the words—*Kṛtī jayati jiṣṇujo gaṇakacakra-cūḍāmaṇiḥ*—i.e. "Brahmagupta excels, as the head-worn gem of all astronomers." So also does Śrīpati praise him[4] prior to Brahmagupta. We can take it that Āryabhaṭa also did that extrapolation.

In this context, the question arises, as to how the ancient Hindu astronomers were able to get the period of a sidereal revolution of Saturn, to a tolerable accuracy, inasmuch as one sidereal revolution of Saturn is nearly 29½ years. The doubt may arise because, to obtain the duration of one sidereal revolution, many of such revolutions must have been observed. This doubt need not be there, because the ancient Hindu astronomers knew the formala $\frac{1}{Y} - \frac{1}{P} = \frac{1}{S}$ though not in this form, where Y stands for the length of an year, P for the duration of the planet's sidereal revolution, and S for the synodic period of that planet all in mean solar days. Since Y and S which are small periods, could be observed accurately P could be calculated to a good amount of accuracy.

Y, i.e. the number of mean solar days in solar year could be observed accurately as follows as was mentioned first by Āryabhaṭa and later by Bhāskara. We are directed to erect a vertical *śaṅku*, i.e. a gnomon on a plane. This *śaṅku* is divided into twelve equal parts called *aṅgulas*. Draw a circle with a radius equal to the length of the *śaṅku;* draw the east-west line through the foot of the *śaṅku*. Bhāskara gives the method of drawing this east-west line correctly, which was primarily given by one Caturvedācārya, a commentator of *Khaṇḍa-khādyaka,* and later reiterated by Śrīpati who preceded Bhāskara. This *Khaṇḍa-khādyaka* is reported to be written by Brahmagupta, but this report is subject to doubt because Brahmagupta criticized Āryabhaṭa right and left and the interpretation given by M. M. Sudhākara Dvivedi that though Brahmagupta happened to criticize Āryabhaṭa in his *Brāhmasphuṭa-siddhānta,* changed his mind later on account of the overwhelming popularity of Āryabhaṭa, and as such chose to write a manual to accord with *Āryabhaṭiyam.* On close examination of the data of *Brāhma-sphuṭa-siddhānta* and *Khaṇḍakhādyaka,* it will be found that some body wrote this *Khaṇḍakhādyaka* and foisted it on Brahmagupta to enhance the credit of Āryabhaṭa.

To draw the east-west line, we are asked to note the point in the morning on the circumference of the circle drawn with the foot on the gnomon as centre, when the shadow of the gnomon equals the radius of that circle. Again we are asked to mark the point on the circumference in the afternoon, when the shadow equals the radius of the circle. The line drawn parallel to the joint of these two points through the foot of the gnomon is the rough east-west line. It is rough because, the declination of the Sun will have changed in between the two points of observation. A correction was prescribed to rectify this, which goes by the name *agrāntara* correction. The amount of correction is given to be

$$\frac{K \triangle (\sin \delta)}{\cos \phi} \text{ } \textit{aṅgulas},$$

where $\triangle$ (sin δ) is the variation of the sine of the declination of the Sun between the two moments. K is equal to $\sqrt{12^2+s^2}$ where s is the length of the shadow, and ϕ is the latitude of the place.[5] We are asked to shift the point pertaining to the afternoon observation on the circumference, when the shadow equals the radius of the circle towards that direction in which the Sun is having motion in declination, i.e. towards north between December 23rd and June 22nd and south between June 22nd and December 23rd.

Now we are asked to note the point roughly about December 23rd, when the Sun's declination equals the obliquity of the ecliptic. It may happen in between two days and the fraction of a day can be roughly obtained. After one year lapses away, again, the point is to be noted about 23rd December. The interval in between these two points of time is the length of a solar year (of course tropical). Observations were carried for a number of years like that and their average gave the actual length of the year. Hindu astronomers took the obliquity to be 24°. Now it is 23°27′. At Brahmagupta's time it should have been greater than 23°30′ since this obliquity has been decreasing. It is noteworthy, that the Hindu mean solar year is just 3.25 minutes of time more than the value given by modern astronomy. The length of a lunation, on the other hand, was computed by noting the time in between two lunar or solar eclipses and dividing that interval by the number of lunations in between. Thus it is also noteworthy that the length of a lunation was also obtained to a great nicety indeed. Since the formula $\frac{1}{P} - \frac{1}{Y} = \frac{1}{S}$ was known to the Hindu astronomers, where P is the length of the sidereal month and S the length of a lunation, P could be obtained very accurately. It was possible to obtain the value of P also by direct observation, so that striking an average it was possible to get the values of P and S very accurately with respect to the Moon.

Similarly noting the intervals between two heliacal settings or risings of planets, and also by direct observation the sidereal periods of Mercury, Venus, and Mars could be obtained. With respect to Jupiter and Saturn alone, whose sidereal periods are far greater, their synodic periods, i.e. the interval between two heliacal risings or settings gave a correct value of the sidereal periods using the formula

$$\frac{1}{Y} - \frac{1}{P} = \frac{1}{S}.$$

Having thus obtained the sidereal periods of all the planets with sufficient accuracy, their L.C.M., so to say, was computed to obtain the period in which an integral number of sidereal revolutions were contained. In the first place, Āryabhaṭa obtained the L.C.M, i.e. the period in which all the planets made an integral number of revolutions as the *Mahāyuga* equal to 4,320,000 years. Thereafter Brahmagupta obtained the period of a Kalpa as that period in which Sun, Moon, Mars, Mercury, Jupiter Venus, Saturn, Moon's apogee, and Moon's node made an integral number of revolutions, thus effecting a change in the number of sidereal revolutions as given by Āryabhaṭa. It is easy to see that Brahmagupta effected this change because, the planetary positions computed did not accord with the observed positions. It was after all a period of one hundred years in between Āryabhaṭa's date of composition namely 499 A.D. and that of Brahmagupta's date of composition 598 A.D. The small divergences perceived between the computed positions and observed positions made Brahmagupta seek the larger period of a *Kalpa* in which the planets made an integral number of revolutions. He computed this period of *Kalpa* and changed the numbers of sidereal revolutions so as to effect accordance between the computed and observed positions at this time. But, it will be noted that the fundamental presumption that the planets must have started at the Hindu Zero-point of the zodiac either in the beginning of a *Mahāyuga* as thought by Āryabhaṭa or at the beginning of the *Kalpa* as construed by Brahmagupta is open to doubt. In the light of modern astronomical data, this presumption stands questioned.*

In conclusion, on account of such a presumption as cited above, as well as a little roughness which could not but be there (on account of lack of instruments which we have today) in the lengths of the sidereal periods of the planets, Hindu astronomers went on moving in a vicious circle, writing new texts to effect accordance between the computed and observed positions. However, we cannot but give them the credit due to them for having persevered diligently for centuries and centuries keeping up the light burning. This creative activity went on upto the time Kṛṣṇa Daivajña who is reported to be born in 1565 A.D. as per M. M. Sudhākara Dvivedi, and who was the court pandit of Jahangir. This great mathematician gave us the *muhūrta* prescribed by him for the coronation of Shahajahan. He wrote a commentary on Bhāskara's *Bījagaṇita* under the caption *Navāṅkura* which exhibits his genius. At the end of the commentary, he wrote a verse which means "Oh! God! Thou alone knowest what stress and strain I have undergone in writing this commentary on Bhāskara's *Bījagaṇita*. Hence I dedicate this to thee alone and to no mortal, however great he may be."

COMPUTATION OF MEAN LONGITUDES OF THE PLANETS

Ancient Hindu astronomers (for example Bhāskara) gave the mean positions of the planets at the beginning of the Kaliyuga at the mid-night between 17th and 18th february 3102 B.C. or at the sunrise on 18th February, which held good for the Laṅkā meridian. This Laṅkā, reported to be the capital of Rāvaṇa is situated on the terrestrial equator. This Laṅkā meridian was taken to be the primary meridian by the Hindu astronomers and was termed *bhūmadhyarekhā*. Nowadays, it is to be noted that this word *bhū-madhyā-rekhā* is applied to the terrestrial equator. In Hindu astronomy the name for the equator is *nirakṣa rekhā*, i.e. the line on which the *akṣa* or latitude is zero. The Laṅkā meridian is reported by Bhāskara to be passing through Ujjain, Kurukṣetra and the north-pole[6]. Ujjain has been famous from times immemorial (Kālidāsa describes this Ujjain in his famous kāvya *Meghasandeśa*). It lies on the meridian 75° east of Greenwich. The latitude and longitude of Laṅkā are therefore 0°, 75° respectively. Since this spot is now deep in the Indian ocean, it is not possible to know, whether this is a mythological or imaginary place or whether the ocean was not there in the distant past (geology says that the ocean also has changing beds). Even today, it is noteworthy that the *pañcāṅga*-computers calculate the mean positions of the planets for this Laṅkā-meridian and thereafter effect the correction called *deśāntara* to obtain the mean positions for their respective places on any day. The *Sūrya-siddhānta* also directs us to calculate the mean positions first for Laṅkā

* However, it may be added that if Jean's theory about the formation of the planetary system were correct, all the planets should have been in a line at the time of their formation.

at midnight.[7] It also says that the week-day begins at the mid-night of this Laṅkā-meridian.[8] Āryabhaṭa, however, directed that the mean longitudes be computed either for the midnight or for the Sun-rise at Laṅkā. On this count, Brahmagupta criticized Āryabhaṭa for giving two systems.

Since computations of the mean planetary positions taking their data at the beginning of the *Kaliyuga,* is very laborious, later on *karaṇa-granthas* came to be written, which may be called manuals for the purpose of easy computation, giving us the mean positions at their respective days. Astronomical treatises which prescribe computation from the beginning of the *Kalpa,* the extrapolated date, when all the planets, their orbital nodes and aphelia all were supposed to have zero longitude are called *siddhāntas;* those which prescribe computation from the beginning of *Kaliyuga* are called *tantras,* whereas those works of recent origin (these are hundred-fold in number) which prescribe computation from their recent respective dates are known as *karaṇa granthas.* The *karaṇa grantha tithi-cakra,* based on *Sūrya-siddhānta,* which has been in vogue in the Andhra Pradesh for the last 571 years, was written by one Narasiṃha, who belonged to a village named Naupurī or Vadapalli in the Godāvary delta called Konasīma. He prescribes computations of the mean positions from the noon of the first day of the 1333 Śālivāhana Śaka (A.D. 1411), as per the *Cāndramāna* or luni-solar reckoning. It is interesting to note his beginning verses which report that one Mallikārjuna wrote a work called *Tithi-cakra* long before him, probably prior to A.D. 1000 (even before Bhāskara's time) which had been in vogue, but got obsolete. Hence, he said that he was writing a fresh *karaṇagrantha,* bringing it into accord with the *Sūrya-siddhānta.* Since our present *Sūrya-siddhānta,* which is now very popular, was not this popular, about A.D. 1000 (for otherwise Śrīpati and Bhāskara would not have written their monumental treatises) it may be surmised that, the *karaṇagrantha* of Mallikārjuna Sūri was not based on the present *Sūrya-siddhānta* but perhaps on some other Siddhānta possibly the *Sūrya-siddhānta* contained in Varāhamihira's *Pañca-siddhāntikā.* One word here about the so called "obsoleteness" of a *karaṇa-grantha* is not out of place here. Since *karaṇa granthas* are but manuals intended for easy computation, they can not but use approximations. These approximations go in course of time beyond the limits of negligibility, so that fresh *Karaṇa granthas* had got to be written again. An example here will clarify the case in point. We have what is called a leap year, in which we add a day to February, which means that we construe that the year consists of 365.25 days. This convention of the leap year therefore grows obsolete in course of time, so that we are obliged to make another convention that A.D. 2000 will be a leap year, but not 2100 A.D. or 2200 A.D. or 2300 A.D. though they are all divisible by four as per the leap-year convention. So is the case with respect to the obsoleteness of *Karaṇagranthas.* This is all by the bye.

To compute the mean planetary positions say at the noon of the first day of this Śālivāhana Śaka year 1904 as per the *cāndramāna* we have to note here that the major Hindu astronomical works used Śālivāhana Śaka, not Vikrama Śaka, which is in vogue in north India, and also used *cāndramāna,* not the *sauramāna,* which is in vogue in Tamil Nādu, Kerala etc. By the major siddhāntas, we mean (1) *Āryabhaṭīyam*

(2) *Pañcasiddhāntikā* of Varāhamihira, (3) Lalla's *Śiṣyadhīvṛddhidam*, (4) *Brāhma-sphuṭa-siddhānta*, (5) Śrīpati's *Siddhānta-śekhara*, and (6) Bhāskara's *Siddhānta-śiromaṇi*. In between Brahmagupta and Śrīpati, there was however another great astronomer named Muñjāla (A.D. 932), whom Bhāskara quotes as having given the correct rate of secular precession not the libration theory postulated in the *Sūrya-Siddhānta*.

All the above-cited siddhānta treatises give the following procedure to obtain the mean positions of the planets at any time. Let us illustrate the procedure basing on Narasiṃha's *Tithi-cakra* cited above, which gave the mean planetary positions on the 1st day of the *cāndramāna* Śaka year of 1333 (i.e. A.D. 1411). On this behalf, we have got to calculate the number of days elapsed after the above date up to the 1st day of this Śālivāhana year 1904 (A.D. 1982) as per *cāndramāna*.

I step

```
 1904
-1333
 ----
  571
×  12
 ----
 6852
    0   months elapsed during this Śaka year of 1904
 ----
 6852   months elapsed after 1333 Śaka year.
```

II step—To calculate the *adhikamāsas*, which occurred in between 1333 and 1904 Śaka years.

```
  6852                248)20556(82
×    3                          220 remainder
 -----
 20556
+   82
+   15
 -----
98)20653(210   Required number of adhikamāsas
        73     remainder
```

III step—To find the *tithis* elapsed from the Ist day of 1333 Śaka to the first day of 1904 Śaka year.

```
   6852   solar months
+   210   adhikamāsas
 ------
   7062   cāndramāsas
×    30
 ------
 211860   tithis
+     1   first tithi of 1904 Śaka year
 ------
 211861
```

IV step—To compute what are called *kṣayāhas*

708)211861(299
69 remainder

211861
+ 299

64)212160(3315 No. of *kṣayāhas.*
0 remainder

V step—To obtain the number of civil days or what are called *sāvanāhas* in between the Ist days of 1333 Śaka year and 1904 Śaka year

212161
— 3315 *kṣayāhas* to be subtracted from *tithis.*

208846

VI step—To ascertain the truth, divide by 7, and seek the remainder to fix the week-day.

7)208846(29792
2 remainder

But we are directed to count from Wednesday, which was the Ist day of 1333 Śaka year, which means that we have by the above remainder 2, Thursday. But since the Ist day of 1904 Śaka *cāndramāna* year happens to be Friday, we are permitted to add (or subtract) one only, no more to adjust to the week day. So, we add one to obtain Friday, so that the number of civil days, which have elapsed in between the two dates is 208846+1=208847.

NOTE :—In the proximity of the occurrence of another *adhikamāsa,* also, we are asked to be careful. If the number of *adhikamāsas* does not correspond to actuality, in the face of the *adhikamāsa,* we are permitted to add one and get the correct number of *adhikamāsas.*

One may wonder, what the rationale behind this procedure is to obtain the number of civil days which have elapsed between the two epochs. Today, we are aware of the Julian days, so that had we known the Julian days corresponding to the two epochs, this laborious process could have been avoided. But since the finding of the Julian day of the Ist day of 1333 Śālivāhana Śaka *cāndramāna* year is equally difficult, we cannot but follow the procedure indicated above. Had the two epochs been solar, we could have easily multiplied the number of years 571 by 365.25875 as per Āryabhaṭa's year or by 365.25844 as per Brahmagupta, Śrīpati and Bhāskara. Today we know the lengths of the sidereal year to be 365.256362 and the tropical year to be 365.2421955.

The rationale behind the computation of the number of civil days between two epochs is a long story, but we have got to note it, to understand our Hindu astronomical works. We know that the concept of the lapse of an year arises to a man of the world by recurrence of seasons which happens in accordance with the tropical year. Similarly the concept of the lapse of a month in olden days, when the modern system of dating was not in vogue, arose to those men, out of the recurrence of full-moon days, or new-moon days. It will be noted here that even today the illiterate men of our country in villages go by these full-moon days or new-moon days only, to understand the lapse of the month. The interval between the moments of two consecutive full-moons or two consecutive new moons, is called a lunation. It need not be added that the moment of full-moon is when the Moon has exactly an elongation of 180° and that the moment of new moon, when the elongtion of the Moon is 360°, i.e. when the Moon is in conjunction with the Sun. The correct length of a lunation as per Modern astronomy is 29.5305881 days. It is noteworthy that this length is given to be 29.5305879 in *Sūrya-Siddhānta* which is correct to six places of decimals. On the other hand, the length of a lunation as given by Brahmagupta, and followed (unwittingly) by Śrīpati and Bhāskara, is not this correct. This means that the author of *Sūryasiddhānta* did improve upon Brahmagupta's data, though he was not such a mathematician as Brahmagupta. Śrīpati and Bhāskara who were good mathematicians, were tempted to follow Brahmagupta, because the latter was a great mathematician. It will be noted further, that the *Sūrya-siddhānta,* apart from its more correct data, betrays lack of depth in other matters, where Bhāskara exhibited his genius. This is by the bye.

Coming to the point, the problem on hand is to find the number of civil days which had elapsed between the first day of 1333 Śaka year and that of the present 1904 Śaka year. In the first place, we note that the number of years which had elapsed is 571. But are these purely solar years? No, because, though the intercalation of *adhikamāsas,* went on effecting an accordance between the *cāndramāna* and *sauramāna,* after the last intercalation, again there has been a divergence between the two. Hence the number 571 is not exactly equal to the number of solar years, but a little different. Nonetheless we are asked to multiply the number 571 by 12 to get the number of solar months. Thus we get the number 6852. Then we are asked to add the number of lunar months to this. Since, we have chosen the epoch namely the first day of the Śaka year 1904, no lunations have elapsed. Suppose we add a few lunar months to the number 6852. Does this not mean a jumbling of matters? No; because, solarity so to say, has been secured upto the last intercalation of an *adhikamāsa*, and construing a few lunations added to a big number of solar months does not affect out procedure intended to calculate the number of *adhikamāsas*. Now the procedure given by Narasiṃha to obtain the number of *adhikamāsas* amounts to saying that the *adhikamāsas* in 6852 months (mostly solar but just a few lunar) $= \dfrac{3\times 6852(1+\frac{1}{248})+15}{98}$. Here evidently $\frac{15}{98}$ is the fraction of *adhikamāsa* accrued upto the first day of 1333 Śaka year, so that $6852\left(3+\dfrac{3}{248}\right)$ gives us the number of *adhikamāsas* in 6852 solar months. The rationale of this may be given as follows. Our ancient Hindu astronomers were conversant with continued fractions as will be seen from Bhâskara'. *Bijagaṇita.* Hence taking the data from *Sūrya-siddhānta* which says that there are 1593336 *adhikamāsas* in 51840000 solar months or what is the same 66389 *adhikamāsas* in 2160000 solar months converting $\frac{66389}{2160000}$ into a continued fraction, we have $\frac{1}{32}\,\frac{1}{+1}\,\frac{1}{+1}\,\frac{1}{+8}\,\frac{1}{+1}\,\frac{1}{+1}\,\frac{1}{+5}$ so that the convergents are $\frac{1}{32}, \frac{1}{38}, \frac{2}{65}, \frac{13}{425}, \frac{15}{488}, \frac{25}{911}.$ Since we know the convergents are alternately greater and smaller than the actual fraction, which is converted into continued fraction, what the ancient Hindu astronomers did was to add the numerators to form the numerator and add the denominators to form the denominator of two successive convergents, which will give a closer approximation to the actual fraction. Thus framing a convergent out of the convergents $\frac{1}{38}$ and $\frac{2}{65}$ we have $\frac{3}{98}$ nearer to the fraction. But by taking $\frac{3}{98}$ as the convergent, the error committed was sought to be rectified. Now going back to the original fraction $\frac{66389}{2160000}$ and considering the difference between it and $\frac{3}{98}$ taken, we have $\frac{66389}{2160000} - \frac{3}{98} = \frac{26122}{98 \times 2160000}$

$$\text{i.e. } \frac{68389}{2160000} = \frac{3}{98} + \frac{26122}{98 \times 2160000} = \frac{3}{98} + \frac{3}{\frac{98 \times 6480000}{26122}}$$

$$= \frac{3}{98} + \frac{3}{98 \times 248} = \frac{3}{98}\left(1 + \frac{1}{248}\right)$$ as given. Thus getting the number of *adhikamāsas,* we are directed to add them, i.e. 210 to the number of elapsed months 6852, so that the lunations elapsed are 7062. In this context, there is one more idea. As per the convention that a lunation which does not contain a *saṃkrānti* (transit of the Sun from one *Rāśi* to another *Rāśi,* where the *Rāśis* are Meṣa, Vṛṣabha etc) it so happens that occasionally two *saṃkrāntis* may occur in the three longer lunar months like Kārtika, Mārgaśīrṣa and Pauṣa. Another convention is therefore necessitated and the convention now made is, that the lunar month is called a *kṣayamāsa,* when two *saṃkrāntis* occur in a lunation. When such a *kṣaya* month occurs, two lunar months are deemed to lapse away in one lunar month. For example let Pauṣa be the lunar month in which two *saṃkrāntis* occur (this is indeed the case during this 1904 Śaka year). Then by convention we are directed to call this *kṣaya* month as *yugalibhūta-pauṣa-māgha māsa,* which means that the lunar month Māgha is also deemed to lapse away simultaneously. In the forenoon Pauṣa *māsa* is supposed to be current and in the afternoon Māgha for ritualistic purposes. In other words, Pauṣa will be running from the previous mid-night up to the noon and Māgha *māsa* current from the noon to the next mid-night. This *kṣaya māsa* is otherwise called *aṃhaspatimāsa.* Then the question arises whether we should not subtract this *kṣaya māsa,* when we are directed to add the *adhikamāsas* to the solar months. The answer is not necessary.' The

reason is, that when such a *kṣaya* month occurs it is preceded and followed by an *adhikamāsa* during the course of one solar year. This happens so because, when the *cāndramāna* overtakes the *sauramāna,* by one lunation, an *adhikamāsa* must occur. In this context Bhāskara says, that when the accrued difference between the luni-solar reckoning which is called *adhimāsa-śeṣa,* at the beginning of the Śaka year amounts to 21 days, during the first five lunations following, the luni-solar reckoning is bound to overtake the solar by the remaining nine days. So that the *cāndramāna* will have overtaken the *sauramāna* by a lunation. Thus during the sixth month, i.e. Bhādrapada there will not be a *saṃkrānti.* Hence by convention it is called an *adhikamāsa.* Under such circumstance, a *saṃkrānti* will have occurred on the last day of Śrāvaṇa, and the next *saṃkrānti* will have occurred on the first day of Āśvayuja. The month in between namely Bhādrapada does not therefore carry a *saṃkrānti* and as such is called an *adhikamāsa.* But the subsequent months, if not Āśvayuja, the next three lunar months being longer than the corresponding solar months, because then, the Sun, being near this perigee, will be moving fast, and will be covering the 30° of the solar month in lesser time, which means that the solar month will be shorter than the lunar month. Then since a *saṃkrānti* has taken place just on the first day of Āśvayuja, then either in Kārtika, or in Mārgaśīrṣa, or at the latest in Pauṣa, which three lunar months are longer than the corresponding solar months, two *saṃkrāntis* are bound to occur (Ref. Fig. below) :

Srāvaṇa α Bhādrapada β Āsvayuja γ Kārtika δ Mārgaśira ϵ Pauṣa ϕ Māgha

A B C D E F G

Note that BC is Bhādrapada, does not carry a *saṃkrānti*; hence Bhādrapada is made and *adhikamāsa.* AB=Srāvaṇa, BC=Bhādrapada, CD=Āsvayuja, DE= Kārtika, EF=Mārgaśira, FG = Pauṣa

α, β, γ, δ, ϵ, ϕ are *saṃkrāntis.* Note that two *saṃkrāntis* occur in Pauṣa. There is the possibility that these two *saṃkrāntis* may occur even in Mārgaśira or even in Kārtika. Again within one year, since two *saṃkrāntis* have occured in Pauṣa, and the latter lunations Māgha, Phālguna, Caitra, Vaiśākha, Jyeṣtha and Āsāḍha are at the latest. Śrāvaṇa are gradually smaller than the corresponding solar months, one of these latter lunations stand to loose a *saṃkrānti,* and as such becomes another *adhikamāsa.*

Since a convention was made that if a lunar month did not contain a *saṃkrānti,* it should be called an *adhikamāsa,* another convention was called for, when two *saṃkrāntis* occurred in one lunar month. The convention made was, that such a month be called a *kṣayamāsa.* Now, the *adhikamāsa* which occurred in Bhādrapada, occurred so because the *cāndramāna* has overtaken the *sauramāna* by one lunation. But on account of the second convention, a lunar month is now suppressed in the name of a *kṣayamāsa.* So the lunar month gained is lost now. But as per calculations, the *cāndramāna* reckoning did gain one lunar month over the solar. So another subsequent lunar month is bound to be devoid of a *saṃkrānti,* so that, we must have an *adhikamāsa* again occurring after the *kṣayamāsa.* The former *adhikamāsa* which occurs in Bhādrapada is called *saṃsarpa māsa* in contradistinction to the appellation *adhikamāsa,* and the second *adhikamāsa* occurring after the *kṣayamāsa* is termed as the regular *adhikamāsa.* In this context, it is worth noting that the *Kṛṣṇa Yajurveda* speaks of this phenomenon of the *kṣayamāsa,* and the preceding *adhikamāsa** signifying that the process of intercalation is as old as the Veda. This should disabuse the minds of those that Hindu astronomy was crude by the time of the *Vedāṅga Jyotiṣa* (in this context the reader is referred to the introduction of the author in his English commentary on Bhāskara's *Siddhānta-śiromaṇi* published by the Kendriya Samskrit Vidyapitha, Tirupati).

The mathematics in the context of these *adhika*-and *kṣayamāsas* runs as follows. We take the data from Bhāskara. He says that 1,593,300,000 *adhikamāsas* occur in a *kalpa* of 4,320,000,000 years or what is the same 5311 *adhikamāsas* in 14400 solar months. Converting $\frac{14400}{5311}$ into a continued fraction we have

$$2\frac{1}{+1}\;\frac{1}{+2}\;\frac{1}{+2}\;\frac{1}{+6}\;\frac{1}{+1}\;\frac{1}{+1}\;\frac{1}{+7}\;\frac{1}{+8}\;\frac{1}{+2}$$

The successive convergents are

$$\frac{2}{1},\ \frac{1}{3},\ \frac{3}{8},\ \frac{19}{7},\ \frac{122}{55},\ \frac{141}{62}.$$

Let us see what these convergents signify. The convergent $\frac{19}{7}$ means that roughly speaking there are 7 *adhikamāsas* during 19 years. This ratio was adopted in the *Romaka siddhānta* of *Pañca-siddhāntikā.* It means that $19\times 12=228$ solar months are equal to 235 lunations. The Metonic cycle described in modern astronomy is based on this equivalence. Given the English date we can calculate the *tithi* as per this equivalence very approximately. In the verse 7 under *adhimāsa nirṇaya, Gaṇitādhyāya,* Bhāskara mentions that the *kṣaya* months may occur in 19 years or 122 years or 141 years. These are seen by the next convergents cited above. The interval between a new-moon and the next *saṃkrānti* goes by the name *śuddhi.* It is the interval gained by the *cāndramāna* over the *sauramāna.* Hence if this year, this *śuddhi* happens to be 21 days, as mentioned by Bhāskara, there is the possibility of the same amount of *śuddhi* occurring after 19 years or 122 years or 141 years respectively more closely. In this context, it will be noted that since Śrīpati was the first astronomer so far as we know, who made a mention of *kṣayamāsa,* some modern interpreters construed that the observance of *kṣayamāsa* came into vogue only after Śrīpati. This is not correct, in the light of the *Kṛṣṇa-yajurveda* text cited. Observance of the *kṣayamāsa* and *adhikamāsa* is therefore as old as the Veda. Simply because *Vedāṅga-Jyotiṣa* gives us a crude presentation of this phenomenon, most modern interpreters are prone to construe, that astronomy was that crude in India in about 1000 B.C. It must be remembered that substandard texts are being written even today, when high

*The fourteen lunations occurring in an year which carries a *kṣayamāsa* are enumerated in 1-4-15 in *Kṛṣṇa Yajurveda Saṃhita* as follows :
Madhu = Caitra, *Mādhaba* = Vaiśākha, *Śukra*=Jyeṣṭha, *Śuci*=Āṣāḍha, *Nabhas* = Śrāvaṇa, *Nabhasya* = Bhādrapada, *Iṣa* = Aśvayuja, *Ūrja* = Kārtika, *Sahaḥ* = Mārgaśira, *Sahasya* = Pauṣa, *Tapa* = Māgha, *Tapasya* = Phālguna, *Saṃsarpa* = the *adhikamāsa* preceding the *kṣayamāsa*, *Aṃhasputi* = *Kṣayamāsa*

standard books are also being written. In this context the reader is referred to the Report of the Calendar Reform Committee (published by the Government of India in 1955) page 250, where the following para occurs: "It will be seen from the above table that according to *Sūryasiddhānta* calculations, one *kṣaya* month occurs on average in 63 years. In rare cases, they occur after 46, 65, 76, 122 years." The figures 63, 46, 76 are wrong, for the following reason.* *Sūrya-Siddhānta* gives that 1,593,336 *adhikamāsas* occur in 4,320,000 solar years or what is the same 66389 *adhikamāsas* occur in 180000 solar years. Converting

$\frac{180000}{66389}$ into a continued fraction we have

$$2 \; \frac{1}{+1} \; \frac{1}{+2} \; \frac{1}{+2} \; \frac{1}{+2} \; \frac{1}{+6} \; \frac{1}{+2} \; \frac{1}{+1}$$ so that the convergents are

$$\frac{2}{1}, \frac{3}{1}, \frac{8}{3}, \frac{19}{7}, \frac{122}{45}, \frac{263}{97}, \frac{385}{142}.$$

Hence a *kṣayamāsa* may occur in 19 years or 122 years or combining the convergents $\frac{19}{7}$ and $\frac{122}{45}$, adding the numerators to form the numerator and the denominators to form the denominator since such a convergent lies nearer the original fraction.

We have $\frac{141}{52}$, so that as Bhāskara said, a *kṣaya* month may occur even in 141 years. On no account, could a *kṣayamāsa* occur in 46,63,65, and 76 years as reported by the Report of the Calendar Reform Committee. In this context, it is interesting to note that Gaṇeśa Daivajña spurred by Bhāskara's prediction of the occurrence of *kṣayamāsa* in the Śaka years 1115, 1256, 1378 (Bhāskara also said that a *kṣayamāsa* had occurred in 974 Śaka) also computed the years in which the *kṣayamāsa* would occur subsequent to his time. The years given by him as per *Sūrya-siddhānta* are 1462, 1603, 1744, 1885, 2167, 2232, 2372, 2392, 2524, 2533, 2655, 2674, 2796, 2815, and that as per *Āryabhaṭīyam* 1482, 1793, 1904, 2129, 2186, 2251 all Śaka years. It will be noted that in 1885 was observed a *kṣayamāsa.*

After the lapse of 19 years as mentioned by Bhāskara during this 1904 Śaka year also, a *kṣayamāsa* is going to occur in the ensuing Pauṣa month. Gaṇeśa predicted this as per *Āryabhaṭīyam.*

In the above calculation, since one *adhikamāsa* occurs in 32½ solar months, and a lunation and a solar month do not differ much from each other, construing that the months obtained namely 6852 as purely solar though they are not so, for *the purposes of calculating the adhikamāsa* is justifiable. After all, it must be noted that out of the 6852 months, there will be just a few lunar months mixed up, which therefore does not matter.

*The author of this paper makes bold to point out that the Report betrays in many places lack of touch with the *Siddhānta* texts.

After thus getting the number of *adhikamāsas,* which have elapsed we are asked to add them to the months 6852 to get the lunar months, which have elapsed. Now we are asked to multiply these lunar months by 30 to get the *tithis* elapsed. Since the epoch we have chosen, is the *first* day of this Śaka Year 1904, we are asked to add one to this, so that we have 211861 as the *tithis,* which have elapsed. The next step is to obtain what are called *kṣayāhas,* which have got to be subtracted from the *tithis* to give us the elapsed number of civil days from the 1st day of 1333 Śaka.

Let us see now what these *kṣayāhas* are, and why we are asked to subtract them from the *tithis* to get the number of civil days. It was already mentioned that a lunation contains 29.5305879 days as per *Sūrya-siddhānta,* i.e. one *tithi* corresponds nearly to one day, but falls short of a day by a fraction .47 roughly or .4694121 correctly. Hence during a period of 30 *tithis* there is a shortage of .4694121 days which means roughly that one *tithi* will be lost approximately in 64 days. There is a convention that a day should be dated by the *tithi* which is current on that day at sunrise. Suppose today a particular *tithi* daśamī is current at sunrise, but lasts just for a few moments. Then the next *tithi* being shorter than a day may lapse away. Thus tomorrow will be dated not by ekādāsī but by dvadasī. Thus we have lost the *tithi* ekādaśī, which is therefore called a *kṣayāha.* One such *kṣayāha* occurs in a period of 64 *tithis.* To compute these *kṣayāhas* accurately the procedure given by Narasimha in the fourth step as cited before, is to divide the *tithis* obtained namely 211861 by 708, and add the quotient to 211861 and then divide it by 64 and take the quotient. The rationale here is as follows. As per *Sūrya-siddhānta* there are 1603000080 *tithis* and 25082252 *kṣāyahas* in a *Mahāyuga.* Hence converting $\frac{1603000080}{25082252}$ into a continued fraction, we have the convergents $\frac{1}{63}, \frac{1}{64}, \frac{11}{703}, \frac{122}{7797}$ etc., We see herefrom that $\frac{1}{64}$ is one convergent and $\frac{122}{7797}$ is a convergent close to the fraction than $\frac{1}{64}$. If x be the number of *tithis,* $\frac{x \times 122}{7797}$ gives the number of *kṣayāhas;* but to avoid multiplication and division by large numbers, the short-cut adopted by Hindu astronomers is to find the difference of the convergents

$$\frac{122}{7797} - \frac{1}{64} = \frac{11}{64 \times 7797}$$

$$\therefore \frac{122}{7797} = \frac{1}{64} + \frac{11}{64 \times 7797} = \frac{1}{64}\left(1+\frac{1}{\frac{7797}{11}}\right)\frac{1}{64} = \left(1+\frac{1}{708}\right)$$

as given by Narasimha. This process virtually amounts to taking the convergent $\frac{122}{7797}$ but the procedure is rendered easier. Thus getting the number of *kṣayāhas* from the *tithis* we have the required number of civil days. We must not be satisfied by the number so got. So, we are asked to divide by 7 and get the remainder. The remainder got is two. But the 1st day of 1333 Śaka year was Wednesday. So by this remainder, we get Thursday. Since the first day of 1904 Śaka year is Friday, we have to add one to adjust the *ahargaṇa* to the week day. Adding or subtracting one is permissible because we are introducing approximations.

Thus far we have got the number of civil days which have elapsed from the 1st day of 1333 Śaka year to the 1st day of this Śaka year 1904. Since we are given the positions of the planets on the 1st day of 1333 Śaka year by Narasimha, the next step is to get the motion of the planets during these 208847 days and add these motions to their positions as given on the 1st day of 1333 Śaka year.

However, we must note that the positions of the planets as given by Narasimha on the 1st day of 1333 Śaka year, are those pertaining to the Laṅkā meridian. For a meridian different from this a correction called *deśāntara* is to be effected in those positions either before computing the mean motion due to 208847 days and adding it to the mean positions given on the 1st day of 1333 Śaka year or after getting the mean positions as on the 1st day of 1904 Śaka year. It does not matter.

The mean motion of a particular planet during an *ahargaṇa* say x (here $x=208847$) we have to proceed as follows. The *Sūrya-siddhānta* says in verse 55 Chapter 1, $x \times R$ gives the motion of the planet in x days, where R is the number of its sidereal revolutions in a *Mahāyuga* and D is the number of days in a *Mahāyuga*. Let us take the case of Saturn for example. *Sūrya-siddhānta* says that Saturn makes 146568 sidereal revolutions during the days of the *Mahāyuga* namely 157791828. The arc moved by Saturn in our *ahargaṇa* of 211861 days is hence $\frac{211861 \times 146568}{157791828}$

This multiplication and division is laborious, so that easy procedures are generally given in the *karaṇagranthas*. In the case of the Moon, $R=57753336$ so that it is indeed more laborious to effect the multiplication. Anyway the procedure is clear to obtain the mean motion of any planet from this formula taking the respective values of R in the case of each planet. Adding the mean motion of each planet during the course of the *ahargaṇa* calculated, to its position as given in the *Karaṇagrantha*, we get the mean planet for the midnight of Laṅkā as per *Sūrya-siddhānta*. The number of revolutions can be clearly omitted, to note the planetary position on the zodiac, even as time is noted from a clock, ignoring the number of revolutions of the hour hand.

Hindu astronomers gave the inclinations of the planetary orbits to the ecliptic. Yet, since these inclinations are generally small, no account is taken with respect to the planets except in the case of the Moon, when a lunar eclipse or solar eclipse is to be computed. In this case the latitude of the Moon will come into the picture.

Since the mean planetary positions are obtained for the Laṅkā meridian, we effect the *deśāntara* correction to obtain their positions for the midnight of the particular place at which its position is required. The next step will be therefore to compute the planetary position for the moment of the sun-rise at the place. This entails finding the lengths of the day and night for the required moment.

NOTE ON THE CALCULATION OF AHARGANA

The method of calculation of *ahargaṇa* from the beginning of *Kaliyuga* upto 28-7-1984, i.e. *Āsāḍha Bahula Amāvāsyā* of this *Raktākṣi Samvatsara* as per *Siddhānta Śiromaṇi* of Bhāskara II is explained as follows :

The number of years elapsed of *Kaliyuga* 5085

$$\begin{array}{r} 5085 \\ 12 \\ \hline 61020 \\ 4 \\ \hline 61024 \end{array}$$

4 Add lunar months elapsed from Caitra

Then as per *Siddhānta Śiromaṇi,* the number of *adhikamāsas* in the *Kaliyuga* whose duration is 4,32000 years, i.e. 5184000 solar months is 159330.
That being so, the *adhikamāsas* which have elapsed from the beginning of Kali are as per rule of three

Solar months		*Adhikamāsas*
5184000	—	159330
61024	—	?

$$= \frac{159330 \times 61024}{5184000} = 1876 \text{ rounding up the fraction } \frac{92}{162} \text{ to unity.}$$

Add these *adhikamāsas* to the elapsed solar months 61024 (construing the four *cāndramāsas* as solar does not affect our calculation)

$$\begin{array}{r} 61024 \\ 1876 \\ \hline 62900 \\ 30 \\ \hline 18876000 \end{array}$$

62900 —*cāndramāsas*

Multiply by 30 to get *tithis.* Elapsed *tithis* from the beginning of Kali.

Now as per the same *Siddhānta Śiromaṇi,* the number of *kṣayāhas* during the duration of the *Kaliyuga* is 2508255, which *Kaliyuga* has 160299000 *tithis.* Hence again by rule of three, if during 1602999000 *tithis,* we have 2508255 *kṣayāhas,* what number of *kṣayāhas* will be there during the course of 1887000 *tithis?*

The answer is

$$\frac{2508255 \times 1887000}{1602999000} = 29526$$

ignoring the small remainder.
Hence subtracting these *tithikṣayas* from the *tithis* 1887000 we have 1857474. Dividing by 7, the remainder is 3.

We are told that this *Kaliyuga* started on Friday, so that the day should be Sunday. But actually the day on 28-7-1984 was Saturday. Since we are given the option to add or subtract unity if the *ahargaṇa* does not accord with the name of the day, subtracting unity from the above, we have the required *ahargaṇa* as 1857473.

THE METHOD OF FINDING THE LENGTH OF THE DAY

In modern astronomy we have the formula $\cos h = -\tan\phi\tan\delta$ to obtain the hour-angle h of the rising Sun for a place of latitude ϕ when the Sun has declination δ. When the Sun is in the northern hemisphere, δ is positive. So far as India is concerned ϕ is positive, so that when δ is positive, since $\cos h$ is negative h will be greater than 90°.

Let $h = 90° + \theta$ so that $\cos(90° + \theta) = -\operatorname{Sin}\theta = -\tan\phi\tan\delta$.

Hence $\sin\theta = \tan\phi\tan\delta$ .. .. .. (1)

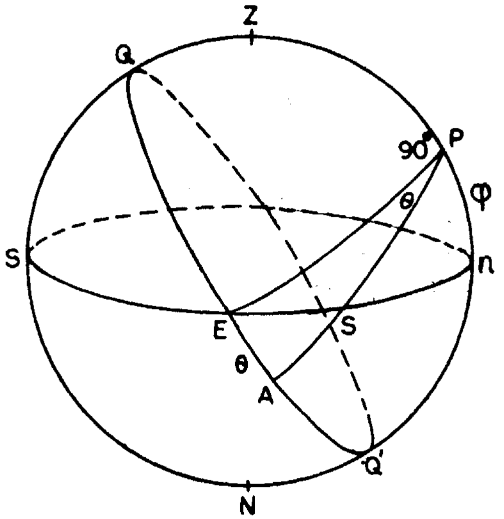

Fig. 6.1 determination of the length of the day

This θ is the arc EA in Fig. 6.1 where S is the rising Sun (when δ is north) and is called *cara* in Hindu astronomy. For a given place ϕ the latitude does not change, whereas δ the declination of the Sun changes from day to day. In Hindu astronomy instead of specifying ϕ in degrees, what is called *palabhā* is used. This *palabhā* is the shadow of a vertical gnomon divided into 12 units called *aṅgulas*, at noon on the equinoctial day. On the equinoctial day at noon the Sun is that Q which is the point of intersection of the celestial equator with the meridian. From Fig. 6.2 $\tan\phi = \frac{S}{12}$. This *palabha* is also given in the same units of *aṅgulas*, so that if

$s = 4\tan\phi = \frac{4}{12} = \frac{1}{3} = \cdot 3333$ so that $\phi = 18°\ 25'$.

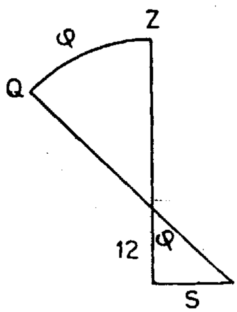

Fig. 6.2 determination of the *palabhā*

Since the *cara* is added to 90°, when δ is positive and subtracted from 90° when δ is negative to obtain the rising hour angle of the Sun, which gives the length of the day, the question resolves itself into our finding this *cara*, for varying declinations only since ϕ is constant for a place. We have the formula sin $\delta =$ sin λ sin ω where sin ω, the obliquity of the ecliptic, is again constant. Since ω was taken to be 24° by the Hindu astronomers, the formula is therefore

$$\text{Sin } \delta = \sin 24° \sin \lambda \qquad \ldots \qquad (2)$$

But λ is measured along the ecliptic from the equinoctial point γ. The Hindu longitude of the Sun is measured from the first point of Aświnī, and the positive arc in between γ and the Hindu Zero point gives what are called *ayanāṃśas*. The Hindu longitude is called the *nirayana* longitude and that measured from γ is called the *sāyana* longitude. Hence before we find δ from formula (2), we have to add the *ayanāṃsas* to the Hindu longitude of the Sun, to get λ.

Hindu astronomy uses what are called *carakhaṇḍas,* taking the equinoctial shadow to be one *aṅgula,* i.e. taking tan $\phi = \frac{1}{12}$. Since sin $\theta =$ tan δ, tan ϕ, sin $\theta =$ tan $\delta \times \frac{1}{12}$ for the *palabhā* of one *aṅgula* and sin $\theta =$ tan $\delta \times \frac{s}{12}$ for a *palabhā* of *s aṅgulas.* Taking $\omega = 24°$, (we may rectify the results taking the present value of ω) following the same procedure for the values 30°, 60° and 90° for λ, have

$$\sin \delta_1 = \sin 24° \sin 30° \qquad \ldots \qquad (3)$$
$$\sin \delta_2 = \sin 24° \sin 60° \qquad \ldots \qquad (4)$$

δ_3 is of course equal to 24°

Taking logarithms

Log sin $\delta_1 = 9.6990 + 9.6093 = 9.3083$
so that $\delta_1 = 11°44'$.
Log sin $\delta_2 = 9.9375 + 9.6093 = 9.5468$
so that $\delta_2 = 20°38'$
From (1) sin $\theta =$ tan δ tan ϕ

$\therefore \sin \theta_1 = \dfrac{\tan 11°\,44'}{12}$, for tan $\phi = \dfrac{1}{12}$ for a *palabhā* of 1 *aṅgula,* and δ_1 corresponding to $\lambda = 30°$, being 11°44′.

Similarly, $\sin \theta_2 = \dfrac{\tan 20°\,38'}{12}$, θ_2 corresponding to $\lambda = 60°$ and

$\sin \theta_3 = \dfrac{\tan 24}{12}$ corresponding to $\lambda = 90°$.

Taking logarithms and proceeding as before, $\theta_1 = 59'$, $\theta_2 = 1°48'$ and $\theta_3 = 2°8'$.

Converting degrees and minutes at the rate of 6′ per *vināḍī* (1 *vināḍi* = 24″ of time). We get :

$\theta_1 = 10$ *vināḍis,* $\theta_2 = 18$ *vināḍis* and $\theta_3 = 21\frac{1}{3}$ *vināḍis.* In other words $\theta_1 = 10$; $\theta_2 - \theta_1 = 8$ and $\theta_3 - \theta_2 = 3\frac{1}{3}$. These 10, 8, and $3\frac{1}{3}$ are called *carakhaṇḍas* corresponding to λ varying between 0° and 30° and varying between 30° and 60° and again between 60° and 90°. For a given place of equinoctial shadow *s aṅgulas,* we have *cara* segments 10, 8 and $2\frac{1}{3}$ multiplied by s''. This proportionality is approximate because the formulae $\sin\theta_1 = \dfrac{\tan 11°44'}{12}$ etc, have not θ_1, θ_2, θ_3 on the left-hand side but $\sin\theta_1$, $\sin\theta_2$, $\sin\theta_3$.

Bhāskara gives in verse 49 of *spaṣṭādhikāra,* that if s be three *aṅgulas,* then the *carakhaṇḍas* are 30, 24, 10, in *vināḍis*. If it be required to find the *carakhaṇḍa* for $\lambda=44°$ (say) we are asked to proceed as we do with the interpolation formula given by him with respect to getting the values of sines in the name of *bhogyakhaṇḍa sphuṭikaraṇa.* It is worth-noting here that this *bhogyakhaṇḍa sphuṭikaraṇa* was originally given by Brahmagupta and this interpolation formula agrees with the quadratic interpolation formula given by Ball in his *Spherical Astronomy* on page 18 which has the form

$$Y = Yo + \frac{x}{h}\,(y_1 - y\,) + \frac{x\,(x - h)}{2h^2}\,(y_2 - 2y_1 + yo)$$

where y_0, y_1, y_2 are three consecutive values of the function of y, h is the constant difference of the arguments. Then for any argument which is greater by x than the first argument, but less than the third argument, the above formula gives the required function. The formal 'rule-of three' is a linear interpolation formula, whereas the *bhogyakhaṇḍasphuṭikaraṇa* is a quadratic interpolation formula.

Adding the *cara vināḍis* to 15 *nāḍis* when δ is positive and subtracting them from 15 *nāḍis* when δ is negative, we have what is called *dinārdha* or half the daytime. Double this gives the duration of the day-time. Subtracting the duration of day-time from 60 *nāḍis,* we get the duration of the night. Having got the duration of half the night, the mean planetary positions are to be rectified to get their positions at sunrise. The *deśāntara saṃskāra* may be applied now or prior to this procedure.

Epicyclic and Eccentric Theories and Planetary Corrections

Having obtained the mean planetary positions on a particular day at sunrise for any place, the next procedure is to rectify these mean positions by applying the necessary corrections called *saṃskāras,* to obtain their True positions at the place. The *saṃskāras* to be applied are of two kinds (1) *manda saṃskāra* (2) *śighra saṃskāra.* With respect to the Sun and Moon, the first *saṃskāra* suffices to obtain their true position,[9] where as, for the five planets called *Tārāgrahas* namely Mercury, Venus, Mars, Jupiter and Saturn both the *saṃskāras* are necessary. The reason for this is simple. The Moon goes in an allipse around the Earth; the Earth being in one focus, whereas the Sun goes relatively round the Earth, the Earth being in one focus (we say relatively because it is the Earth going round the Sun, in an ellipse, the Sun being in one focus as per Kepler's first law). The mean planetary positions signify that we have presumed that the planets are going in a circle, as a first approximation. The *manda-karma* is therefore intended to get the corresponding position in the ellipse from the position in the circle (though not stated so).

It will be noted however, that Hindu astronomical texts do not say that the planets are going round in ellipses. They say that they go in an epicycle, whose centre moves along the mean circular orbit from west to east. This theory is called the epicyclic theory. There is another theory called the Eccentric theory, which says that the planet goes in a circle whose centre is not the Earth but a different point other than the Earth. The distance of this point from the Earth, is said to be equal to the radius of the epicycle. We shall presently see how both the theories are identical.

In the first place, we shall consider the eccentric theory. Bhāskara says here "The centre of the celestial sphere coincides with the centre of the Earth. The circle in which the planet goes does not have its centre coinciding with the centre of the Earth." Hence, astronomers prescribed what is called *bhujaphalam* (otherwise called equation of centre in the case of *mandaphala*, whereas in the case of the five *tārāgrahas*, this *bhujaphala* besides connoting equation of centre or *mandaphala*, also stands for the correction required to reduce their heliocentric positions to the geocentric, as a correction to be made to obtain the true positions from the mean."[10]

Eccentric Theory

Let A be the centre of the Earth; let P be the mean planet construed to go round in a circle whose centre is A. (Fig. 6.3) Let B be the centre of the circle, in which the planet actually moves. It must be noted that the radii of these two circles are equal.

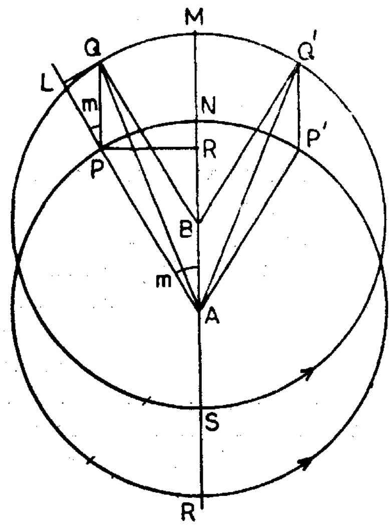

Fig. 6.3 Eccentric model for planetary manda

The circle (A) is the mean orbit, whereas the circle (B) is the true orbit. The mean planet moves on the mean orbit called by some as the deferent. The position of the true planet in the eccentric circle (B) namely Q will be such that PQ is parallel and equal to AB. In fact, PQ is drawn parallel to AB, and $AP = BQ$ by construction. By simple geometry we can prove that $PQ=AB$. Also we can see that when B is vertically above A, and when the radii of the two circles are equal, every corresponding point of the circle (A) will be vertically moved by the same distance AB. Thus $PQ=AB$ and they are also parallel. Join AQ to cut the circle (A) in T, which is said to represent the true planet.Evidently when the planet is at N in the mean orbit, the planet is at M, in the eccentric, which point is called the *mandocca*. When the planet is at R in the mean orbit the planet is at S in the eccentric. Thus M and S correspond to the apogee and perigee in the case of the *mandaphala* correction (otherwise called equation of centre). When the planet is at M in the eccentric, the position of the true planet coincides with N, the mean planet, so that the *mandaphala* correction is zero. Simlarly the true planet at R being in the same direction as AS, there also the *mandaphala* is zero. We know that the equation of centre at apogee and perigee becomes Zero similarly. When the mean planet is at P, since the true planet is at T, the correction *mandaphala* PT is negative. When the mean planet is at P' on the right, the true planet will be at T' so that the correction $P'\ T'$ is positive. The angle NAP is called the *mandakendra,* which corresponds to mean anomaly of modern astronomy. Thus when the mean anomaly lies between 0° to 180°, the equation of centre is negative, being equal to zero both when the mean anomaly is 0° and 180°. On the other hand, when the mean anomaly lies between 180° and 360°, the equation of centre is positive. In modern astronomy, it must be noted that m the mean anomaly is measured from the perigee, so that we have the reverse sign. Bhāskara and Śrīpati construe the mean anomaly in the case of the equation of centre as planet's longitude minus the longitude of the *mandocca* or apogee. In the case of what is called *śighraphala,* intended to get the geocentric planet from the heliocentric, they construe the longitude of what is called *śighrocca* minus the longitude of the planet as the *kendra*. (or mean anomaly). To use the word mean anomaly for the *kendra* here is rather awkward in modern Astronomy, but if we construe the *kendra* as the argument from which, we calculate the *śighraphala,* it will be alright. In the *Sūrya-siddhānta* however, for the sake of uniformity the *kendra* or argument from which either *mandaphala* or *śighraphala* is to be computed is taken as the longitude of the *ucca* (*mandocca* or *sīghrocca*) minus the longitude of the planet. In this case sin m will be evidently positive between the values 0° to 180° of the *kendra* and negative between 180° to 360°. It is only a matter of convention, about which, we need not bother. However, the definition of *kendra* given by Śrīpati and Bhāskara have a greater significance, which will be elaborated later. In modern astronomy the equation of centre s is given by $2e\sin m + \frac{5}{4}e^2 \sin 2m$.

The arc PT (Ref Fig. 6.3) is approximately taken to be equal to QL the perpendicular dropped from Q on AP produced. This approximation is permissible because the difference will not be much in the case of the equation of centre. Since triangle APK and PQL are similar, we have

$$\frac{QL}{PQ} = \frac{PK}{AP} = \sin m$$

$\therefore\ QL = PQ \sin m = PT$ (approximately)

$=$ *mandaphala* or equation of centre comparing this with the modern formula, neglecting the second term in the equation of centre (for the present) $2e \sin m = PQ \sin m$

$\therefore\ PQ = 2e$

since $PQ \sin m$ will be maximum when $m = 90°$

$2e = PQ$ is called the *antyaphala*, i.e. maximum equation of centre. The word eccentric circle is used because the centre B of the circle in which the True planet is supposed to move does not coincide with the centre of A of the circle which is the mean orbit.

Epicyclic Theory

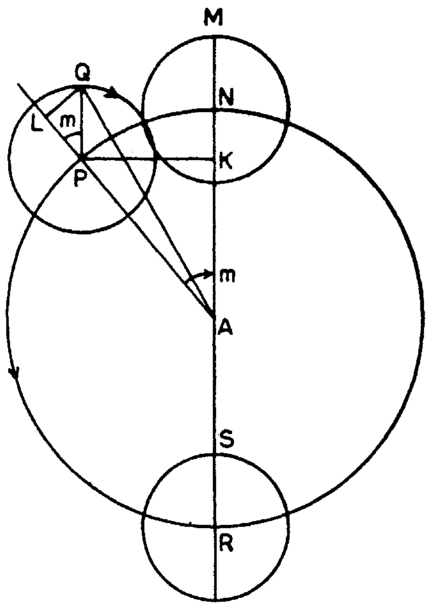

Fig. 6.4. Epicyclic model for planetary *manda* corrections.

In Fig. 6.4, the circle (A) is the mean orbit. Let P be the mean planetary position. The small circle (P) is called an epicycle. The position of the true planet Q is always taken to be such that PQ is paralled to AN. The length of PQ being equal to the length of AB in Fig. 6.3 the distance between the two centres of the mean orbit and the true orbit, it is evident that two theories the eccentric and the epicyclic give one and the same position of the True planet. Here also the triangles, APK and PQL are similar so that taking QL equal to the arc PT approximately as before $QL=PQ$ sin m. Here also M corresponds with the perigee of the orbit.

In view of the epicyclic theory, Hindu astronomers gave instead of the value of PQ the radius of the epicycle, the circumference of the epicycles in the case of each of the planets. It appears peculiar to note that the circumferences are given in degrees or what is the same the radii also in degrees, so that the equation of centre may be got in degrees direct. The epicycle in the case of the equation of centre is termed *mandanīcoccavṛtta,* where the word *manda* pertains to the *mandaphala* or equation of centre and the word *nīca-ucca-vṛtta* signifies that the word *nīca* should be taken for perigee and the word *ucca* for *apogee,* because they are respectively nearer and farther from the Earth. The words *manda,* and *śīghra* occurring in the case of reducing the heliocentric longitudes to the geocentric have further significance, which will be explained shortly. Another point deserves notice in this context. *Sūrya-siddhānta,* and *Siddhānta-śekhara* of Śrīpati make the dimensions of the epicycles variable, whereas Bhāskara and Kamalākara, the author of *Siddhānta-tattvaviveka,* (1606 AD) do not.

Śīghra Karma Or The Correction To Reduce The Heliocentric Positions To The Geocentric

It is very interesting to note here, that, though the Hindu astronomers construe implicitly or explicitly that all the planets go round the Earth yet as per Bhāskara's statement the centre of the eccentric circle does not coincide with the centre of the Earth. The centre of the eccentric circle, so taken, it has to be noted, coincides with the centre of the Sun as the mathematics involved will reveal to us. Bhāskara gives us in verses 23-25 *Gaṇitādhyāya,* that the peripheries of the *śīghra* epicycles of Mars, Mercury, Jupiter, Venus and Saturn are respectively 243°40′, 132°, 68°, 258° and 40°. He later adds, that these dimensions are variable to some extent and specifies, where and to what extent they are variable.

As already stated, that *śīghraphala* is prescribed only for the planets Mars, Mercury, Jupiter, Venus and Saturn. This is so because, they revolve not around the Earth, but round the Sun. Hence this *śīghraphala* is intended to reduce the heliocentric positions to the geocentric.

The following table shows how there is a beautiful accordance between the Hindu Astronomy and modern astronomy in this. This itself is a proof that the point about which these five planets revolve, as postulated by Bhāskara to be some other point than the centre of the Earth is no other than the Sun. So, though it was not mentioned in so many words in Hindu astronomy, that these planets are revolving round the Sun, the mathematics which goes into computing the *śīghraphala* clearly reveals that the Hindu theory was also heliocentric. Then the question arises as to how the *śīghraphala* came to be formulated so correctly under a supposed geocentric theory. The following argument can be adduced.

TABLE 6.5. *Dimensions of śighra epicycles and their ratios with reference to the deferent circle*

Planet	Periphery of the *Śighra* epicycle	Periphery of the deferent	Ratio	Value in modern astronomy taking Earth's ortital radius to be unity
Mercury	132°	360	$\frac{132}{360} = .37$	.387
Venus	258°	360	$\frac{258}{360} = .716$	.723
Mars	$243\frac{2}{3}$	360	$\frac{360}{243\frac{2}{3}} = 1.5$	1.52
Jupiter	68°	360	$\frac{360}{68} = 5.3$	5.2
Saturn	40°	360	$\frac{360}{40} = 9$	9.6

In the case of the Sun and Moon, it was discovered that the true positions were slightly different from the computed mean positions, except at two points (namely the perigee and apogee) where they were found to be the same. These two points were noted correctly. Later, it was discovered that the Sun and Moon were being drawn towards the apogee on either side of the apse line. So the *Sūrya-siddhānta* says significantly "the planets are being attracted towards the *mandocca*, i.e. the apogee, which is an unseen entity" (verses 1 and 2 Chapter-2). Not only mention is made of the *mandocca*, but also of *śighrocca*, and the nodes playing a part in this attraction.

Noting carefully, that the divergence between the computed mean position and the observed true position was a maximum at a distance of 90° from the *mandocca* on either side, this maximum was called *antyaphala* (meaning etymologically the maximum). The amounts of divergence in between, were then discovered to vary not as m but $\sin m$. Hence the *mandaphala* or equation of centre came to be formulated very easily in the case of the Sun and the Moon.

But when it came to the case of the five planets Mars etc., the divergence between the computed mean positions and the true positions was observed to be very great.

Then it was observed that this divergence was a minimum when these planets were in conjunction or opposition with the Sun. Soon it was discovered that the Sun was playing a part in this divergence. Since Mercury and Venus were found to be in oscillation about the mean position of the Sun, whereas the other planets Mars, Jupiter and Saturn were going round the sky, a differentiation was made between Mercury and Venus on the one side, and the other planets on the other. Also since in course of time, due to the oscillation of Mercury and Venus about the Sun, the number of sidereal revolutions of Mercury and Venus about the Earth (not about the Sun) were taken to be those of the Sun. Hence peculiarly indeed, the Sun's mean position is taken to be the mean position of the planets Mercury and Venus. Then their elongation becomes now the *śighraphala,* which is to be added or subtracted from the Sun's mean position to get their geocentric positions. In the case of Mars, Jupiter and Saturn i.e, the Superior planets as they are called in modern astronomy, it must be noted that their mean positions construed unknowinghly as geocentric by the Hindu astronomers, are indeed heliocentric. This we can realize in two ways : (1) Even if actual observations made with respect to these planets, irrespective of their retrograde motion, the mean geocentric sidereal period will be the same as the mean heliocentric sidereal period, for the simple reason that the Earth's orbit is contained within the orbits of these planets (of course a number of observations of the sidereal periods must be made to strike a mean) (2) or again if the sidereal periods were calculated from the formula

$\frac{1}{Y} - \frac{1}{P} = \frac{1}{S}$ (of course is not the formula used in this form, but it was used indirectly as follows. The difference of the sidereal revolutions of the Sun and those of the planets Mars, Jupiter and Saturn were noted to constitute the synodic revolutions of these planets) they are to be noted to be heliocentric and not geocentric.

Hence "the mean planet" in the case of the planets Mars, Jupiter and Saturn, it should be noted, is the mean heliocentric position and *not at all the mean geocentric position.* Interpreters of Hindu astronomy went wrong here, in construing the 'mean planet' in the case of the superior planets, as the geocentric mean planet. The very fact that the *śighra* correction applied to these mean planets is no other than the difference of the heliocentric and geocentric position should disabuse the minds of those interpreters. In other words by applying the *śighra saṃskāra* we are reducing the heliocentric positions to geocentric positions. This correction was observed to be zero when the planet (i.e. one of the superior planets) is in conjuction or opposition. Hence the Sun plays the same part here in the *śighra* correction as the apogee while computing the equation of centre, so that the Sun is spoken of as the *śighrocca* of these planets. In contradistinction, with respect to Mercury and Venus, since the mean Sun is taken to be their mean planet, the part played by the *mandocca* in the computation of the equation of centre, is now played by the heliocentric position of the planets Mercury or Venus. Hence, though it sounds odd, the heliocentric Mercury or the heliocentric Venus are spoken of as their own *śighroccas.* All this is connoted by the following verse "*Kujajīvaśaniṇāmtu raviḥ śighrocca ṇā makaḥ; jna śukrayoḥ grahaḥ Sūrya bhavet tau śighraṇamakau*", i.e. in the case of the superior planets, the

Sun plays the part of the *śighrocca,* whereas in the case of Mercury and Venus, their heliocentric positions are spoken of as the positions of the *śighroccas,* whereas the mean sun is spoken of as their mean planet.

This is corroborated by the verse 8 Chapter-2 of *Sūryasiddhānta,* as well as the verse of *Gaṇitādhyāya* of Bhāskara, which says that the geocentric revolutions of the nodes of the orbits of the planets Mercury and Venus are got by increasing their (heliocentric) revolutions by the number of their synodic revolutions. In other words, the terms *śighroccas* applied to the heliocentric Mercury and Venus, indirectly implies that their motion is heliocentric and not geocentric.

Under this perspective, let us see the method of computing the *śighra-saṃskāra,* i.e. the correction to be applied to reduce the heliocentric positions to the geocentric. Let us take first the case of the superior planets, that of Jupiter for example. In modern astronomy, the equation of centre is applied to the heliocentric true planet and later the heliocentric true planet is reduced to the geocentric true planet. In Hindu astronomy, the heliocentric mean planet (though it is construed as the geocentric) when corrected for the equation of centre is called the *mandasphuṭagraha* and when then the *śighra* correction is applied we get the geocentric true or *sphuṭagraha* as it is called. However, in this a particular procedure is prescribed by Bhāskara, and another particular procedure by the *Sūrya-siddhānta* to obtain the true geocentric positions.

Bhāskara says that the mean star-planets except Mars are to be rectified for the equation of centre first. It is called the *mandasphuṭagraha.* Now subtract the longitude of this *mandasphuṭa* from the longitude of the *śighrocca.* The resultant is called the *śighra* anomaly. From this argument, obtain the *śighraphala* and apply it to the *mandasphuṭa.* The result is again taken to be the *mandasphuṭa* so that the equation of centre is again derived for this and applied to the *original mean planet* Then again, correct it for *śighraphala* and repeat the process till a constant value is obtained as the true planet. But with respect to Mars, let the mean planet be corrected for half of the equation of centre. Then derive the *śighraphala* and effect half the correction to the previous result. Now derive the equation of centre and effect the entire correction from the resulting *mandasphuṭa* and then derive the *śighraphala* and effect the whole correction. The result then gives the true planet.

Sūrya-siddhānta on the other hand, prescribes a uniform procedure to all the star-planets. In the first place, half of the *śighraphala* is the correction to the mean planet. Taking the result to be the mean planet, derive the equation of centre and effect half of this equation of centre to the previous result. Now consider the result to be the mean planet, compute the equation of centre and effect the whole correction. Taking the result to be the *mandasphuṭagraha,* compute the *śighraphala* and effect the whole correction to obtain the true planet.

As per modern astronomy, there is no connection between the equation of centre and the subsequent *śighraphala.* The former is to get the true heliocentric position.

Then the *śighraphala* is to reduce the heliocentric true planet to the geocentric. According to this, the processes prescribed by Bhāskara under the name of *āgama* and *upalabdhi*, i.e. tradition and accordance, between the computed position and the observed, or that prescribed by the *Sūrya-siddhānta,* seem to be beating about the bush. However, we shall revert to this later to see some rationale behind these procedures.

Now we shall try to interpret the *śighraphala* as no other than the correction to be effected to reduce the heliocentric position of the planet to the geocentric. In the first place, we shall consider this with respect to the Hindu epicyclic theory. In the case of the equation of centre, construing the arc PT as roughly equal to QL did not matter because of the smallness of the equation of centre (Fig. 6.5).

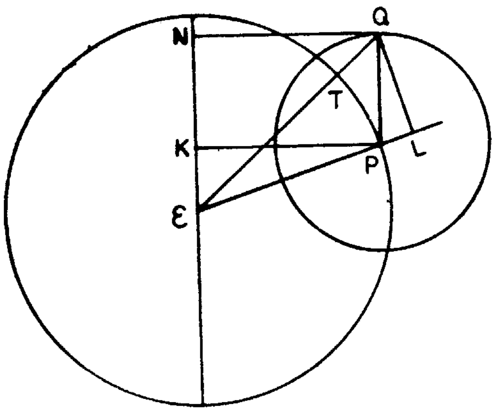

Fig. 6.5. Computation of *śighrakarṇa*.

But in the case of the *śighraphala* which is generally of a larger value, we cannot do so. Hence EQ called the *śighrakarṇa* is computed first. Bhāskara says that, this EQ (called K here) is got as follows

$K^2 = EQ^2 (EP+PL)^2 + QL^2$ .. .. (5)

or $= EN^2 + QN^2$ .. .. (6)

Here $EP=R$ (called *trijyā*) PQ is called *antyaphala* or maximum *śighraphala*. QL is termed *bhujaphala* and PL as *koṭiphala*. Also $QN=PK$ is called *bhuja*, and EK is termed *koṭi*.

Hence what Bhāskara says amounts to

$K^2 = (\textit{trijyā} + \textit{koṭiphala})^2 + \textit{bhujaphala}^2$ .. .. .. (5)

or $= (\textit{koṭi} + \textit{bhujaphala})^2 + \textit{bhuja}^2$ .. .. .. (6)

(5) reduces to $K^2 = R^2 + a^2 \pm 2\, R\, \textit{koṭiphala}$. (6) reduces to

$K^2 = R^2 + a^2 \pm 2a \times \textit{koṭi}$

where $EP=R$, and $PQ=a$.

The proof is simple.

We shall derive these formulae from the modern heliocentric figure also. We shall consider the case of a superior planet, say Jupiter, for example.

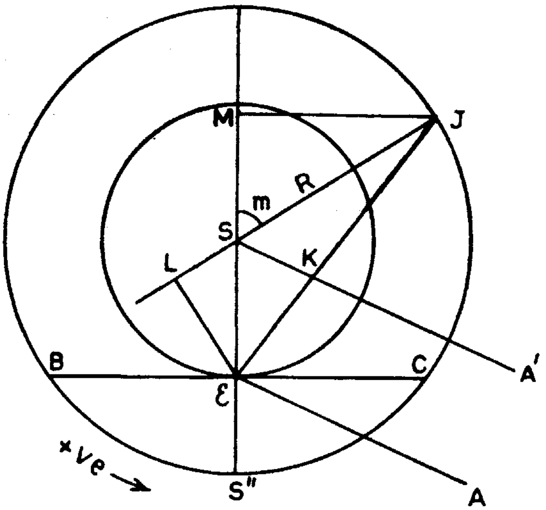

Fig. 6.6

The same formula are derivable from the heliocentric Fig. 6·6 also as follows.

In fig. 6.6 $K^2=EJ^2=(ES+SM)^2+JM^2$ or $EL^2 + (LS+SJ)^2$

$\therefore$ $K^2=a^2+koṭi^2+2a\ koṭi+bhuja^2$ or $bhujaphala^2+koṭiphala^2+trijyā^2$ $+2\ trijyā \times koṭiphala$.

Note that $koṭi^2+bhuja^2=R^2$ and $bhujaphala^2+koṭiphala^2=a^2$.

Now we shall find the value of *śighraphala* both in the epicyclic fig. 6.5 and the heliocentric fig. (6.6). In fig. (6.5) construing PT, which is the *sighraphala* as a chord parallel to LQ.

$$\frac{PT}{LQ}=\frac{ET}{EQ} \text{ approximately.}$$

$$\therefore\ PT=\frac{ET\times LQ}{EQ}=\frac{R\times LQ}{K} \quad \text{But } \frac{LQ}{PK}=\frac{PQ}{EP}.$$

$\therefore$ Substituting for LQ

$$\therefore\ PT=\frac{R}{K}\times\frac{PQ}{R}\times PK=\frac{a\times pK}{K} \quad \text{But} = H \text{ Sin } m \text{ (Hindu Sine)}$$

where m is called the *śighra* anomaly.

$\therefore$ PT = *śighraphala*$=a$ sin m where k is called the *śighrakarṇa*. In the heliocentric figure (6.6)

$$\frac{\sin EJS}{A}=\frac{\sin m}{K} \quad \therefore\ \sin EJS=\frac{a}{K}\sin m.$$ Multiplying both sides by R

$$H\sin EJS=\frac{a}{K}\ H\ \sin m$$

as before, where *H* sine signifies the Hindu sine. Thus we see that *a* plays the part of *antyaphala,* i.e. *PQ* of the Epicyclic figure. As a matter of fact, keeping the epicycle fixed, and rotating the centre E in Fig (6.5) around *P,* we have the heliocentric figure. In other words, both the figures lead to the same value of *śighraphala.* Or we can again see that the triangle *EPQ* of Fig. (6.5) is congruent to the triangle *SJE* of fig. (6.6), which means that we get the same result from the epicyclic figure as we get from the heliocentric figure.

We have still to establish further correspondence between figures (6.5) and (6.6). Let us consider the case of the superior planet *J* (Jupiter) which procedure holds good for Mars and Saturn as well. In fig. (6.6) the heliocentric longitude of Jupiter with respect to the zero-point of Hindu zodiac namely the first point of Aświnī asterism is given by the angle $A'SJ$ (Note that the geocentric direction *EA* and the heliocentric direction SA' are almost parallel because of the great distance of Aświnī. The geocentric direction of Jupiter is given by the angle *AEJ*.

Now $AEJ—A'SJ=E\ J\ S$,
i.e. the difference between the two directions is the angle *EJS* which is therefore the *śighraphala* correction to be effected to the heliocentric direction to get the geocentric direction. Here in this case *J* is on the right-hand side of *ES,* so that the correction is positive (on the left-hand side of *ES,* the correction would be negative). In other words the correction will be positive or negative according as $180° < m < 360°$ or $0° < m < 180°$. Here '*m*', called the *śighra* anomaly, is equal to $A\ E\ S—A'\ S\ J=$ geocentric direction of the Sun minus heliocentric longitude of the planet. The Sun is spoken of as the *śighrocca* therefore with respect to the superior planets, bearing the analogy of the *mandocca* in finding the equation of centre.' *m*' the argument, which was called the *manda-kendra* or the argument from which the equation of centre was computed, there the situation being,

heliocentric longitude of the planet—longitude of the aphelion or *mandocca* =*manda kendra.*

Here, in the case of *śighraphala,* '*m*' is given by geocentric longitude of the Sun minus heliocentric longitude of the planet. The words *manda* and *śighra* will be seen to have etymological significance that they have slower or faster motion as compared with the planet. Hence we have shown the planet ahead or behind respectively in the two cases. The *mandaphala* was negative when $0°<m<180°$ and positive when $180°<m<360°$, so that in both the cases, we find the planet attracted towards the *ucca* (whether *śighrocca* or *mandocca*). Hence Bhāskara says as also *Sūryaisiddhānta,* that the *ucca* attracts the planet towards it. In particular Bhāskara says "*ucco hi ākarṣako bhavati*".

Bhāskara has a significant statement: "*kakṣā-madhyaya tiryagrekha prativṛtta sampate, madyaiva gatiḥ spaṣṭā, param phalam tatra khetasya*". "The *mandaphala* or *śighraphala* will be maximum positively or negatively at the points of intersection of the eccentric circle with the line passing through the centre of the *kakṣāvṛtta* or the deferent." This is clear in fig. 6.3 in the context of the *mandaphala* in the position p' of the true planet where the angle M A P=90° the maximum equation of centre being equal to PO the radius of the epicycle otherwise called *antyaphala*. Similarly, if we draw the eccentric circle in the case of the *śighraphala* also (the same figure holds good in this case also) we can see that the *śighraphala* also will be maximum at that spot. This fact is also borne out in the heliocentric fig. 6.6 (the heliocentric figure is not drawn in the case of the equation of centre because in modern astronomy, we have the ellipse whereas in Hindu astronomy we do not have such an ellipse but only the same epicyclic or eccentric theory) where when J is at the points A or B, the angle E J S will be a maximum because

$$\frac{\sin EJS}{a} = \frac{\sin JES}{R} \therefore \sin EJS = \frac{a}{R} \sin JES$$

$$\therefore \; EJS \text{ will be maximum with } J\,E\,S = 90^\circ \quad .. \quad .. \quad (7)$$

Bhāskara's verse quoted above says further that the *madhyagati* or the velocity of the mean planet in the case of the equation of centre and that of the *mandaspuṭagraha* in the case of the *śighra* correction will be equal to the velocity of the *mandasphuṭagraha* or the planet corrected for the equation of centre and the true planet corrected for *śighraphala* in the two cases respectively. This is easy to say from equations, already discussed, where, in the first case the velocity of the *mandocca* or the aphelion almost being negligible.

$(PQ \sin m)$ when $m=0$ will be zero. Hence the difference in the equation of centre which is maximum at that point will be zero (the criterion of maximum). Similarly in equation 7, $\triangle\,(a \sin J\,E\,S)$ When $J\,E\,S=90°$, is equal to zero, (again the criterion of the maximum). In the case of the equation of centre, because we took arc $PT=QL$ approximately, the *mandagati* or the velocity of the *mandasphuṭagraha*$=\triangle\ (PQ \sin m)=PQ \cos m\,\delta m$. This is given by Bhāskara in verse 37 of *Spaṣṭādhikāra*, when $M_1 + E_1 = M_2$, i.e. longitude of the mean planet plus equation of centre is equal to the longitude of the *mandasphuṭagraha*. But $\triangle M_2 = \triangle\ (M_1+E_1) = \triangle\ M_1 + \triangle\ E_1 = 0+0$ because the mean motion $\triangle$ is constant and $\triangle\ E_1 = 0$ because E_1 is maximum. Similarly $M_1+E_2=M_2$ or the longitude of the *mandasplutagraha*+*śighraphala*=true planet.

$$\therefore \triangle\ (M_2) = \triangle\ M_1 + \triangle\ E_2. \text{ When } \triangle\ E_2=0 \text{ because } E_2 \text{ is a maximum,}$$
$$\triangle\ M_2 = \triangle\ M_1$$

i.e. the true motion is equal to the motion of the *mandasphuṭagraha* as stated by Bhāskara.

Śighragatiphala—from equation (7)

$$\sin EJS = \frac{a}{R} \sin JES$$

$$\therefore \triangle\ (\sin EJS) = \frac{a}{R} \triangle\ (\sin JES) = \textit{śighragatiphala} \text{ as it is called. But } JES \text{ being}$$

the elongation of the true planet and this being not measured in practice by the Hindu astronomers, Bhāskara derives the *śighragatiphala* in an ingenious way, which exhibits his genius. This *śighragatiphala* is very important because we have got to know when the planet retrogrades.

Before we seek to find the *śighragatiphala* let us take up the procedure laid down to rectify the inferior planets Mercury and Venus. We take Venus and the procedure is the same with respect to Mercury also.

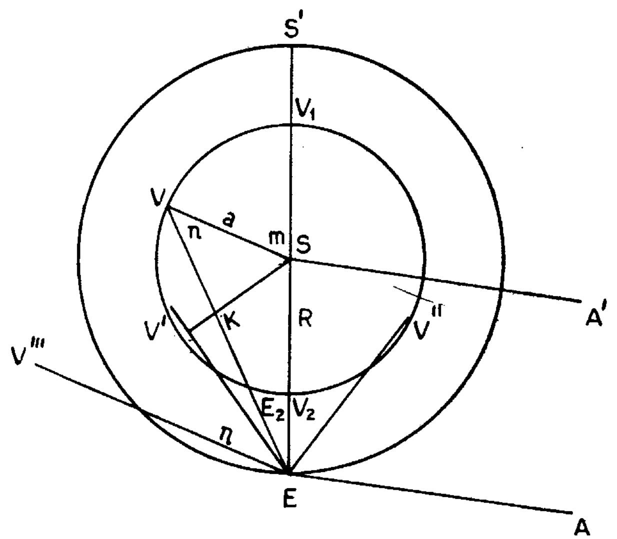

Fig. 6.7 Correction for inferior planets Mercury and Venus.

E = Earth; S = Sun, V = Venus.
Take $SV = a$ and $SE = R$. EA is the geocentric direction of the Hindu Zero-point A; SA' is the heliocentric direction of the same point. V' = position of V at max. elongation. 'm' is here the *śighra* anomaly.

Since, it was construed that the *madhyagraha* of the inferior planet is the Sun, inasmuch as the inferior planet oscillates about the mean position of the Sun as seen from the Earth, and since V the heliocentric Venus was construed as the *śighrocca* of Venus, to accord with the method postulated with respect to the superior planets.

Longitude of *śighrocca*—longitude of the mean planet

$= \textit{śighrakendra} = m$ or $A'\hat{S}V - A'\hat{S}S' = S'\hat{S}V = m.$

From the triangle ESV, $\dfrac{a}{K} = \dfrac{\sin S\hat{E}V}{\sin E\hat{S}V}$

$$\therefore \text{Sin } S\hat{E}V = \frac{a}{K} \sin E\hat{S}V = \frac{a}{K} \sin m.$$

This is the same as we got in the case of a superior planet. Hence $S\hat{E}V$ is termed here as the *śighraphala,* which is no other than the elongation of the planet. This will be evidently a maximum when EV' is tangential to the inner circle. Then $\sin S\hat{E}V = \frac{a}{R}$ = a constant so that the maximum *śighraphala* will be equal to $\sin^{-1} \frac{a}{R}$ which is the same as the maximum elongation. From the table shown before, under the caption 'Elements of the Solar system' taken from Ball's *Spherical Astronomy* in the case of Mercury the maximum elongation of Mercury is $\sin^{-1}$ $.387 = 22°48'$ and in the case of Venus $\sin^{-1}$ $.7233 = 47°12'$ (ofcourse taking the orbits to be coplanar circles.)

$$\text{Since } A\hat{E}S + S\hat{E}V = A\hat{E}V,$$

therefore, the mean longitude of the Sun plus the *śighraphala* equals the geocentric longitude of the planet (Venus or Mercury). K is calculated as was done in the case of the superior planets. In the case of the inferior planets, since they move faster than the Earth, or what means the same, they move faster than the Sun, relatively speaking, so the heliocentric Venus is termed *śighrocca*, justifying the word *śighra*. As per the word *ucca*, it has a double significance: (1) when V occupies the position V_1 in Fig. 6.7, Venus will be farthest from the Earth; under this perspective when Venus is at V_2 it is nearest the Earth, but the word *ucca* has no significance here; (the word *śighranīca* is not used in the Hindu astronomical works); (2) more than the first significance, since in the equation of centre we said longitude of *graha*-longitude of *mandocca*=*mandakendra*, similarly here heliocentric longitude of Venus (or its *śighrocca*)—longitude of the mean Sun (i.e. mean planet)=*śighrakendra*. Thus the word *ucca* is significant so far as its position gives us the mean anomaly or the *śighra* anomaly (the word anomaly here is an odd usage).

Thus in the case of an inferior planet, its elongation is the *śighraphala*.

The Hindu epicyclic theory also leads us to the value *śighraphala* $= \frac{a}{K} \sin m$, as in the case of the superior planets. However in the case of the superior planets, the epicycle corresponds to the orbit of the Earth, and the circle got rotating the point E about the centre of the epicycle will be the orbit of the planet. The centre of the epicycle here coincides with the point S of the heliocentric figure 6.7. Looking at the dimensions of the epicycles both with respect to the superior and inferior planets, and looking at the table cited before from Ball's *Astronomy*, it is clear that the *śighraphala* is no other than the correction required to reduce the heliocentric longitude to the geocentric in both the cases of the superior and inferior planets.

Śighragatiphala: As mentioned before, this is a very important concept, which gives us the knowledge when a planet retrogrades. Bhāskara says in verse 39 Chapter-II *Gaṇitādhyāya*

$$\delta l - \frac{H\sin(90—E)\,\delta m}{K} = \delta S,$$

where δl is daily mean motion of the *śighrocca*, δm=daily mean motion of the *śighra* anomaly, E=*śighraphala*, and δS=true geocentric motion of the planet. If δS is negative the planet is in retrograde motion. K is the *śighrakarṇa* mentioned before. From fig. 6.6, *sphuṭagati* or the true daily motion of the planet Jupiter is $\triangle(A\hat{E}J) = \delta S$.

$$\text{But } \triangle\ A\hat{E}J = \triangle\ A\hat{E}S - \triangle\ J\hat{E}S.$$

$\triangle\ A\hat{E}S$ is called the *śighrabhukti* of Jupiter or the true daily motion of the *śighrocca* of Jupiter or what is the same the true daily motion of the Sun. Hence, to find $\triangle AEJ$, we have to find $\triangle(J\hat{E}S)$, because $\triangle AES$ is known. In fig. 6.6, we have connoted $J\hat{E}S$ as $\hat{n}$, whereas m, i.e. the angle $J\hat{S}M$ in the figure is called the *śighrakendra*, this angle $\hat{n}$ is called *sphuṭa śighrakendra*. In this context the verse given by Bhāskara is very significant, as well as its proof which is highly complicated and as such reflects on his genius. The verse deserves to be quoted and is

Phalāṃśa khāṅkāntara śiñjinighnī,
Drāk-kendra bhuktiḥ śrutihṛt viśodhyā
Swaśighra bhukteḥ, sphuṭakheta bhuktiḥ
Seṣam ca vakrā viparita śuddhau

Meaning:—phalāṃśa=the degrees of the *śighraphala*, *khāṅka*=90° (*kha*=*o*, *aṅka*=9; in Hindu numbering we proceed from the right to the left as is said *aṅkānām vāmato gatiḥ*, so that, the number becomes not 09 but 90°). Thus *phalāṃśa khāṅkāntara* means ($90°-E_2$) where E_2 is the *śighraphala*. The word *śiñjinī* means the Hindu sine. We are asked therefore to multiply (the word *ghnī* means being multiplied) H Sin ($90-E_2$) by *drāk-kendra bhukti*, i.e. the daily motion of the *śighrakendra*, i.e. δm. So we get $H \sin (90-E_2)\ \delta m$. This is directed to be *śrutihṛt*, i.e. to be divided by the *karṇa* K. Thus far we have $\dfrac{H \sin (90-E_2)}{K}$. This is directed to be *viśodhyā*, i.e. subtracted from *svaśighra bhukti*, i.e. the daily motion of the *śighrocca*, i.e. of the Sun. The result will be δS, i.e. the true daily motion of the planet given by

$\triangle\ (AEJ)$. So, $\delta S = \delta l - \dfrac{H \cos E_2\ \delta m}{K}$, where δl is the daily motion of the Sun.

Here one point is to be noted. After getting the *manda-sphuṭagati* or the daily motion of the planet rectified for the equation of centre (let us call this M_1) $M_1+E_2=S$, where E_2 is the *śighraphala* equal to $\dfrac{a \sin m}{K}$ as obtained before, and S the true planet.

$\delta s = \delta M_1 + \delta E_2$. Here δM_1 could be got easily being the daily motion of the planet rectified for the equation of centre. In fact, $\delta M_1 = \delta M + \delta E_1$ where δM is the mean daily motion of the planet and δE_1 is the daily variation of the equation of centre, δM is known and $\delta E_1 = \delta\ (a \sin m) = a \cos m\ \delta m$, which is known easily. (Note here that, we do not effect what is called *karṇānupāta* in the case of the equation of centre, i.e. we do not divide by K. Since the equation of centre is small, we effect this *karṇānupāta* in the case of the *śīghraphala*, because this *śīghraphala* is far greater than the equation of centre).

Coming to the point, since

$M_1+E_2=S$, δM_1 being known easily, is it not enough to get δE_2 or $\delta \dfrac{(a \sin m)}{K}$?

Bhāskara says significantly that in $\dfrac{a \sin m}{K}$ both m and K are variable from day to day, and therefore to get the differential of $\dfrac{a \text{ Sin } m}{K}$ is not easy (Remember here though Bhāskara knew that δ (sin m) $=\cos m \; \delta m$ which formula he used very often, he was not aware of the formula $\delta \dfrac{(u)}{v} = \dfrac{v\,\delta u - u\,\delta v}{v^2}$ of modern calculus. Bhāskara was five centuries older than Newton and to have gauged that δ (sin m) $= \cos m \; \delta m$ was indeed creditable on his part). So Bhāskara says "*mahā matimadbhiḥ kendragatiḥ eva spaṣṭī kṛtā*", i.e. great intellects rectified 'm' of fig. 6.6 instead of seeking $\delta \dfrac{(a \sin m)}{K}$ where m and K are both variable. In other words, they sought to find 'n' which is called *sphuṭakendragati*, in the place of finding δm called *madhyakendra gati*. (Incidentally note that he was paying a compliment to the genius of Brahmagupta by using the word 'great intellects').

Now we shall seek the proof of the formula given by Bhāskara, using the heliocentric figure first and then understand his own proof in the eccentric theory.

From fig 6.6 $K \cos n - R \cos m = a$ (using the elementary trigonometric formula in a triangle namely b bos $C + C \cos B = a$. Taking differentials from the above,

$-K \sin n \; \delta n + \cos n \; \delta k + R \sin m \; \delta m = 0$ .. .. .. (8)

because a is a constant. But we have

$K^2 = R^2 + a^2 + 2\,R\,a \cos m$. So that $2K\delta K = -2\,R\,a \sin m \; \delta m$. .. (9)

Eleminating δk between (8) and (9)

$$-K \text{ Sin } n\,\delta n - \frac{Ra \sin m\,\delta m \times \cos n}{K} + R \sin m\,\delta m = 0.$$

$$\text{i.e. } K \sin n\,\delta n = R \sin m\,\delta m \left(1 - \frac{a}{K} \cos n\right)$$

$$= R \sin m\,\delta m \; \frac{(K - a \cos n)}{K}$$

But $K - a\cos m = R\cos E$

$$\therefore\ \delta n = \frac{R\sin m\,\delta m \times R\cos E}{K^2\sin n}\quad \text{But } R\sin m = K\sin n$$

$$\therefore\ \delta n = \frac{R\cos E\ \delta m}{K} = \frac{H\cos E\,\delta m}{K}$$

as given by Bhāskara. It is worth-while for the reader to go into the argument adduced by Bhāskara (in his own words).

In the above, we have given a proof based on the modern heliocentric figure just to show how Bhāskara could go into intricate mathematics. Before we see his own proof, it yet remains for us to note the treatment of the minor planets Mercury and Venus.

We have seen that the true geocentric longitude of the inferior planet is got by adding to the *mandasphuṭagraha* or the mean planet rectified for the equation of centre, the *śighraphala* which is no other than the elongation of the planet given by $\frac{a}{K}\sin m$.

Next the question arises, as to whether the procedure laid down to get the *śighragatiphala* in the case of the inferior planets also is the same as that laid down in the case of the superior planets. There is a slight difference, however, though the formula $\frac{H\cos E\ \delta m}{K}$ holds good here also. There difference is with respect to the *sphuṭa-kendra,* The *sphuṭa-kendra* in this case (Ref. Fig. 6.7) is the angle between the heliocentric direction of Venus namely SV or EV and the geocentric direction of V namely Ev, marked equal to n in the figure since in this case also

$m+E_2=n$ as in fig. 6.6, E_2 of course, having a different connotation, and since E_2 here also has the same formula namely $\frac{a}{K}\sin m$, the same mathematics holds good here also to get the true daily motion of the planet.

Now we shall give Bhāskara's proof with respect to the *śighragatiphala,* formerly proved and obtained as $\frac{H\cos E\ \delta m,}{K}$ taking the heliocentric fig. 6.7. Now we shall trace the steps of Bhāskara from his eccentric fig. 6.8, given below, and ascertain the veracity of his statement that the rectified *kendragati* δn is equal to $\frac{H\cos E\ \delta m.}{K}$

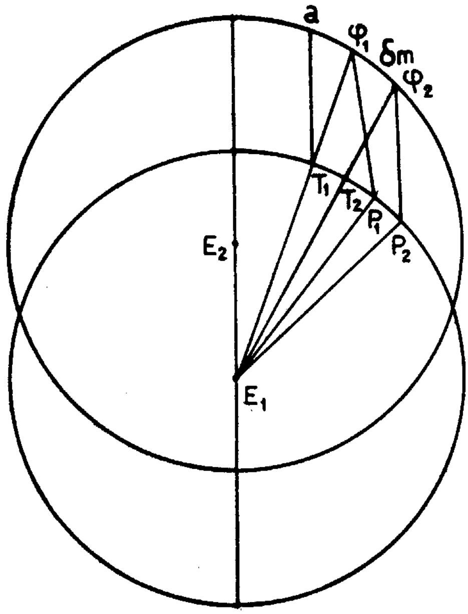

Fig. 6.8 Bhāskara's proof with respect to *śighragatiphala*

In the fig 6.8 above P_1, P_2 and Q_1, Q_2 are the positions of the mean planet in the *kakṣāvṛtta* and *prativṛtta* on two consecutive days. Let T_1, T_2 be the true planets on the two consecutive days. Evidently P_1T_1, and P_2T_2 are the value of the *śighraphala* on the two days. Draw a parallel through T_1 to E_1A_2 to meet the *prativṛtta* at *a*. A_1T_2—A_1T_1= the difference of the *sphuṭakendras* on the two days, which is equal to T_1T_2 is called *sphuṭakendragati*. Since *śighra—sphuṭagraha=sphuṭakendra, śighragati—sphuṭagrahagati= sphuṭa-kendragati* or what is the same *sphuṭagrahagati=śighragati—sphuṭakendragati*. Now the problem is to find P_2T_2—P_1T_1 which is called *sphuṭakendragati* (as named previously). It will be noted that *madhyakendragati* is equal to Q_1Q_2 in Fig. 6.8. In the figure P_1T_1 is the *śighraphala* on the first day which is equal to a Q_1, P_2T_2 is the *śighraphala* on the second day.

In the figure considering Q_1Q_2, i.e. δm as an increment in the *śighraphala* aQ_1 (in the figure it may not appear so. δm is small compared to aQ_1) equal to $\frac{a}{K}$ Sin m=sin E we are hence required to find sin $(E+\delta m)$ — sin E where δm is small compared to E.

$$\sin(E+\delta m)=\sin E\cos\delta m+\cos E\sin\delta m.$$ Put $\cos\delta m=1$ and $\sin\delta m=\delta m$

$$\therefore\ \sin(E+\delta m)=\sin E+\cos E\ \delta m.$$

Since this cos E δm is on the *prativṛtta*, we have to reduce it to *kakṣāvṛtta* by multiplying by R and dividing by K. Hence the required quantity

$$=\frac{R\cos E\ \delta m}{K}=\frac{H\cos E\ \delta m}{K}$$ as is given by Bhāskara.

In this context, we find Bhāskara criticizing Lalla for having used the word *āśu-cāpa,* by which Lalla meant *āśu-phala-cāpa* and not *āśu-kendra-cāpa* as misread by Bhāskara. So the credit may also go partly to Lalla, who was indeed a great astronomer contemporary to Brahmagupta.

It remains for us to establish a correspondence between Bhāskara and modern astronomy in this point, because, we have to assure ourselves that the Hindu method of finding at what point a planet retrogrades is also correct. Assuming coplanar concentric circles,
let S=Sun, E=Earth, J=Jupiter, m=*śighra* anomaly, Ev and Ev' drawn tangents to ES and EJ. Let angle $V\hat{E}V''=\theta$. Let Jupiter appear stationary as viewed from the Earth in this position, so that it is the beginning point of retrograde motion. Let $E\,S\,J=\phi$ and $EJS=\xi$. Let u and v be the velocities of the Earth and Jupiter (linear) let a and R be the orbital radii as before. Since the relative velocity of Jupiter with and respect to the Earth is zero,

$$u\cos\theta+v\cos\xi=0\ \therefore\ \frac{u}{v}=\frac{-\cos\xi}{\cos\theta}.$$

But from triangle ESJ, $R\cos\phi+K\cos\theta=a$ and

$$a\cos\phi+K\cos\xi=R\quad\therefore\ \frac{\cos\xi}{\cos\theta}=\frac{R-a\cos\phi}{a-R\cos\phi}$$

$$\therefore\ \frac{-u}{v}=\frac{R-a\cos\phi}{a-R\cos\phi}.\ \text{So that}\cos\phi=\frac{au+Rv}{av+Ru}.$$

$$\text{But } m=180-\phi\quad\therefore\ \cos m=-\left(\frac{au+Rv}{av+Ru}\right)$$

Here *spaṣṭagati* is equal to zero. Hence in the formula,

$$\textit{spaṣṭagati}=\textit{śighragati}-\frac{H\cos E\,\delta m}{K},$$

$$\textit{śighragati}=\frac{H\cos E\,\delta m}{R}.$$

The angular velocities of the Earth and Jupiter are respectively $\frac{u}{a}$ and $\frac{v}{R}$ so that the Sun's apparent velocity also is u/a.
δm=*kendragati*=Sun's apparent velocity minus Jupiter's heliocentric velocity.

$$=\frac{u}{a}-\frac{v}{R}.$$

$$\therefore\ \textit{śighragati}=\frac{u}{a}=\frac{H\cos E}{K}\left(u/a-\frac{v}{R}\right)$$

$$\therefore\ \frac{u}{a}\left(\frac{H\cos E-1}{K}\right)=\frac{H\cos E}{K}\times\frac{v}{R}$$

$$\therefore\ \frac{u}{a}\left(\frac{H\cos E-K}{K}\right)=\frac{H\cos E}{K}\times\frac{v}{R}.$$

But $H \cos E = R \cos E$ (Hindu Trigonomety) $= R \cos \xi$ (here ξ standing for E) and $R \cos \xi - K = - r \operatorname{Cos} \theta$

$\therefore u \cos \theta + v \cos \xi = 0$ which accords with the modern method as derived before. From this follows

$$\cos m = -\left(\frac{au + Rv}{av + Ru}\right) \text{ as said before.}$$

Substituting the relative values of a and R, we have the respective values of m for each planet, when it becomes stationary. We could more easily use equation $u \cos \theta + v \cos \xi = 0$, noting $\theta = m - \xi$. We shall next make here some points of observation pertinent to the context.

We have $M_1 + E_1 = M_2$ and $M_2 + E_2 = S$ where M_1 is the original mean planet, E_1=equation of centre, M_2=*mandasphuṭagraha*, E_2=*śighraphala*, S= true planet.

$\delta M_2 + \delta E_2 = \delta S$, which means *mandasphuṭagati* plus *śighragatiphala* (not *śighraphala*)=*spaṣṭagati*. Now let E_2 be maximum, so that $\delta E_2 = 0$, Then $\delta M_2 = \delta S$. This means that at the points B and C of the heliocentric figure 6.6, the *śighraphala* is maximum and the *mandasphuṭagati* is equal to the true motion. This line BC corresponds to the line mentioned by Bhāskara in the verse.

Kakṣā madhyagatiryak-rekhā prativṛtta-sampāte,
Madhā eva gatiḥ spaṣṭā, param phalam tatra khetasya

This is the line drawn through the centre of the *kakṣāvṛtta* to cut the *prativṛtta* in the eccentric figure.

(2) Another point of observation is that since

$\delta M_2 + \delta E_2 = 0$ means that the true velocity of the planet is zero. So δM_2 must cancel δE_2 when the planet is stationary. In Fig. 6.6 $E_2 = 0$ at $\acute{S}$; it becomes positive as J moves from S' to C in the clockwise direction (this is so because we have kept ES in a constant direction and as S has a greater positive velocity, J takes a clockwise motion). Then E_2 will have a positive maximum at C. From C to the point diametrically opposite to S, the positive E_2 becomes zero. From this diametrically opposite point as it moves clockwisely to B, it grows negatively to a maximum at B. As J then moves from B clockwisely, the negative maximum gradually gets nullified at S'. In other words, in the arc $C\,S'$ taken anticlockwise, since E_2 increases positively, δE_2 will be positive and this δE_2 becomes zero at C. Since δM_2 is always positive, $\delta M_2 + \delta E_2 = 0$ in the arc $S'C$. Now δE_2 becomes negative as J proceeds clockwisely to B. At some point in between J on this arc and the point S', $\delta E_2 + \delta M_2$ becomes zero. This means that as a superior planet is near opposition it becomes retrograde. Similarly on the arc $S''B$ taken clockwise, in between S'' and B, $\delta M_2 + \delta E_2$ will be zero, and thereafter becomes positive. Hence on the arc $C'D'$, where C' and D' are symmetrically situated with respect S'', the superior planet is retrograde.

In the case of the Inferior planets (Ref. Fig. 6.7) they reach a maximum elongation on either side of the Sun. Suppose the Sun is in advance, so that the inferior planet will be a morning star; here the planet retrogrades to a maximum elongation and then after sometime, becoming stationary, the elongation decreases to zero. Thereafter the planet overtakes the Sun and becomes visible in the western sky as an evening star. The planet now has a direct motion, and attains a maximum elongation on the right of ES in fig. 6.7. Then the planet becomes retrograde. Thus the Inferior planet is retrograde on the arc $V' V''$ taken anticlockwise and direct on the arc $V'' V'$ taken also anticlockwise. While at V_1 and V_2 it sets in they rays of the Sun. This setting in the rays of the Sun either for the Inferior planet or the Superior planet, is called heliacal setting and emerging from the rays of the Sun is called heliacal rising. It is easy to see that a superior planet sets in only in conjunction whereas an inferior planet sets both at the superior and inferior conjunctions. Which means that it sets both in the East and the West. The superior planet on the other hand sets only in the West heliacally and goes round the Sun the entire circle or the entire synodic period. The period of heliacal setting in the case of an inferior planet at the inferior conjunction is far longer than that at the superior conjunction, because the planet and the Earth are moving in the same anticlockwise direction at the inferior conjunction, whereas at the superior conjunction, their velocities add.

In conclusion, it may be mentioned that the data given by Bhāskara with respect to the stationary points, and heliacal rising and setting accord very well with the data given in modern astronomy. This is indeed remarkable, because the Hindu astronomers using the epicyclic and eccentric theories, and under the explicit geocentric theory (which is in fact not at all geocentric as the mathematics on the previous pages has revealed to us) achieved much more than could be expected of them at that distant past as long ago as Āryabhaṭa's time, many a century before Copernicus. In the wake of the mathematics expounded in this paper, nobody has the right to say that the Hindu theory was geocentric. What Copernicus achieved was, to read in between the mathematics and discover the heliocentric theory, which was formerly implicit.