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PHASES OF THE MOON, RISING AND SETTING OF PLANETS AND STARS AND THEIR CONJUNCTIONS

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INTRODUCTION

It has been known from time immemorial that the Moon is intrinsically a dark body but looks bright as it is lighted by the Sun. There is an oft-quoted statement in the *Yajurveda*[1] which describes the Moon as sunlight. As the Moon revolves round the Earth its lighted portion that faces us is seen by us in successively increasing or diminishing amounts. These are called the phases of the Moon.

When the Sun and Moon are in the same direction, the face of the Moon which is turned towards us is completely dark. It is called new-moon and marks the beginning of the light fortnight. When the Moon is 12 degrees ahead of the Sun, it is seen after sun-set in the shape of a thin crescent. As the Moon advances further this crescent becomes thicker and thicker night after night. When the Moon is 180 degrees away from the Sun, the Moon is seen fully bright. It is called full-moon. The light fortnight now ends and the dark fortnight begins. The phases are now repeated in reverse order until the Moon is completely dark at the end of the dark fortnight when the Sun and Moon are again in the same direction.

Vaṭeśvara says :

"The Sun's rays reflected by the Moon destroy the thick darkness of the night just as the Sun's rays reflected by a clean mirror destroy the darkness inside a house.[2]

"In the dark and light fortnights the dark and bright portions of the Moon (gradually) increase as the Moon respectively approaches and recedes from the Sun."[3]

"On the new-moon day the Moon is dark; in the middle of the light fortnight, it is seen moving in the sky half-bright; on the full moon day it is completely bright as if parodying the face of a beautiful woman."[4]

"The crescent of the Moon appears to the eye like the creeper of Cupid's bow, bearing the beauty of the tip of the Ketaka flower glorified by the association of the black bees, and giving the false impression of the beauty of the eyebrows of a fair-coloured lady with excellent eyebrows,"[5] "When the measure of the Moon's illuminated part happens to be equal to the Moon's semi-diameter, the Moon looks like the forehead of a lady belonging to the Lāṭa country (Southern Gujarat)."[6]

Similar statements appear in the writings of Varāhamihira and other Indian astronomers.

Phase and Sita

In modern astronomy the phase of the Moon is measured by the ratio of the central width of the illuminated part to the diameter. In Indian astronomy it is generally measured by the width of the illuminated part itself which is called *sita* or *śukla*. The width of the unilluminated part, which is equal to 'the Moon's diameter *minus* the *Sita*', is called *Asita*.

The *Pūrva-khaṇḍakhādyaka* of Brahmagupta, which summarizes the contents of Āryabhaṭa I's astronomy based on midnight day-reckoning, gives the following approximate rule to find the *sita* in the light half of the month:

"The difference in degrees between the longitudes of the Sun and Moon, divided by 15, gives the *śukla* (in terms of *aṅgulas*)."[7]

Stated mathematically, it is equivalent to the following formula:

$$sita = \frac{M - S}{15} \text{ } a\dot{n}gulas.,$$

where S and M denote the longitudes of the Sun and Moon respectively, in terms of degrees. This formula may be obtained by substituting

$$\text{Moon's diameter} = 12 \text{ } a\dot{n}gulas$$

in the general formula :

$$sita = \frac{(M - S) \times \text{Moon's diameter}}{180}. \tag{1}$$

Bhāskara I (629), a follower of Āryabhaṭa I, who claims to have set out in his works the teachings of Āryabhaṭa I, however, gives the following rule:

"(In the light fortnight) multiply (the diameter of) the Moon's disc by the R versed-sine of the difference between the longitudes of the Moon and the Sun (when less than 90°) and divide (the product) by the number 6876: the result is always taken by the astronomers to be the measure of the *sita*. When the difference between (the longitudes of) the Moon and the Sun exceeds a quadrant (i.e. 90°), the *sita* is calculated from the R sine of that excess, increased by the radius.

"After full moon (i.e. in the dark fortnight), the *asita* is determined from the R versed-sine of (the excess over six or nine signs, respectively, of) the difference between the longitudes of the Moon and the Sun in the same way as the *sita* is determined (in the light fortnight)."[8]

That is to say :

(i) In the light fortnight *(śukla-pakṣa)*

$$sita = \frac{R \text{ versin } (M—S) \times \text{Moon's diameter}}{6876},$$

if $M—S \leqslant 3$ signs, i.e. if it is the first half of the fortnight; and

$$= \frac{[\, R + R \sin (M—S—90°) \,] \times \text{Moon's diameter}}{6876},$$

if $M—S > 3$ signs, i.e. if it is the second half of the fortnight.

(ii) In the dark fortnight *(kṛṣṇa-pakṣa)*

$$asita = \frac{R \text{ versin } (M—S—180°) \times \text{Moon's diameter}}{6876},$$

if $M—S > 6$ signs, i.e. if it is the first half of the fortnight; and

$$= \frac{[\, (R + R \sin (M—S—270°) \,] \times \text{Moon's diameter}}{6876},$$

if $M—S > 9$ signs, i.e. if it is the second half of the fortnight.

Bhāskara I's contemporary Brahmagupta (628) gives the following rule, which is a via media between the above two rules:

"One half of the Moon's longitude minus Sun's longitude, multiplied by the Moon's diameter and divided by 90, gives the *sita*. This is the first result.

"When the Moon's longitude minus Sun's longitude, reduced to degrees, is less than or equal to 90°, take the *R* versed-sine of that; and when that exceeds 90°, take the *R* sine of the excess over 90° and add that to the radius. Multiply that by the measure of the Moon's diameter and divide by twice the radius (i.e. by 2×3438 or 6876). This is another result. The former result gives the *sita* in the night and the latter in the day. One half of their sum gives the same during the two twilights."[9]

That is :

$$\text{(i) } sita \text{ for night} = \frac{[\, (M—S)/2 \,] \times \text{Moon's diameter}}{90},$$

$M—S$ being in degrees.

$$\text{(ii) } sita \text{ for day} = \frac{R \text{ versin } (M—S) \times \text{Moon's diameter}}{2R},$$

if $M—S \leqslant 90°$; or

$$= \frac{[\, R + R \sin (M—S—90°) \,] \times \text{Moon's diameter}}{2R},$$

if $M - S > 90°$. ($R = 3438$)

(iii) *sita* for twilights $= \dfrac{\textit{sita} \text{ for day} + \textit{sita} \text{ for night}}{2}$.

These formulae obivously relate to the light half of the month.

Vaṭeśvara[10] (904) and Śrīpati[11] (1039) have followed Brahmagupta. Lalla[12] gives the two results stated by Brahmagupta, treating them as alternative. But whereas his commentator Mallikārjuna Sūri (1178) interprets them as alternative rules, his other commentator Bhāskara II (1150) makes no distinction between the rules of Brahmagupta and Lalla and interprets them in the light of Brahmagupta's rules. Bhāskara II has also attempted to explain why different formulae were prescribed for day, night and twilights. He says: "The first *sita*, being based on arc, is gross. This is to be used in the graphical representation of the Moon in the night, because then there is absence of the accompaniment of the Sun's rays. The second *sita*, being based on *R* sine, is accurate. This is to be used in the graphical representation of the Moon in the day, because then the Moon's rays being overpowered by the Sun's rays are not bright. During the twilights, the *sita* should be obtained by taking their mean value, because then the characteristic features of the day and night are medium."[13]

Āryabhaṭa II[14] (c. 950) and Bhāskara II[15] have prescribed the first result of Brahmagupta for all times. The method given in the *Sūrya-siddhānta*[16] is also essentially the same. The difference exists in form only.

According to Bhāskara II[17] the *sita* amounts to half when the Moon's longitude minus Sun's longitude is 85° 45′, not when it is 90° as presumed by the earlier astronomers. This means that he understood that the *sita* varies as the elongation of the Earth from the Sun (as seen from the Moon) and not as the Moon's longitude minus Sun's longitude. However, he has not stated this fact expressly, nor has he attempted to obtain the Earth's elongation from the Sun. Instead, he has applied a correction to the Moon in order to get the correct value of the *sita*.[18]

The astronomers who succeeded Bhāskara II have calculated the *sita* from the actual elongation of the Moon from the Sun and not from the difference between the Moon's longitude and the Sun's longitude. Since the actual elongation of the Moon from the Sun was the same as the angular distance between the discs of the Sun and Moon, these astronomers have called it *bimbāntara* ("the disc interval") and have calculated the *sita* by using it in place of the Moon's longitude minus Sun's longitude.

The *sita* really varies as the versed sine of the elongation of the Moon from the Sun (or more correctly as the versed sine of the elongation of the Earth from the Sun as seen from the Moon), not as the elongation of the Moon from the Sun (as measured on the ecliptic). So Brahmagupta's first result is gross and has been rightly criticized by the author of the *Valana-śṛṅgonnati-vāsanā*. "Brahmagupta and others (who have followed him)" says he, "have not considered the nature of the arc relation."[19] The rule given by Bhāskara I, however, is fairly good for practical purposes.

Special Rules

Muñjala (932), the author of the *Laghu-mānasa*, gives the following ingenious rule :

> "The number of *karaṇas* elapsed since the beginning of the (current) fortnight diminished by two and then (the difference obtained) increased by one-seventh of itself, gives the measure of the *sita* if the fortnight is white or the *asita* if the fortnight is dark."[10]

That is :

$$sita = (K—2)\ (1+1/7)\ a\dot{n}gulas,$$

where K denotes the number of *karaṇas* elapsed in the light fortnight, the diameter of the Moon being assumed to be 32 *aṅgulas.*

As the Moon is visible only when it is at the distance of 12 degrees from the Sun, i.e. when 2 *karaṇas* have just elapsed, so the proportion is made here with $180-12=168$ degrees, instead of 180 degrees. If M and S denote the longitudes of the Moon and the Sun in terms of degrees, the proportion implied is: "When $M—S—12$ degrees amount to 168 degrees, the measure of the *sita* is 32 *aṅgulas,* what will be the measure of the *sita* when $M—S—12$ degrees have the given value?" The result is:

$$sita = \frac{(M—S—12) \times 32}{168} = \left(\frac{M—S}{6} — 2\right) (1+1/7)$$
$$= (K — 2)\ (1+1/7)\ a\dot{n}gulas.$$

Similar is the rule stated by Gaṇeśa Daivajña (1520) :

> "The number of *tithis* elapsed in the light fortnight diminished by one-fifth of itself gives the measure of the *sita.*[11]

That is :

$$sita = (1—1/5)\, T\ a\dot{n}gulas,$$

T being the number of *tithis* elapsed in the light fortnight and the Moon's diameter being assumed to be equal to 12 *aṅgulas.*

Gaṇeśa Daivajña has applied proportion with the *tithis* elapsed in the light fortnight. His proportion is: "When on the expiry of 15 *tithis* the *sita* amounts to 12 *aṅgulas,* what will it amount to on the expiry of T *tithis?*" The result is :

$$sita = \frac{12\ T}{15} = (1—1/5)\ T\ a\dot{n}gulas.$$

Both the above rules are approximate.

It will be noticed that Gaṇeśa Daivajña's formula is the same as the first result of Brahmagupta. The difference is in form only.

Graphical Representation of the Sita

The Indian astronomers have also stated rules to exhibit the *sita* graphically. It enabled them to know which of the two lunar horns was higher than the other at the time of the Moon's first visibility, the knowledge of which is of importance in natural astrology.

Bhāskara I and other early astronomers have exhibited the *sita* by projecting the Sun and Moon in the plane of the observer's meridian. They have first constructed a rightangled triangle *MAS*, in which *S* denotes the projection of the centre of the Sun, *M* the projection of the centre of the Moon, and *MA* the projection of the altitude-difference of the Sun and Moon, all in the plane of the observer's meridian. *AS*, the horizontal side of this triangle, is called the base; *MA*, the vertical side, the perpendicular or upright; and *MS*, the hypotenuse.

Describing how the construction of the *sita* is to be done at sun-set in the first quarter of the lunar month, Bhāskara I says :

"Lay off the base from the Sun in its own direction. (Then) draw a perpendicular-line passing through the head and tail of the fish-figure constructed at the end (of the base). (This) perpendicular should be taken equal to the *R* sine of the Moon's altitude and should be laid off towards the east. The hypotenuse-line should (then) be drawn by joining the ends of that (perpendicular) and the base.

"The Moon is (then) constructed by taking the meeting point of the hypotenuse and the perpendicular as centre (and the semi-diameter of the Moon as radius); and along the hypotenuse (from the point where it intersects the periphery of the Moon) is laid off the *sita* towards the interior of the Moon.

"The hypotenuse (indicates) the east and west directions; the north and south directions should be determined by means of a fish-figure. (Thus are obtained the three points, viz.) the north point, the south point, and a third point obtained by laying off the *sita*. (Now) with the help of two fish-figures constructed by the method known as *triśarkarāvidhāna* draw the circle passing through the (above) three points. Thus is shown, by the elevation of the lunar horns which are illuminated by the light between two circles, the Moon which destroys the mound of darkness by her bundle of light."[12]

Fig. 8.1 illustrates how the construction is made at sunset in the first quarter of the month. *A S* and *M* are the projections of the centres of the Sun and Moon in the plane of the observer's meridian, and *MA* the projection of the Moon's altitude in the same plane.

The triangle *MAS* is right-angled at *A*, *SA* is called the base, *MA* the perpendicular or upright, and *MS* the hypotenuse of this triangle. In the present case the base

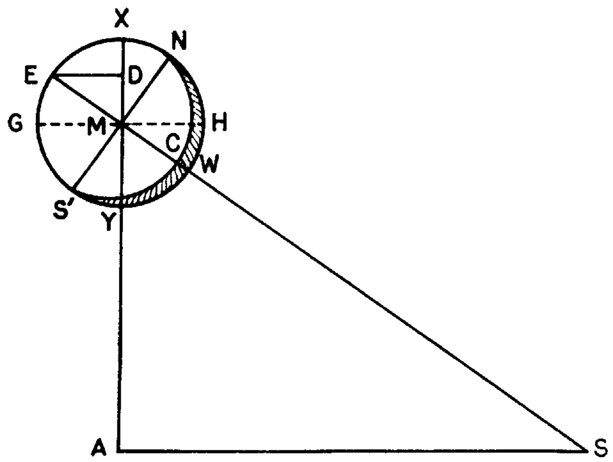

Fig. 8.1.

lies to the south of the Sun and the upright to the east of the base. The circle centred at *M* is the Moon's disc, i.e. the projection of the Moon's globe in the plane of the observer's meridian. The point *W* where *MS* intersects it is the west point of the Moon's disc. *E, N* and *Ś* are the east, north and south points. *WC* is the *Sita* which has been laid off, in the present case, from the west point *W* towards the interior of the Moon's disc. *NCŚ* is the circle drawn through *N, C* and *Ś*. The shaded portion of the Moon's disc between the circles *NWŚ* and *NCŚ* is the illuminated part of the Moon's disc; the remaining part of the Moon's disc does not receive light from the Sun and remains dark *(asita)* and invisible.

Let *GH* be drawn perpendicular to *MA* through *M*. Then the Moon's horn which is intersected by it lies to the north of the upright *MA*. This is the higher horn. The elevation of this higher horn is measured by the angle *NMH*. The other horn, viz. *CŚW,* which is not intersected by *GH* is the lower one. It lies to the south of the upright *MA*. So in the present case the northern horn is the higher one.

If the figure be held up with *MA* in the vertical position, the Moon in the sky will look like the shaded portion in the figure. This is what was intended.

The author of the *Sūrya-siddhānta* has followed the method of Bhāskara I. Describing the construction of the *sita,* at sun-rise in the last quarter of the month, he says:

> "Set down a point and call it the Sun. From it lay off the base in its own direction. From the extremity of that lay off the upright towards the west. Next draw the hypotenuse by joining the extremity of the upright and the point assumed as the Sun. Taking the junction of the upright and the hypotenuse as centre and the semi-diameter of the Moon at that time as the radius, draw the Moon's disc. Now with the help of the hypotenuse (assumed as the east-west line), first deter-

mine the directions (relative to the Moon's centre). From the point where the hypotenuse intersects the Moon's disc lay off the *sita* towards the interior of the Moon's disc. Between the point at the extremity of the *sita* and the north and south points draw two fish-figures. From the point of intersection of the lines going through them (taken as centre) draw an arc of a circle passing through the three points. As the Moon looks (in the figure) between this arc and the eastern periphery of the Moon's disc, so it looks in the sky that day. If the directions are determined with the help of the upright, the horn which is intersected by the line drawn at right angles to the upright through the Moon's centre is the higher one. The shape of the Moon should be demonstrated by holding up the figure keeping the upright in a vertical position."[23]

The construction given by Lalla is more general. He says :

"Take a point on the level ground and assume it to be the Sun. From this point lay off the base in its own direction (north or south). From the point reached lay off the upright. If the Moon is in the eastern hemisphere, the upright should be laid off towards the western direction; if the Moon is in the western hemisphere, it should be laid off towards the east. The hypotenuse should then be drawn by joining the extremity of the upright and the point assumed as the Sun. The Moon's disc should (then) be drawn by taking the junction of the hypotenuse and the upright as the centre. The hypotenuse-line here goes from west to east. The remaining (north and south) directions should be determined by means of a fish-figure. All this should be drawn very clearly with chalk. From the west point lay off the *sita* in the light fortnight or the *asita* in the dark fortnight (towards the interior of the Moon's disc). Taking the point thus reached, as also the north and south points (on the Moon's disc) as the centre draw two fish-figures. Where the mouth-tail lines of these fish-figures meet. taking that as the centre draw a neat circle passing through the *sita*-point inside the Moon's disc to exhibit the illuminated portion of the Moon. The direction in which the *aṅgulas* of the base have been laid off gives the direction of the depressed horn; the other horn is the elevated one."[14]

Āryabhaṭa II and Bhāskara II have omitted the construction of the triangle *MAS*. They have drawn the Moon directly with any point in the plane of the horizon as centre. Then they draw the direction-lines, i.e. the east-west and north-south lines. Assuming the north-osuth line as the same as the line *XY* of Fig. 8.1, they lay off *ED* (drawn orthogonally to the upright) which they call *digvalana* ("direction-deflection"). Having thus obtained the point *E* they draw *EW*, the line joining the Sun and Moon. After this their procedure is the same as that of Bhāskara I. The *digvalana ED* is evidently equal to

$$\frac{SA \times \text{Moon's diameter}}{MS}$$

which follows from the comparison of the similar triangles *MED* and *MAS*. See Fig. 8.1.

Brahmagupta does not project the Sun and Moon in the plane of the observer's meridian or any other plane. He keeps them where they are. So in the triangle *MAS*, which he constructs, *M* and *S* denote the actual positions of the centres of the Sun and Moon; *AS* is parallel to the north-south line of the horizon, and *MA* is perpendicular from *M* on this line. The Moon's disc is drawn in the plane of *MAS* with *M* as the centre. The laying off of the *sita* and the construction of the inner boundary of the *sita* is done as before.

Brahmagupta has been followed by Lalla and Śrīpati. Vaṭeśvara too follows Brahmagupta except in the case of sun-set or sunrise where he follows Bhāskara I.

Bhāskara II has pointed out a fallacy in the method of Brahmagupta. He says: "When the *sita* of the Moon is graphically shown in the way taught by him, using his base and hypotenuse, the lunar horns (shown in the figure) will not look like those seen in the sky. This is what I feel. Those proficient in astronomy should also observe it carefully. For, at a station in latitude 66°, the ecliptic coincides with the horizon and when the Sun is at the first point of Aries and the Moon at the first point of Capricorn the Moon appears vertically split into two halves by the observer's meridian and its eastern half looks bright. But this is not so in the opinion of Brahmagupta, for his base and upright are then equal to the radius. Actually, the tips of the lunar horns fall on a horizontal line when there is absence of the base, and on a vertical line when there is absence of the upright. Brahmagupta's base and upright then are both equal to the radius. Or, be as it may; I am not concerned. I bow to the great."[25]

Gaṇeśa Daivajña does not see any utility of the *parilekha* (graphical representation of the Moon). When from the direction of the *(dig) valana* itself, one can know which horn is high and which low, then, asks he, what is the use of the *parilekha?*[26]

The Visible Moon

In the present problem, we are concerned with the actual Moon and not with its calculated position on the ecliptic. The Indian astronomers have found it convenient to use, in place of the actual Moon, that point of the ecliptic which rises or sets with the actual Moon. This point of the ecliptic is called "the visible Moon" *(dṛśya-candra)*. This is derived from the calculated true Moon by applying to the latter a correction known as the visibility correction *(dṛkkarma* or *darśana-saṃskāra)*. The early astronomers, from Āryabhaṭa I to Bhāskara II, have applied two visibility corrections, viz. the *ayana-dṛkkarma* and the *akṣa-dṛkkarma*. The former is the portion of the ecliptic that lies between the secondaries to the ecliptic and the equator going through the actual Moon, and the latter is the portion of the ecliptic that lies between the horizon and the secondary to the equator going through the actual Moon, the actual Moon being supposed to be on the horizon.

Āryabhaṭa I gives the following rule for deriving the above-mentioned visibility corrections :

"Multiply the *R* versed-sine of the Moon's (tropical) longitude (as increased by three signs) by the Moon's latitude and also by the (*R* sine of the Sun's) greatest declination and divide (the resulting product) by the square of the radius: (the result is the *ayana-dṛkkarma* for the Moon). When the Moon's latitude is north, it should be subtracted from or added to the Moon's longitude, according as the Moon's *ayana* is north or south (i.e. according as the Moon is in the six signs beginning with tropical sign Capricorn or in the six signs beginning with the tropical sign Cancer); When the Moon's latitude is south, it should be added or subtracted (respectively)."[27]

"Multiply the *R* sine of the latitude of the local place by the Moon's latitude and divide (the resulting product) by the *R* sine of the co-latitude: (the result is the *akṣa-dṛkkarma* for the Moon). When the Moon is to the north (of the ecliptic), it should be subtracted from the Moon's longitude (as corrected for the *ayana-dṛkkarma*) in the case of the rising of the Moon and added to the Moon's longitude in the case of the setting of the Moon; when the Moon is to the south (of the ecliptic), it should be added to the Moon's longitude (in the case of the rising of the Moon) and subtracted from the Moon's longitude (in the case of the setting of the Moon)."[28]

If β be the Moon's latitude and M the Moon's tropical longitude, then the above rules are equivalent to the following formulae:

$$\textit{ayana-dṛkkarma} = \frac{R \text{ versin } (M+90°) \times \beta \times R \sin 24°}{R \times R} \tag{1}$$

and

$$\textit{akṣa-dṛkkarma} = \frac{R \sin \phi \times \beta}{R \cos \phi} \tag{2}$$

ϕ being the latitude of the place and 24° being the Indian value of the Sun's greatest declination.

The same formulae occur in the works of Lalla,[29] Vaṭeśvara[30] and Śrīpati.[31]

These formulae are approximate and were modified by the later astronomers. Brahmagupta[32] replaced (1) by the better formula :

$$\textit{ayana dṛkkarma} = \frac{R \sin (M + 90°) \times \beta \times R \sin 24°}{R \times R}.$$

This formula reappears in the *Mahā-siddhānta*[33] of Āryabhaṭa II in the form :

$$\textit{ayana-dṛkkarma} = \frac{R \cos M \times \beta \times R \sin 24°}{R \times R}.$$

Śripati, while retaining the use of the *R* versed-sine, has improved (1) by multiplying it by 1800 and dividing by the *asus* of the rising of the sign occupied by the Moon.[34] (The *asus* are the minutes of arc of the equator).

Bhāskara II has criticized the use of the R versed-sine and has applauded Brahmagupta for replacing the R versed-sine by the R sine. He has also given the following new formulae:[35]

$$\text{(i) } \textit{ayana-dṛkkarma} = \frac{R \sin (\textit{ayanavalana}) \times \beta}{R \cos \delta} \times \frac{1800}{T}$$

where the *ayanavalana* is the angle between the secondaries to the equator and the ecliptic going through the Moon, δ the Moon's declination, and T the time of rising (in *asus*) of the sign occupied by the Moon.

$$\text{(ii) } \textit{ayana-dṛkkārma} = \frac{R \sin (\textit{ayanavalana}) \times \beta}{R \cos (\textit{ayanavalana})}$$

Formula (2) was modified by Bhāskara II. For his modified formulae the reader is referred to his *Siddhānta-śiromaṇi* (Part I, ch. vii, vss. 3, 6-8 and Part II, ch. ix, vs. 10).

Altitudes of Sun and Moon

To determine the Sun's altitude for the given time one has to know the Sun's ascensional difference and the earthsine. The Sun's ascensional difference is the difference between the times of rising of the Sun at the equator and at the local place. It is measured by the *asus* (minutes of equator) lying between the hour circle through the east point (called the six O'clock circle) and the hour circle through the rising Sun. The formula used to obtain it is :

$$R \sin c = \frac{R \sin \phi \times R \sin \delta \times R}{R \cos \phi \times R \cos \delta}$$

or, in modern notation,

$$\sin c = \tan \phi \tan \delta,$$

where c denotes the ascensional difference, δ the declination, and ϕ the latitude of the place.

The Sun's declination is obtained by the formula :

$$R \sin \delta = \frac{R \sin \lambda \times R \sin 24^\circ}{R}$$

where λ is the Sun's tropical longitude and 24° the Indian value of the obliquity of the ecliptic.

The earthsine is the R sine of c reduced to the radius of the diurnal circle and is obtained by the formula :

$$\text{earthsine} = \frac{R \sin c \, R \cos \delta}{R} = \frac{R \sin \phi \, R \sin \delta}{R \cos \phi}.$$

The Sun's ascensional difference and the earthsine being thus known, the Sun's altitude can be determined. Bhāskara I gives the following rule to find the Sun's altitude when the time elapsed since sunrise in the forenoon or to elapse before sunset in the afternoon is known :

"The *ghaṭīs* elapsed (since sun-rise) or those to elapse (before sun-set), in the first half and the other half of the day (respectively), should be multiplied by 60 and again by 6: then they (i.e. those *ghaṭīs*) are reduced to *asus*. (When the Sun is) in the northern hemisphere, the *asus* of the Sun's ascensional difference should be subtracted from them and (when the Sun is) in the southern hemisphere, the *asus* of the Sun's ascensional difference should be added to them. (Then) calculate the *R* sine (of the resulting difference or sum) and multiply that by the day-radius (i.e. by $R \cos \delta$). Then dividing that (product) by the radius, operate (on the quotient) with the earthsine contrarily to the above (i.e. add or subtract the earthsine according as the Sun is in the northern or southern hemisphere). Multiply that (sum or difference) by the *R* sine of the co-latitude and divide by the radius: the result is the *R* sine of the Sun's altitude."[36]

"When the Sun's ascensional difference cannot be subtracted from the given (time reduced to) *asus*, reverse the subtraction (i.e. subtract the latter from the former) and with the *R* sine of the remainder (proceed as above). In the night the *R* sine of the Sun's altitude should be obtained contrarily (i.e. by reversing the laws of addition and subtraction)."[37]

That is, when the Sun is in the northern hemisphere,

$$R \sin a = \frac{[\{R \sin (T \mp c)\ R \cos \delta)\}\ /R \pm \text{earthsine}]\ R \cos \phi,}{R}$$

where a denotes the Sun's altitude, δ the Sun's declination, T the time elapsed since sunrise in the forenoon or to elapse before sun-set in the afternoon (reduced to *asus*), c the Sun's ascensional difference (in *asus*), and ϕ the local latitude, the sign $+$ or $\sim$ being chosen properly, depending on the Sun's position.

In Fig. 8.2, which represents the celestial sphere for a place in latitude ϕ, $S'ENW$ is the horizon, E, W, N and S' being the east, west, north and south points; Z is the zenith. RER' is the equator and P its north pole. S is the Sun and LSM its diurnal circle. VU' is the Sun's rising-setting line and EW the east-west line. SA is the perpendicular from the Sun on the plane of the horizon and SB on the rising-setting line; AB is perpendicular to the rising-setting line. C is the point where AB intersects EW. SA is the R sine of the Sun's altitude, AB is the Sun's *śaṅkutala*, CB the Sun's *agrā*, AC the Sun's *bhuja*, and SB the Sun's *iṣṭahṛti*.

It can be easily seen that

$$SB = \{R \sin (T - c)\ R \cos \delta\}\ /R + \text{earthsine},$$

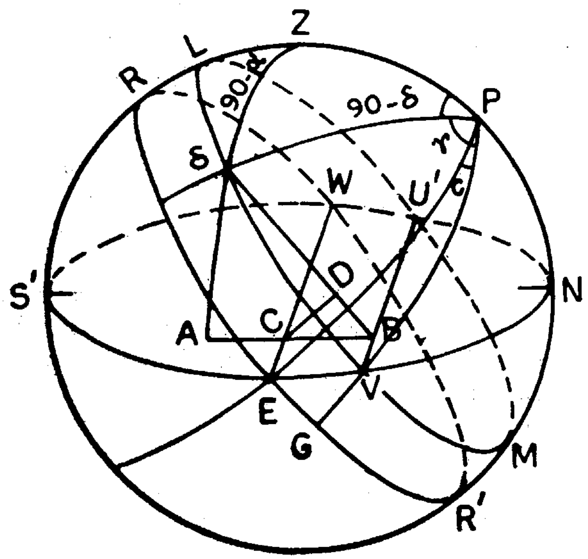

Fig. 8.2.

so that from the triangle SAB, right-angled at A, in which $\angle SAB = 90° - \phi$, we easily have

$$SA \text{ or } R \sin a = \frac{SB \times R \cos \phi}{R}$$

$$= \frac{[\{R \sin (T - c)\ R \cos \delta\}/R + \text{earthsine}]\ R \cos \phi}{R}.$$

Using modern spherical trigonometry, the rationale of this rule is as follows :

In Fig. 8.2, $\angle VPS = T$ and $\angle VPE = c$, so that $\angle EPS = T - c$, and likewise $\angle ZPS = 90° - (T - c)$. Now in the spherical triangle ZPS, we have $ZS = 90° - a$, $ZP = 90° - \phi$, $SP = 90° - \delta$, and $\angle ZPS = 90° - (T - c)$. Therefore, using cosine formula, we have

$$\cos ZS = \cos ZP \cos SP + \sin ZP \sin SP \cos \angle ZPS$$

or $\sin a = \sin \phi \sin \delta + \cos \phi \cos \delta \sin (T - c)$, and multiplying by R and re-arranging,

$$R \sin a = \frac{[\{R \sin (T - c)\ R \cos \delta\}/R + R \sin \phi\ R \sin \delta / R \cos \phi]\ R \cos \phi}{R}$$

$$= \frac{[R \sin (T - c)\ R \cos \delta\}/R + \text{earthsine}]\ R \cos \phi}{R}.$$

This is true when the Sun is in the northern heimsphere and above the horizon. Similar rationale may be given when the Sun is in the southern hemisphere or below the horizon.

The Moon's altitude is obtained in the same way. But in this case one has to use the Moon's true declination, i.e. the declination of the actual Moon. For this, in the present context, the early Indian astronomers, from Āryabhaṭa I to Bhāskara II, use the following approximate formula :

$$\text{Moon's true declination} = \delta \pm \beta,$$

where δ is the declination of the Moon's projection on the ecliptic and β the Moon's latitude.

In place of the time elapsed since sun-rise or to elapse before sunset, one has to use the time elapsed since moon-rise or to elapse before moon-set. The methods used to find the time of moon-rise or moon-set will be described in the next chapter.

Base and Upright

The base SA of the triangle MAS (in Fig. 8.1) is equal to the difference or sum of the Sun's *bhuja* and the Moon's *bhuja*. In case the Sun and the Moon are both above the horizon, the difference is taken provided the Sun and Moon are on the same side of the east-west line; otherwise the sum is taken. The *bhuja* of a heavenly body is defined by the distance of its projection on the plane of the horizon from the east-west line, so that

bhuja = distance of projection from the east-west line

= distance of projection from the rising-setting line (called *śaṅkutala* or *śaṅkvagra*) ± distance between the east-west and rising-setting lines (called *agrā*)

= *śaṅkutala* ± *agrā*.

In Fig. 8.2, S is the Sun. A is the Sun's projection on the plane of the horizon, AB is the Sun's *śaṅkutala*, CB is the Sun's *agrā*, and AC the Sun's *bhuja*. It is evident from the figure that in this case

$$AC = AB - CB$$

i.e. Sun's *bhuja* = Sun's *śaṅkutala* — Sun's *agrā*.

The Sun's *śaṅkutala* AB is obtained from triangle SAB (of Fig. 8.2) by using the sine relation :

$$AB \text{ or Sun's } \textit{śaṅkutala} = \frac{SA \times R \sin \angle ASB}{R \sin \angle SBA}$$

$$= \frac{R \sin a \times R \sin \phi}{R \cos \phi}.$$

Bhāskara I says :

"The R sine of the Sun's altitude multiplied by the R sine of the latitude and divided by the R cosine of the latitude is the (Sun's) *śaṅkvagra,* which is always to the south of the rising-setting line."[38]

The Sun's *agrā* is obtained thus: In Fig. 8.2, let CD be the perpendicular from C to SB. Then in the triangle CDB, right-angled at D, CB = Sun's *agrā,* $CD = R \sin \delta$, $\angle CBD = 90°—\phi$, so that

$$CB \text{ or Sun's } agr\bar{a} = \frac{AD \times R \sin \angle CDB}{R \sin \angle CBD}$$

$$= \frac{R \sin \delta \times R}{R \cos \phi}$$

Brahmagupta says :

"The R sine of the declination multiplied by the radius and divided by the R sine of the co-latitude is the *agrā* which lies east-west in the plane of the horizon."[39]

The Moon's *bhuja* is obtained in the same way, using the Moon's true declination. The difference or sum of the Moon's *bhuja* and Sun's *bhuja* finally gives the base.

When the calculations are made for sunset, the Sun's *agrā* itself is the Sun's *bhuja.* In that case, the difference or sum of the Moon's *bhuja* and the Sun's *agrā* gives the base.

Bhāskara I says :

"From the *asus* intervening between the Sun and Moon (corrected for the visibility corrections) and from the Moon's earthsine and ascensional difference, determine the R sine of the (Moon's) altitude; and from that find the (Moon's) *śaṅkvagra,* which is always south (of the rising-setting line of the Moon).

"The R sine of the difference or sum of the (Moon's) latitude and declination according as they are of unlike or like directions is (the R sine of) the Moon's true declination. From that (R sine of the Moon's true declination) determine her day-radius, etc. Then multiply (the R sine of) the Moon's (true) declination by the radius and divide by (the R sine of) the co-latitude: then is obtained (the R sine of) the Moon's *agrā.*

"If that (R sine of the Moon's *agrā*) is of the same direction as the (Moon's) *śaṅkvagra,* take their sum; otherwise, take their difference. Thereafter take the difference of (the R sine of) the Sun's *agrā* and that (sum or difference), if their directions are the same; otherwise, take their sum: thus is obtained the *base* (*bāhu* or *bhuja*)."[40]

The difference of the R sines of the Moon's altitude and the Sun's altitude during the day or their sum during the night, obviously, gives the upright, When the calculations are made for sun-set, the R sine of the Moon's altitude itself is the upright, as the Sun then is on the horizon and its altitude is zero.

The base and upright obtained in the above way are according to those astronomers who, like Bhāskara I, project the Sun and Moon in the plane of the meridian. Brahmagupta and his followers, who keep the Sun and Moon where they are, obtain their base and upright, which shall be called Brahmagupta's base and upright, thus:

Brahmagupta's base $= b \pm b'$

Brahmagupta's upright $= \sqrt{(k \pm k')^2 + (R \sin a \pm R \sin a')^2}$, where b, b'; k, k'; a, a' are the *bhujas*, uprights and altitudes of the Sun and Moon respectively, derived in the manner described above.

It will be noted that whereas Brahmagupta's upright differs in length from that of Bhāskara I, his base is exactly equal to that of the latter.

RISING AND SETTING OF PLANETS AND STARS

Heliacal Rising and Setting of the Planets

When a planet gets near the Sun, it is lost in the dazzling light of the Sun and becomes invisible. The planet is then said to set heliacally. Sometimes later the planet comes out of the dazzling light and is seen again. It is then said to rise heliacally. In the case of the Moon, a special term *candra-darśana* ("Moon's first appearance") is used for its heliacal rising.

Brahmagupta says :

"A planet with lesser longitude than the Sun rises in the east, in case it is slower than the Sun; in the contrary case, it sets in the east. A planet with greater longitude than the Sun rises in the west, in case it is faster than the Sun; and sets in the west, in case it is slower than the Sun."[41]

The author of the *Sūrya-siddhānta* says :

"Jupiter, Mars, and Saturn, when their longitude is greater than that of the Sun, go to their setting in the west; when it is lesser, to their rising in the east; so likewise Venus and Mercury, when retrograding. The Moon, Mercury, and Venus, having a swifter motion, go to their setting in the east when of lesser longitude than the Sun; when of greater, go to their rising in the west."[42]

Vaṭeśvara's account is fuller and more explicit:

"A planet with lesser longitude (than the Sun) rises in the east if it is slower than the Sun; and sets in the east if it is faster than the Sun; whereas a planet with greater longitude (than the Sun) rises in the west if it is faster than the Sun, and sets in the west if it is slower than the Sun.

"The Moon, Venus, and Mercury rise in the west, whereas Saturn, Mars, and Jupiter and also retrograding Mercury and Venus rise in the east. These planets set in the opposite direction."[43]

Similar statements have been made by Lalla,[44] Āryabhaṭa II,[45] Śrīpati,[46] Bhāskara II[47] and others.

The distances from the Sun at which the heliacal rising or setting occurs is not the same for all the planets. It depends upon the size and luminosity of the planet. The larger or more luminous it is, the lesser will be its distance from the Sun at the time of its rising or setting.

The Indian astronomers state the distances of the planets from the Sun at the time of their first visibility ("rising") or last visibility ("setting") in terms of time-degrees, i.e. in terms of time, between the time of rising or setting of the planet and that of the Sun, converted into degrees by the formula :

60 *ghaṭis* or 24 hours = 360 degrees.

Āryabhaṭa I says :

"When the Moon has no latitude it is visible when situated at a distance of 12 degrees (of time) from the Sun. Venus is visible when 9 degrees (of time) distant from the Sun. The other planets taken in the order of decreasing sizes (viz. Jupiter, Mercury, Saturn, and Mars) are visible when they are 9 degrees (of time) increased by two-s (i.e. when they are 11, 13, 15 and 17 degrees of time) distant from the Sun."[48]

The same distances have been given in the *Āryabhaṭa-siddhānta* and the *Khaṇḍakhādyaka,*[49] and by Brahmagupta,[50] Lalla,[51] Vaṭeśvara,[52] and Śrīpati.[53] Those given by Āryabhaṭa II,[54] Bhaskara II,[55] and by the author of the *Sūrya-siddhānta*[56] slightly differ in one or two cases.

Regarding Venus and Mercury, Brahmagupta says :

"Owing to its small disc, Venus (in direct motion) rises in the west and sets in the east at a distance of 10 time-degrees (from the Sun); and owing to its large disc, the same planet (in retrograde motion) sets in the west and rises in the east at a distance of (only) 8 time-degrees (from the Sun). Mercury rises and sets in a similar manner when its distance (from the Sun) is 14 time-degrees (in the case of direct motion) or 12 time-degrees (in the case of retrograde motion)."[57]

So has been said by the author of the *Sūrya-siddhānta*[58] and Śrīpati.[59]

Time-degrees for heliacal rising and setting

Celestial body	Time-degrees according to Āryabhaṭa I	Brahmagupta	Lalla and Vaṭeśvara	Āryabhaṭa II	*Sūrya-siddhānta* and Bhāskara II
Moon	12°	12°	12°	12°	12°
Mars	17°	17°	17°	17°	17°
Mercury	13°	13° (mean), 14°	13°	13°	14°
Mercury (retro)		12°	12°	12°30′	12°
Jupiter	11°	11°	11°	12°	11°
Venus	9°	9° (mean) 10°	9°	8°	10°
Venus (retro)		8°	8°	7°30′	8°
Saturn	15°	15°	15°	15°	15°

To find the day on which a planet is to rise or set heliacally in the east or west, the Indian astronomers proceed as follows: In the case of rising or setting in the east, they first calculate for sunrise the longitudes of the Sun and the planet, the latter being corrected by the visibility corrections for rising. Then, using the table giving the times of rising of the signs for the local place, they calculate the time of rising of the portion of the ecliptic lying between the Sun and the corrected planet. This they convert into time-degrees, and then find the difference between these time-degrees and the time-degrees for rising or setting of the planet under consideration. If the planet is in direct motion they divide this difference by the degrees of difference between the daily motions of the Sun and the planet; and if the planet is in retrograde motion they divide that difference by the degrees of the sum of the daily motions of the Sun and the planet. The quotient obtained gives the days elapsed since or to elapse before the rising or setting of the planet in the east.

In the case of rising or setting in the west, they first calculate for sun-set the longitudes of the Sun and the planet, the latter corrected by the visibility corrections for setting. Both these longitudes are increased by six signs. Then, using the table giving the times of rising of the signs for the local place, they calculate the time of rising of the portion of the ecliptic lying between the Sun as increased by six signs and the corrected planet as increased by six signs. This they convert into time-degrees, and find the differenee between these time-degrees and the time-degrees for rising or setting of the planet under consideration. If the planet is in direct motion they divide this difference by the degrees of difference between the daily motions of the Sun and the planet; and if the planet is in retrograde motion they divide that difference by the degrees of the sum of the daily motions of the Sun and the planet. The quotient obtained gives, as before, the days elapsed since or to elapse before the rising or setting of the planet in the west.

Bhāskara I says :

"(When the planet is to be seen) in the east, (its) visibility should be announced by calculating the time (of rising of the part of the ecliptic between the Sun and the planet corrected by the visibility corrections) by using the time of rising at the local place of that very sign (in which the Sun and the planet are situated); (when the planet is to be seen) in the west, (its) visibility should be announced by calculating the time (of setting of the part of the ecliptic between the Sun and the planet) by using the time of rising of the seventh sign at the local place."[60]

Lalla describes the method as follows :

"(If the heliacal rising or setting of a planet) on the western horizon is considered, the true longitude of the Sun and the *dṛggraha* (i.e. the planet corrected by the visibility corrections for setting) should each be increased by six signs.

"Find the *asus* of rising of the untraversed part of the sign occupied by the planet with lesser longitude and the *asus* of rising of the traversed part of the sign occupied by the planet with greater longitude. To the sum of the two add the *asus* of rising of the intervening signs. The result divided by 60 gives the time-degrees of the planet's distance from the Sun. If these time-degrees are lesser than the time-degrees stated for the rising or setting of the planet, it must be understood that the planet is invisible.

"Find the difference expressed in minutes between the calculated time-degrees and the time-degrees for rising or setting of the planet. Divide it by (the minutes of) the difference of the daily motions of the Sun and the planet if they are moving in the same direction, or by (the minutes of) the sum if they are moving in opposite directions. The quotient gives the days elapsed since or to elapse before the rising or setting of the planet, which is to be understood by the following consideration.

"When the setting of a planet is considered, if the time-degrees for rising or setting of the planet are greater than the calculated time-degrees, know then that the planet has set heliacally before the number of days found above; if the former is lesser, the planet will set after so many days. When the rising is considered, then in the former case the planet will rise after the days calculated and in the latter case the planet has risen before the days calculated."[61]

Vaṭeśvara explains the method thus :

"In the case of rising in the east, find the *asus* of rising of the untraversed part of the sign occupied by the planet (computed for sunrise and corrected by the visibility corrections for rising), those of the traversed part of the sign occupied by the Sun (at sunrise); and in the case of setting in the west, in the reverse order.[61] Add them to the *asus* of rising of the intervening signs. (Then are obtained the *asus* of rising of the part of the ecliptic lying between the planet corrected by the visibility corrections and the Sun at sunrise. These divided by 60 give the time-degrees between the planet corrected by the visibility corrections and the Sun).

"(To obtain the time-degrees corresponding to the traversed and untraversed parts), one should multiply the untraversed and traversed degrees by the *asus* of rising of the corresponding signs and divide (the products) by 30 and 60 (i.e. by 1800). The time-degrees divided by 6 give the corresponding *ghaṭis*.

"When the time-degrees between the planet corrected by the visibility corrections and the Sun are greater than the time-degrees for the planet's rising or setting, it should be understood that the planet has already risen (heliacally); if lesser, the rising has not yet taken place.

"Divide their difference by the daily motion of the Sun diminished by the daily motion of the planet when the planet is in direct motion, and by the daily motion of the Sun increased by the daily motion of the planet when the planet is in retrograde motion: the result is the time (in days) which have to elapse before the planet will rise or set or elapsed since the rising or setting of the planet."[63]

Similar rules have been given by the other Indian astronomers.[64]

Heliacal Rising and Setting of the Stars

The stars having no motion rise in the east and set in the west. The distance from the Sun at which they rise or set heliacally, according to the Indian astronomers, is 14 time-degrees or $6\frac{1}{3}$ *ghaṭis*. In the case of Canopus (Agastya) this distance is 12 time-degrees or 2 *ghaṭis* and in the case of Sirius (Mṛgavyādha) it is 13 time-degrees or $2\frac{1}{6}$ *ghaṭis*. This is so because Canopus and Sirius are bright stars.

The point of the ecliptic which rises on the eastern horizon exactly when a star rises on the eastern horizon, is called the star's *udayalagna;* and the point of the ecliptic which rises on the eastern horizon exactly when a star sets on the western horizon, is called the star's *astalagna*. Similarly, the point of the ecliptic occupied by the Sun when a star rises heliacally is called the star's *udayārka* or *udayasūrya;* and the point of the ecliptic occupied by the Sun when a star sets heliacally is called the star's *astārka* or *astasūrya*.

The positions of the stars are given in terms of their polar longitudes. So only one visibility correction, viz. the *akṣa-dṛkkarma,* has to be applied to them. When the *akṣa-dṛkkarma* for rising is applied to the polar longitude of a star, one gets the star's *udayalagna;* and when the *akṣa-dṛkkarma* for setting is applied to the polar longitude of a star and six signs are added to that, one gets the star's *astalagna.*

The *udayārka* for a star is obtained by calculating the *lagna* (the rising point of the ecliptic), by taking the Sun's longitude as equal to the star's *udayalagna* and the time elapsed since sunrise as equal to the *ghaṭīs* of the star's distance from the Sun at the time of its heliacal rising. The *astārka* for a star is obtained by calculating the *lagna*, by taking the Sun's longitude as equal to the star's *astalagna* and the time to elapse before sunrise as equal to the *ghaṭīs* of the star's distance from the Sun at the time of its heliacal visibility, and adding six signs to that.

Taking the case of Canopus and Sirius, Brahmagupta says :

"From the *udayalagna* of Canopus calculate the *lagna* at two *ghaṭīs* after sunrise by means of the times of rising of the signs (at the local place). The result is the *udayasūrya* of Canopus. Again from the *astalagna* (of Canopus) calculate the *lagna* at two *ghaṭīs* before sunrise, and add six signs to it. The result is the *astasūrya* of Canopus.

"In the same manner the *udayasūrya* and *astasūrya* of Sirius may be found. In this case $2\frac{1}{6}$ *ghaṭīs* should be used.

"Similarly, the *udayasūrya* and *astasūrya* of other stars should be calculated. In this case $2\frac{1}{3}$ *ghaṭīs* should be used.

"Canopus, Sirius or any of the (other) stars rises or sets according as its *udayasūrya* or *astasūrya* is the same as the true Sun."[65]

Lalla says :

"On account of the motion of the provector wind, the rising of a star occurs with the rising of its *udayalagna,* and the setting of a star occurs with the rising of its *astalagna.*

"Two *ghaṭīs* plus one-third of a *ghaṭī* is the time-distance of a star from the Sun at the time of its heliacal rising or setting; that for Sirius, it is two *ghaṭīs* plus one-sixth of a *ghaṭī;* and that for Canopus, it is two *ghaṭīs.*

"The star whose *udayalagna* increased by the result due to that time-distance (i.e. by the arc of the ecliptic that rises in that time) happens to be equal to the Sun's longitude (at that time), rises heliacally; and the star whose *astalagna* diminished by the result due to that time-distance and also by six signs, happens to be equal to the Sun's longitude (at that time), sets heliacally."[66]

So also says Vaṭeśvara :

"When the longitude of the Sun is equal to the longitude of the star's *udayalagna* as increased by the result obtained on converting the 14 time-degrees for the star's heliacal rising or setting into the corresponding arc of the ecliptic (which rises at the local place in that time), the star rises heliacally; and when the longitude of the Sun is equal to the longitude of the *astalagna* as diminished by the result due to the time-degrees for the star's heliacal rising or setting and by half a circle, the star sets heliacally.

"When the star's *udayalagna* or *astalagna* is at a lesser distance from the Sun, the star is invisible; in the contrary case, the star is visible."[67]

A similar statement has been made by Bhāskara II.[68]

As regards the duration of a star's visibility or invisibility, Brahmagupta says:

"A star is visible as long as the Sun lies between its *udayasūrya* and *astasūrya;* otherwise, it is invisible.

"Find the difference between the star's *udayasūrya* and the Sun, or between the *astasūrya* and the Sun. Express the difference in minutes. Divide each difference by the daily motion of the Sun. The result gives respectively the number of days passed since the heliacal rising of the star and those which will pass before the star sets heliacally."[69]

Lalla says :

"As long as the Sun is between the star's *udayārka* and *astārka,* so long is the Sun heliacally visible, provided that the star's declination diminished or increased by the local latitude according as they are of like or unlike directions, is less than 90°.

"As long as the Sun is between the star's *astasūrya* and *udayasūrya,* so long is the star heliacally invisible."

"The difference between the two (i.e. the star's *udayasūrya* minus the star's *astasūrya*) expressed in minutes, when divided by the true daily motion of the Sun, gives the days (for which the star is invisible)."[70]

Śrīpati says :

"As long as the Sun is between the star's *udayasūrya* and the star's *astasūrya,* so long is the star heliacally visible; and as long as the Sun is between the star's *astasūrya* and the star's *udayasūrya,* so long is the star invisible. The star, however, is seen as long as its zenith distance is less than 90°."[71]

Vaṭeśvara :

"Subtract the star's *astārka* from the star's *udayārka* and reduce the difference to minutes. Divide these minutes by the minutes of the Sun's daily motion.

Then is obtained the number of days during which the star remains set heliacally."[72]

Stars Always Visible Heliacally

The stars which are far away from the ecliptic do not fall prey to the dazzling light of the Sun. Such stars are always visible heliacally. The author of the *Sūrya-siddhānta* says :

"Vega (Abhijit), Capella (Brahmahṛdaya), Arcturus (Svātī), α Aquilae (Śravaṇa), β Delphini (Śraviṣṭhā), and λ Pegasi (Uttara-Bhādrapada), owing to their (far) northern situation, are not extinguished by the Sun's rays."[73]

These stars have large latitudes and in their case the *astasūrya* exceeds the *udayasūrya*. The latter is the condition for a star's permanent heliacal visibility.

Brahmagupta says :

"The star whose *udayārka* is smaller than its *astārka* is always visible."[74]

Lalla says :

"The star, whose *astārka* is greater than its *udayārka,* never sets heliacally."[75]

So also say the other Indian astronomers.[76]

Diurnal Rising and Setting

The rising of the heavenly bodies every day on the eastern horizon is called the diurnal rising of those heavenly bodies. Similarly the setting of the heavenly bodies on the western horizon is called their diurnal setting. It is this rising or setting that is meant when one talks of sun-rise or sun-set, moon-rise or moon-set.

The rising of the Sun does not present any difficulty, because it is taken as the starting point of time measurement. The rising and setting of the Moon are indeed of importance to the Indian astronomers. All astronomical works deal with them and give rules to find the time of moon-set or moon-rise in the light and dark fortnights of the month.

Bhāskara I gives the following rules to find the time of moon-set or moon-rise : "In the light fortnight, find out the *asus* due to oblique ascension (of the part of the ecliptic) intervening between the Sun (at sun-set) and the (visible) Moon (at sun-set treated as moon-set) both increased by six signs, and apply the method of successive approximations. This gives the duration of the visibility of the Moon (at night) (or, in other words, the time of moon-set)."[77]

"Thereafter (i.e. in the dark fortnight) the Moon is seen (to rise) at night (at the time) determined by the *asus* (due to oblique ascension) derived by the method of successive approximations from the part of the ecliptic intervening between the Sun as increased by six signs and the (visible) Moon as obtained by computation, (the Sun and the Moon both being those calculated for sun-set)."[78]

Further he says :

"(In the light half of the month) when the measure of the day exceeds the *nāḍis* (due to the oblique ascension of the part of the ecliptic) lying between the Sun and the (visible) Moon (computed for sun-set), the moonrise is said to occur in the day when the residue of the day (i.e. the time to elapse before sun-set) is equal to the *ghaṭis* of their difference."[79]

"(In the dark half of the month) find out the *asus* due to the oblique ascension of the part of the ecliptic lying from the setting Sun up to the (visible) Moon; and therefrom subtract the length of the day. (This approximately gives the time of moon-rise as measured since sun-set). Since the Moon is seen (to rise) at night when so much time, corrected by the method of successive approximations, is elapsed, therefore the *asus* obtained above should be operated upon by the method of successive approximations."[80]

"Or, determine the *asus* (due to the oblique ascension of the part of the ecliptic lying) from the (visible) Moon at sun-rise up to the rising Sun; then subtract the corresponding displacements (of the Moon and the Sun) from them (i.e. from the longitudes of the visible Moon and the Sun computed for sun-rise); and on them apply the method of successive approximations (to obtain the nearest approximation to the time between the visible Moon and the Sun computed for moon-rise, i.e. between the risings of the Moon and the Sun). The Moon,. . . ., rises as many *asus* before sun-rise as correspond to the *nāḍis* obtained by the method of successive approximations."[81]

Bhāskara I has given the details of the implied processes of successive approximations also.

Vaṭeśvara gives the methods of finding out the time of moon-rise and moon-set thus :

"In the light half of the month the calculation of the time of rising of the Moon in the day is prescribed to be made from the positions of the Sun and the (visible) Moon (for sun-rise) in the manner stated before; and that of the time of setting of the Moon at the end of the day (i.e. at night) from the positions of the Sun and the (visible) Moon (for sun-set), both increased by six sings.

"In the dark half of the month, the (time of) rising of the Moon, when the night is yet to end, should be calculated by the process of iteration from the positions of the Sun and the (visible) Moon (for sun-rise); and in the light half of the month, the (time of) rising of the Moon, when the day is yet to end, should be calculated by the process of iteration from the position of the Sun (for sun-set) increased by six signs and the position of the (visible) Moon (for sun-set).

"In the dark half of the month, the time of setting of the Moon, when the day is yet to elapse, should be obtained from the positions of the Sun and the (visible) Moon (for sun-set), each increased by six signs.

"In the light half of the month, the same time (of setting of the Moon), when the night is yet to elapse, should be obtained from the positions of the (visible) Moon (for sun-rise) increased by six signs and the position of the Sun (for sun-rise)."[82]

Similar methods have been prescribed by the other Indian astronomers also.

The phenomenon of moon-rise on the full moon day is of special importance and the Indian astronomers have dealt with this topic separately. Bhāskara I says :

"If (at sun-set) on the full moon day the longitude of the Moon (corrected for the visibility corrections for rising) agrees to minutes with the longitude of the Sun (increased by six signs), then the Moon rises simultaneously with sun-set. If (the longitude of the Moon is) less (than the other), the Moon rises earlier; if (the longitude of the Moon is) greater (than the other), the Moon rises later.

"(In the latter cases) multiply the minutes of the difference by the *asus* of the oblique ascension of the sign occupied by the Moon and divide by the number of minutes of arc in a sign, and on the resulting time apply the method of successive approximations (to get the nearest approximation to the time to elapse at moon-rise before sun-set or elapsed at moon-rise since sun-set)."[83]

Lalla says :

"If the true longitude of the Moon, (corrected for the visibility corrections for rising), is the same as the true longitude of the Sun at sun-set, increased by six signs, the Moon rises at the same time as the Sun sets; if greater, it rises later; and if less, it rises before sun-set.

If the true longitude of the Moon, (corrected for the two visibility corrections for setting) and increased by six signs, is the same as the true longitude of the Sun while rising, the Moon sets at that time; if greater, it sets after, and if less, before sun-rise."[84]

So also says Vaṭeśvara :

"When the true longitude of the Moon (for sun-set), (corrected for the visibility corrections for rising), becomes equal to the longitude of the Sun (for sun-set), increased by six signs, then the Moon, in its full phase, resembling the face of a beautiful lady, rises (simultaneously with the setting Sun), and goes high up in the sky, glorifying by its light the circular face of the earth freed from darkness, making the lotuses close themselves and the water lilies blossom forth.

"On the full-moon day, at evening, the Sun and the Moon, stationed in the zodiac at the distance of six signs, appear on the horizon like the two huge gold bells (hanging from the two sides) of Indra's elephant."[85]

Time-interval From Rising To Setting

In the case of the Sun, the time-interval from rising to setting is called the duration of sunlight or the duration of the day. Similarly, the time-interval from setting to rising is called the duration of the night. These are obtained by the formulae:

duration of day $=$ 2 (15 *ghaṭis* $\pm$ *ghaṭis* of Sun's ascensional difference)

duration of night $=$ 2 (15 *ghaṭis* $\pm$ *ghaṭis* of Sun's ascensional difference),

the upper of lower sign being taken according as the Sun is to the north or south of the equator.

Brahmagupta says :

"15 *ghaṭis* respectively increased and diminished when the Sun is in the northern hemisphere, or respectively diminished and increased when the Sun is in the southern hemisphere, by the *ghaṭis* of the Sun's ascensional difference, and the results doubled, give the *ghaṭis* of the durations of the day and night, respectively."[86]

Lalla says :

"When the Sun's ascensional difference expressed in *ghaṭis* is respectively added to and subtracted from 15 *ghaṭis*, and the results are doubled, the lengths of night and day are obtained, provided the Sun is in the southern hemisphere beginning with Libra. The same give the lengths of day and night, if the Sun is in the northern hemisphere beginning with Aries."[87]

So also has been stated by Śrīpati,[88] Bhâskara II,[89] and other Indian astronomers.

The duration from moon-rise to moon-set is called the length of the Moon's day and the duration from moon-set to moon-rise, the length of the Moon's night. The former is obtained by the formula:

length of Moon's day $=$ time of rising of the untraversed portion of the sign occupied by the Moon's *udayalagna* $+$ time of rising of the traversed portion of the sign occupied by the Moon's *astalagna*+time of rising of the intermediate signs.

Vaṭeśvara says :

"The Moon's *udayalagna* increased by six signs gives the Moon's *astalagna*. Find the oblique ascension of that part of the ecliptic that lies between the two (i.e. between the Moon's *udayalagna* and *astalagna*) with the help of the oblique ascensions of the signs: (this gives the length of the Moon's day). The difference between half of it and 15 *ghaṭīs* is the Moon's ascensional difference."[90]

Vaṭeśvara's method of finding the Moon's *astalagna* is gross. For, here the motion of the Moon from moon-rise to moon-set has been neglected. The correct rule is: First find out the Moon's true longitude for the time of moon-rise; then increase it by half the Moon's daily motion; then apply to it the visibility corrections for setting; then and six signs to that: the result thus obtained will be the Moon's *astalagna.*

In the case of a planet or a star the length of the day is defined and obtained as in the case of the Moon.

Āryabhaṭa II gives the following rule to get a planet's *astalagna:*

"Calculate the true longitude of the planet for the time of its rising, apply to it one-half of the planet's daily motion, then correct it by the visibility corrections for the western horizon (i.e. for setting), and then add six signs to it. (The result is the planet's *astalagna*). Now find the time of rising of the travereed part of the decan occupied by it, to it add the time of rising of the untraversed part of the decan occupied by the planet's *udayalagna,* as also the times of rising of the intervening decans. The result is the length of the planet's day. Using this length of the planet's day, again calculate the true longitude of the planet for the time of its setting, and iterate the above process. Thus will be obtained the accurate longitude of the visible planet on the western horizon. That increased by six signs is the planet's *astalagna.*"[91]

The planet's *udayalagna* and *astalagna* being known, the length of the planet's day is obtained by adding together the time of rising of the untraversed portion of the decan (or sign) occupied by the planet's *udayalagna,* the time of rising of the traversed portion of the decan (or sign) occupied by the planet's *astalagna,* and the times of rising of the intervening decans (or signs).

In the case of the stars too the method used to find the length of the day is the same. The stars being fixed, their *udayalagna* and *astalagna* remain the same for years. Āryabhaṭa II says: "In the case of Canopus, the Seven Sages and the stars (in general) the *udayalagna* and the *astalagna* remain invariable for some years. Not so is the case with the evermoving planets, the Moon etc., because of their inconstancy."[92]

Stars That Do Not Rise Or Set (Circumpolar Stars)

The stars whose declination is greater than or equal to the co-latitude of the place do not rise or set at that place. If the declination is north, these stars are always visible at the place; if south, they are always invisible there.

Bhāskara II says :

"The stars for which the true declination, of the northern direction, exceeds the co-latitude (of the local place), remain permanently visible (at that place). And the stars such as Sirius and Canopus etc. for which the true declination, of the southern direction, exceeds the co-latitude (of the local place), remain permanently invisible (at that place)."[93]

CONJUNCTION OF PLANETS AND STARS

Conjunction Of Two Planets

Samāgama and Yuddha ("Union And Encounter")

When two planets have equal longitudes they are said to be in conjunction. This conjunction of two planets is given different names depending on the participating planets. When the conjunction of a planet takes place with the Sun, it is called *astamaya* (setting of the planet); when with the Moon, it is called *samāgama* (union); and when any two planets, excluding the Sun and Moon, are in conjunction, it is called *yuddha* (encounter).

Viṣṇucandra says :

"Conjunction (of a planet) with the Sun is called *astamaya* (setting); that with the Moon, *samāgama* (union); and that of Mars etc. with one another, *yuddha* (encounter)."[94]

Brahmagupta says :

"Conjunction (of two planets), in which the Sun and Moon do not take part, is called *yuddha* (encounter); that of Mars etc. with the Moon, *samāgama* (union); and that with the Sun, *astamaya* (setting). (In the case of encounter) the planet that lies to the north of the other is the victor; but Venus is the victor (even) when it is to the south of the other."[95]

According to the *Sūrya-siddhānta* :

"Of the star-planets (Mars etc.) there take place, with one another, *yuddha* (encounter) and *samāgama* (union); with the Moon, *samāgama* (union); with the Sun, *astamaya* (setting)."[96]

"(In an encounter) Venus is generally the victor, whether it lies to the north or to the south (of its companion)."[97]

The conjunction of two star-planets Mars etc. which has been defined above as *yuddha* (encounter), is further classified into five categories, depending on the distance between them at the time of their conjunction. Let d be the distance between their centres at the time of their conjunction, and s the sum of their semi-diameters.

Then the conjunction is called :

1. *Ullekha* (external contact), when $d = s$;
2. *Bheda* (occultation), when $d<s$;
3. *Aṃśu-vimarda* ("pounding or crushing of rays, friction of rays), when $d>s$;
4. *Apasavya* (dexter) when $d>s$ but $<1°$ and one planet is tiny;
5. *Samāgama* (union), when $d>s$ and also $>1°$ and the planets have large discs.

The *Sūrya-siddhānta* says :

"The conjunction of two star-planets is called *ullekha* (external contact), when they touch each other (externally); *bheda* (occultation), when there is overlapping; *aṃśu-vimarda* (pounding or crushing of rays, or friction of rays), when there is mingling of rays of each other; *apasavya-yuddha* (dexter), when one planet has tiny disc and the distance between the two is less than one degree; *samāgama* (union), when the discs of the planets are large and the distance between them is greater than one degree."[98]

The *Sūrya-siddhānta* further says :

"In the *apasavya yuddha* (dexter encounter) the star-planet which is tiny, destitute of brilliancy, and covered (by the rays of the other), is the defeated one. (In general) the star-planet which is rough, colourless, struck down, and situated to the south, is the vanquished one. That situated to the north is the victor if it is large and luminous; that situated to the south too is the victor if it is powerful (i.e. large and luminous). When two star-planets are in proximity, there is *samāgama* (union) if both are luminous; *kūṭa* (confrontation) if both are small in size; and *vigraha* (conflict, or fight) if both are struck down. Venus is generally the victor whether it is to the north or to the south (of the other).[99]

The *Bhārgavīya* says :

"Hostility should be foretold when there is *apasavya* (dexter); war when there is *raśmi-saṃkula* (melee of rays); ministerial distress when there is *ullekha* (external contact); and loss of wealth when there is *bheda* (occultation)."[100]

Conjunction In Celestial Longitude (Kadambaprotiya-Yuti)

Āryabhaṭa I and his staunch follower Bhāskara I have dealt with the conjunction of the planets in celestial longitude (i.e. along the circle of celestial longitude or secondary to the ecliptic) and have given rules to find the time when such a conjunction occurs.

Bhāskara I says :

"If one planet is retrograde and the other direct, divide the difference of their longitudes by the sum of their daily motions; otherwise (i.e. if both of them are either retrograde or direct), divide the same by the difference of their daily motions; thus is obtained the time in terms of days etc. after or before which the two planets are in conjunction (in longitude). The velocity of the planets being different (from time to time), the time thus obtained is gross. One, proficient in the science of astronomy, should, therefore, apply some method to make the longitudes of the two planets agree to minutes. Such a method is possible from the teachings of the precepter or by day to day practice."[101]

"In the case of Mercury and Venus, subtract the longitude of the ascending node from that of the *śighrocca:* (thus is obtained the longitude of the planet as diminished by the longitude of the ascending node). The longitudes (in terms of degrees) of the ascending nodes of the planets beginning with Mars (i.e. Mars, Mercury, Jupiter, Venus and Saturn) are respectively 4, 2, 8, 6, and 10, each multiplied by 10.

"The greatest latitudes, north or south, in minutes of arc, (of the planets beginning with Mars) are respectively 9, 12, 6, 12, and 12, each multiplied by 10. (To obtain the *R* sine of the latitude of a planet) multiply (the greatest latitude of the planet) by the *R* sine of the longitude of the planet minus the longitude of the ascending node (of the planet) (and divide by the "divisor" defined below).

"The product of the *mandakarṇa* and the *śighrakarṇa* divided by the radius is the distance between the Earth and the planet: this is defined as the "divisor".

"Thus are obtained the minutes of arc of the latitudes (of the two planets which are in conjunction in longitude).

"From these latitudes obtain the distance between those two given planets (which are in conjunction in longitude) by taking their difference if they are of like directions or by taking their sum if they are of unlike directions. The true distance between the two planets, in minutes of arc, being divided by 4 is converted into *aṅgulas*.

"Other things should be inferred from the colour and brightness of the rays (of the two planets) or else by the exercise of one's own intellect."[102]

The method prescribed by Āryabhaṭa I in his work employing midnight day-reckoning was also practically the same. Brahmagupta has summarized it as follows:

"Divide the difference between the longitudes of the two planets (whose conjunction is under consideration) by the difference of their daily motions, if they are both direct or both retrograde, or by the sum of their daily motions, if one is direct and the other retrograde. The result is in days. If the slower planet is ahead of the other (and if both the planets are direct), the conjunction is to occur after the days obtained; if the quicker planet is ahead of the other, the conjunction has already occurred before the days obtained.

"Multiply the difference between the longitudes of the two planets by their own daily motions and divide (each product) by the difference or sum of their daily motions, as before. Subtract each result from the longitude of the corresponding planet, if the conjunction has already occurred, and add, if it is to occur, provided the planet is in direct motion. If it is retrograde, reverse the order of subtraction and addition. The planets will then have equal longitudes.

"From the longitudes of the two planets made equal up to minutes of arc, subtract the longitudes of their own ascending nodes (in the case of Mars, Jupiter and Saturn). In the case of Mercury and Venus, the longitude of the ascending node should be subtracted from the *śighrocca* of the planet. Multiply the *R* sine of that by the greatest latitude of the corresponding planet and divide by the last *karṇa* ("hypotenuse for the planet"): the result is the latitude of the planet.

"Take the difference or sum of the latitudes of the two planets (which are in conjunction in longitude) according as they are of like or unlike directions. Then is obtained the distance between the planets (at the time of their conjunction in longitude)."[103]

Occultation (bheda-yuti)

When the distance between the two planets in conjunction in longitude falls short of the sum of their semi-diameters the lower planet covers partly or wholly the disc of the higher planet. The situation is analogous to the solar eclipse where the Moon eclipses the Sun. In such a case the lower planet is treated as the Moon and the upper one as the Sun, and all processes prescribed in the case of a solar eclipse are gone through in order to obtain the time of contact and separation, immersion and emersion, etc.

Vateśvara says :

"When the distance between the two planets (which are in conjunction) is less than half the sum of the diameters of the two planets, there is occultation *(bheda)* of one planet by the other. The eclipser is the lower planet. All calculations (pertaining to this occultation), such as those for the semi-duration etc. are to be made as in the case of a solar eclipse.

"When the Moon occults a planet, the time of conjunction should be reckoned from moon-rise and for that time one should calculate the *lambana* (parallax-difference in longitude) and the *avanati* (parallax-difference in latitude). In case one planet occults another planet, the time of conjunction should be reckoned from the (occulted) planet's own rising and for that time one should calculate the *lambana* and the *avanati*."[104]

The whole process has been explained by Bhaṭṭotpala as follows :

"The planet which lies in she lower orbit is the occulting planet (or the occulter); it is to be assumed as the Moon. The planet which lies in the higher orbit is the occulted planet; it is to be assumed as the Sun. Then, assuming the time of conjunction (of the two planets) as reckoned from the rising of the occulted planet as the *tithyanta,* calculate the *lagna* for that *tithyanta,* with the help of (the longitude of) the occulted body, which has been assumed as the Sun, and the oblique ascensions of the signs. Subtracting three signs from that, calculate the corresponding declination (i.e. the declination of the *vitribha*).[105] Taking the sum of that (declination) and the local latitude when they are of like directions, or their difference when they are of unlike directions, calculate the *lambana* (for the time of conjunction) as in the case of a solar eclipse. When the longitude of the planets in conjunction is greater than (the longitude of) the *vitribha,* subtract this *lambana* from the time of conjunction; and when the longitude of the planets in conjunction is less than (the longitude of) the *vitribha,* add this *lambana* to the time of conjunction; and iterate this process: this is how the *lambana* is to be calculated. Then from the longitude of the *vitribhalagna* which has got iterated in the process of iteration of the *lambana,* severally subtract the ascending nodes of the two planets, and therefrom calculate the celestial latitudes of the two planets, as has been done in the case of the solar eclipse. Then taking the sum or difference of the declination of the *vitribhalagna,* the latitude of the *vitribhalagna,* and the local latitude, each in terms of degrees, (according as they are of like or unlike directions), in the case of both the planets. Then applying the rule: "Multiply the R sine of those degrees of the sum and difference by 13 and divide by 40: the result is the *avanati,*" calculate the *avanatis* for the two planets. Then calculate the latitudes of the occulted and the occulting planets in the manner stated in the chapter on the rising and setting of the heavenly bodies, and increase or decrease them by the corresponding *avanatis* according as the two are of like or unlike directions: the results are the true latitudes (of the occulted and occulting planets). Take the sum or difference of those true latitudes according as they are of unlike or like directions. The result of this is the *sphuṭa-vikṣepa.*

Having thus obtained the *sphuṭa-vikṣepa,* one should see whether there exists eclipse-relation between this *sphuṭa-vikṣepa* and the diameters of the discs of the two planets. If the *sphuṭa-vikṣepa* is less than half the sum of the diameters of the two planets, this relation does exist; if greater, it does not. The totality of the occultation should also be investigated as before. Then, (severally) subtract the square of the *sphuṭa-vikṣepa* from the squares of the sum and the difference of the semi-diameters of the occulted and occulting planets, and take the square roots (of the results). Multiply them by 60 and divide by the difference or sum of the daily motions of the planets as before: then are obtained the *sthityardha* and the *vimardārdha*, (respectively). They are fixed (by the process of iteration) as in the case of a solar eclipse. The *sthityardha* and *vimardārdha* having been obtained in this way, they should be corrected by the *lambana* obtained by the process of iteration. Then the time of apparent conjunction should be declared as the time of the middle of the occultation; this diminished and increased by the *(spārśika* and *maukṣika) sthityardhas,* (respectively), the times of contact and separation (of the two planets); and the same diminished and increased by the *(spārśika* and *maukṣika) vimardārdhas* (respectively), the times of immersion and emersion."[106]

Bhāskara II explains the same as follows :

"When there is *bheda-yuti,* then one should compute the *lambana* etc. as in the case of a solar eclipse. There, the lower of the two planets is to be assumed as the Moon and the upper one as the Sun. Why are they so assumed? To compute the *lambana* etc. But the *lagna,* which is obtained in order to find the *vitribhalagna,* is to be computed from the actual Sun, not from the assumed Sun. For what time is the *lagna* calculated from the Sun? For the time of conjunction (in longitude of the two planets). What is meant is this : On the day the conjunction (of the two planets) takes place, find the *ghaṭis* of the night elapsed at the time of conjunction. Therefrom calculate the Sun as increased by six signs, and therefrom the *lagna.* Then calculate the *vitribha* and then the corresponding *śaṅku* (i.e. *R* sine of the altitude of the *vitribha*). Then, applying the rule: "Multiply the R sine of the difference of that *vitribha* and the assumed Sun by 4 and divide by the radius, and so on," calculate the *lambana* and *nati,* as before. Then correct the time of conjunction by that *lambana.* But the *lambana* etc. should be applied only when the two planets are fit for observation. In this *vheda-yuti,* the north-south distance between the planets is the latitude; and the direction of the latitude is that in which the assumed Moon lies, as seen from the assumed Sun. Now is stated the peculiarity in the case of *parilekha* (graphical representation of the occultation). When the lower planet, which has been assumed as the Moon, is slower or retrograde, then one should understand that the contact (of the two planets) occurs towards the east and the separation towards the west. In the contrary case, one should understand that the contact occurs towards the west and the separation towards the east. We have stated here the (notable) points of difference in the case of *Bheda-yoga:* there is no other difference in the procedure."[107]

Conjunction Along The Circle Of Position (Samaprotīma-Yuti)

Conjunction in longitude, though theoretically sound and perfect, suffered from one practical setback, viz. that there being no star at the pole of the ecliptic such a conjunction could not be observed with precision and so the calculated time of its occurrence could not be confirmed by observation. Brahmagupta noted that the stars

Citrā (Spica) and Svātī (Arcturus), which, though of unequal longitudes, were seen daily to be in conjunction along the circle of position *(samaprota-vṛtta)*. This conjunction was easily observable and agreement between computation and observation in this case could be established. Brahmagupta therefore gave preference to conjunction along the circle of position over conjunction in longitude.

To obtain the time when two planets are in conjunction along the circle of position, Brahmagupta first finds the time of their conjunction in longitude and then he derives how much earlier or later conjunction along the circle of position takes place. He states two rules for the purpose, one gross and the other approximate.

Brahmagupta's Gross Rule

Brahmagupta's gross rule runs as follows :

"Find the *udayalagna* (the rising point of the ecliptic at the time of rising of the planet) and also the *astalagna* (the setting point of the ecliptic at the time of setting of the planet) of the two planets equalized up to minutes of arc (i.e. for the time of their conjunction in longitude). Then find the *ghaṭis* of the day-lengths of the two planets by adding together the times of rising at the local place of (1) the untraversed part of the *udayalagna,* (2) the traversed part of the *astalagna* as increased by six signs, and (3) the intervening signs, (in each case). If out of the two planets (in conjunction in longitude), the planet with lesser *udayalagna* is such that its *astalagna* increased by six signs is smaller than the other planet's *astalagna,* increased by six signs, one should understand that the conjunction of the two planets along the circle of position is to occur;[108] if greater, one should understand that the conjunction of the two planets along the circle of position has already occurred.

"Now (in the case of both the planets) multiply the minutes of the difference between the planet's *astalagna* plus six signs and the *udayalagna* by the *ghaṭis* of the planet's own day-length. The result (in each case) should be taken as negative or positive according as the *astalagna* plus six signs is smaller or greater than the *udayalagna.* In case these results are both negative or both positive, divide the minutes of the difference between the planets' own *udayalagnas* by the difference of the two results; in case one result is positive and the other negative, divide the same minutes by the sum of the two results. (This gives the time, in terms of *ghaṭis,* to elapse before or elapsed since the conjunction of the two planets along the circle of position, at the time of their conjunction in longitude). By these *ghaṭis* multiply the minutes of the difference between the planet's *udayalagna* and *astalagna,* the latter increased by six signs, and divide by the *ghaṭis* of the planet's own day-length. By the resulting minutes increase or diminish the planet's own *udayalagna* according as it is smaller or greater than the planet's (own) *astalagna* plus six signs: then is obtained the planets' common longitude at the time of their conjunction along the circle of position. In case it is less than the *udayalagna* for that time in the night or greater than the *udayalagna* plus six signs, the two planets will be seen (in the sky) in conjunction along the circle of position."[109]

Śrīpati, following Brahmagupta, has stated this rule in his *Siddhānta-śekhara.*[110] But Lalla and Vaṭeśvara have omitted it.

Brahmagupta's Approximate Rule

The second rule of Brahmagupta which is intended to give better time of conjunction (of two planets along the circle of position) than the first rule (stated above), runs as follows :

"Multiply the duration of day for the planet with greater day-length by the time (in *ghaṭis*) elapsed (at the time of conjunction in longitude) since the rising of the planet with smaller day-length and divide by the duration of day for the planet with smaller day-length. When the resulting time is greater than the time elapsed (at the time of conjunction in longitude) since the rising of the planet with greater day-length, (it should be understood that) the conjunction of the two planets (along the circle of position) has already occurred; when less, (it should be understood that) the conjunction of the two planets (along the circle of position) is to occur.[111]

"The difference of the two times (in terms of *ghaṭis*) is the "first". A similar result derived from the two planets, diminished or increased by their motion corresponding to "arbitrarily chosen *ghaṭis*"[112] (as the case may be), is the "second". When the "first" and the "second" both correspond to conjunction past or to occur, divide the product (of the *ghaṭis*) of the "first" and the "arbitrarily chosen *ghaṭis*" by the *ghaṭis* of the difference between the "first" and the "second"; in the contrary case (i.e. when out of the "first" and the "second", one corresponds to conjunction past and the other to conjunction to occur), divide the product by (the *ghaṭis* of) the sum of the "first and the "second". The resulting *ghaṭis* give the *ghaṭis* elapsed since or to elapse before the conjunction along the circle of position, at the time of conjunction in longitude, depending upon whether the "first" relates to conjunction past or to occur.

"The conjunction of two planets, along the circle of position, takes place when the result (in *ghaṭis*) obtained on dividingby the *ghaṭis* of the day-length of one planet, the product of the *ghaṭis* elapsed since the rising of that planet and the *ghaṭis* of the day-length of the other planet, is equal to the *ghaṭis* elapsed since the rising of the other planet."[113]

This latter rule of Brahmagupta has been adopted by Lalla[114] and Śrīpati.[115]

Alternative Form Of Brahmagupta's Approximate Rule

Brahmagupta has stated his approximate rule in the following alternative form also ;

"Multiply the *nāḍis* of the duration of day for the planet with smaller day-length by the *ghaṭis* elapsed since the rising of the planet with greater day-length and divide by the *ghaṭis* of the duration of day for the planet with greater day-length: the result is in terms of *nāḍis*. When these *nāḍis* are less than the *ghaṭis* elapsed since the rising of the planet with smaller day-length, (it should be understood that) conjunction (along the circle of position) of the two planets has already occurred; when greater, (it should be understood that) conjunction is to occur. Assume the difference of the two, in terms of *ghaṭis*, as the "first". Now multiply the daily motion of each planet by "the arbitrarily chosen *ghaṭis*" and divide each product by 60: add the result to or subtract it from the longitude of the corresponding planet according as the conjunction has occurred or is to occur. Then obtain the difference similar to the "first" and call it "second". When both the differences, the "first" and the "second", correspond either to conjunction past or to conjunction to occur, divide the product of the "first" and the "arbitrarily chosen *ghaṭis*" by the difference of the "first" and the "second"; in the contrary case (i.e. when out of the "first" and the "second", one corresponds to conjunction past and the other to conjunction to occur), divide that product by the sum of the "first" and the "second". The resulting *ghaṭis* give the *ghaṭis* elapsed since or to elapse before the conjunction of the two planets (along the circle of position), depending upon whether the "first" relates to conjunction past or to conjunction to occur. If by applying the above rule once conjunction of the two planets is not arrived at, the rule should be iterated (until one does not get the conjunction of the two planets)."[116]

This alternative form of Brahmagupta's approximate rule has been adopted by Vaṭeśvara who states it as follows :

"Multiply the duration of day for the planet with smaller day-length by the time (in *ghaṭis*) elapsed since the rising of the planet with greater day-length, and divide by the duration of day for the planet with greater day-length. When the resulting time is greater than the time elapsed since the rising of the planet with smaller day-length, (it should be understood that) the conjunction (along the circle of position) of the two planets is to occur; in the contrary case, (it should be understood that) the conjunction has already occurred.

"The difference of the two times (in terms of *ghaṭis*) is the "first". A similar difference derived from the "*ghaṭis* arbitrarily chosen" (for *ghaṭis* elapsed since or to elapse before conjunction) is the "second". When both the "first" and the "second" correspond either to conjunction past or to conjunction to occur, divide the product of the "first" and the "arbitrarily chosen *ghaṭis*" by the *ghaṭis* of the difference between the "first" and the "second"; in the contrary case (i.e. when out of the "first" and the "second", one corresponds to conjunction past and the other to conjunction to occur), divide that product by (the *ghaṭis* of) the sum of the "first" and the "second". The resulting *ghaṭis* give the *ghaṭis* elapsed since or to elapse before the conjunction of the two planets (along the circle of position), depending on whether the "first" relates to conjunction past or to occur."[117]

Munīśvara has criticised conjunction along the circle of position advocated by Brahmagupta, for the reason that the time of such a conjunction will differ from place to place, and so it will create confusion in making astrological predictions. See *Siddhānta-sārvabhama, Bharahayuti*, vs. 15, p. 543.

Āryabhaṭa II's Rule

Āryabhaṭa II gives the following rule to find the time of conjunction in celestial longitude and that of conjunction along the circle of position :

"Divide the difference (in minutes) between the longitudes of the two planets (whose conjunction is under consideration) by the difference between the daily motions (of the two planets), provided they are both direct or both retrograde; if one of the planets is retrograde (and the other direct), divide by the sum of the daily motions (of the two planets); the result gives the days elapsed since the conjunction of the two planets, in case the faster planet is greater than the other, and also if the planet with lesser longitude is retrograde (and the other direct). When both the planets are retrograde, the case is contrary to what happens when both the planets are direct. The two planets should then be calculated for the time of conjunction. Then the two planets become equal in longitude.

When conjunction suitable for observation (i.e. along the circle of position) is required, then the two planets should be corrected for the *ayana-dṛkkarma* and *akṣa-dṛkkarma* also. The time when they become equal in longitude, is certainly the time of conjunction (along the circle of position)."[118]

Indications of this rule occur in the *Sūrya-siddhānta*[119] and the *Vaṭeśvara-siddhānta*[120] also. According to Kamalākara, a staunch follower of the *Sūrya-siddhānta*, however, the conjunction of the planets and stars taught in the *Sūrya-siddhānta* is in celestial longitude.[121]

Conjunction In Polar Longitude (Dhruvaprotīya-Yuti)

Bhâskara II has given rules for conjunction in celestial longitude as well as conjunction in polar longitude. But as there is no star at the pole of the ecliptic conjunction in celestial longitude does not, says he, create confidence in the observer; while there being one at the pole of the equator conjunction in polar longitude better for observation. However, conjunction of two planets, in his opinion, really occurs when the two planets are nearest to each other and this happens when the two planets are in conjunction in celestial longitude only.[122] He has given no credit to conjunction along the circle of position probably because it was not universal. He has not even mentioned this conjunction.

Bhāskara II's rule for the conjunction of two planets in polar longitude rus thus :

"Divide the minutes of the difference between the longitudes of the two planets by the difference of their daily motions (if both planets are direct or both retrograde); if one of them is retrograde (and the other direct), divide by the sum of the daily motions (of the planets): the result is the number of days elapsed since the conjunction of the two planets provided the slower planet has lesser longitude than the other, or if, one planet being retrograde, its longitude is lesser than that of the other. If otherwise, the conjunction occurs after the days obtained. If both the planets are retrograde, the result is contrary to that for direct planets. (This gives approximate time for conjunction. To get accurate time, proceed as follows :)

"(Calculate the longitudes of the planets for the time of conjunction and) apply the *ayana-dṛkkarma* (to them). Iterate the process until the time of conjunction is not fixed. When this is done, the two planets lie on the same great circle passing through the poles of the equator. The planets are then said to be in conjunction in the sky. If the *ayana-dṛkkarma* is not applied, the planets lie on the same secondary to the ecliptic."[123]

2. CONJUNCTION OF A PLANET AND A STAR

The conjunction of a planet and a star is treated in the same way as the conjunction of two planets and the rules in the two cases are similar. The only remarkable difference is that the stars, unlike the planets, are supposed to be points of light having no diameter and fixed in position having no eastward daily motion.

Bhāskara I says :

"All planets whose longitudes are equal to the longitude of the junction-star of a *nakṣatra*[124] are seen in conjunction with that star. (Of a planet and a star) whose longitudes are unequal, the time of conjunction is determined by proportion."[125]

"The distance between a planet and a star (when they are in conjunction) is determined from (the sum or difference of) their latitudes.[126]

Brahmagupta says :

"If the longitude of a planet is less than the longitude *(dhruvaka)* of a star, their conjunction is to occur; if greater, their conjunction has already occurred. If the planet is retrograde, reverse is the case. The rest is similar to that stated in the case of the conjunction of two planets.[127]

Lalla says:

"If the longitude of a planet is greater than the longitude of the junction-star of a *nakṣatra,* their conjunction has already taken place; if less, it will take place. If the planet is retrograde, the contrary is the case. The rest is similar to that in the case of the conjunction of two planets."[128]

A similar statement has also been made by Vaṭeśvara,[129] Āryabhaṭa II,[130] Śrīpati,[131] the author of the *Sūrya-siddhānta,*[132] and others.

In the case of occultation, Brahmagupta says :

"When a planet is on the same side of the ecliptic as the junction star of a *nakṣatra,* the planet will occult the junction star if its true latitude is greater than the latitude of the junction star minus the semi-diameter of the planet or less than the latitude of the junction star plus the semi-diameter of the planet.[133]

The occultation of a star by the Moon was considered important. So the occultation of certain prominent stars was specially noted and recorded by the Indian astronomers.

Bhāskara I says :

"The Moon, moving towards the south of the ecliptic, destroys (i.e. occults) the Cart of Rohiṇī (the constellation of Hyades), when its latitude amounts to 60 minutes; the junction star of Rohiṇī (i.e. Aldebaran), when its latitude amounts to 256 minutes; (the junction star of) Citra (i.e. Spica), when its latitude amounts to 95 minutes; (the junction star of) Jyeṣṭhā (i.e. Antares), when its latitude amounts to 200 minutes; (the junction star of) Anurādha,[134] when its latitude amounts to 150 minutes; (the junction star of) Śatabhiṣak (i.e., λ Aquarii), when its latitude amounts to 24 minutes; (the junction star of) Viśakha,[135] when its latitude amounts to 88 minutes; and (the junction star of) Revatī (i.e., Zeta Piscium), when its latitude vanishes. When it moves towards the north (of the ecliptic), it occults the *nakṣatra* Kṛttikā (i.e. Pleiades), when its latitude amounts to 160 minutes; and the central star of the *nakṣatra* Maghā, when it assumes the greatest northern latitude. These minutes (of the Moon's latitude)..are based on actual observation made by means of the Yaṣṭi instrument (i.e. the Indian telescope)."[136]

Brahmagupta says :

"The planet whose south latitude at 17° of Taurus exceeds 2°, occults the Cart of Rohiṇī.[137] The Moon, when it has the maximum north latitude, occults the third star of Maghā; when it has no latitude, it occults Puṣya, Revatī and Śatabhiṣak."[138]

Lalla says :

"The Moon, situated in the middle of the *nakṣatra* Rohiṇī, occults the Cart of Rohiṇī, when its southern latitude amounts to 2°40′; (the junction star of) the *nakṣatra* Rohiṇī, when its southern latitude is 4°30′; the middle of the *nakṣatra* Maghā, when its north latitude amounts to 40°30′; and the *nakṣatras* Revatī, Puṣya and Śatabhiṣak, when is devoid of latitude."[139]

Vateśvara says :

"The planet, whose latitude at 17° of Taurus amounts to $1\frac{1}{2}$ degrees south, occults the Cart of Rohiṇī. The Moon with its (maximum) latitude south (i.e., 4°30′ S) covers the junction star of Rohiṇī."[140]

Śrīpati similarly says :

"The planet whose southern latitude at 17° of Taurus exceeds 2° certainly occults the Cart of Rohiṇī. The Moon with its longitude equal to that of (the junction star of) Maghā occults the third star of Maghā, when it has maximum (north) latitude; and the *nakṣatras* Śatabhiṣak, Revatī and Puṣya when its longitude is equal to their longitudes."[141]