

RITUAL GEOMETRY IN INDIA AND ITS PARALLELISM IN OTHER CULTURAL AREAS

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The ritual origin of Indian geometry has been well established. The perpetual daily sacred fires and the optional fires were placed on altars of various shapes. These altars were constructed on a specified area having normally five layers, each made of fixed number of bricks. The tradition is very old in India, and important details of information are available in the *Samhitās*, *Brāhmaṇas* and the class of writings called *Śulbasūtras*. As to the reason which might have induced the ancient Indians to devise all these strange shapes, *Rgveda* says, 'May the altars be raised for our happiness', *Taittirīya Samhitā* says, 'He, who desires heaven, may construct falcon-shaped altar, for falcon is the best flyer among the birds'. These may appear to be superstitious fancies but led to important contribution in geometry and mathematics, because of their conviction in social value system. The construction of altars having drawn on a base of different figures — square, circle, semi-circle, isosceles trapezium, triangles, rhombus, falcon or tortoise shape and others led to the development of various geometrical figures, their transformations and calculation of areas involving many Pythagorean relations with rational and irrational numbers leading to its general statement, approximation of the value of $\sqrt{2}$ and others. Tackling of geometrical and mathematical problems with irrational numbers was indeed a unique achievement of the early Indians. They had not only developed the technical terms like *dvikaraṇi* ($\sqrt{2}$), *dvītiyākaraṇi* ($1/\sqrt{2}$), *tr-karaṇi* ($\sqrt{3}$), *trītiyā-karaṇi* ($1/\sqrt{3}$), *pañca-karaṇi* ($\sqrt{5}$) and *pañcama-karaṇi* ($1/\sqrt{5}$) etc. but actually understood the significance. Their handling of the problems for addition, subtraction, multiplication and division with these numbers shows that both rational and irrational numbers were tackled on the same footing keeping them on a number line. The Greeks, Egyptians, Babylonians and the Chinese were also concerned with rectangles, triangles, circles, pyramids and others. The main purpose of the paper is to discuss the background and antiquity of Indian tradition, the origin of some of the important concepts and their parallelism with those developed in other cultural areas on a broader perspective.

1. INTRODUCTION

Thibaut¹ in 1876 and Burk² in 1901 referred to the Sacred Book of the East and the *Sulbasutras* containing the use of large number of triples in connection with the construction of altars and hinted at the non-Greek origin of the Pythagorean geometry. Neugebauer³ kept silent on geometry of the Indian till 1945, when he discovered a set of Pythagorean triples in the Babylonian cuneiform text and remarked 'What is Pythagorean in the Greek tradition had better be called Babylonian. The geometry of the East had found no place even in Van der Waerden's book, *Science Awakening*

(1954; new edition 1961). As to the reason for this omission, Seidenberg (1962) pointed out⁴, "We are all subject to the ancient Greek rationalist ideology which is apt to scorn priestly works, and this may have caused the general neglect of the *Śulbasūtras*. It is true that those who maintained the priority of Indian geometry may have claimed too much when they said that Greek geometry came from India. What they should have said was that Indian geometry and the Greek geometry derive from a common source". Van der Waerden⁵ in 1980 while making an analysis of the Chinese triples made exactly the same recommendation. He mentioned, "If one compares the Nine Chapters with Babylonian texts, one sees that the Chinese text is more systematic and richer in geometrical content. Several problems and methods which are present in the Chinese texts are absent from the numerous extant Babylonian texts. For instance, the Chinese text tells us by what method Pythagorean triples can be found, whereas in Babylonian texts we find a list of such triples. Therefore it seems reasonable to assume that the common source is not Babylonian, but Pre-Babylonian". Seidenberg has explained the psyche behind the suppression or omission but at the same time hints at a common source. Van der Waerden picks up this hypothesis and suggests a common source i.e. Pre-Babylonian. What has actually been attempted is an hypothetical reconstruction to prove that they follow from a common source on the basis of Neolithic monuments found in England, Scotland and Ireland. It does not appear to be meaningful and a happy trend. If this is allowed to put forth in India and China similar reconstruction would not be difficult. What would have been interesting in history of science, to my mind, was further analysis of important records of all the old cultures which have made them distinct and to search for new records if any. My intention in this appear is to show some of the characteristic features of early ritual geometry and how the concept of irrational numbers had been developed in India in tackling the problems of triples involving length, breadth and diagonal of right triangle alongwith general statement. Lastly, a parallelism of approaches will be made to appreciate the Indian contribution which is distinct from those of others.

2. ALTARS OR *Citis*

The ritual connection of Indian geometry, as elaborated by Thibaut and Burk has been intensely discussed by Datta,⁶ Seidenberg,⁷ Sen & Bag⁸ and others. The ceremonies were performed on the top of the altars either in sacrificer's house or on a nearby plot of ground. The altar is a specified raised area, generally made of bricks for keeping the fire. The fire altars were of two types, namely: the perpetual (*nitya*, daily) and optional (*kāmya*, wish fulfilment). The perpetual fires were constructed on a smaller area of one *sq. puruṣa* and the optional fires were constructed on a bigger area of $7\frac{1}{2}$ *sq. puruṣa* or more, each having minimum of five layers of bricks. The perpetual fires had 21 bricks and optional fires had 200 bricks in each layer in the first construction, and the number of bricks became more in the subsequent constructions with various other restrictions. The measurement of area and the construction of bricks for optional fires were a laborious process and the ceremonies continued for a long time. The whole family of the organiser had to reside by the side of the optional fire altar, for which another class of structure known as *mahāvedi* and other related vedis

were made. The tradition is very old in India and details of information are available from the *Samhitās*, *Brāhmaṇas* and the class of writings called *Śulbasūtras*. The information available in different *Śulbasūtras* are tabulated below:

Altar	Shape/Horizontal section	Area	Reference ⁹
I. Perpetual Fire Altar:			
i) <i>Āhavanīya</i>	Square	One sq. <i>Vyāyāma</i>	Bśl. 3.1-5; 7.4; 7.5
ii) <i>Gārhapatya</i>	Circles, square	————	Āśl. 4.4
iii) <i>Dakṣināgni</i>	Semi-circle	————	Kśl. 1.11
II. Vedis:			
i) <i>Mahāvedi</i> or <i>Saumikyā vedi</i>	Isosceles trap. a = face = 24 <i>padas</i> b = base = 30 <i>padas</i> c = height = 36 <i>padas</i> Isosceles trap.	972 sq. <i>padas</i>	Bśl. 4.3 Bśl. 3.11-13 Āśl. 5.1-8 Kśl. 2.11; 2.12
ii) <i>Sautrāmaṇi vedi</i>	(a = 24/√3 b = 30/√3 c = 36/√3) or (a = 8√3, b = 10√3 c = 12/√3)	324 sq. <i>padas</i>	
iii) <i>Paitṛkī vedi</i>	i) Isosceles trap. a = 8, b = 10, c = 12 ii) a square having four corners in four coordinate directions	108 sq. <i>padas</i>	
iv) <i>Prāgvamśa</i>	Rectangle 16 × 12 or 12 × 10	192 or 120 sq. <i>prakramas</i>	

Altar	Shape/Horizontal section	Area	Reference ⁹
III. <i>Optional Fire Altars:</i>			
i) <i>Caturasra- śyenacit</i>	Hawk bird with sq. body, sq. wings and sq. tail.	$7\frac{1}{2}$ sq. <i>puruṣa</i>	Bśl. 8.1-19, 9. 1-10; Āśl. 8.2-15.7
ii) <i>Vakrapakṣa- vyastapuccha śyena</i>	Hawk bird with bent wings and out-spread tail.	$7\frac{1}{2}$ sq. <i>puruṣa</i>	Bśl. 10.1-20; 11.1-13 Bśl. 11.1-13
iii) <i>Kankacit</i>	Hawk bird with curved wings & tail.	$7\frac{1}{2}$ sq. <i>puruṣa</i>	Bśl. 12.1-8; Āśl. 21.1-21.
iv) <i>Alajacit</i>	Alaja bird with curved wings and tail.	$7\frac{1}{2}$ sq. <i>puruṣa</i>	Bśl 13.1-6; Āśl. 21.1
v) <i>Prauga</i>	Triangle	$7\frac{1}{2}$ sq. <i>puruṣa</i>	Bśl. 14.1-8; Āśl. 12.4-12.
vi) <i>Ubhayataprauga</i>	Rhombus	$7\frac{1}{2}$ sq. <i>puruṣa</i>	Bśl. 15.1-6 Āśl. 12.7-12;
vii) <i>Rathacakracit</i>	Circle	$7\frac{1}{2}$ sq. <i>puruṣa</i>	Bśl. 15.1-6; Āśl. 12.9-12.
viii) <i>Droṇacit</i>	Trough	$7\frac{1}{2}$ sq. <i>puruṣa</i>	Bśl. 17.1-12; 18.1-15 Āśl. 13.4-13.
ix) <i>Śmaśānacit</i>	Isoceles trap.	$7\frac{1}{2}$ sq. <i>puruṣa</i>	Bśl. 19.1-11 Āśl. 14.7-14.
x) <i>Kūrmacit</i>	Tortoise	$7\frac{1}{2}$ sq. <i>puruṣa</i>	Bśl. 20.1-21; 21. 1-13

While giving the details of these altars, the *Śulbasūtras* use the word *vijñdyate* (known as per tradition), *vedervijñdyate* (known as per vedic traditions) etc. very often.¹⁰

3. BRICKS

The altar makers used to manufacture bricks of various sizes for optional fires to get the desired shape with 200 bricks not more nor less in the first construction, fixed for each, covering an areas of $7\frac{1}{2}$ sq. *puruṣa*. For *nitya agni* of course, small number of bricks were required. It was indeed a tremendous exercise, sometimes the area of $7\frac{1}{2}$ sq. *puruṣa* was covered with 125 bricks and the instruction was given to replace original bricks by its half, quarter or one-eighth bricks to fulfil the desired number of 200 bricks in each layer. The bricks were also used as units for measuring, calculating and verifying the accuracy of measured area. The square bricks *caturthī* (one-fourth), *pañcamī* (one-fifth), *ṣaṣṭī* (one-sixth), and their sub-divided bricks were manufactured and each was named separately and details of dimension both in rational and irrational numbers were given. These bricks were made by a special class of specialists. No burnt bricks or bricks with cleavage were used for the purpose, only sun-dried bricks were used for the purpose. The *caturthī*¹¹ and *pañcamī*¹² bricks and dimensions of the sub-divisions are listed below:

- I. *Caturthī* (one-fourth, square brick, size 30 *ahg.* × *ahg.* Five types were available:
- caturthī* (square quarter) = 30×30 (*ahg.*)
 - ardhā* (triangular half) = $30 \times 30 \times 30\sqrt{2}$ (*ahg.*)
 - trasra pādyā* (triangular quarter) = $30 \times 15\sqrt{2} \times 15\sqrt{2}$ (*ahg.*)
 - caturasra pādyā* (four sided quarter) = $22\frac{1}{2} \times 15 \times 15\sqrt{2} \times 15\sqrt{2}$ (*ahg.*)
 - haṃsamūkhī* (pentagonal half brick) = $15\sqrt{2} \times 7\frac{1}{2} \times 15 \times 7\frac{1}{2} \times 15\sqrt{2}$ (*ahg.*)
- II. *Pañcamī* (one-fifth square brick, $\frac{1}{5}$ *pu.* × $\frac{1}{5}$ *pu.*; 24 *ahg.* × *ahg.*)
- pañcamī* (one-fifth sq. brick) = 24×24 (*ahg.*)
 - adhyardhā-pañcamī* (rectangular brick, side longer by one-half) = 24×36 (*ahg.*)
 - pañcamī-sapādā* (rectangular brick, side longer by one-quarter) = 24×30 (*ahg.*)
 - pañcamī-ardhā* (triangular half bricks) = $24 \times 24 \times 24\sqrt{2}$ (*ahg.*)
 - pañcamī-pādyā* (triangular quarter bricks) = $24 \times 12\sqrt{2} \times 12\sqrt{2}$ (*ahg.*)
 - adhyardhārdhā* (triangular half brick of *adhyardhā*) = $36 \times 24 \times 12\sqrt{13}$ (*ahg.*)
 - dirghapādyā* (triangular quarter bricks of *adhyardhā* with larger base) = $36 \times 6\sqrt{13} \times 6\sqrt{13}$ (*ahg.*)
 - śulapādyā* (triangular quarter brick of *adhyardhā* with shorter base) = $24 \times 6\sqrt{13} \times 6\sqrt{13}$ (*ahg.*)
 - ubhayī* (triangular brick when half brick of no. (g) & (h) are attached i.e. $30 \times 12\sqrt{2} \times 6\sqrt{13}$ (*ahg.*))
 - pañcamī-aṣṭamī* (one-eighth triangular brick of *pañcamī*) = $12 \times 12 \times 12\sqrt{2}$ (*ahg.*)
- III. Similarly there were various other bricks which were prepared to meet the

requirements of covering the *agni* of specified size. *Baudhāyana*^{12A} says, for falcon shaped fire altar $187\frac{1}{2}$ *pañcamī* bricks cover $7\frac{1}{2}$ *sq. puruṣa* i.e. $187\frac{1}{2} \times (1/5 \times 1/5) = 7\frac{1}{2}$ *sq. puruṣa*, out of these bricks $3\frac{1}{2}$ *pañcamī* bricks were recommended for-head, 52 for body, 117 for two wings and 15 for tail. It was further suggested that for filling quota of 200 bricks, a few bricks were replaced by its half and quarter without breaking their symmetry. There were various other similar example. Further their bricks were considered as small square units for verifying the area specified for each figure. The lay out of altar will give some idea about the shape, type of bricks and the diagrams of the specified size.

4. ANTIQUITY OF THE TRADITION

The earliest enumeration of the three *nitya-agni* in the *Rgveda*¹³ imply the *gārhapatya*, the *āhavanīya* and the *dakṣiṇāgni*. The *Rgveda*¹⁴ also refers to optional fire altar in the form of *śyena*. It also refers,¹⁵ 'Let the priest decorate altar (*vedi*), let them kindle the fire to the east', 'May the measure-lengths of the sacrificial post be to our felicity, may the sacred grass be (strewn) for our happiness, may altar be (raised for) our happiness'. Let the *gārhapatyaciti* be constructed with 21 bricks according to *Taittirīya Samhitā*.¹⁶ Similar passages are found in the *Maitrāyani Samhitā*, *Kāṭha Samhitā*¹⁸ and *Kapisthala Samhitā*¹⁹. The *Saumikī vedi* has been described with face 24 *prakramas*, base 30 *prakramas* and height 36 *prakramas* in the *Taittirīya Samhitā* and in other texts.²⁰ Regarding optional fire, the *Taitt. Samhitā* gives various details. It says 'He should pile a *śyaenacit* who desires the heaven, the *śyena* was the best flyer among the birds; verily becoming *śyena*, he flies to the world of heaven.... He should pile in the form of *alaja* bird with four furrows who desires support, there are four quarters, verily he finds support in the quarters. He should pile in the form of triangle (*praugacit*) who has rival, verily he repels his rival. He should pile in the form of a rhombus (*ubhayata prauga*) who desires, 'May I annihilate the rivals I have and those I shall have', verily he annihilates the rivals, he has and those he will have. He should compile in the form of chariot wheel (*rathacakracit*), who has foes, the chariot is a thunderbolt, verily he hurls the thunderbolt at his rival. He should pile in the form of a wooden trough (*dronacit*) who desires food; in a wooden trough food is kept, verily he gains food together with its place of birth. He should pile one that has to be collected together, who desires cattle, verily he becomes rich in cattle. He should pile one in a circle (*paricāyya*) who desires a village, verily he gets possession of the village. He should pile in the form of '*śmaśānacit* who desires the place where his forefathers have gone; verily he attains the place where his forefathers have gone'. It further says²², 'He should pile (the fire) of a 1000 bricks upto knee-deep when first piling *saptavidha agni* ($7\frac{1}{2}$ *puruṣa*), this world is commensurate with 1000, verily he conquered this world. He should pile it of 2000 upto naval deep, when piling a second time. He should pile it of 3000 upto neck-deep when piling further". The expert was also called *agnicit* (construction of the *agni*) appearing in the *Taitt. Sam.*²³ According to *Śathapatha Brāhmaṇa*, the *kāmyaciti* ($7\frac{1}{2}$ *sq. puruṣa*) was to be constructed first as *saptavidha*, then by the increment of 1 *sq. puruṣa* in succession *ekasatidha agni* ($101\frac{1}{2}$ *sq. puruṣa* would be constructed). *Baudhāyana*, *Āpastamba* follow the same tradition. It accepted

'heavens' for builders of *śyenacit*, the world of the supreme spirit (*Brahmaloka* for *kūrmacit*, the destroyer of enemies for *rathacakracit* and so on²⁵. Baudhāyana and Āpastamba refer to *ekavidha* ($1\frac{1}{2}$ sq. *puruṣas*); *dvividha* ($2\frac{1}{2}$ sq. *puruṣas*) and so on. The *ekavidha*, *dvividha* do not have wings and tails, only *saptavidha* ($7\frac{1}{2}$ sq. *puruṣas*) *agni* is the first to be constructed. Thereafter altars of bigger size are obtained by increasing the area by 1 sq. *puruṣa*²⁶

Many a conjecture have been made by scholars²⁷ like Buhler (1879), Macdonell (1904), Thibaut (1899), Keith (1914), Kane (1968), Rāmagopal (1959) placing the time of *Śulbasūtras* varying from 800 B.C. to 200 B.C. The sequence of Baudhāyana, Āpastamba, Pānīnī, Kātyāna, Patañjali were considered by Kane and Ramgopal carefully who concluded that the principal sūtras were composed between c. 800 B.C. and 500 B.C. Whatever be the date of principal sūtras like Baudhāyana and Āpastamba *Śulbasūtras*, the details show the evidence of altar construction occurring in all strata of Vedic literature and the earliest reference goes back to the time of the *R̥gveda*. The *R̥gveda* has been dated 2000-1500 B.C. by Whitney and even earlier by Jacobi²⁸. Further, the study of chronology becomes irrelevant when we think that there was a long tradition and the tradition may be old by 200 years or 2000 years. It would be difficult to interpret anything unless we are definite. The construction of geometrical figures on the ground with the help of peg and cord, verification of area and obtaining desired shapes, the nature of treatment is very fundamental and is primary in nature.

5. NATURE OF KNOWLEDGE

One can guess the nature of geometrical and mathematical knowledge which could originate from such altar constructions. However, the *Śulbasūtras* have summarised some of these knowledges. The order in which these have been given is summarized below as per *Baudhāyana Śulbasūtra*. Almost the same order has been maintained by the *Āpastamba Śulbasūtra* though it belongs to a different school. How this is possible? The scholars were ordered to maintain strict secrecy of the occurrence of important results from other schools. It indicates that they follow from the same older source. The older sources of *Samhitās* and *Brahmaṇas*, the way these appear in the *Śulbasūtras*. There is no doubt that most of the result had followed from the earlier tradition.

5.1 Give various units of linear measurement used for the purpose. *Units*²⁹: 1 *prādeśa* = 12 *angulas*, 1 *pada* = 15 *ang.*, 1 *iṣā* = 188 *ang.*, 1 *akṣa* = 104 *ang.*, 1 *yuga* = 86 *ang.*, 1 *jānu* = 32 *ang.*, 1 *śamyā* = 36 *ang.*, 1 *bāhu* = 36 *ang.*, 1 *prakrama* = 2 *pada*, 1 *aratni* = 2 *prādeśa*, 1 *puruṣa* = 5 *aratni*, 1 *vyāyāma* = 4 *aratni*, 1 *ang.* = 14 *anus* = 34 *tilas* = $\frac{3}{4}$ inch approx. Āpastamba³⁰ gives the same units. The units like *ang.*, *pada*, *prakrama*, *prādeśa*, *bāhu*, *aratni* carry long tradition and have been used earlier in *Samhitā* and *Brahmaṇic* literature in the same sense as these have been used in the *Śulbasūtras*.

5.2. Knowledge of rational numbers like $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{3}{2}$, $\frac{5}{12}$, $\frac{7}{2}$,

$8\frac{1}{2}$, $9\frac{1}{2}$, etc. were used in a decimal notations, and their fundamental operations like addition subtraction, multiplication, division were known.

5.3. Describes construction of square, rectangle, isosceles trapezium with use of cord and peg.

Baudhāyana gives two methods of construction of square,³¹ Āpastamba³² gives four methods of construction of square, two being different and so on.

5.4. Gives ideas of irrational numbers³³ e. g., diagonal of a square of unit length is $\sqrt{2}$ (i.e. $1,1\sqrt{2}$), diagonal of a rectangle of sides $\sqrt{2}$, and 1 is $\sqrt{3}$ (i.e. $\sqrt{2}, 1, \sqrt{3}$) followed by general statement of the theorem of the square on the diagonal. For easy verification Baudhāyana also suggested triples expressed in rational number, (3, 4, 5), (12, 5, 13), (15, 8, 17) ($7, \sqrt{24}, 25$), (12, 35, 37), and (15, 36, 39,). The same triples appear in the *Āpastamba Śulbasūtra*³⁴. The various other triples also appeared in the Kātyāyana and Mānava *Śulbasūtra*. A few examples are: (1, 3, $\sqrt{10}$), (2, 6, $\sqrt{40}$), (1, $\sqrt{10}, 11$), (188, $78\frac{1}{3}; 2032/3$), (6, $2\frac{1}{2}, 55/12$), (10, $4\frac{1}{6}, 105/6$) and so on. There are 25 triples given in the *Śulbasūtras*.

5.5. Construction of a square as the sum or difference of two squares.³⁵

5.6. Transformation³⁶ of square into a rectangle, isosceles trapezium, and rectangle into a square, triangle, rhombus.

5.7. Calculation of the value of $\sqrt{2} = 1 + 1/3 + 1/3.4 - 1/3.4.34$ (approx.)³⁷.

In decimal notations $\sqrt{2} = 1.41, 42, 56$, the modern value being 1.41, 42, 13..... This shows that the value is correct to four places of decimal.

5.8. i) Transformation of square (side = s) into a circle³⁸ (diameter = d) i.e. $d/s = 1/3 \times (2 + \sqrt{2})$

ii) Circle into square:³⁹ *First method:* $s/d = 1 - 1/8 + 1/8.29 - 1/8.29.6 + 1/8.29.6.8$
Second method: $s/d = 1 - 2/15$

5.9. Area of square = $a.a = a^2$; area of the rectangle = $a.b$; area of the triangle = $1/2$ of the rectangle = $1/2 a.b$, where a and b are two consecutive sides. Area of the isosceles trapezium = $1/2 c (a + b)$ where a and b are the parallel sides and c is the distance between the parallel sides. Though the formulae have not been mentioned but the result are evident from the construction of different *agnis* having the same area and from the transformation of the one figure into another. For example (Fig. 1) for *caturasra śyenacit*, the body = 4 square *puruṣas*, two wings = $2 \times (1 + 1/5) \times 1 = 12/5$ sq. *puruṣas*, 1 tail = $(1 + 1/10) \times 1 = 11/10$ sq. *puruṣas*, total = $4 + 12/5 + 11/10 = 7\frac{1}{2}$ sq. *puruṣas*.

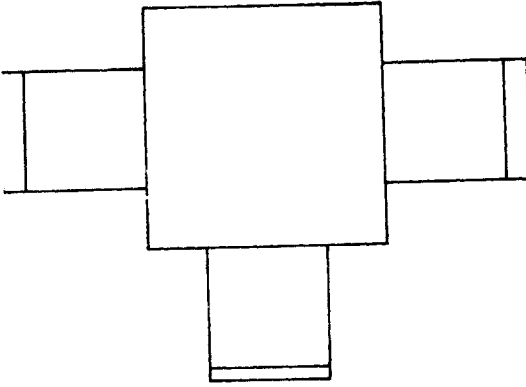


FIG. 1(a). Caturasraśyenacit (Plan)

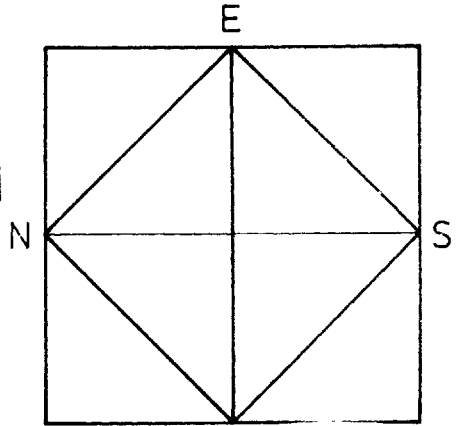


FIG. 1(b). Paitrkī vedi (Plan)

5.10. Gives details of bricks and construction of compulsory and optional sacrificial fires and altars.

6. KNOWLEDGE OF IRRATIONAL (OR SURD) NUMBERS AND THEOREM OF SQUARE ON THE DIAGONAL

6.1. For construction of square, rectangle, circle, isosceles trapezium, various technical terms *prācī* (east-west line), *pārśvamānī* (line parallel to east-west line), *tiryahmānī* (line parallel to north-south line i.e. line perpendicular to east-west line), *askṣṇayā* (diagonal of rectangle), *karānī* (producer), *dvikarānī* ($\sqrt{2}$), *trīkarānī* ($\sqrt{3}$), *dvitīyākarānī* ($\sqrt{2/2} = 1/\sqrt{2}$), *trītīyākarānī* ($\sqrt{3/3} = 1/\sqrt{3}$) etc. were used. It is said by Baudhāyana, Āpastamba and other *śulbakāras* that the diagonal of the square of unit length is $\sqrt{2}$, diagonal of the rectangle of breadth $\sqrt{2}$ and height 1 is $\sqrt{3}$, the diagonal of a rectangle of breadth $\sqrt{10}$ and height 1 is $\sqrt{11}$. They have also expressed the diagonal of sides 3 and 1 as $\sqrt{10}$ and so on (vide Fig. 2.).

In the figures, $OA = \sqrt{2}$, $OB = \sqrt{3}$, $OC = \sqrt{4} \dots OE = \sqrt{10}$ and of the numbers of like 1, $\sqrt{2}$, $\sqrt{3}$, 3, $\sqrt{10}$, 4 etc. are represented on the same number line and they had also clear idea that the $\sqrt{2}$ is the greater than 1 and the $\sqrt{3}$ is the greater than $\sqrt{2}$. They had also attempted to approximate the value of $\sqrt{2} = 1 + 1/3 + 1/3.4 - 1/3.4.34$ by geometrical method (vide item no. 5.7.).

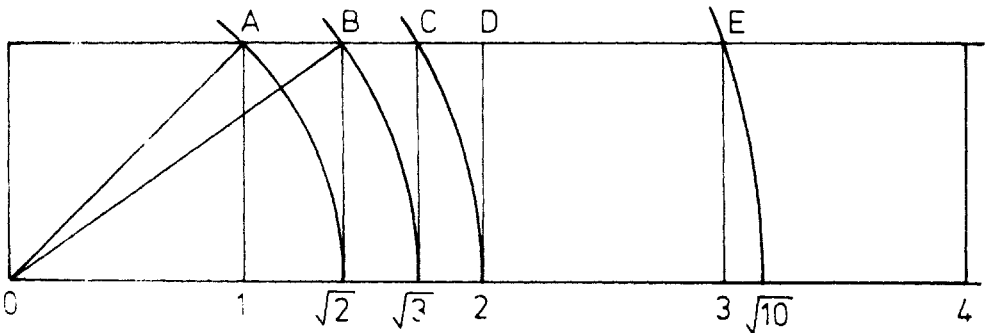


FIG. 2. Number line

As regards statement on the square of the diagonal, question has been raised whether Baudhāyana and other *śulbakāra* understood the truth of the general statement made by them. Many a conjecture have been made by Heath, Burk, Muller, Thibaut, Datta and others about the possible proofs of the theorem. I have not discussed any of these proofs, rather I have tried to re-examine systematically the statements made by Baudhāyana which in my mind gives enough support for a general statement of the theorem.

6.2. Baudhāyana, *sūtra* 1.9 states that the diagonal of square produces double its area and it is $\sqrt{2}$. This is clear from the *Paīrkivedi* in which the *vedi* is oriented towards cardinal directions E,F,N, and S. Baudhāyana was a great professional and an expert in laying bricks and verifying their areas. The following diagrams drawn on the basis of the hints available in the *Śulbasūtras* justify possibly the truth of the statement both geometrically and arithmetically.

Geometrical explanation:

Fig.3 (a) shows $AB^2 = I + II + III + IV = AC^2 + BC^2$

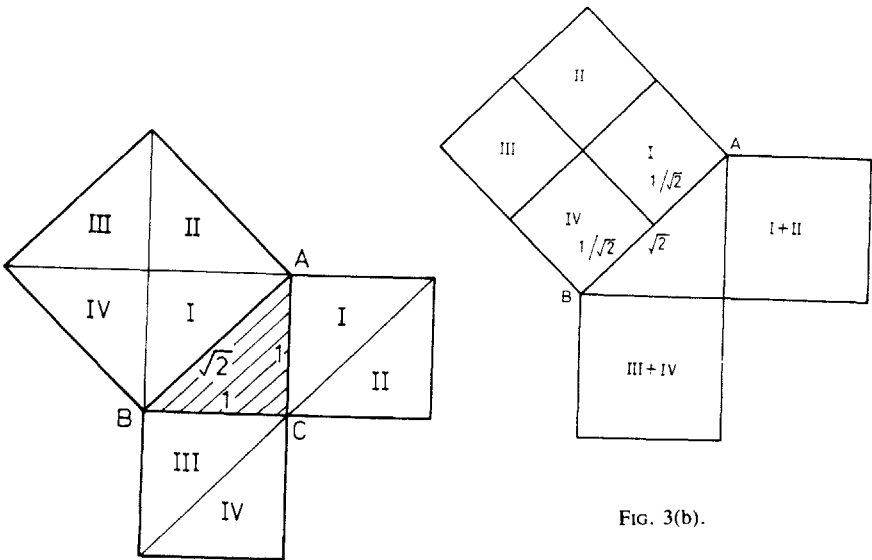


FIG. 3(b).

FIG. 3(a).

Arithmetical explanation:

Fig. 3 (b) shows that the diagonal $\sqrt{2}$ has been divided into two parts each being $\sqrt{2}/2 = 1/\sqrt{2}$. Obviously the square on the diagonal breaks into four square units, each having an area of $1/\sqrt{2} \times 1/\sqrt{2} = 1/2$ and the two squares together becomes equal to one unit. Obviously $AB^2 = (I + II) + (III + IV) = 1 + 1 = AC^2 + BC^2$. Since the *śulbakāras* could construct a square as the sum of two squares. They had no problem in comparing and verifying that both geometrical and arithmetical attempts lend to the same result, $AB^2 = AC^2 + BC^2$

6.3. Baudhāyana's next *sūtra* 1.10 — 1.11 justify the same statement from the diagonal of a rectangle. It states that the diagonal of a rectangle of breadth $\sqrt{2}$ and height 1 is $\sqrt{3}$. The truth of this statement also follows from the similar diagram given below. The geometrical diagram is in the pattern of the construction of *syenaciti* in which wings and tail are adjusted for fulfilling the area of $7\frac{1}{2}$ sq. *puruṣa*. For arithmetical proof Baudhāyana asks us to divide the diagonal $\sqrt{3}$ into nine square units (*navamastu bhumerbhāgo*). It was not clear originally why Baudhāyana, in connection with the statement of theorem on the square of diagonal has directed to divide the diagonal into nine square units. The picture will be cleared from the following diagrams.

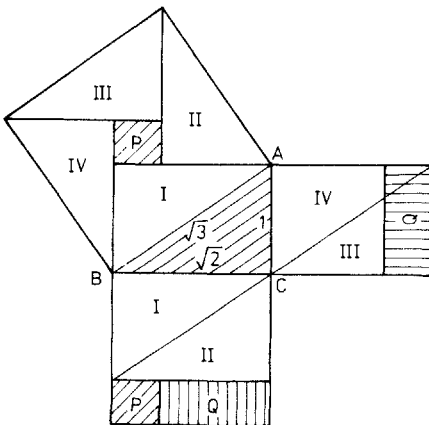


FIG. 4(a).

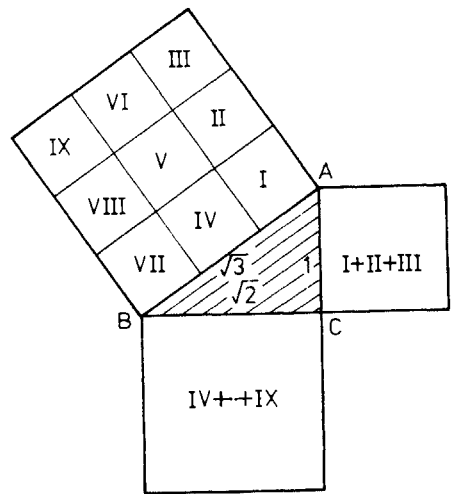


FIG. 4(b).

Geometrical explanation:

Fig. 4 (a) shows that $AB^2 = I + II + III + IV + \text{area P (side} = \sqrt{2})$, From BC^2 a strip (side $\sqrt{2}$) is cut off. From the cut off strip, square area P (side $\sqrt{2}$) is kept aside and the remaining rectangular strip is transferred and joined to the area AC^2 . Obviously,

$$BC^2 = I + II + \text{area P} + \text{area Q}$$

$$AC^2 = III + IV - \text{area Q}$$

$$BC^2 + AC^2 = I + II + III + IV + \text{area P}$$

$$\therefore AB^2 = BC^2 + AC^2$$

Arithmetical explanation:

Fig. 4 (b) shows that the diagonal $\sqrt{3}$ has been divided into three parts, each being $\sqrt{3}/3 = 1/\sqrt{3}$. Obviously the square of the diagonal breaks into nine square units, each having area $1/\sqrt{3} \times 1/\sqrt{3} = 1/3$. Three square units together becomes equal one unit in area and six square units become $= 1/3 \times 6 = 2$ square unit area. Hence $AC^2 = (I + II + III) + (IV + V + \dots + IX) = 1 \text{ unit} + 2 \text{ units} = AC^2 + BC^2$. This shows that *śulbakār's* directive of breaking the diagonal into nine square units is quite meaningful which establishes the theorem of the square on the diagonal of a rectangle.

6.4. The Baudhāyana *śulbasūtra* 1.12 gives the general statement of the theorem of the square on the diagonal which states 'that the areas of the square produce separately by the length and breadth of a rectangle together equal the area of the square produced by the diagonal'.

Next *sūtra* 1.13 says that this can easily be understood for rectangle having sides (3,4,5), (12,5,13), (15,8,17), (7,24,25), (12,35,37), (15,36,39). Items 6.1 to 6.4 show the mode of treatment in the *Śulbasūtras* which undoubtedly hints that an attempt was made systematically to understand the theorem of square on the diagonal of a square of rectangle, having rational and irrational values of sides.

7. TRIPLES IN OTHER CULTURAL AREAS

Pythagorean triple (3,4,5) is found in almost in all the old cultures. The relation $x^2 + y^2 = z^2$, where x,y,z are the length, breadth and the hypotenuse of a rectangle was discovered by the Babylonians and Egyptians much before Pythagoras. According to Van der Waerden, the Chinese followed a more or less the same Babylonian algebraic technique in a more developed form. According to him the Chinese type is more geometrical than the Babylonian though types of treatment are the same. Let us now discuss these in a nutshell.

7.1. *The Babylonian & the Chinese:*

Neugebauer and Sach's translations of Babylonian text (BM 34568) gives 19

problems involving $X^2 + Y^2 = z^2$ and Plimpton (No. 322) gives 15 such triples in sexagesimal numbers. The script of the text is old Babylonian and according to them it is supposed to belong to the period 1900 B.C. to 1600 B.C. The problems have been divided by Van der Waerden ⁴⁰ into 13 types as follows:

Type I	: Given x and y, to find z
Type II	: Given y and z to find x
Type III	: Given $x = a$, and $z - y = d$, to find y and z
Type IV	: Given $x - y = d$, $z = c$, find x & y
Type V	: Given $z - x = a$ and $z - y = b$ to find x, y, and z
Type VI	: Given $x = a$, and $t + z = b$, to find x and z
Type VII	: Given $x = a$, $y - z = p/q$ y, to find y and z.
Type VIII	: Given $x = a$, $y = b$ to find the side s of the inscribed square.
Type IX	: Given $x = a$, $y = b$, to find the diameter of the inscribed circle
Type X	: Given $x + y = s$ and $xy = F$, to find x&y.
Type XI	: Given $y - z = d$, $xy = F$, to find x&y
Type XII	: Given $x + y = s$ and $z = c$ to find x & y.
Type XIII	: Given $x + y + z = s$ and $xy = F$ to find z.

The Plimpton text 322 contains only a list of Pythagorean triples and the text (BM 34568) has also used the triples (3, 4, 5), (5, 12, 13), (8, 15, 17), (20, 21, 29).

The Chinese used the problems which have used much more geometrical terms but the types and the mode of solutions are more elaborate and similar to those of the Babylonian. It has used the same triplets as mentioned above and also the triple (7,24,25). It is not uncommon if we think these appeared in *Chiu Chang Suan Shu* (Nine chapters on the Mathematical Art, composed between 50 B.C. and 100 A.D.), about 2000 years later than the Babylonian. The Chinese might have received and reviewed the Babylonian materials and developed them in their own style.

7.2. The Egyptian:

The Egyptian had also some knowledge of such triples. The Egyptian rope stretcher played an important role in the foundation ceremony of temples. The Egyptian manuscripts of Middle Kingdom (2000 to 1800 BC.) appear to be purely arithmetical works taking geometrical materials as background for arithmetical computation. In the Berlin Papyrus (No. 6619) is published the problem as follows:

“A square and a second square whose side is one-half and one-fourth of that of the first square, have together an area of 100, show how to calculate this”

Taking two unknown x & y, equations may be written in the form

$$y = \frac{3}{4}x \text{ and } x^2 + y^2 = z^2 = 100$$

This gives the solutions of the problem

$x = 8, y = 6$ and $z = 10$, The triple (6,8,10)
is linked with the wellknown triple (3,4,5).

7.3. The Greeks:

Seidenberg has shown that the Pythagorean legend (5th century B.C.) 'sacrificing an OX in honour of the discovery (3,4,5)' or the Delian legend of Apollo (260-200 B.C.) "the way to fight a plague is to double an altar" or the Heron's use (62 A.D.) of peg and cord for the construction right angles' have similarities with Indian traditions. Cantor⁴¹ hold for twenty years the opinion that the geometry of the *śulbasūtras* was a derivative of Alexandrian knowledge and finally renounced it in 1904 when he was convinced with Schroeder's suggestion that 'the Indian peg and cord construction go back to atleast 10th century B.C.' Of course, Cantor preferred Egypt and not India as a source of Greek geometry, though the reason has not been explained.

8. CONCLUDING REMARKS

From the discussion that follows, it is clear that Indian tradition of altar construction is old, original and primary in nature. It is not yet clear whether the Indian discovered the basic knowledge of the triple of their own or got it from other sources. The Babylonian appears to have known the relation $x^2 + y^2 = z^2$, because their pattern of tackling the problem appear to have been used for teaching purposes and no history of original development is available. The Chinese followed the Babylonian and shows originality and simplicity in tackling some of their problems. The Egyptian, Indian & the Greeks may have some links at some stages because of the use of cord and peg method. But Indian technique is distinct because of its concept of both rational and irrational numbers and their application for verifying the truth of theorem of square of the diagonal, Van der Waerden⁴² has proposed the idea of Pre-Babylonian tradition playing a central role on the basis of *Neolithic geometry and algebra* invented somewhere in Central Europe is just to hold all the traditions as carrier of the same tradition without proper acknowledgement to contributions to each culture. Indian history or irrational numbers and their attempt to use them for theorem of square on the diagonal of square or rectangle is distinct from solutions of other cultures and deserve recordings in world history of mathematics.

ABBREVIATIONS

- | | |
|---------------------------------|-------|
| 1. <i>Āpastamba Śulbasūtra</i> | — Āśl |
| 2. <i>Baudhāyana Śulbasūtra</i> | — Bśl |
| 3. <i>Kapisthala Samhitā</i> | — KPS |
| 4. <i>Kāṭha Samhitā</i> | — KS |
| 5. <i>Kātyāyana Śulbasūtra</i> | — Kśl |
| 6. <i>Maitrāyanīya Samhitā</i> | — MS |

- | | |
|-------------------------------|-------|
| 7. <i>Mānava Śulbasūtra</i> | — Mśl |
| 8. <i>Ṛgveda</i> | — Ṛg. |
| 9. <i>Śatpatha Brāhmaṇa</i> | — SBR |
| 10. <i>Taittirīya Samhitā</i> | — TS |

For edition of Āśl, Bśl, Kśl, Mśl. See item no. 8 under Notes and References.

NOTES AND REFERENCES

- ¹Thibaut, G. 'On the *Śulbasūtras*, *Journal of the Asiatic Society of Bengal*, 44, no. 3, 227-275, 1875; *Śulbasūtra* of Baudhāyana, *The Pandit*, Old Series, 9 & 10, 1874-75; n.s., 1, Benaras, 1876-77.
- ²Burk, A. 'Āpastamba' *Śulbasūtra*', edited and published with notes and comments, *Zeitschrift d. deutsch morgenländischen Gesellschaft*, 55, 543-591, 1901; 56, 327-391, 1902. Burk pointed out (55, p. 564) that the triple (8, 15, 17) satisfy the basic property of Pythagorean triple but are not amongst the triples ascribed to the Pythagorean.
- ^{3a}Neugebauer, O. *Vorlesungen über Geschichte der Antiken Mathematischen Wissenschaften*, Berlin, 1934;
- ^{3b}Neugebauer, O. & Sachs, A. (ed.) — *Mathematical Cuneiform Texts* (American Oriental Series — Vol. 29), New Haven, p. 41, 1945.
- ⁴Seidenberg, A. 'The Ritual Origin of Geometry', *Archive for History of Exact Sciences*, 1, 488-527, 1962, see p. 489. Seidenberg believes in ritual origin of geometry. According to him (p.521) temple building was a ritual and stretching cord was a ritual, so we have geometrical rituals going back to times of about 3500 B.C.
- ⁵Van der Waerden, B.L. 'On the Pre-Babylonian Mathematics I', *Archive for History of Exact Sciences*, 23, 1-25, 1980, vide p.3.
- ⁶Datta, B. *The Science of Śulba*, Calcutta University, 1932.
- ⁷Seidenberg, A., *Ibid.* Vide *Notes and References*, No. 4.
- ⁸Sen, S. N. & Bag, A.K. *The Śulbasūtras* (Baudhāyana, Āpastamba, Kātyāyana and Mānava), text with English translation and commentary, Indian National Science Academy, New Delhi, 1983. The references of all śulba texts (Bśl, Āśl, Kśl, Mśl) have been used from this edition.
- ⁹Vide *Notes and Reference* No. 8.
- ¹⁰Bśl. 3.1, 3.9-3.11, 4.3, 6.6, 7.4, 8.3, 8.6, 16.1, 17.1, 19.1, 20.1, 21.10, 21.11; Āśl 5.1, 5.8, 6.1, 6.3, 6.7, 6.11, 7.1, 7.2, 7.3, 8.1, 8.7, 1.10, 11.1, 12.7, 12.9, 13.4, 13.8, 14.1, 14.6, 20.11, 21.3, 21.6 & so on.
- ¹¹Bśl 10.2-10.3
- ¹²Bśl 11.5-11.6
- ^{12a}Bśl 11.2, 11.3
- ¹³Ṛg. 1.15, 12; 6.15.19, 5.11.2
- ¹⁴Ṛg. 1.164.52, 10.14.5, 1.58.5, 1.4.7, 2.2.4, 6.3.7.
- ¹⁵Ṛg. 7.35.7.
- ¹⁶TS. 5.2. 3.5.
- ¹⁷MS. 3.2.3.
- ¹⁸KS. 20.1
- ¹⁹KPS. 32.3
- ²⁰TS. 6.2.4.5; MS. 3.8.4; KS. 25.3; KPS. 3.8.6.
- ²¹TS. 5.4. 11. 1-2
- ²²TS. 5.6 8. 2-3
- ²³TS. 5.2.5.5-6; 5.7.6.1.
- ²⁴SBR. 10.2.3. 17-18.
- ²⁵Bśl. 8.1, 20.1; Āśl. 12.9, 14.1, 14.4., 15. 1. 18.1 etc.
- ²⁶Āśl. 8.3-8.5.
- ²⁷Buhler, G. *Sacred Book of the East*, 14, pp. xliiii. 2, pp. xxx; Macdonell, A. *Bṛhaddevaśā* ed. & tr. into English, 2 parts, Harvard Oriental Series, nos. 5 & 6; Cambridge. 1904, vide p. xxii; Thibaut. G.

'*Astronomie, Astrologie und Mathematik*' (in Grundriss d. Indo-Arischen Philologie u. Alter, Vo. 3:9) Strassburg, 1899; Keith, A. B. *Veaa of the Black Yajurveda*, Harvard Oriental Series No.s 18 & 19, 1914; Ramagopal, *India of Vedic Kalpasūtras*, pp. 88-100, New Delhi, 1959, Kane. P.V. History of Dharma Sāstra, 1 (1) pp. 73-75, Revised and enlarged, Bhandarkar Oriental Series, 1968.

²⁸Thiabaut, G. *The Pandit* n.s. 1, p. 769.

²⁹Bśl. 1.3.

³⁰Āśl. 6.5, 15.4.

³¹Bśl. 1.4-1.5, 1.6-1.8

³²Āśl. 1.7, 2.1.

³³Bśl. 1.9-1.13

³⁴Āśl. 1.5, 2.2, 2.3, 5.3, 5.5.

³⁵Bśl. 2.1-2.2; Āśl. 2.4, 2.8.

³⁶Bśl. 2.3-2. 11; Āśl. 2.7, 2.8, 3.3.

³⁷Bśl. 2.12; Āśl. 2.9.

³⁸Bśl. 2.9; Āśl. 3.2; Kśl. 3.11.

³⁹Bśl. 2.19-2.11; Āśl. 3.3; Kśl. 3.12.

⁴⁰Vide *Notes and References* No. 5.

⁴¹Cantor, M. 'Über die älteste indische Mathematik' Archiv d. Math. u. Phys. 8, 1904.

⁴²Van der Waerden, B.L. 'On Pre-Babylonian Mathematics II' *Archive for History of Exact Sciences*, 23, 27-46, 1980; vide p. 24; Van der Waerden believes in two way direction of Pre-Babylonian Mathematics, viz. i) For teaching purposes, collections of algebraic and geometrical problems with solutions are developed in the manner of the extant Chinese, Babylonian and Egyptian collection and ii) Ritual purposes, geometrical constructions were developed in ceremonial meeting places (hengses and temples) and altars satisfying certain geometrical conditions. Of course, he remains silent about the origin.