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DEVELOPMENT OF ASTRONOMICAL OBSERVATION IN VEDIC AND POST-VEDIC INDIA*

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The earliest written records of astronomical knowledge in India are found in Vedic literature. I have shown in the first section of this paper that the gradual development of astronomy in accordance with the transition of mode of production can be traced in Vedic literature. There is no record of astronomical instruments in Vedic literature proper and only naked eye observations must have been made in Vedic period.

I have discussed two earliest astronomical instruments in India, the gnomon and the clepsydra, in the second section of this paper. They are mentioned in *Vedāṅga* literature and some other post-Vedic literature. I have analyzed their records, particularly the records of shadow-length of the gnomon, and shown that they are based on actual observations in North India, and that there is no reason to assume foreign influence in *Vedāṅga* astronomy.

1. EARLY DEVELOPMENT OF ASTRONOMY AND ASTRONOMICAL OBSERVATION IN INDIA

(i) Development of Astronomy in Vedic India

We can easily suppose that any primitive society, whether pastoral or agricultural, had certain astronomical knowledge, because it is indispensable to determine seasons, and sometimes directions. This is, of course, applicable to the Indus civilization (*c.* 2500 BC-1700 BC?)[1], whose agriculture and excellent town planning suggest a fairly developed astronomical knowledge. However, the Indus script has not been deciphered, and there is no means to estimate the development of astronomy in the Indus civilization at this moment. Some people are attempting to interpret some signs in Indus seals as signifiers of asterisms[2], but they are still at the stage of hypothesis. We also come across a passage in the *Śatapatha-brāhmaṇa*[3] (I.6.1.2-3), which suggests the astronomical knowledge of non-Aryan agriculturists. It reads:

"......the Seasons went to the Asuras, the malignant, spiteful enemies of the gods.

Those (Asuras) then throve in such a manner that they (the gods) heard of it; for even while the foremost (of the Asuras) were still ploughing and sowing, those behind them were already engaged in reaping and threshing:

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indeed even without tilling the plants ripened forthwith for them" (Translated by Eggeling)[4].

This shows that Asuras, who might have been non-Aryan agriculturists, had a regular calendar which was necessary for their agricultural activities. In this connection, we can note that the relationship between Asuras in Vedic literature and Indus civilization and proto-materialism has been suggested by Debiprasad Chattopadhyaya, although many references to the Asuras in ancient literature are not all of the same nature[5].

It is estimated that the appearance of Aryans in India may be attributed to *c.* 1600 BC or even a little earlier[6]. The earliest document of them is the *Ṛg-veda* whose composition is supposed to be started about 1500 BC or so[7]. Among the Books (*Maṇḍalas*) of the *Ṛg-veda*, the Family Books (Books II-VII) are considered to be the earliest portions[8]. At this stage, economy was predominantly pastoral, but agriculture was also known[9]. It seems that the Aryans were already pastoral before their migration to India, because names of cattle are common to the European and Asiatic Aryans, but names of cultivable plants are hardly common[10].

Already in the early portion of the *Ṛg-veda* (VII.103.7-9), certain calendrical knowledge has been recorded.

> "Like Brahmans at the *Soma* libation, at the *Atirātra* sacrifice, you are now croaking around the replenished lake (throughout the night), for on that day of the year you frogs are everywhere about, when it is the day of the setting in of the rains.
>
> They utter a loud cry, like Brahmans when bearing the *Soma* libation, and reciting the perennial prayer: like manifest priests with the *gharma* offering, they hid (in the hot weather) perspiring (in their holes), but now some of them appear.
>
> These leaders of rites observe the institutes of the gods, and the (appropriate) season of the twelvemonth; as the year revolves, and the rains return, then, scorched and heated, they obtain freedom (from their hiding-places)". (Translated by Wilson)[11].

In the above hymn, we can see that there was a kind of calendrical knowledge which was connected with rainfall and *Soma*-rite. Frogs cry at the rainy season regularly every year, and similarly priests also performed *Soma*-rites regularly, according to their knowledge of seasons.

In this connection, we can note the similarities between some terminologies in Vedic and Avestan, for example, *yajña* in Vedic and *yasna* in Avestan for sacrifice[12]. The *soma (haoma* in Avestan) was celebrated both in ancient India and Iran[13]. These facts suggest that some concepts of ritual can be traced back to proto-Indo-Iranian. However, ancient Persian calendar was quite different from ancient Indian calendar. Avestan year consisted of 12 months of 30 days each and additional 5 days[14]. Therefore, ancient Persian month was not synodic, and a year constantly consisted of 365 days. Although the schematic 12-month year of 360 days also appears in Vedas[15], ancient Indian month was practically synodic, and the intercalary month was used in order to adjust to the solar year. So, we can guess that Vedic astronomy gradually developed in India with the transition of the mode of production. Especially, the importance of accurate calendar must have increased with the development of agriculture. As D.D. Kosambi pointed out[16], annual monsoon, which comes at approximately fixed time annually, is very important for Indian agriculture, and naturally a good calendar was required.

Let us proceed to the later *Ṛg-vedic* period. It is said that the date of the *Ṛg-veda* may be *c.* 1500-1000 BC and Books I and X of the *Ṛg-veda* are admittedly late[17].

In this period, the intercalary month was, most probably, known. The *Ṛg-veda* (I.25.8) reads:

> "He, who accepting the rites (dedicated to him), knows the twelve months and their productions, and that which is supplementarily engendered." (Translated by Wilson)[18].

Quoting Sāyana's commentary, the translater H.H. Wilson remarked:

> "*Vedā ya upajāyate,* who knows what is *upa,* additionally or subordinately, produced. The expression is obscure, but, in conncetion with the preceding, *Veda māso... dvādaśa,* who knows the twelve months, we cannot doubt the correctness of Scholiast's conclusion, that the thirteenth, the supplementary or intercalary month of the Hindu lunisolar year, is alluded to; 'that thirteenth or additional month which is produced of itself, in connection with the year'"(Wilson)[19].

According to this interpretation, the intercalary month was known to *Ṛg-vedic* people. Then, how did they regulate their calendar? It seems that they regulated their calendar by the observation of the moon. Let us see three hymns from the hymns of bridal of Sūryā (a daughter of Savitṛ) and Soma. The *Ṛg-veda* (X.85.2) reads:

> "By *Soma* the *Ādityas* are strong; by *Soma* the earth is great; *Soma* is stationed in the vicinity of these *Nakṣatras.*" (Translated by Wilson)[20].

The *Ṛg-veda* (X.85.5) reads:

"When, O god, they quaff thee, then dost thou renew thyself again; *Vāyu* is the guardian of *Soma*, the marker of years and months". (Translated by Wilson)[21].

In the above hymns, it is seen that *Ṛg-vedic* people noticed that the moon travels among *nakṣatras* (lunar mansions, i.e. 27 or 28 asterisms along the ecliptic), and considered the moon as the 'marker' of their calendar. *Soma* is epithet of the moon. Although the complete list of *nakṣatras* is not given in the *Ṛg-veda*, some of them are mentioned by name. The *Ṛg-veda* (X.85.13) reads:

"*Sūryā's* bridal procession which *Savitṛ* despatched has advanced; the oxen are whipped along in the *Maghā* (constellation); she is borne (to her husband's house) in the *Arjunī* (constellation)." (Translated by Wilson)[22].

Here, names of *Maghā* (actually, *Aghā* in the original *Ṛg-veda*), and *Arjunī* (i.e. Phalgunī) are mentioned. They indicate, most probably, the positions of the moon.

In this period, one year was divided into three seasons[23]. The *Ṛg-veda* (X.90.6) reads:

"When the gods performed the sacrifice with *Puruṣa* at the offering, then Spring was its ghi (*ājya*), Summer the fuel, and Autumn the oblation." (Translated by Wilson)[24].

Here, the names of Spring (*vasanta*), Summer (*grīṣma*), and Autumn (*śarad*) appear. The *Ṛg-veda* also knows the Rainy season (*prā-vṛś*) and Winter (*hīma, hemanta*)[25].

It seems that the daytime was divided into five parts in *Ṛg-vedic* age. The *Ṛg-veda* (V. 76.3) reads:

"Whether you come at the (milking time) of the cattle *[saṁgava]*, at the dawn *[prātar]* of day, at noon *[madhyaṁdina]*, when the sun is high, or by day or by night, (come) with plicitous protection: the drinking of the *soma* has not now extended beyond the *Aśvins*." (Translated by Wilson)[26].

In the above hymn, the first three of the five-fold division of daytime in later Vedic literature[27] *(prātar, saṁgava, madhyaṁdina, aparāhṇa,* and *sāyāhna)* are mentioned.

Let us proceed to the later Vedic *Śrutis (= Saṁhitās* and *Brāhmaṇas)*, i.e.the *Saṁhitās* of the *Sāma-veda, Yajur-veda,* and *Atharva-veda,* and the *Brāhmaṇas*. Vedic Aryans advanced towards east, and these later Vedic *Śrutis* were composed in the land of Kurus and Pañcālas, i.e. "the major portion of western Uttar Pradesh, almost the whole of Haryana, and the neighbouring parts of the Panjab and Rajasthan"[28]. Roughly, they are supposed to be composed in *c.* 1000-500 BC. The lower date of the *Brāhmaṇas* may *be c.* 600-500 BC, and the upper limit of the earliest *Brāhmaṇas,* which do not include the *Śatapathabrāhmaṇa* and the *Aitareya-brāhmaṇa,* excluding its first five books, may be 800 BC or so[29]. The earliest *Upaniṣads* may be placed about 500 BC or so[30]. The *Upaniṣads* are also included in the Vedic *Śrutis* in wider sense (i.e. *Saṁhitā, Brāhmaṇa, Āraṇyaka,* and *Upaniṣad).* We should also note that the society had become essentially agricultural in this stage[31]. The chief cereal in *Ṛg-vedic* age was barley, which ripens in sixty days and does not require much rain[32]. In later Vedic *Śrutis,* several kinds of pulses and rice, which are absent in the *Ṛg-veda,* are mentioned[33]. The cultivation of new cereals must have required a more accurate calendar.

For the convenience of later discussions, the relationship between the recensions of the *Saṁhitās* and *Brāhmaṇas* is depicted in Table 1.

Table 1.

	Saṁhitās	*Brāhmaṇas*
Ṛg-veda	*Śākala*-recension	*Aitareya* *Kausītaki (= Śāṅkhāyana)*
Sāma-veda	*Kauthuma*-recension *Rāṇāyanīya*-recension *Jaiminīya*-recension	*Pañcaviṁśa* *(= Tāṇḍyamahā),* etc. *Jaiminīya*
Black *Yajur-veda*	*Kāṭhaka-saṁhitā* *Maitrāyanī-saṁhitā* *Taittirīya-saṁhitā*	 *Taittirīya*
White *Yajur-veda*	*Vājasneyi-saṁhitā* *(Mādhyaṁdina* and *Kāṇva* recensions)	*Śatapatha* (Also two recensions)
Atharva-veda	*Śaunaka*-recension (= Current recension) *Paippalāda-saṁhitā*	 *Gopatha*

In later Vedic *Śrutis,* more advanced astronomy than that of the *Ṛg-veda* is mentioned. The complete list of 27 *nakṣatras* is given in the *Taittirīya-saṁhitā* (IV.4.10)[34]. while the *Atharva-veda* (XIX.7)[35] gives a list of 28 *nakṣatras.*

The *nakṣatras* were used to indicate the position of the full moon. The word "*Tiṣyā-pūrṇamāsa*" (full moon day of the month *Tiṣya*) occurs in the

Taittirīya-saṁhitā (II.2.10.1)[36], and the words "*Phalgunī-pūrṇamāsa*" and "*Citrā-pūrṇamāsa*" occur in the *Taittirīya-saṁhitā* (VII.4.8.1-2)[37]. Here, the name of *nakṣatras* is used to indicate month. However, the seasonal names of the months are usually used in literature belonging to the *Yujurveda*[38]. The *Taittirīya-saṁhitā* (IV.4.11.1)[39], etc. of the Black *Yajur-veda* and the *Vājasneyi-saṁhitā* (XIII.25; XIV. 6;15;16;27; XV.57)[40] of the White *Yajur-veda* give the names of the months as given in Table 2.

Table 2

Spring *(vasanta)*	(1) *Madhu* (2) *Mādhava*
Summer *(grīṣma)*	(3) *Śukra* (4) *Śuci*
Rainy *(varṣā)*	(5) *Nabha* (6) *Nabhasya*
Autumn *(śarad)*	(7) *Iṣa* (8) *Ūrja*
Winter *(hemanta)*	(9) *Saha* (10) *Sahasya*
Cool *(śiśira)*	(11) *Tapa* (12) *Tapasya*

These names of the months are not found in the *Ṛg-veda*. One year is divided into 6 seasons in the above case, but is usually divided into 5 seasons also in the *Yajur-veda, Atharva-veda,* etc.[41]. In the case of the latter, *hemanta* and *śiśira* were combined into one season, "*hemanta-śiśira*". It is interesting that the growth of the division of the seasons from *Ṛg-vedic* three to later five was explained by Zimmer as indicating the advance of the Vedic Indians towards the east[42]. The above list starts from spring, and, as S.B. Dikshit supposed[43], a year must have commensed from spring in *Yajur-vedic* age. That the sun goes south for six months and north for six months is already mentioned in the *Taittirīya-saṁhitā* (VI.5.3)[44]. The word "*ayana*" in the sense of a half year, however, does not appear in the *Saṁhitās* or *Brāhmaṇas*, and only appears in the *Upaniṣads*[45].

The intercalary month is explicitly mentioned as *aṁhasaspati, saṁsarpa,* or *malimluca* in the *Yajur-veda*[46]. The *Vājasneyi-saṁhitā* (VII.30; XXII. 31)[47] calls it *aṁhasaspati,* and the *Taittirīya-saṁhitā* (I.4.14)[48] calls it *saṁsarpa.* The *Atharva-veda* (V.6.4)[49] calls it *sanisrasa.* The last reference quite clearly mentions it as an intercalary month.

".... Weakling *(sanisrasa)* by name art thou, the thirteenth month, Indra's house". (Translated by Whitney)[50].

The rule of intercalation is, however, not mentioned in Vedic *Śrutis*. We can only notice that five-year cycle is mentioned in *Śruti* literature, although four, three, two, or even six-year cycle also appears[51]. In any case, names of each year of the five-year cycle are mentioned in the *Vājasneyi-saṁhitā* (XXVII.45)[52], *Taittirīya-saṁhitā* (V.5.7.3-4)[53], etc. as: *Saṁvatsara, Parivatsara, Idāvatsara, Idvatsara* (or *Iduvatsara*), and *Vatsara;* or some other variants. Here, we can see a forerunner of the five-year *yuga* of later *Vedāṅga* period.

As regards the commencement of a month, it seems that both *pūrṇimānta* (ending with full moon) and *amānta* (ending with new moon) are permissible[54]. The words *"pūrva-pakṣa"* (the first fortnight) and *"apara-pakṣa"* (the second fortnight) appear in the *Taittirīya-saṁhitā* (III.4.9.6.)[55] and some *Brāhmaṇas.* According to the *Nirukta* (XI.6), the *"pūrva-pakṣa"* is the waxing half (i.e. *śukla-pakṣa*) and the *"apara-pakṣa"* is the waning half (i.e. *kṛṣṇa-pakṣa*)[56].

Although the word *"muhūrta"* appears in the *Ṛg-veda* in the sense of "moment"[57], the *muhūrta* as one-thirtieth of a day first appears in the *Brāhmaṇas*. The *Taittirīya-brāhmaṇa* (III.10.1.1-3; 10.9.7)[58] gives the name of each *muhūrta*. The *Śatapatha-brāhmaṇa* (XII.3.2.5)[59] gives further divisions of time as follows:

1 year = 10800 *muhūrtas*
1 *muhūrta* = 15 *kṣipras*
1 *kṣipra* = 15 *etarhis*
1 *etarhi* = 15 *idānis*
1 *idāni* = 15 *prāṇas* (breathings), etc.

In the Vedic India, the regular calendar was symbolized in rituals. The *Taittirīya-saṁhitā* (I.6.10.3) reads:

"When the year is completed he should thus with these (*Vyāhṛtis*) perform the setting down; verily with the Brahman he surrounds the year on both sides. He who is undertaking the new and full moon *[darśapūrṇamāsa]* and four monthly offerings *[cāturmāsya]* should set in place the oblations with these *Vyāhṛtis* ...." (Translated by Keith)[60].

Here, the new and full moon and four monthly offerings, both of which require a regular calendar, are mentioned. The growth of large-scale ritual presupposes certain social surplus, and, according to Sharma[61], the *Yajur-vedic* rituals are mostly the product of the pre-iron agriculture phase. The main objective of the rituals was to "assert the authority of the king over the peasants and kinsmen"[62]. Therefore, it is easily supposable that the regular calendar, which is indispensable to agriculture, was one of the symbols of the authority. The rituals are divided into *Śrauta*-rituals and *Gṛhya*-rituals (domestic rituals). The *Śrauta*-rituals are usually divided into *Havir-yajñas* and *Soma*-offerings[63]. According to Gautama's *Dharma-sūtra* (VIII.19-20)[64], the *Havir-yajñas* include seven rituals: the *Agnyādheya*, the *Agnihotra*, the *Darśapaurṇamāsas*, the *Āgrayaṇa*, the *Cāturmāsyas*, the *Nirūḍhapaśubandha*, and the *Sautrāmaṇī*; and the *Soma*-offerings include seven rituals: the *Agniṣṭoma*, the *Atyagniṣṭoma*, the *Ukthya*, the *Ṣoḍaśin*, the *Atirātra*, and the *Aptoryāma*.

The following passage in the *Śatapatha-brāhmaṇa* (I.6.3.35-36) clearly shows that some of those rituals were the symbol of the regular calendar:

"After Prajāpati had created the living beings, his joints *(parvan)* were relaxed. Now Prajāpati, doubtless, is the year, and his joints are the two junctions of day and night (i.e. the twilights), the full moon and new moon, and the beginning of the seasons.

He was unable to rise with his relaxed joints; and the gods healed him by means of these *havis*-offerings: by means of the *Agnihotra* they healed that joint (which consists of) the two junctions of day and night, joined that together; by means of the full-moon and the new-moon sacrifice they healed that joint (which consists of) the full and new moon, joined that together; and by means of the (three) *Cāturmāsyas* (seasonal offerings) they healed that joint (which consists of) the beginning of the seasons, joined that together". (Translated by Eggeling)[65].

We should also note that the *Āgrayaṇeṣṭi*, one of the *Havir-yajñas*, is the offering of first-fruits, rice in *śarad*, (autumn), barley in *vasanta* (spring), and millet crop (*śyāmāka*) in rainy season[66].

When the *Jyotiṣa-vedāṅga* was composed, probably the rituals had already highly developed for their own sake, and the author of the *Jyotiṣa-vedāṅga* tells as if the sacrifice is the final purpose of astronomy. However, the development of astronomy was related to practical activities, such as agriculture, and the sacrifice was only the symbol of the authority over those activities.

Mention may be made of an astronomical concept which is absent in the Vedic *Saṁhitās* and *Brāhmaṇas*. The *tithi* as the thirtieth part of a synodic month does not appear in the *Saṁhitās* or *Brāhmaṇas*[67], and only found in the *Gṛhya-sūtras*[68], and the *Jyotiṣa-vedāṅga*. The current text of the *Aitareya-brāhmaṇa* (32.9)[69] has the following sentence:

यां पर्यस्तमियादभ्युदियादिति सा तिथिः ।

Dikshit[70] conjectured that this is a definition of the *tithi* as the "period of time during which the moon sets and rises again", although this definition is quite different from that of later period. Keith, on the contrary, considered that this is a corruption of the following sentence of the *Kauṣītaki-brāhmaṇa* (III.1)[71].

यां पर्यस्तमयमुत्सर्पेदिति स स्थितिः

"The period is that when (the sun) sets near or rises towards the moon." (Translated by Keith)[72].

If Keith's interpretation is correct, the *tithi* is absent in Vedic *Śrutis*.

As regards the planets, some people conjectured that they are mentioned in the *Saṁhitās* and *Brāhmaṇas*[73], but other people rejected this[74]. It is difficult to judge which is correct, but it is not surprising if Vedic people knew planets, although planetary astronomy is not mentioned even in the *Jyotiṣa-vedāṅga*[75]. One planet which is supposed by several people to be mentioned in early Vedic *Śrutis* is Jupiter as *Bṛhaspati*[76]. The word *"graha"* in the sense of planet appears in the *Maitrāyaṇī-upaniṣad* (VI.16)[77].

(ii) The Absence of the Description of Astronomical Instruments in Vedic *Śrutis*

The development of astronomy, which can be traced in Vedic literature, cannot be achieved without astronomical observation. However, no description of astronomical instrument occurs in the Vedic *Śrutis* proper. Some people claimed to find references of astronomical instruments in the Vedic *Śrutis*, but they are based on misunderstandings. Let us examine them one by one.

(a) *The alleged Quadrant in the Ṛg-veda*

Tilak[1] thought that he found an astronomical instrument in a hymn of the *Ṛg-veda* where a solar eclipse is referred to. The *Ṛg-veda* (V.40.5-6)[2] reads:

यत् त्वा सूर्य स्वर्भानुस्तमसाविध्यदासुरः ।
अक्षेत्रविद् यथा मुग्धो भुवनान्यदीधयुः ॥5॥
स्वर्भानोरध यदिन्द्र माया अवो दिवो वर्तमाना अवाहन् ।
गूळ्हं सूर्यं तमसापव्रतेन तुरीयेण ब्रह्मणाविन्ददत्रिः ॥6॥

"When, *Sūrya*, the son of *Asura Svarbhānu* [a name of *Rāhu*], overspread thee with darkness, the worlds were beheld like one bewildered, knowing not his place.

When, *Indra*, thou wast dissipating those illusions of *Svarbhānu* which were spread below the sun, then *Atri*, by his fourth sacred prayer *[turīyeṇa brahmaṇā]*, discovered the sun concealed by the darkness impeding his functions". (Translated by Wilson)[3].

Rejecting Sāyana's traditional interepretation of *"turīyeṇa brahmaṇā"* as "the fourth verse or mantra", Tilak wrote as follows:

"I would rather interpret *turīyeṇa brahmaṇā* to mean "by means of *turīya"*. *Turīya* is mentioned in modern astronomical works as a name for an instrument called quadrant (*Siddhānta Śiromaṇi,* X.15), and though we may not suppose the same instrument to have existed in the old Vedic days, yet there seems to be no objection to hold that it may have meant *some* instrument of observation. The word *brahma* is no doubt used to denote a *mantra,* but it may also mean knowledge *or* the means of acquiring such knowledge" (Tilak)[4].

Although this interpretation is sometimes referred to[5], the existence of the Quadrant in Vedic age, when the concept of the altitude or zenith distance was absent, cannot be supposed. Moreover, any early astronomical literature, the *Jyotiṣa-vedāṅga* or works of Āryabhaṭa or Varāhamihira, do not mention the Quadrant. The earliest work which mentions the Quadrant is the *Brāhma-sphuṭa sidhānta* (AD 628) of Brahmagupta. Therefore, the interpretation of Tilak is untenable, and there is no reason to reject the traditional interpretation of Sāyana, etc.

(b) *The alleged Gnomon in the Atharva-veda*

Das wrote that *"dvādaśāṅgula-śaṅku"* (12-*aṅgula* Gnomon) is mentioned in the *"Atharva-veda"*[6]. However, any description of the Gnomon cannot be traced in the *Atharva-veda-saṁhitā.* I suspect that what S.R. Das meant by *"Atharva-veda"* was actually the *Ātharvaṇa-Jyotiṣa* (a *jyotiṣa*-literature which is attributed to the *Vedāṅga* of the *Atharvaveda*), where the word *"dvādaśāṅgula-śaṅku"* certainly occurs. S.R. Das referred to the *Bhāratīya Jyotiḥśāstra* of S.B. Dikshit in the foot-note of his paper in connection with the above topic, but Dikshit only mentions the Gnomon in the *Atharva-jyotiṣa*[7], and not in the *Atharva-veda* itself. As Bhagavad Datta, the editor of the Lahore edition of the *Ātharvaṇa-jyotiṣa,* states, the *Ātharvaṇa-jyotiṣa* "appears to be a new work"[8], and someone might have compiled this work "by extracting certain material from the original law book of Manu,.... and later on adding to it some new and quite recent notions"[9]. Therefore, it is quite wrong to say that the Gnomon is mentioned in the *"Atharva-veda"*.

(c) *The alleged Gnomon in the Aitareya-brāhmaṇa*

There is a passage in the *Aitareya-brāhmaṇa* which is supposed to imply the observation of the summer solstice. Sengupta wrote that they "probably observed the noon-shadow of a vertical pole"[10]. Sometimes, this is referred to as the earliest Gnomon in India[11], but there is no evidence of the use of the Gnomon in the *Aitareya-brāhmaṇa* itself. Let us see the original text of the *Aitareya-brāhmaṇa* (XVIII.4)[12].

एकविंशमेतदहरुपयन्ति विषुवन्तं मध्ये संवत्सरस्य। एतेन वै देवा एकविंशेनाऽऽदित्यं स्वर्गाय लोकायोदयच्छन्। स एष इत एकविंशः। तस्य दशावस्तादहानि दिवाकीर्यस्य भवन्ति दश परस्तान्मध्य एष एकविंश उभयतो विराजि प्रतिष्ठित उभयतो हि वा एष विराजि प्रतिष्ठितस्तस्मादेषो ऽन्तरेमाल्लँोकान्यन्न व्यथते। -------------

"They perform the *Ekaviṁśa* day, the *Viṣuvant*, in the middle of the year; by the *Ekaviṁśa* the gods raised up the sun to the world of heaven; it is here the *Ekaviṁśa; below the Divākīrtya* are ten days, ten above; in the middle is the *Ekaviṁśa* resting on both sides in the *Virāj*, for on both sides does he find support in the *Virāj*. Therefore, he going between these worlds does not shake. ....." (Translated by Keith)[13].

P.C. Sengupta interpreted this passage that the *Viṣuvant* day which means the middle day of the year was the summer solstice day, because the Vedic year-long sacrifice was begun in the earliest times on the day following the winter solstice. He further interpreted that the above passage shows that the sun was observed by the Vedic people to remain stationary for 21 days near the summer solstice[14]. It may be so, but, as far as we can judge from the text itself, there is no reason to suppose that the Gnomon-shadow was observed by them. Sengupta also suggested that Vedic people might have observed the sun's amplitude. I think that this is quite possible.

As we shall see later, the descriptions of astronomical instruments first appear in *Vedāṅga* literature.

(iii) Naked Eye Observations in Vedic India

We have seen that Vedic people noticed several heavenly phenomena, although they may not have used any astronomical instrument. Let us now examine their method of naked eye observations. The ritual was one of the occasions of the astronomical observation. Especially, the *Darśapūrṇamāsas* were the occasions to observe the new and full moon, and the *Cāturmāsyas* were the occasions to ascertain the season. Our source to investigate these observations is the *Śrauta-sūtra*[1], the manual of rituals.

The *Śāṅkhāyana-śrauta-sūtra* (I.3.3-6) reads:

"There are two days of full moon and two days of new moon.

Thee two days of full moon are: (1) the day on which the moon appears full about the setting of the sun, and (2) the day on which (it appears full) after the setting of the sun.

The two days of new moon are: (1) the day on which they remark "tomorrow it will not be visible" and (2) the day on which it is not visible." (Translated by Caland)[2].

Here, we can see that the date of new and full moon was determined very carefully. Especially the date of full moon was determined by the observation of the rising time of the moon as well as the lunar phase. Therefore, the date of full moon could be determined fairly accurately. It is also seen from the determination of new moon that the vanishing crecent moon was carefully observed at dawn. This fact also ascertains that the Vedic definition of new moon was, like the modern definition, the conjunction of the sun and the moon, and not the first visible moon.

Let us see the description of the *Cāturmāsyas*. The *Śāṅkhāyana-śrauta-sūtra* (III. 13.1-2) reads:

> "On full-moon day in the month Phālguna are begun the four-monthly sacrifices (*cāturmāsyas*), or full-moon day in Caitra". (Translated by Caland)[3].

Here, we can see that the season was basically determined by the position of the full moon.

By the way, we have seen that the northward and southward movement of the sun was well noticed by Vedic people, and there is a passage in the *Aitareya-brāhmaṇa*[4] (belonging to the *Ṛg-veda*) which seems to imply the observation of the summer solstice in connection with a year-long sacrifice. There are also similar passages in the *Kauṣītaki-brāhmaṇa*[5], which is also a *Brāhmaṇa* belonging to the *Ṛg-veda*. The *Kauṣītaki-brāhmaṇa* (XIX.3) reads:

> On the new moon of Māgha he rests, being about to turn northwards; these also rest, being about to sacrifice with the introductory *Atirātra;* thus for the first time they obtain him; on him they lay hold with the *Caturviṁśa;* this is why the laying hold rite has its name. He goes north for six months; him they follow with six-day periods in forward arrangement. Having gone north for six months he stands still, being about to turn southwards; these also rest, being about to sacrifice with the *Viṣuvant* day; thus for the second time they obtain him. He goes south for six months; him they follow with six-day[6] periods in reverse order. Having gone south for six months he stands still, being about to turn north; these also rest, being about to sacrifice with the *Mahāvrata* day; thus for the third time they obtain him. In that they obtain him thrice, and the year is in the three ways arranged, verily (it serves) to obtain the year. ....." (Translated by Keith)[7].

From the above quotation, it is seen that the sun was considered to move constantly northwards or southwards, but to be stationary around the solstices. Although some people supposed the use of the Gnomon for the determination of the solstices in the age of the *Brāhmaṇas*, it is unlikely[8], because there is no account which implies the use of the Gnomon in the *Brāhmaṇas*. It is more likely that the movement of the sun was observed by the position of the rising (or setting) point of the sun. From the movement of the rising point of the sun, its "northward" or "southward" movement is clearly observed. And also, the sun looks stationary around the solstices by this observation. If we assume that the latitude of the observer is, say, 30°N, the rising point[9] changes about 0°.46 per day around the equinoxes. However, it requires about ten days from (or until) the solstice to move this amount of angular distance from (or up to) the rising point at the solstice. Since the apparent diameter of the sun is 0°.53, about one-half of a degree was perhaps the least distinguishable angular distance of the change of the rising point of the sun.

There is an interesting passage in the *Taittirīya-brāhmaṇa,* which implies the observation of the heliacal rising of a *nakṣatra*. Tilak noticed this passage, and wrote that "this passage is very important, as it describes the method of making celestial observations in old times"[10]. Let it describes the original text. The *Taittirīya-brāhmaṇa* (I.5.2.1)[11] reads:

> यत्पुण्यं नक्षत्रम्। तद्बट् कुर्वीतोपव्युषम्। यदा वै सूर्य उदेति। अथ नक्षत्रं नैति। यावति तत्र सूर्यो गच्छेत्। यत्र जघन्यं पश्येत्। तावति कुर्वीत यत्कारी स्यात्। पुण्याह एव कुरुते। एवं ह वै यज्ञेषुं च शतद्युम्नं च मात्स्यो निरवसाययांचकार।

> "The auspicious *nakṣatra* should be ascertained at dawn. When the sun has risen, one does not arrive at the *nakṣatra*. When the sun reaches the place where [the *nakṣatra* was] seen lastly (*jaghanyam*)[12], one should do the work which he has to do. An auspicious day is made indeed. Thus, Yajñeṣu, Śatadyumna, and Mātsya (names of *Ṛṣis*) have established like this".

From this passage, it is seen that the heliacal rising of a *nakṣatra* was observed by later Vedic people. To conclude, we can say that several kinds of naked eye observations were carried out by later Vedic people.

(iv) **Astronomy in Vedāṅga Period**

Before we proceed to the discussion of astronomical instruments in ***Vedāṅga*** literature, it will be convenient to summarizę the system of astronomy in ***Vedāṅga*** period and some related topics.

The ***Vedāṅga*** (limbs of the ***Veda***) is a class of work regarded as auxiliary to the ***Veda***. It consists of six divisions, namely, *Śikṣā* (phonetics), ***Chandas*** (metrics), ***Vyākrana*** (grammar), *Nirukta* (etymology), *Jyotiṣa* (astronomy), and ***Kalpa*** (ceremonial). The ***Kalpa*** further consists of four divisions, namely, ***Śrauta-sūtras*** (public ceremonial), *Gṛhya-sūtras* (domestic ceremonial), ***Dharma-sūtras*** (law), and ***Śulba-sūtras*** (texts of the method of the construction of the altar).

The main source of astronomy of this period is the *Jyotiṣa-vedāṅga* (= *Vedāṅga-jyautiṣa*). The *Śulba-sūtras* contain geometrical knowledge, and cardinal directions are determined astronomically there. Astronomical knowledge similar to the *Jyotiṣa-vedāṅga* is also found in the *Artha-śāstra*, early Buddhist and Jaina texts, and the *Paitāmaha-siddhānta* quoted in the *Pañca-siddhāntikā* (XII) of Varāhamihira.

The *Jyotiṣa-vedāṅga* has two recensions, namely, the *Ṛg-vedic* recension or the *Ārca-jyotiṣa* and the *Yajur-vedic* recension or the *Yājuṣa-jyotiṣa*. The *Yājuṣa-jyotiṣa* has a commentary of Somākara, whose date is unknown. Somākara's commentary is not so helpful for understanding the text. There is another text called *Ātharvaṇa-jyotiṣa* (= *Ātma-jyotiṣa* or *Atharvavedīya-jyautiṣa*)[1], but it is quite different from other two recensions, and is a very late work. Bhagavad Datta supposed that it was compiled by somebody by extracting from the original law book of Manu and adding some new and quite recent notions[2]. Therefore, the *Ātharvaṇa-jyotiṣa* should be considered as an independent work rather than the third text of the *Jyotiṣa-vedāṅga*.

Both *Ṛg-vedic* and *Yajur-vedic* recensions of the *Jyotiṣa-vedāṅga* were published by Weber[3] in 1862. Thibaut[4] wrote a paper in 1877, and explained some obscure verses. S.B. Dikshit also explained some more verses in his book[5] in 1896. Lālā Choṭe Lāl (alias Bārhaspatya) also wrote a paper in 1907, and tried to interpret all the verses[6]. In 1907, Sudhākara Dvivedin[7] published both recensions with Somākara's commentary on the *Yajur-vedic* resension, and Dvivedin's own Sanskrit commentary on both recensions. Tilak[8] wrote a note in 1914, which was published posthumously. In 1936, Shamasastry[9] published the text, following the *Yajur-vedic* recension and adding the verses of the *Ṛg-vedic* recension to relevant places, with his own Sanskrit commentary and English translation. Shamasastry referred to several Jaina astronomical texts, and interpreted the *Jyotiṣa-vedāṅga* in his own way. Both the recensions along with a Bengali translation were published by Sīteśacandra Bhaṭṭācārya[10] in 1974. He basically followed Dvivedin's reading. Finally, an edition and English translation of T.S. Kuppanna Sastry was posthumously published by Sarma[11].

The *Jyotiṣa-vedāṅga* begins as follows (*Ṛg-vedic* recension vss. 1 and 3; *Yajur-vedic* recension vss.1-2.)

पञ्चसंवत्सरमयं युगाध्यक्षं प्रजापतिम् ।
दिनर्त्वयनमासाङ्गं प्रणम्य शिरसा शुचिः ॥
ज्योतिषामयनं पुण्यं प्रवक्ष्याम्यनुपूर्वशः ।
सम्मतं ब्राह्मणेन्द्राणां यज्ञकालार्थसिद्धये ॥

"Saluting Prajāpati with a bow, who is the embodiment and supervisor of the five-year *yuga* and has the day *(dina)*, season *(ṛtu)*, half-year *(ayana)*, and month *(māsa)* as his limbs, I, being purified (*śuci*)[12], describe the correct movement of the heavenly bodies, which is accepted by highest Brahmans, for the sake of the determination of the proper time for sacrifices".

Here, we can recall that the day, month, and season are represented by three kinds of sacrifices, i.e. the *Agnihotra, Darśapūrṇamāsas* and *Cāturmāsyas,* which are mentioned as healers of joints of Prajāpati in the *Śatapatha-brāhmaṇa* (I.6.3.35-36). The *ayana* can also be represented by the sacrifice of *Viṣuvant* day (summer solstice) and *Mahāvrata* day (winter solstice) as was suggested in the *Kauṣītaki-brāhmaṇa* (XIX.3).

The calendrical system of the *Jyotiṣa-vedāṅga* can be summarized as follows:

1 *yuga*	=	5 years
	=	60 solar months
	=	61 *sāvana*-months = 1830 *sāvana*-days (civil days)
	=	62 synodic months = 1860 *tithis*
	=	67 sydereal months
	=	1835 sydereal days
1 year	=	2 *ayanas*
	=	6 *ṛtus* (seasons)
	=	12 solar months
	=	366 *sāvana*-days
	=	372 *tithis*

Here, a solar month *(saura-māsa)* is defined as one-twelfth of a year, a *sāvana*-day is a civil day from sunrise to sunrise, a *sāvana*-month is defined as 30 *sāvana*-days, and a *tithi* is defined as one-thirtieth of a synodic month. It is clear from these relations that there are two intercalary months in a *yuga.*

The beginning of a *yuga* is explained as follows (*Ṛg-vedic* recension vss. 5-6; *Yajur-vedic recension vss. 6-7):*

स्वराक्रमेते सोमार्कौ यदा साकं सवासवौ।
स्यात्तदादि युगं माघस्तपः शुक्लो ऽयनं ह्युदक्॥
प्रपद्येते श्रविष्ठादौ सूर्यचन्द्रमासावुदक्।
सार्पार्धे दक्षिणोर्कस्तु माघश्रावणयोः सदा।

"When the sun and moon rise together with *Vāsava* (= the *nakṣatra Dhaniṣṭhā*) in the sky, begins a *yuga,* the waxing half (*śukla*) of the *Māgha*-month that is *Tapas*-month, and the northward course of the sun (*udagayana*).

When the sun and moon situate at the first point of *Śraviṣṭhā (= Dhaniṣṭhā),* the sun starts to move northwards. When the sun is at the middle point of *Sarpa*

(= the *nakṣatra Āśleṣā)*, it starts to move southwards. They always happen in *Māgha*-month and *Śrāvaṇa*-month respectively".

Sub–division of time is as follows:

1 day = 30 *muhūrtas* = 60 *nāḍikās* = 603 *kalās*.
1 *kalā* = 124 *kāṣṭhās*.

The nāḍikā is used in connection with the Clepsydra, which I shall examine in the next section.

One *nakṣatra*, which is one twenty-seventh of the ecliptic, is subdivided into 124 *bhāṁśas*. It should be noted that the *nakṣatras* in the *Jyotiṣa-vedāṅga* are not actual constellations, but 27 equal segments of the ecliptic.

In the winter solstitial day, the length of daytime is considered to be 12 *muhūrtas*, and that of night 18 *muhūrtas*. The length of daytime is considered to change linearly, i.e. (12 + 2/61 n) *muhūrtas*, where *n* is the number of day after (or before) the winter solstice[13].

The *Artha-śāstra*[14], traditionally attributed to Kauṭilya (= Cāṇakya), a minister of Candragupta Maurya (throned in 321 BC), also contains the description of a similar calendrical system. The date of the actual composition of the text is controversial, and even the third century AD can be suggested as its date, but R.P. Kangle tried to show that there is no reason to regard that the text is not the work of Kauṭilya himself[15].

The *Artha-śāstra* also mentions the five-year *yuga* which has two intercalary months[16]. The five-year *yuga* system is the same as that of the *Jyotiṣa-vedāṅga*, and the *yuga* appears to have ended with winter solstice. At one place, the *Artha-śāstra* also mentions "the year of work" (*karma-saṁvatsara*), which ends with the full-moon day of *Āṣāḍha*-month[17]. In the *Artha-śāstra*, the summer solstice was considered to occur in *Āṣāḍha*-month[18]. Naturally, the equinoxes occur in *Caitra*-month and *Āśvayuja*-month[19]. However, the *Artha-śāstra* states that the sun's northward course (*uttarāyaṇa*) starts from *Śiśira*-season (i.e. from *Māgha*-month), and the sun's southwards course (*dakṣiṇāyana*) starts from *Varṣa*-season (i.e. from *Śrāvaṇa*-month)[20], that is, one month after *Āṣāḍ*ha-month. This statement looks strange at first sight, but Ganapati Sastri[21] rightly pointed out that the summer solstice was considered to be the last day of *Āṣāḍha*-month (*āṣāḍhamāsānte*) and the sun starts to move southwards from *Śrāvaṇa*-month (i.e. from the next day of the summer solstice). We can also recall that the sun was considered to be stationary around the solstices in the *Brāhmaṇas*. This theory would naturally have produced an idea that the solstice should precede the beginning of the *ayana*.

The *Śārdūlakarṇāvadāna*[22], one of the Buddhist Sanskrit texts, also mentions a similar calendrical system. It was translated twice into Chinese in the third century AD. One Chinese translation is the *Madengqie-jing*[23] translated by Zhu Lüyan[24] and Zhi Qian[25]. The other Chinese translation is the *Shetoujian-taizi-ershiba-xiu-jing*[26] translated by Zhu Fahu[27]. There is also a Tibetan translation[28], translated by Ajita-śrībhadra and Śākya-'od.

This text describes *nakṣatras* at length. Regarding the intercalation, the text reads as follows[29]:

तत्र तृतीये वर्षे ऽधिको मासो युज्यते ।

"There, in the third year, an intercalary month is inserted".

Evidently, the five-year *yuga* is implied here. Perhaps, one sentence, such as "And the second intercalary month is inserted in the fifth year", has dropped out[30]. Its Tibetan translation mentions the five-year cycle as follows[31]:

"For each five-year period, months are intercalated".

In the Sanskrit text of the *Śārdūlakarṇāvadāna*, the summer solstice occurs in the first month of rainy season (*varṣa*), when the new moon is in *Puṣya*, and the full moon is in *Śrāvaṇa*[32]. The autumnal equinox occurs in the fourth month of rainy season, when the new moon is in *Citra*, and the full moon is in *Kṛttikā*[33]. The winter solstice occurs in the third month of winter (*hemanta*), when the new moon is in *Pūrvāṣāḍhā*, and the full moon is in *Maghā*[34]. The vernal equinox occurs in the second month of summer (*grīṣma*), when the new moon is in *Aśvinī*, and the full moon is in *Viśākhā*[35].

Several jaina Prakrit texts[36] also describe the astronomical knowledge of this period. The canon of Śvetāmbara sect of Jainism is traditionally said to have been edited by the Council of Valabhī (the 5th century AD)[37]. The canon of Śvetāmbara sect is divided into the following classes: The *Aṅgas*, the *Upāṅgas*, the *Prakīrṇakas*, the *Chedasūtras*, the *Mūlasūtras*, and the *Nandī* and *Anuyogadvāra*. The main source of astronomy is the *Sūriyapannatti* (= *Sūrya-prajñapti*)[38], the *Jambuddīva-pannatti* (= *Jambudvīpa-prajñapti*), and the *Canda-pannatti* (= *Candraprajñapti*)[39], the 5th, 6th and 7th *Upāṅga* respectively. The contents of the *Candra-prajñapti* are practically the same as the *Sūrya-prajñapti*. That the calendrical system of these Jaina texts is practically the same as that of the *Jyotiṣa-vedāṅga* is seen, for example, in the astronomical constants given in the *Candra-prajñapti*[40], where a solar year is 366 days, a synodic month is $29 \frac{32}{62}$ days, etc.

A year starts from the first month of the rainy season, but the summer solstice falls in the last month of the year, or the fourth month of summer[41]. These texts give interesting information on the Gnomon-shadow, which I shall examine in the next section. The *Uttarajjhayaṇa* (= *Uttarādhyayana*)[42], the first *Mūla-sūtra*, also gives information on the Gnomon-shadow.

Another important Jaina Prakrit text is the *Jyotiṣkaraṇḍaka*[43]. According to Jagadīś-candra Jain[44], it was composed by somebody before the Council of Valabhī (the 5th century AD). It has a description of the Clepsydra.

The *Paitāmaha-siddhānta* quoted in the *Pañca-siddhāntikā* (XII)[45] also describes the calendrical system, which is the same as that of the *Jyotiṣa-vedāṅga*. The epoch of this *Paitāmaha-siddhānta* is Śaka 2 (AD 80). Although this *Paitāmaha-siddhānta* does not contain any information on astronomical instruments, it is quite important for us, because it implies that the calendrical system of the *Jyotiṣa-vedāṅga* was in use in the first century AD.

(v) Date and Place of the *Jyotiṣa-vedāṅga*

Now, les us discuss the date and place of the composition of the *Jyotiṣa-vedāṅga*, because this problem must be considered in connection with the development of early observational astronomy in India.

The date of the composition of the *Jyotiṣa-vedāṅga* was sometimes estimated to be the 12th century BC or so[1] by the calculation based on the amount of the precession. Let us briefly review the latest attempt of Kuppanna Sastry[2]. He firstly calculated the date of the *Jyotiṣa-vedāṅga*, from the following two facts:

(1) In the *Jyotiṣa-vedāṅga*, the winter solstice is situated at the first point of *Dhaniṣṭhā*.

(2) In the *Pañca-siddhāntikā* and the *Bṛhat-saṁhitā* of Varāhamihira, the winter solstice is situated at *Uttarāṣāḍhā* 1/4, and Varāhamihira's time is *c*. AD 530.

The difference of these two positions of the winter solstice is 23°20′. Since the winter solstice shifts 1° during about 72 years due to the precession, the *Jyotiṣa-vedāṅga* is about 1860 years earlier than Varāhamihira. So, the date of the *Jyotiṣa-vedāṅga* must be *c*. 1150 BC. Kuppanna Sastry calculated the date by a different method also, and obtained a similar result[3].

This method, however, cannot be relied upon much for several reasons. Firstly, there is no certainty that the definition of the position of the *nakṣatras* on the celestial sphere[4] in *Jyotiṣa-vedāṅga* is exactly the same as that of Varāhamihira or other later astronomers. Moreover, there must have been some observational errors. At least we should allow a quarter *nakṣatra's* error in the position of the solstice[5], which causes about 240 years' ambiguity. And also, the date of the solstice cannot be accurate, because the number of days in a *yuga* (5 years) is inaccurate by 3.8 days. So, there will be at least 4 days' error in the date of the solstice, which causes about 280 years' ambiguity. From the above considerations, we will have to admit at least 500 or 600 years' inaccuracy, or even more, in the date of the *Jyotiṣa-vedāṅga* determined from the amount of the precession.

Now let us consider historical circumstance of the *Jyotiṣa-vedāṅga*. As we have seen, astronomy as well as several rituals had not developed much in *Ṛg-vedic* age. Therefore, the date of the *Jyotiṣa-vedāṅga* would not be earlier than *Ṛg-vedic* age. And also, it may be supposed that the *Jyotiṣa-vedāṅga* is not earlier than later Vedic age (*c.* 1000-600/500 BC), because full-developed astronomical system of the *Jyotiṣa-vedāṅga* cannot be seen in the Vedic *Saṁhitās* and *Brāhmaṇas*. On the other hand, the *Jyotiṣa-vedāṅga* must have existed when the *Artha-śāstra* (traditionally attributed to the 4th century BC, but the actual composition may be of a later date) was composed. And also, Dikshit[6] suggested that the *Jyotiṣa-vedāṅga* might have been earlier than Pāṇini, because, for example, the measure of *āḍhaka*, which was used to denote the measurement of the clepsydra in the *Jyotiṣa-vedāṅga*, occurs in the *Aṣṭādhyāyī* (V.1.53)[7] of Pāṇini. Unfortunately, the date of Pāṇini is again in dispute[8], although it is sometimes attributed to 500-400 BC or so[9]. So, from historical considerations, the most probable date of the *Jyotiṣa-vedāṅga* may be placed sometime around the 6th-4th century BC, although we cannot fix it definitely.

Pingree[10] also suggested the date of the *Ṛg-vedic* recension of the *Jyotiṣa-vedāṅga* as the 5th or 4th century BC. This date itself appears to be reasonable at first sight, but Pingree's argument is based on quite a doubtful hypothesis. I shall examine his argument, because it concerns the earliest Indian astronomical instruments also. Pingree's argument is that the system of the *Ṛg-vedic* recension of the *Jyotiṣa-vedāṅga* was formed under Mesopotamian influence during the Achaemenid occupation of the Indus Valley between *c.* 513 and 326 BC. Pingree interpreted that the "day" in the *Ṛg-vedic* recension is not the civil day, but the sidereal day. So, he conjectured that one solar year was 366 sidereal days, i.e. 365 civil days. The *Yajur-vedic* recension[11] clearly states that the number of sidereal days (lit. rising of *Śraviṣṭhā*) in a *yuga* is the number of "days" plus five, and this statement shows that the "day" is the civil *(sāvana)* day. However, Pingree argued that this is due to the misunderstanding of the compiler of the *Yajur-vedic* recension, and this recension is a much later work. Pingree, however, does not show any understandable reason for his argument that the "day" in the *Ṛg-vedic* recension is sidereal, except that 365-day year is the same as the Egyptian-Persian year[12]. Pingree also argued that the *Paitāmaha-siddhānta* of the *Viṣṇudharmottara-purāṇa* and the *Āryabhaṭīya*[13] also express the length of their *yugas* in sidereal days[14]. However, Pingree's argument is not justified. In the case of the *Āryabhaṭīya*, the use of sidereal days is due to the rotating earth theory of Āryabhaṭa. And also, it is not believable that the *Paitāmaha-siddhānta* of the *Viṣṇudharmottara-purāṇa* is earlier than Āryabhaṭa. We should also keep in mind that sidereal days were never used for civil purpose in ancient India. Pingree conjectured that the 365-day year was introduced into India through Persia, but I shall show that the day in both recensions of the *Jyotiṣa-vedāṅga* is definitely the civil day, and there is no similarity between the calendrical system of the *Jyotiṣa-vedāṅga* and that of ancient Egypt or Mesopotamia.

As we have seen, the calendrical system of the *Jyotiṣa-vedāṅga* was used for the determination of the dates of sacrifices. We have also seen that the new and full moon was carefully observed by Vedic people at the time of *Darśapūrṇamāsas,* and the date of the full moon was determined by the lunar phase and the moon's rising time. Therefore, the new and full moon days could be determined fairly accurately. On the contrary, we have seen that the determination of the solstitial days was not so accurate. And also, much accuracy of the determination of the solstices and equinoxes was not required for the seasonal sacrifices, because the season was determined by the position of the full moon, and not by the direct observation of the sun's position. Now, the five-year *yuga* consists of 62 symodic months, or 67 sidereal months. The modern accurate values of them are:

62 synodic months = 62 × 29.5306 days = 1830.90 days,
67 sidereal months = 67 × 27.3217 days = 1830.55 days.

Therefore, one *yuga* should be 1830 civil days; otherwise a panic would have happened at the time of *Daraśapūrṇamāsas.* If Pingree's argument is true, one *yuga* becomes 1825 civil days, which causes about 5 days' error in the determination of the new and full moon days, by no means a permissible error for sacrifices. On the contrary, five days' difference in the determination of seasons would not cause a panic. This inaccuracy can be adjusted by comparison with the actual season at every beginning of *yugas.* That the same calendrical system is recorded in several different texts, Brahmanic, Jaina, Buddhist, and Political, shows that it was widely used; hence it could predict at least the date of the new and full moon correctly. It will be clear now that one *yuga* was 1830 civil days, and a solar year was 366 civil days in the *Jyotiṣa-vedāṅga.* There was no such calendar in ancient Egypt or Mesopotamia. And also, the five-year cycle of intercalation is not found in ancient Egypt or Mesopotamia.

The rule for the calculation of the length of daytime in the *Jyotiṣa-vedāṅga* also supports the conclusion that the "day" was civil, and not sidereal, because the concept of the "length of daytime" has physical meaning for a civil day only. I shall examine this rule in connection with the place of the composition of the *Jyotiṣa-vedāṅga.* Pingree argued that the clepsydra and the gnomon were introduced into India from Mesopotamia, but we shall see later that the descriptions of Indian instruments are somewhat different from those of Mosopotamian instruments.

Now, let us examine the place of the composition of the *Jyotiṣa-vedāṅga*. The *Jyotiṣa-vedāṅga (Ṛg-vedic* recension vs.7; *Yajur-vedic* recension vs.8) reads:

घर्मवृद्धिरपां प्रस्थः क्षपाह्रास उदग्गतौ ।
दक्षिणे तौ विपर्यासः षण्मुहूर्त्ययनेन तु ॥

"The increase of daytime and decrease of nighttime is [the time equivalent of] one *prastha* of water [in the clepsydra per day] during the northward course [of the sun]. They are in reverse during the southward course. [The total difference is] 6 *muhūrtas* during an *ayana* (half year)."

The *Jyotiṣa-vedāṅga* (*Ṛg-vedic* recension vs. 22; *Yajur-vedic* recension vs. 40) also reads:

यदुत्तरस्यायनतो गतं स्याच्छेषं तथा दक्षिणतो ऽयनस्य ।
तदेकषष्ट्या द्विगुणं विभक्तं सद्वादशं स्याद्दिवसप्रमाणम् ॥

"[The number of days] elapsed in the northward course or remaining in the southward course is doubled, divided by sixty one, and added to twelve. The result is the length of daytime [in terms of *muhūrtas*]."

These rules show that the length of daytime was considered to change linearly, and to be given by the following linear zig-zag function:

$$\text{The length of daytime} = \left(12 + \frac{2}{61}\,n\right) \textit{muhūrtas},$$

where *n* is the number of days after, or before, the winter solstice. The length of daytime is, for example, 15, 16, 17 *muhūrtas* at the vernal equinox, after one solar month, and after two solar months respectively, and is 18 *muhūrtas* at the summer solstice. From the fact that the proportion of daytime and nighttime is 2:3 at the solstice by this formula, the place of the composition of the *Jyotiṣa-vedāṅga* is usually estimated about 35°N or so[15]. Some people, notably Pingree[16], conjectured that this value was borrowed from Mesopotamia, although 35°N was not an inaccessible place for ancient Indian people. The historical circumstance of the *Jyotiṣa-vedāṅga,* however, obliges us to think that the aforesaid formula was not obtained by the interpolation from the observation at the solstices, but rather was obtained by the extrapolation from the observation of the change of the length of daytime around the equinoxes. We have seen that the sun was considered in the *Brāhmaṇas* to move constantly during its northward and southward courses, but to be stationary around the solstices. Therefore, this theory naturally has produced an idea that the linear function should be obtained from observations, around the equinoxes. Practically, there are two possibilities, which give almost the same results. (1) Let us assume that the formula was extrapolated from one *muhūrta's* difference of the length of daytime during one solar month after the equinox. The most suitable latitude[17] for this observation is about 27°N. (2) Let us assume that the formula was extrapolated from two *muhūrtas'* difference of the length of daytime during two solar months after the equinox. The most suitable latitude[18] for this observation is about 29°N.

These two assumptions give nearly the same result, because the length of daytime changes almost linearly during two months or so after the equinox. These results clearly show that the linear function of the length of daytime is based on observations at the latitude around 27-29°N. In order to make it more intelligible, the actual length of daytime at 35°N, 29°N, and 27°N, and the *Jyotiṣa-vedāṅga's* linear zig-zag function are together graphed in Fig. 1. This result is not contradictory to the latitude estimated from the shadow length[19] in the *Artha-śāstra*, which also gives the same linear function of the length of daytime.

In conclusion, we can say that the *Jyotiṣa-vedāṅga* was composed in North India (around 27-29°N) sometime around the 6th-4th century BC.

2. The First Astronomical Instruments: The Gnomon and the Clepsydra

(i) The Determination of Cardinal Directions by the Gnomon

The earliest Indian literature which mentions the gnomon is the *Kātyāyana-śulba-sūtra*, which describes the determination of cardinal directions by the gnomon.

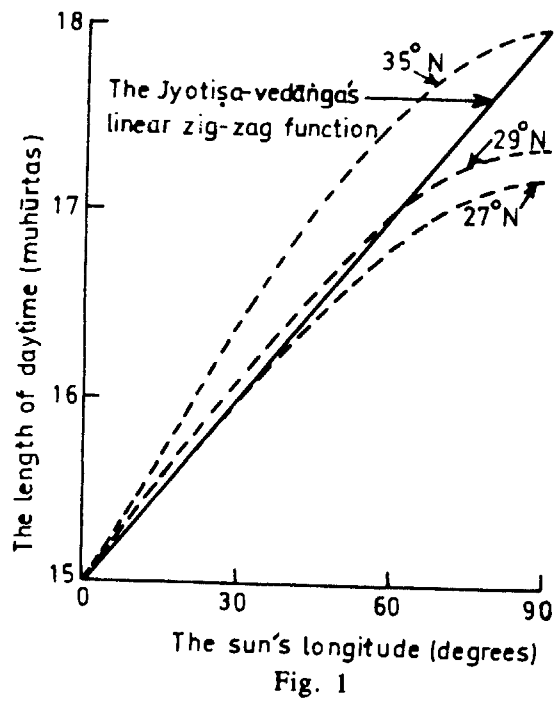


Fig. 1

The *Śulba-sūtras* are the texts of the method of construction of the altar, and belong to the *Kalpa-sūtras,* one of the classes of the *Vedāṅga* literature. We have four full-extant texts of the *Śulba-sūtras*[1], i.e. the *Baudhāyana-, Āpastamba-, Kātyāyana-* and *Mānava-śulbasūtra.*

It was necessary to determine cardinal directions in order to construct the altar. The *Baudhāyana-* and *Āpastamba-śulbasūtra* do not explain the method for the determination of directions, but the *Kātyāyana-śulbasūtra* explains the use of the gnomon for the determination of directions, and the *Mānava-śulbasūtra* explains the determination of directions by the observation of rising stars. Of course, exact directions can be determined by the gnomon.

The *Kātyāyana-śulba–sūtra* (1.2)[2] reads:

समे शङ्कुं निखाय शङ्कुसम्मितया रज्ज्वा मण्डलं परिलिख्य यत्र
लेखयोः शङ्क्वग्रच्छाया निपतति तत्र शङ्कू निहन्ति सा प्राची।

"Having put a gnomon *(śaṅku)* on a level ground, and having described a circle with a cord whose length is equal to the gnomon, two pins *(śaṅkū)* are placed on each of the two points where the tip of the gnomon-shadow touches [the circle in the forenoon and afternoon respectively]. This [line joining the two points] is the east-west line *(prācī)*".

This method is the well known "Indian Circle" method, as called by al-Bīrūnī[3]. On any desired day[4], the east-west direction (see Fig. 2) can be determined by joining two points (E′ and W′ in the figure) where the tip of the shadow of the gnomon (AB) touches the circle (NESW).

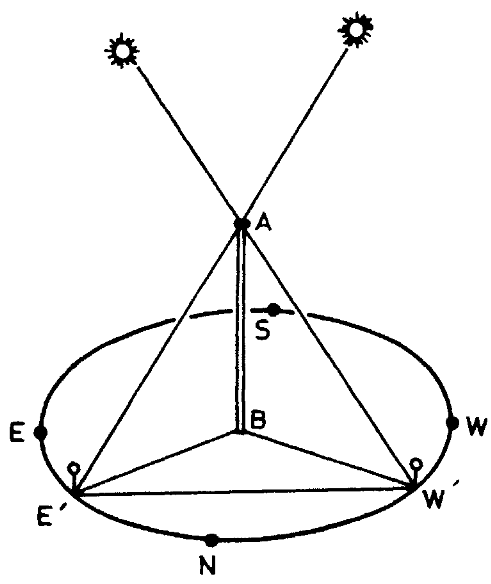


Fig. 2

The same method was also described in later astronomical texts and architectural texts.

Although the *Āpastamba-śulba-sūtra* itself does not contain the method of the determination of directions, its commentaries by Kapardi[5] and Karavinda[6] explain the method for the determination of cardinal directions by the gnomon. The method is the same as that of the *Kātyāyana-śulba-sūtra*. Karavinda quotes not only the *sūtra* of the *Kātyāyana-śulba-sūtra* (I.2-3), but also a verse of the *Laghu-bhāskarīya* (III.1) of Bhāskara I.

(ii) **Annual Variation of the Gnomon-shadow**

The annual variation and diurnal variation of the gnomon-shadow are described in the *Artha-śāstra*, etc. Let us take up the annual variation first.

(a) *The Artha-śāstra*

The *Artha-śāstra* (II.20.41-42)[1] described the annual variation of the gnomon-shadow as follows:

आषाढे मासि नष्टच्छायो मध्याह्नो भवति । 41 ।
अतः परं श्रावणादीनां षण्मासानां द्व्यङ्गुलोत्तरा माघादीनां द्व्यङ्गुलावरा छाया इति । 42 ।

"41. In the month of *Āṣāḍha*, the midday loses shadow.
42. After that, in the six months beginning with *Śrāvaṇa*, the shadow (at midday) increases by two *aṅgulas* in each month and in the six months beginning with Māgha, it decreases by two *aṅgulas* in each month." (Translated by Kangle)[2].

The height of the gnomon *(chāyāpauruṣa)* has been mentioned as 12 *aṅgulas* in the text[3].

From the above quotation, it is seen that the summer solstice falls in *Āṣāḍha*-month, and the midday shadow increases from *Śrāvaṇa*-month, which is the next month of *Āṣāḍha*-month. Ganapati Sastri rightly explained this difference in his Sanskrit commentary[4], which can be rendered into English as follows:

"'In the month of *Āṣāḍha*', i.e. at the end of the month of *Āṣāḍha*, 'the midday' 'loses shadow', i.e. there is no shadow. 'After that', i.e. afterwards, 'in the six months beginning with *Śrāvaṇa*', up to the end [of the month] of *Pauṣa*, 'the shadow increases by two *aṅgulas*', i.e. the shadow gradually becomes longer two by two *aṅgulas*. It means that it is two *aṅgulas* at the end of *Śrāvaṇa*, it is four *aṅgulas* at [the end of] *Bhādrapada*, it is six *aṅgulas* at [the end of] *Āśvayuja*, it is eight *aṅgulas* at [the end of] *Kārttika*, it is ten *aṅgulas* at [the end of] *Mārgaśīrṣa*, and it is twelve *aṅgulas* at [the end of] *Pauṣa*....."

The above commentary is quite clear. Now, let us estimate the probable latitude of the place of this observation[5].

(1) Since the shadow length was considered to be zero at the summer solstice, the most suitable latitude is about 23°.7N.

(2) Since the shadow length was 6 *aṅgulas* at equinoxes, the most suitable latitude is about 26°.6N.

(3) Since the shadow length was 12 *aṅgulas* at the winter solstice, the most suitable latitude is about 21°.3N.

Now, let us calculate the actual shadow length at the summer solstice (A), at equinox (B), and at the winter solstice (C) in terms of *aṅgulas*, at 21°.3N, 23°.7N, and 26°.6N (Table 3).

Table 3

Latitude	A	B	C
21°.3N	−0.5	4.7	12.0
23°.7N	0.0	5.3	13.0
26°.6N	0.6	6.0	14.5

From Table 3, we can only say that the observation was done somewhere in North India, and the rule is not accurate enough to say something more.

(b) *The Śārdūlakarṇāvadāna*

The *Śārdūlakarṇāvadāna* gives a list of the length of the midday shadow of the 16-*aṅgula* gnomon for each month[6]. The text for the first month of rainy season is, for example, as follows:

वर्षाणां प्रथमे मासे पुष्यनक्षत्रममावास्यां भवति, श्रावणा पूर्णमास्याम्। अष्टादशमुहूर्तो दिवसो भवति। द्वादशमुहूर्ता रात्रिः। षोडशाङ्गुकाष्ठस्य मध्याह्ने ऽर्धाङ्गुलायां छायायामादित्यः परिवर्तते। आषाढा रात्रिं नयति। मृगशिरसि आदित्यो गतो भवति।

"In the first month of rainy season, [the moon is at] *Puṣya-nakṣatra* at new moon, and *Śrāvaṇā* at full moon. The length of daytime is 18 *muhūrtas*, and the length of nighttime is 12 *muhūrtas*. The sun turns at midday when the shadow of the 16-*aṅgula* gnomon *(kāṣṭha)* is a half *aṅgula*. *Āṣāḍhā* conducts night. The sun has come to *Mṛgaśiras*"[7].

The data for midday shadow length of the 16-*aṅgula* gnomon in the *Śārdūlakarṇāvadāna* are given in Table 4.

Table 4

Season	Month No.	Shadow length *(aṅgulas)*
	1	0.5
Rainy	2	2
(Varṣa)	3	4
	4	6
	1	8
Winter	2	10
(Hemanta)	3	12
	4	10
	1	8
Summer	2	6
(Grīṣma)	3	4
	4	2

This shadow table is somewhat strange, because 12-*aṅgula* shadow of 16-*aṅgula* gnomon at the winter solstice implies the latitude of the observation as about 13°N, while 0.5-*aṅgula* shadow at the summer solstice implies the latitude as north of the Tropic of Cancer. The 16-*aṅgula* gnomon at 13°N will produce 3-*aṅgula* shadow towards south at the summer solstice. Moreover, the stage of this story is the bank of Gaṅgā *(Gaṅgātaṭa)*[8]. Therefore, I suspect that this table was originally meant for 12-*aṅgula* gnomon, and the phrase "16-*aṅgula* gnomon" was mistakingly interpolated later. If so, this shadow table is basically the same as that of the *Artha-śāstra*, except the data at the summer solstice.

The Tibetan translation of the *Śārdūlakarṇāvadāna* gives the length of the shadow of 6-*aṅgula* gnomon[9]. The length of the shadow is 6 *aṅgulas* at the winter solstice[10], and 0.5 *aṅgula* at the summer solstice[11], in the Tibetan translation. Unfortunately, it is not sure that these data are not later interpolations, because the date of solstices, etc. differs from the Sanskrit text.

One Chinese translation, *Madangqie-jing*, gives different data which were probably interpolated somewhere in northern latitude.

(c) *The Jaina texts*

The *Sūrya-prajñapti* (X.10), the *Jambudvīpa-prajñapti* (IX.17-19), and the *Candra-prajñapti* (X.10) give the same list of the shadow length of every month[12].

The Sanskrit rendering by Ghāsīlālajī Mahārāja of the text of the shadow length of, for example, the first month of rainy season in the *Candra-prajñapti* is as follows[13]:

तावत् वर्षाणां प्रथमं मासं कति नक्षत्राणि नयन्ति ? तावत् चत्वारि नक्षत्राणि नयन्ति, तद्यथा – उत्तराषाढा, अभिजित्, श्रवणः, धनिष्ठा। उत्तराषाढा चतुर्दश अहोरात्रान् नयति, अभिजित् सप्त अहोरात्रान् नयति, श्रवणः अष्ट अहोरात्रान् नयति, धनिष्ठा एकम् अहोरात्रं नयति। तस्मिंश्च खलु मासे चतुरङ्गुलया पौरुष्या छायया सूर्यः अनुपरावर्त्तते। तस्य खलु मासस्य चरमे दिवसे द्वे पदे चत्वारि च अङ्गुलानि पौरुषी भवति।

"How many *nakṣatras* conduct the first month of rainy season? Four *nakṣatras* conduct, namely, *Uttara-āṣāḍhā*, *Abhijit*, *Śravaṇa*, and *Dhaniṣṭhā*. *Uttara-āṣāḍhā* conducts 14 days, *Abhijit* conducts 7 days, *Śrāvaṇa* conducts 8 days, and *Dhaniṣṭhā* conducts 1 day. In this month, the sun turns by 4 *aṅgulas* of *pauruṣī*-shadow. On the last day of the month, the *pauruṣī* [-shadow] is 2 *padas* and 4 *aṅgulas*."

The *pauruṣī*-shadow means the shadow of a gnomon in terms of gnomon-length. One *pada* is apparently 12 *aṅgulas*. The relationship between the two figures of the shadow length is not clear from the text itself. In any case, the data for all months are presented in Table 5[14].

Table 5

Season	Month	"The sun turns" (*aṅgulas*)	"On the last day" (*padas-aṅgulas*)
	1	4	2-4
Rainy	2	8	2-8
(*Varṣa*)	3	12	3-0
	4	16	3-4
	1	20	3-8
Winter	2	24	4-0
(*Hemanta*)	3	20	3-8
	4	16	3-4
	1	12	3-0
Summer	2	8	2-8
(*Grīṣma*)	3	4	2-4
	4	0	2-0

Lishk and Sharma[15] interpreted that the data "with which the sun moves" means the "total variation of noon-shadow length" from the shortest noon shadow, and the data "on the last day of the month" is the noon-shadow length. If so, the shadow length will be 4 *padas* at the winter solstice, and 2 *padas* at the summer solstice. Their interpretation is, however, impossible, because there is no place on the earth where the shadow length at the winter solstice is double the shadow length at the summer solstice in the same direction[16].

A better interpretation has been suggested by Hermann Jacobi. Jacobi[17] suggested that the length of the gnomon is 24 *aṅgulas*, and the data by which "the sun turns" are the length of the midday shadow. The data "on the last day" are the length of the shadow at the end of the first quarter of daytime. This interpretation seems to be correct, because it is harmonious with other texts, such as the *Artha-śāstra*. According to this interpretation, the midday shadow length is zero at the summer solstice, and is equal to the gnomon length at the winter solstice. The shadow length is longer by the gnomon length than the midday shadow before (or after) a quarter of daytime. It is harmonious with Jaina theory of diurnal variation of the gnomon shadow, which I shall discuss in the next section.

A similar data of the shadow length as the above data "on the last day" is found in the *Uttarādhyayana-sūtra*. The *Uttarādhyayana-sūtra* (XXVI. 13-14) reads:

> "In the month *Āṣāḍha*, the *pauruṣī* is 2 *padas;* in the month *Pauṣa*, four; in the months *Caitra* and *Āśvayuja*, three.
>
> [The *pauruṣī*] increases or decreases an *aṅgula* every week, two *aṅgulas* every fortnight, four *aṅgulas* every month." (Originally translated by Hermann Jacobi; modified by me following Jacobi's later paper)[18].

The *Samavāyāṅga-sūtra*[19] also gives similar fragmental data. For example, the *Samavăyāṅga-sūtra* (XXIV.4)[20] reads:

> उत्तरायणगते णं सूरिए चउवीसंगुलियं पोरिसियछायं णिव्वत्तइत्ता णं णिअट्टति ।
>
> "When the *uttarāyana* is completed (i.e. at the summer solstice), the sun produces 24-*aṅgula pauruṣī*-shadow, and turns back".

The *Samavāyāṅga-sūtra* (XXXVI.4)[21] reads:

> चेत्तासोएषु णं मासेसु सइ छत्तीसंगुलियं सूरिए पोरिसीछायं निव्वत्तइ ।
>
> "The sun produces a *pauruṣī* shadow of 36 *aṅgulas* length in the month of *Caitra* and *Āśvina* each." (Translated by S.S. Lishk and S.D. Sharma)[22].

There are some other data also in the *Samavāyāṅga-sūtra*.

The data of the *Uttarādhyayana-sūtra* and the *Samavāyāṅga-sūtra* are the same as the data "on the last day" in the *Candra-prajñapti*, etc., i.e. the data which are supposed to be meant for the end of the first quarter of daytime.

(d) *Concluding Remarks*

All the above-mentioned data on seasonal variation of the gnomon shadow follow a simple linear zig-zag function. The shadow length is zero at the summer solstice, since then the shadow length increases linearly, and the shadow length becomes the same as the gnomon length at the winter solstice. This function was also stated in the *Pañca-siddhāntikā* (II. 9-10)[23] of Varāhamihira.

As we have examined in connection with the data of the *Artha-śāstra,* this formula seems to have been obtained by observation in North India, although this formula is not exact enough to locate the exact latitude of the place of the observation.

Let us briefly examine the alleged relationship between this Indian formula and Mesopotamian astronomy. David Pingree supposed that the *Artha-śāstra* employed "a Babylonian linear zig-zag function to obtain the lengths of noon-shadows"[24] in each month, although the zero-length of the summer solstice had been adjusted to Indian conditions. Pingree also stated that the one gnomon-length of the noon-shadow at the winter solstice is also "given in *mul Apin*"[25]. However, Pingree's hypothesis seems to be based on an incorrect interpretation of the *mul Apin*. According to O. Neugebauer[26], the *mul Apin* gives a table where the length of shadow is measured in cubits. Weidner and van der Waerden had interpreted that the text refers to a gnomon of 1 cubit length, but Sachs and Neugebauer have shown that it is wrong, and the text only means to measure in cubits. Moreover, according to Neugebauer, the length of the moon shadow seems to be 5/6 cubits, rather than one cubit, for all cases, i.e. winter solstice, equinox, and summer solstice[27]. Therefore, the seasonal variation of the moon shadow is not implied in the *mul Apin*.

Some Greek shadow tables, whose earliest one was written in around 200 BC, mention the noon shadow which linearly changes from 8 feet to 2 feet from the winter solstice to the summer solstice[28]. It is meant for a man's shadow which is measured by his own feet. So, the exact height of the "man" is not given in the texts, although Neugebauer suggested 6 feet or 7 feet as a possible height[29]. I would like to note one thing regarding the Greek shadow table. If we assume that the height of the "man" was 6 feet, the difference between the maximum and minimum shadow lengths is one gnomon length. This is somewhat similar to the Indian shadow table. Therefore, one might suspect a kind of cultural exchange between them[30]. In this respect, it is almost impossible to suppose that the Greek table influenced the Indian table. The assumption "a gnomon-length difference of the shadow length between the winter and summer solstices" nicely fits in Indian latitude, but it is far from the truth in Greek latitude. (See Fig. 3)[31]. Moreover, the Indian shadow table was basically meant for an artificial gnomon whose height is definite.

From the above considerations, it may be safely concluded that the Indian shadow table of seasonal variation was produced in North India by actual observations without explicit foreign influence.

(iii) **Diurnal Variation of the Gnomon-shadow**

The data on diural variation of the gnomon-shadow in early Indian texts are classified into two classes. Let us examine them one by one.

(a) *The Artha-śāstra and the Jaina texts*

The *Artha-śāstra* and the Jaina texts give the similar data of diurnal variation of the gnomon shadow.

The *Artha-śāstra* (II.20.39-40)[1] reads:

छायायामष्टपौरुष्यामष्टादशभागश्छेदः षट्पौरुष्यां चतुर्दशभागः, त्रिपौरुष्यामष्टभागः[2], द्विपौरुष्यां षड्भागः, पौरुष्यां चतुर्भागः, अष्टाङ्गुलायां त्रयो दशभागाः, चतुरङ्गुलायां त्रयो[3] ऽष्टभागाः, अच्छायो मध्याह्न इति । 39 ।
मध्याह्न इति । 39 ।

"39. When the shadow (of the gnomon) is eight *pauruṣas*, one-eighteenth part (of the day) is past, when six *pauruṣas*, one-fourteenth part (is past), when three *pauruṣas*, one-eighth part, when two *pauruṣas*, one-sixth part, when one *pauruṣa*, one-fourth part, when eight *aṅgulas*, three-tenth part (is past), when four *aṅgulas*, three-eighth part, (and) when there is no shadow, it is midday.

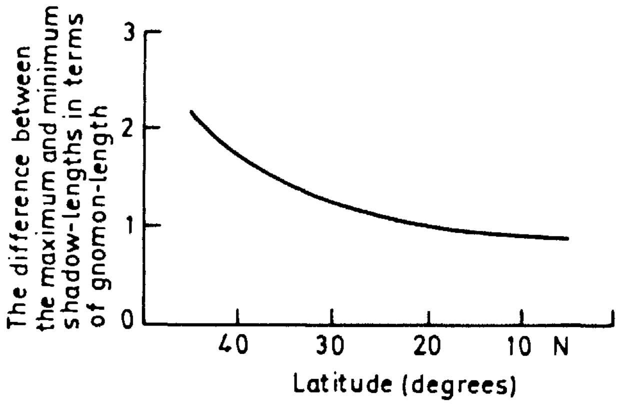


Fig. 3

40. When the day has turned, one should understand the remaining parts in like manner". (Translated by Kangle)[4].

The above data are presented in Table 6.

Table 6

Time elapsed	Shadow-length
1/18 of daytime	8 *pauruṣas*
1/14 ,,	6 ,,
1/8 ,,	3 ,,
1/6 ,,	2 ,,
1/4 ,,	1 ,,
3/10 ,,	8 *aṅgulas*
3/8 ,,	4 ,,
1/2 ,,	0 ,,

Before analysing data in Table 6, it will be convenient to see a passage in a Jaina text which gives a similar list.

The Sanskrit rendering by Ghāsīlālajī Mahārāja of the *Candra-prajñapti* (IX.4)[5] reads as follows:

वयं पुनरेवं वदामः सूर्यः सातिरेकैकोनषष्टिपौरुषीं छायां निर्वर्त्तयति। तावत् अपार्धपौरुषी खलु छाया दिवसस्य किं गते वा शेषे वा ? तावत् त्रिभागे गते वा शेषे वा। तावत् पौरुषी खलु छाया दिवसस्य किं गते वा शेषे वा ? तावत् चतुर्भागे गते वा शेषे वा। तावत् **द्व्यर्धपौरुषी** खलु छाया दिवसस्य किं गते वा शेषे वा ? तावत् पञ्चभागे गते वा शेषे वा। एवम् अर्धपौरुषीं क्षिप्त्वा 2 पृच्छा। दिवसस्य भागं क्षिप्त्वा 2 व्याकरणं यावत् तावत् अपार्धैकोनषष्टिपौरुषी खलु छाया दिवसस्य किं गते वा शेषे वा ? तावत् एकोनविंशतिशतभागे गते वा शेषे वा। तावत् एकोनषष्टिपौरुषी खलु छाया दिवसस्य किं गते वा शेषे वा ? द्वाविंशतिसहस्रभागे गते वा शेषे वा। तावत् सातिरेकैकोनषष्टिपौरुषी खलु छाया दिवस्य किं गते वा शेषे वा ? तावत् नास्ति किञ्चिद् गते वा शेषे वा।

"We are further telling. The sun produces a shadow of more than 59 *pauruṣīs*. When the shadow is half *pauruṣī*, how much part of daytime has passed or remaining? Then, one third [of daytime] has passed or is remaining. When the shadow is one *pauruṣī*, how much part of daytime has passed or is remaining? Then one fourth has passed or is remaining. When the shadow is one and a half *pauruṣī*, how much part of daytime has passed or is remaining? Then one fifth has passed or is remaining. Like this, giving half by half *pauruṣī*, the question is asked. Giving the part of daytime, the explanation is given. When the shadow is 58 1/2 *pauruṣī*, how much part of daytime has passsed or is remaining? Then 1/119 has passed or is remaining. When the shadow is 59 *paurusīs*, how much part of daytime has passed or is remaining. Then 1/1022 has passed or is remaining[6]. When the shadow is more than 59 *paurusīs*, how much part of daytime has passed or is remaining? Then no part has passed or is remaining."

The above data may be tabulated as per Table 7.

Table 7

Shadow-length *(pauruṣī)*	Time elapsed or remaining
0.5	1/3 of daytime
1	1/4 "
1.5	1/5 "
58.5	1/119 "
59	1/1022 "

The last data is an unexpected one. Since the context implies that the denominator of the part of daytime increases one by one when the length of the shadow increases half by half *pauruṣī*, the expected data would be 1/120 of daytime for 59 *pauruṣī* shadow.

In any case, Hermann Jacobi[7] rightly pointed out that the data in the *Sūrya-prajñapti*, which is basically the same as the *Candra-prajñapti*, follow the following formula:

The shadow-length = (n/2 − 1) *pauruṣī*,

when 1/n of daytime has passed or is remaining. Jacobi also pointed out that the data in the *Artha-śāstra* also follow the same formula, but he wrongly thought that the variable *n* was meant for natural number only and the data of 8 and 4 *aṅgula* shadow in the *Artha-śāstra* had not been derived from the above formula.

As regards the data in the *Artha-śāstra*, Abraham[8] perfectly explained that they follow the following fourmula:

$$\frac{d}{2t} = \frac{s}{g} + 1, \qquad \text{.... (1)}$$

where t/d is the fraction of daytime which has elapsed since surrise or is remaining until sunset, and *s* is the shadow of the gnomon of length *g*. This formula itself is mathematically equivalent to Jacobi's formula. Abraham further pointed out that the above formula is a special case of the formula in the *Yavana-jātaka* (LXXIX.32)[9] and the *Pañca-siddhāntika* (IV.48-49)[10], which can be expressed as follows:

$$\frac{d}{2t} = \frac{s - s'}{g} + 1, \qquad \dots (2)$$

where s' is the noon shadow.

Evidently, the *Artha-śāstra's* data was meant for the summer solstitial day, when the midday shadow was considered to be zero. That the *Artha-śāstra's* formula is fairly good on the summer solstitial day can be seen[11] in Fig. 4, where the *Artha-śāstra's* formula is compared with the actual shadow length. It is not certain whether the above-mentioned generalized formula (2) was used in the time of the *Artha-śāstra* or is a later invention. We shall examine this generalized formula again in the section giving concluding remarks.

We should note that the shadow length is one gnomon-length longer than the midday shadow at the end of the first quarter of daytime according to the generalized formula (2). This is exactly what we have seen [12] in the Jaina table of the seasonal variation of the gnomon-shadow interpreted by Hermann Jacobi. Therefore, the formula (2) seems to have been used by ancient Jainas.

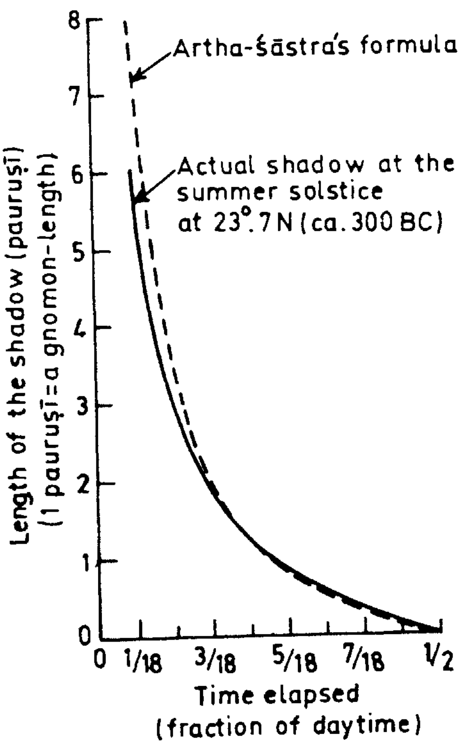


Fig. 4

(b) *The Śārdūlakarṇāvadāna and the Atharva-jyotiṣa*

The *Śārdūlakarṇāvadāna* and the *Arharva-jyotiṣa* give a similar data of the diurnal variation, which, apparently, does not follow a simple algebraic formula. Al-Bīrūnī also recorded a similar data.

The *Śārdūlakarṇāvadāna*[13] reads as follows:

आदित्य उदयति षण्णवतिपौरुषायां छायायां चतुरोजा नाम मुहुर्तो भवति। षष्टिपौरुषायां छायायां श्वेतो नाम मुहूर्तो भवति।-----

"The sun has risen. When the shadow is 96 *pauruṣīs,* it is the *muhūrta* called *caturojā.* When the shadow is 60 *pauruṣīs,* it is the *muhūrta* called *śveta.* ....."

The data for all 15 *muhūrtas* in daytime is tabulated in Table 8[14].

Table 8

No.	Shadow-length	Name of *muhūrta*
1	96	*caturojā*
2	60	*śveta*
3	12	*samṛddha*
4	6	*śarapatha*
5	5	*matisamṛddha*
6	4	*udgata*
7	3	*sumukha*
8	–	*vajraka*
9	3	*rohita*
10	4	*bala*
11	5	*vijaya*
12	6	*sarvarasa*
13	12	*vasu*
14	60	*sundara*
15	96	*parabhaja*

The shadow-length is given in terms of *pauruṣīs* (gnomon-length) in the text[15], but it is, most probably, a mistake, because the actual shadow is not so much long. Probably, the original data was meant to be measured in terms of *aṅgulas,* and the gnomon-length was 12 *aṅgulas.*

The text for the 8th *muhūrta* is as follows[16]:

स्थिते मध्याह्ने वज्रको नाम मुहूर्तो भवति।

"[The shadow is] stationary at midday. It is the *muhūrta* called *vajraka*"

We shall investigate the meaning of "stationary" after examining other sources.

The *Atharva-jyotiṣa*[17] gives a series of data on the shadow of 12-*aṅgula* gnomon *(dvādaśāṅgulam ucchaṅkum)* for each *muhūrta*. For example, the *Atharva-jyotiṣa* (I.1.6) reads as follows:

नवति षडङ्गुलाश्चैव प्रतीचीं तां प्रकाशयेत् ।
पुरस्तात्सन्धिवेलायां मुहूर्तो रौद्र उच्यते ।। 6 ।।

"The [shadow] is cast towards west at length of 96 *aṅgulas*. In the beginning, at dawn, the *muhūrta* is called *raudra*."

The data for all 15 *muhūrtas* in daytime is tabulated in Table 9.

Table 9

No.	Shadow-length *(aṅgulas)*	Name of *muhūrta*
1	96	*raudra*
2	60	*śveta*
3	12	*maitra*
4	6	*sārabhaṭa*
5	5	*sāvitra*
6	4	*vairāja*
7	3	*viśvāvasu*
8	–	*abhijit*
9	3	*rauhiṇa*
10	4	*bala*
11	5	*vijaya*
12	6	*nairṛta*
13	12	*vāruṇa*
14	60	*saumya*
15	maximum	*bhaga*

It is seen that this is basically the same as the list of the *Śārdūlakarṇavadāna*. The text of the *Atharva-jyotiṣa* (I.1.8c-d)[18] for the 8th *muhūrta* is as follows:

मध्याह्ने अभिजिन्नाम यस्मिन् छाया प्रतिष्ठिता ।

"At noon, when [the *muhūrta* is] called *abhijit*, the shadow is stationary".

Dikshit considered that the given numerical data in the *Atharva-jyotiṣa* are actual shadow-lengths, and wrote that "it cannot be said that the shadow at noon is zero length, but it must be shorter than 3 *aṅgulas*"[19].

However, al-Bīrūnī offers a different interpretation of this series of numerical data in his *India* (Table 10).

Table 10

The *muhūrtas* which have elapsed *before* noon....	1	2	3	4	5	6	7	
How many digits the shadow in question is larger than the noon-shadow	96	60	12	6	5	3	2	0
The *muhūrtas* which have elapsed *after* noon.	14	13	12	11	10	9	8	

(Translated by E.C. Sachau)[20]

Here, al-Bīrūnī explained that the data is the excess shadow *(ziyādatu ẓ-ẓilli)* of a "person" *(shakhṣ)* larger than the midday shadow *(fay'u z-zawāli)* in terms of digits (*iṣba'* = finger). As regards its source, al-Bīrūnī simply wrote that it is from "some of their (Hindu's) metrical compositions".

Although the last figures of al-Bīrūnī's *India* slightly differ from the data in the *Śārdūlakarṇāvadāna* and the *Atharva-jyotiṣa*, it is clear that they are based on the same idea. It will be reasonable to suppose that these data are chiefly meant for an equinoctial day, because the daytime consists of 15 *muhūrtas*. (Al-Bīrūnī's table has 14 *muhūrtas*, but it is not contradictory if we interpret that the data are meant for the last moments of *muhūrtas*.) Al-Bīrūnī conjectured[21] that the *muhūrtas* are like the "temporal hours" (twelve equal parts of the daytime and twelve equal parts of the nighttime, both of which change their length according to seasons). This conjecture is, however, impossible, because Indian *muhūrta* is a definite length of time, i.e. one thirtieth of a day.

In the early Indian texts, the length of the shadow on an equinoctial day is 6 *aṅgulas* when the gnomon-length is 12 *aṅgulas*. Therefore, the above data of the shadow table must be really the excess shadow larger than the midday shadow, rather than the actual shadow-length. The data around the noon are too short for the actual shadow-length.

The data is not so bad, but it is not smooth. (See Fig. 5)[22]. The rationale of this data is still not clear[23].

The data on diurnal variation of the shadow, which does not follow a linear equation, was also used in the Greek shadow table[24] which we have discussed in

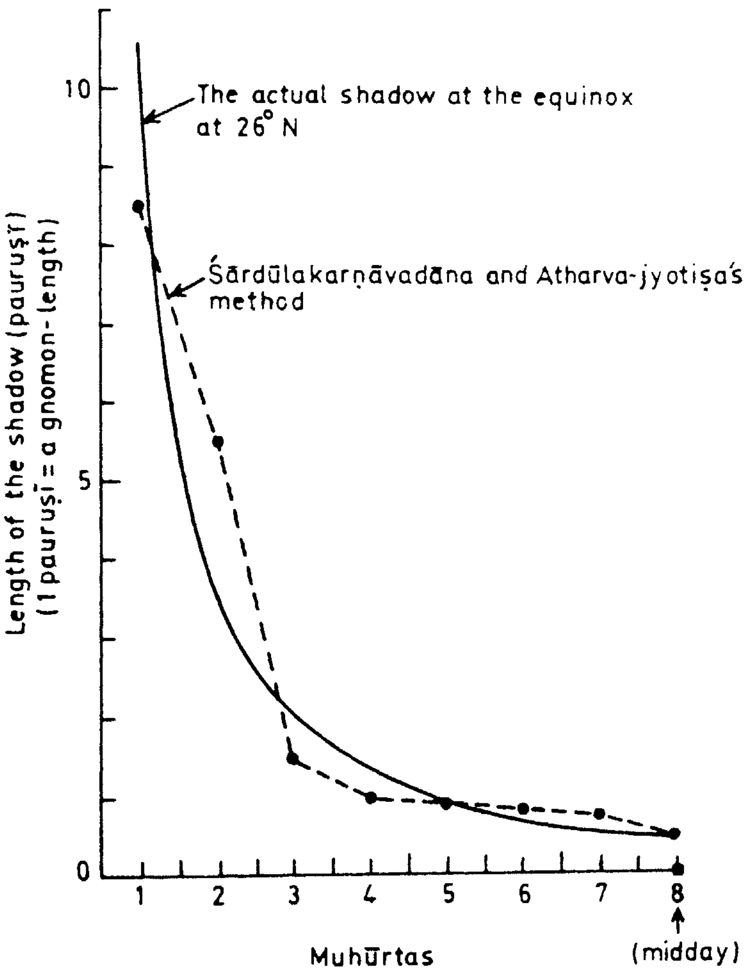


Fig. 5

connection with the seasonal variation of the shadow. Here, the length of the shadow is given for each "hour" in different seasons. The "hours" are seasonal, not equinoctial, hours. The shadows for the hours before and after noon increase by 1, 2, 3, 4, and 10 (= 1 + 2 + 3 + 4) feet in any month (expressed in zodiacal signs)[25]. So, the excess shadow-length larger than the midday shadow is: 1, 3, 6, 10, and 20 feet for the successive hours. Here, any direct relationship with the Indian shadow table cannot be seen. Therefore, the Indian and Greek shadow tables are probably independent.

(c) *Concluding Remarks*

We have seen that there were two classes of the texts on the diurnal variation of the gnomon-shadow in ancient India. Among them, the data in the *Artha-śāstra*, etc. follows the following formula:

$$\frac{d}{2t} = \frac{s}{g} + 1, \qquad \ldots (1)$$

where t/d is the fraction of daytime which has elapsed since sunrise (or is remaining until sunset), s the shadow of the gnomon of length g. A generalized formula has been mentioned in the *Yavana-jātaka* and the *Pañca-siddhāntikā* as follows:

$$\frac{d}{2t} = \frac{s - s'}{g} + 1, \qquad \ldots (2)$$

where s' is the length of the midday shadow.

David Pingree conjectured that the Indian theory of the diurnal variation of the gnomon-shadow had been influenced by the Mesopotamian theory. Pingree wrote as follows:

> ....Though the methods of computation of these several tables differ, it is noteworthy that the shadow is assumed in *mul Apin* to equal 8 gnomon-lengths after 1/2 seasonal hour on the day of the summer solstice or 45 minutes of an equinoctial hour, whereas both the *Artha-śāstra* and the *Śārdūlakarṇāvadāna* state that the shadow equals 8 gnomon-lengths after 1 *muhūrta* or 48 minutes of an equinoctial hour. It seems plausible, then, that the Sanskrit texts represent an adaptation of a lost Mesopotamian scheme reduced to expression in Indian units of time-measurement, and further modification to produce a shadow-length of 0 at noon". (Pingree)[26].

Although this assumption of Pingree seems to be based on his misinterpretation of the *mul Apin*[27], we will have to examine the possibility of the alleged relationship between Mesopotamian and Indian shadow tables.

Otto Neugebauer has shown that all data in the shadow table of the *mul Apin* can be obtained from the equation:

$$t = c/s, \qquad \ldots (3)$$

where t is the time after sunrise, counted in time degrees (1 day = 360°), s the length of the shadow in terms of cubits, and c is a constant. The constant c is 60 at the winter solstice, 75 at the equinoxes, and 90 at the summer solstice[28]. This constant c is, in fact, uniquely determined by the length of daytime. According to Neugebauer's interpretation, the length of the noon shadow is always 5/6 cubit, independent of the seasons. It is quite strange, and Neugebauer wrote: "I do not see a plausible model of a sundial that would explain such a norm"[29].

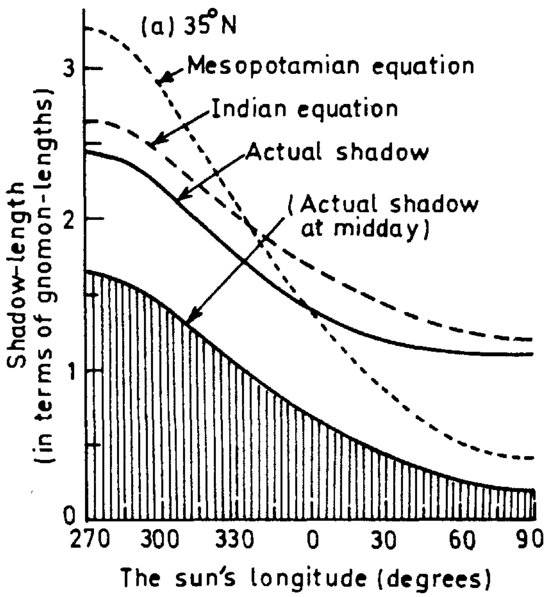
As we shall see later, Eq. (3) can be understood as a theoretical synthesis of diurnal variation of the shadow at a particular day and seasonal variation of the duration of daytime.

Let us now compare Eqs. (2) and (3). It is clear at first sight that they are not mathematically equivalent. What is more important is the difference of idea behind these formulae. Eq. (2) implies that the excess shadow at the time after a certain fraction of daytime since sunrise is constant and independent of seasons. Eq. (3) implies that the shadow length at the time after a certain fraction of daytime since sunrise is proportional to the length of the midday shadow.

Let us take up a simple case, the time after a quarter of daytime since sunrise. This is evidently the most important moment in order to examine the diurnal variation of the shadow. According to Eq. (2), the shadow-length is one gnomon-length longer than the midday shadow. According to Eq. (3), the shadow-length is double the length of the midday shadow. I have plotted the shadow-length of a gnomon of length 1 at a quarter of daytime after sunrise according to Eq. (2) (i.e. the midday shadow plus one) and Eq. (3) (i.e. double the midday shadow) as well as the actual shadow-length for different seasons in Fig. 6, for the latitude 35°N (a typical latitude of Mesopotamia) and 23°.7N (a typical latitude of India)[30].

From Fig. 6 (a) (for 35°N), it is seen that the Mesopotamian Eq. (3) is good around the equinoxes at the latitude 35°N, while the Indian Eq. (2) gives wrong value throughout the year. Eq. (3) has raison d'être in Mesopotamia, and it is plausible that it was a theoretical synthesis of the actual diurnal variation of the shadow at the equinoctial day and the theory of the seasonal variation of the duration of daytime. On the contrary, it is not possible to suppose that the Indian Eq. (2) originated in Mesopotamia.

From Fig. 6 (b) (for 23°.7N), it is seen that the Mesopotamian Eq. (3) is of no use there, because it gives zero-length throughout the day at the summer solstice. On the contrary, the Indian Eq. (2) is excellent around the summer


(a) 35°N
Mesopotamian equation
Indian equation
Actual shadow
(Actual shadow
at midday)
Shadow-length
(in terms of gnomon-lengths)
0
1
2
3
270
300
330
0
30
60
90
The sun's longitude (degrees)

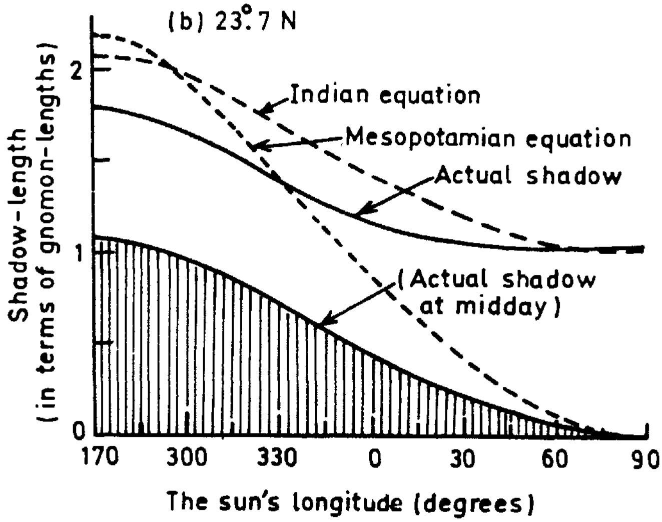

(b) 23°.7 N
Indian equation
Mesopotamian equation
Actual shadow
(Actual shadow
at midday)
Shadow-length
(in terms of gnomon-lengths)
0
1
2
170
300
330
0
30
60
90
The sun's longitude (degrees)

Fig. 6

solstice. This is exactly what we expected, because Eq. (2) must have originated in the data at the summer solstice, as we have seen in the *Artha-śāstra*.

From the above considerations, it is now clear that the Indian Eq. (2) on diurnal variation of the shadow-length was derived from observations at the summer solstice in North India without explicit foreign influence.

(iv) **The Clepsydra**

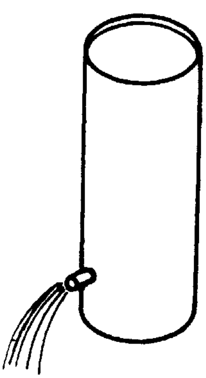
There are three types of clepsydra:

Type (1): Outflow type. This is a cylinder with a hole near its bottom from which water flows out (See Fig. 7).

Type (2): Floating bowl. This is a hemispherical bowl which has a hole at its bottom from which water flows in. It sinks after a certain time interval (See Fig. I-8).

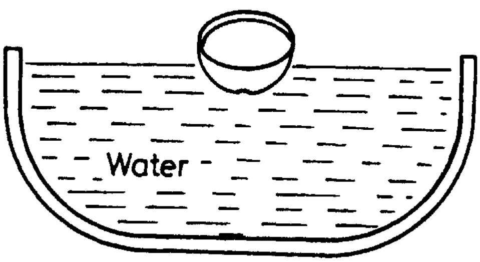
Type (3): Inflow type. Water constantly inflows to a cylindrical receiver from another water tank. Time can be read from the amount of water in the receiver. This is the common type in pre-modern China, and several devices were made in China in order to keep water-flow constant (See Fig. 9).

Among the above three types, the first two types are found in Sanskrit texts. Let us examine early Sanskrit accounts of the clepsydra one by one.



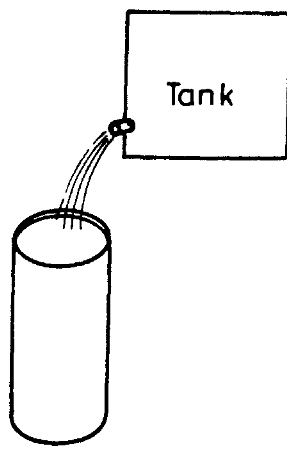
Outflow type

Fig. 7



Floating bowl

Fig. 8



Inflow type

Fig. 9

(a) *The Jyotiṣa-vedāṅga*

The *Jyotiṣa-vedāṅga* is the earliest Sanskrit text which refers to the clepsydra[1], although the shape of the clepsydra is not described there.

The *Jyotiṣa-vedāṅga* (*Yajur-vedic* recension vs. 24)[2] reads:

पलानि पञ्चाशदपां धृतानि तदाढकं द्रोणमतः प्रमेयम् ।
त्रिभिर्विहीनं कुडवैस्तु कार्यं तन्नाडिकायास्तु भवेत् प्रमाणम् ॥ 24 ॥

"Fifty *palas* of water which is held [in a vessel] is one *āḍhaka* (= 16 *kuḍavas*)[3]. From this, the *droṇa* (= 4 *āḍhakas*) is derived. This (one *droṇa*) should be lessened by three *kuḍavas*. It is the amount [of water in the clepsydra] for one *nāḍikā*".

The commentator, Somākara[4], mentions a copper vessel *(tāmraghaṭa)* with a hole at its bottom, pierced by a 3-*aṅgula* needle, from which water flows out.

S.B. Dikshit[5] also wrote that a *nāḍikā* was the time needed for the remaining water (1 *droṇa* minus 3 *kuḍavas*) to flow out from the hole of the vessel. J.F. Fleet supported this view, and wrote that the water-clock in the *Jyotiṣa-vedāṅga,* the *Artha-śāstra,* etc. worked "by emptying itself in the course of a *nāḍikā*"[6], and was not a "floating" type which was mentioned in later Siddhāntas. As regards the substance of the clepsydra in the *Jyotiṣa-vedāṅga,* etc., Fleet wrote that "the ordinary earthern water-jar *(kumbha, ghaṭa)* served the purpose"[7].

On the contrary, R. Shamasastry wrote that the vessel was "floated upon water" and after a *nāḍikā,* "it sank making a sound soon after it was filled with water entering into it through the hole at its bottom"[8].

It is difficult to say which is correct, but Dikshit and Fleet's investigation is harmonious with other early texts which we shall see below.

Before examining other texts, we should see a verse of the *Ṛg-vedic* recension. The *Jyotiṣa-vedāṅga* (*Ṛg-vedic* recension vs. 17)[9] reads:

नाडिके द्वे मुहूर्तस्तु पञ्चाशत्पलमाषकम् ।
माषकात् कुम्भको द्रोणः कुटपैर्वर्धते त्रिभिः ॥ 17 ॥

"Two *nāḍikās* are equal to one *muhūrta*. Fifty *palas* are equal to one *māṣaka*. From the *māṣaka*, the *kumbhaka* or *droṇa* is there. It is increased by 3 *kuṭapas*".

This text is not harmonious with the text of the *Yajur-vedic* recension which we have discussed. Weber[10] pointed out that the *"māṣaka"* might have been a corruption of the *"āḍhaka"*, but he wrote that the last phrase "increased by 3 *kuṭapas*" is not clear.

S.B. Dikshit, referring to Bhaṭṭotpala's commentary on the *Bṛhat-saṁhitā*, wrote that the text should be read as follows[11]:

नाडिके द्वे मुहूर्तस्तु पञ्चाशत्पलमाढकम् ।
चतुर्भिराढकैर्द्रोणः कुटपैर्वर्धते त्रिभिः ॥

"Two *nāḍikās* are equal to one *muhūrta*, 50 *palas* are equal to one *āḍhaka*, and 4 *āḍhakas* are equal to one *droṇa*. This is 3 *kuḍava* larger [than one *nāḍī*]"[12].

Sudhākara Dvivedin emended the text as follows[13]:

नाडिके द्वे मुहूर्तस्तु पञ्चाशत्पलमाढकम् ।
आढकात् कुम्भको द्रोणः कुटपैर्वर्धति त्रिभिः ॥

"Two *nāḍikās* are equal to one *muhūrta*, and 50 *palas* are equal to one *āḍhaka*. From the *āḍhaka*, the clepsydra *(kumbhikā)* [is measured]. One *droṇa* is 3 *kuḍava* larger [than the volume of the clepsydra]"[14].

Fleet tried to read the text with the least emendation, and emended and translated the text as follows[15]:

नाडिके द्वे मुहूर्तस्तु पञ्चाशत्पलमाढकम् ।
आढकात् कुम्भको द्रोणः कुटपैर्वर्धते त्रिभिः॥

"Two *nāḍikās* [are] a *muhūrta:* an *āḍhaka* consists of fifty *palas:* from the *āḍhaka* [there should be filled] a water-jar [to the extent of] a *droṇa;* it is too much by three *kuṭapas.*" (Translated by Fleet)[16].

Fleet's translation is understandable, and Dvivedin's interpretation is also quite intelligible.

(b) *The Artha-śāstra*

The *Artha-śāstra* (II.20.35)[17] reads:

सुवर्णमाषकाश्चत्वारश्चतुरङ्गुलायामाः कुम्भच्छिद्रमाढकमम्भसो वा नालिका।

"Or, a hole in a jar (with a dimension) of four *māṣakas* of gold made four *aṅgulas* in length, (with) an *āḍhaka* of water (running through it) measures one *nālikā*." (Translated by Kangle)[18].

Sastri[19] commented in his Sanskrit commentary which can be rendered into English as follows:

"'Four *māṣakas* of gold made four *aṅgulas*': by a metaphorical expression of two identical numbers, a hollow tube *(nāla-śalākā)* through which water flows, whose weight is 4 *māṣakas* and length is 4 *aṅgulas,* is meant. 'A hole in a jar': a hole, which is the same [as the tube in diameter], of a vessel which holds water. 'An *āḍhaka* of water': an *āḍhaka* of water which flows from the jar. A word *'iti'* (thus) may be supplied here. 'Thus also, one *nālikā*': thus, by the three-fold combination also, one *nālikā* is determined. From the jar, through the hole where the aforesaid tube *(śalākā)* is connected, an *āḍhaka* of water flows out from the hole in a certain time-span, and this time-span is one *nālikā*. This is the meaning"[20].

Venkatanathacharya's Sanskrit commentary is also similar[21]. According to R.P. Kangle[22], a Malayalam commentary *Bhāṣāvyākhyāna* also commented that the gold is to be turned into a hollow tube, apparently for fixing in the jar, through which the water is to flow out. However, Kangle commented that "unless the thickness of the gold leaf out of which the tube is made is also stated, the diameter would vary from tube to tube"[23]. Kangle has also written: "the idea is, gold 4 *māṣakas* in weight is made into a wire 4 *aṅgulas* long, the wire's thickness representing the measure of the hole in the jar through which the water is to flow out"[24], therefore, the "wire itself is of no further use in the *nālikā*"[25].

Let us see some other texts.

(c) *The Śārdūlakarṇāvadāna*

The *Śārdūlakarṇāvadāna* reads as follows[26].

नाडिकायाः पुनः किं प्रमाणम्? तदुच्यते – द्रोणं सलिलस्यैकम्। तद्धरणतो द्वे पलशते भवतः। नालिकाछिद्रस्य किं प्रमाणम्? सुवर्णमात्रम्। उपरि चतुरङ्गुला सुवर्णशलाका कर्तव्या। वृत्तपरिमण्डला समन्ताच्चतुरस्रा आयता। यदा चैवं शीर्येत तत् तोयं घटस्य तदैका नाडिका। एतेन नाडिकाप्रमाणेन विभक्ते द्वे नाडिके एको मुहूर्तः। एतेन भो ब्राह्मण त्रिंशन्मुहूर्ताः, यै रात्रिदिवसा अनुमीयन्त इति॥

"Again, what is the measure of a *nāḍikā?* That is told. One *droṅa* of water. As for its contents, they are 200 *palas*. What is the measure of the hole of the tube *(nālikā)?* One *suvarṇa* (= 16 *māṣas*) in weight. Further, a golden 'needle' *(śalāka)* (i.e. tube?) whose length is 4 *aṅgulas* should be made. An oblong rectangle is to be [rolled up into] a perfectly round circumference (i.e. tube?). The time-span during which the water of the vessel *(ghaṭa)* falls out is one *nāḍikā*[27]. Two *nāḍikās*, determined by this measure of *nāḍikā*, are one *muhūrta*. Like this, oh Brahman, there are 30 *muhūrtas*, by which the nights and days are measured".

The text has some ambiguity, and it is not impossible to consider that the word *"nālikā"* (tube) is a corruption of the word *"nāḍikā"* (measurement of time), and the "needle" was only used to pierce a hole. However, it seems to me that it is more natural to think that the golden "needle" was actually a tube.

Its Tibetan translation does not give a definite solution regarding this point, but a Chinese translation *Madengqie-jing*[28] mentions a cylindrical tube (*yuan tong*)[29] below the vessel. Unfortunately, the Chinese translation contains later interpolations, or rather improvements suitable for Chinese condition, and it cannot be used as a definite testimony for the interpretation of the Sanskrit original.

(d) *The Jyotiṣkaraṇḍaka*

The *Jyotiṣkaraṇḍaka (gāthā* nos. 10(ii)-14, and 28-29)[30] reads:

तीसे पुण संठाणं छिड्डुं उदगं च वोच्छामि ॥ 10 ॥

दालिमपुप्फागारा लोहमयी नालिगा उ कायव्वा
तीसे तलंमि छिद्दं छिद्दपमाणं च मे सुणह ॥ 11 ॥

छन्नउयमूलवालेहिं तिवस्सजायाए गयकुमारीए।
उज्जुकयपिंडिएहि उ कायव्वं नाडियाछिद्दं ॥ 12 ॥

अहवा दुवस्सजायाए गयकुमारीएँ पुच्छवालेहिं ।
बिहिं बिहिं गुणेहिं तेहि उ कायव्वं नाडियाछिद्दं ॥ 13 ॥

अहवा सुवण्णमासेहिं चउहिं चतुरंगुला कया सूई।
नालियतलंमि तीए उ कायव्वं नालियाछिद्दं ॥ 14 ॥

उदगस्स नालियाए हवंति दो आढगा उ परिमाणं।
उदगं च इच्छियव्वं जारिसगं तं च वोच्छामि ॥ 28 ॥

एयस्स उ परिकम्मं कायव्वं दूसपट्टपरिपूतं।
मेहोदयं पसन्नं सारइयं वा गिरिनईणं ॥ 29 ॥

"10 (ii): Further, I explain its *(nālikā's)* shape, hole, and water.

11: A copper clepsydra (*nālikā*), whose shape is like a flower of pomegranate (*dālima*)[31], should be made. A hole is at its bottom. Listen to my measurement of the hole.

12: The hole of a *nāḍikā* should be made by a straight collection of 96 hairs of the tail of a 3-year-old girl-elephant.

13: Otherwise, the hole of a *nāḍikā* should be made by a double-number (i.e. 192) hairs of the tail of a 2-year-old girl-elephant.

14: Otherwise, the hole of a *nālikā* should be made at the bottom of the clepsydra (*nālikā*) by a 4-*aṅgula* needle made of 4 *māṣas* of gold.

28: The amount of water of a *nālikā* is 2 *āḍhakas*. I also explain the desirable water.

29: Its purification should be made. It is purified by a piece of cloth. Otherwise, rainwater or clear autumnal mountain-torrent [may be used]"[32].

In the above text, there is one ambiguity about of the meaning of *nālikā* or *nāḍikā*, which means a measurement of time as well as the clepsydra. The clepsydra in this text seems to be a floating type like the clepsydra in later *Siddhāntas*, and not an early outflow type which we have seen in the *Artha-śāstra*, etc. The shape of the clepsydra in the *Jyotiṣkaraṇḍaka* is "like a flower of pomegranate", and this is similar to the usual floating type clepsydra which is like a hemispherical bowl. The substance is also specified as copper just like later *Siddhāntas*.

(e) *Purāṇas*

Some *Purāṇas* also have description of the clepsydra. Fleet[33] has pointed out that it is found in the *Vāyu-*, *Viṣṇu-*, *Brahma-*, and *Bhāgavata-purāṇa*.

Among them, the *Vāyu-*, *Viṣṇu-*, and *Brahma-purāṇa* give similar sentences in more or less corrupt forms.

The *Vāyu-purāṇa* (100.219(ii)-221(i))[34] reads:

तस्थानेनाम्भसा (सां) चापि पलान्यथ त्रयोदश ॥ 219 ॥
मागधेनैव मानेन जलप्रस्थो विधीयते
एते चाप्युदकप्रस्थाश्चत्वारो नालिको घटः ॥ 220 ॥
हेममाषैः कृतच्छिद्रैश्चतुर्भिश्चतुरङ्गुलैः ।

The *Viṣṇu-purāṇa* (6.3.7(ii)-8)[35] reads:

उन्मानेनाम्भसस्सा तु पलान्यर्द्धत्रयोदश ॥ 7 ॥
मागधेन तु मानेन जलप्रस्थस्तु स स्मृतः ।
हेममाषैः कृतच्छिद्रश्चतुर्भिश्चतुरङ्गुलैः ॥ 8 ॥

The *Brahma-purāṇa* (124.7(ii)-8)[36] reads:

उन्मानेनाम्भसः सा तु पलान्यर्धत्रयोदश ॥ 7 ॥
हेममाषैः कृतच्छिद्रा चतुर्भिश्चतुरङ्गुलैः ।
मागधेन प्रमाणेन जलप्रस्थस्तु स स्मृतः ॥ 8 ॥

It is seen that the above texts are basically the same, but are given in more or less corrupt forms. Fleet found that the "absolutely correct"[37] text had been given by Viṣṇucandra in his commentary on the *Viṣṇu-purāṇa* (6.3.7-9(i)) as a quotation from the *Vāyu-purāṇa*. The text quoted by Fleet is as follows[38]:

उन्मानेनाम्भसश्चापि पलान्यर्धत्रयोदश ।
मागधेन तु मानेन जलप्रस्थो विधीयते ।
एते चाप्युदकप्रस्थाश्चत्वारो नाडिकाघटः ।
हेममाषैः कृतच्छिद्रश्चतुर्भिश्चतुरङ्गुलैः ॥

"Moreover, by the measure of water [it is determined thus]: twelve and a half *palas;* [this] is laid down to be a *prastha* of water by the Māgadha measure: four of these *prasthas* of water [make up] the water-jar for the *nāḍikā;* it has a hole made by four *māṣas* gold [drawn out to the length of] four *aṅgulas.*" (Translated by Fleet)[39].

Fleet[40] rightly pointed out that the above clepsydra is exactly the same as that of the *Artha-śāstra,* because 4 *prasthas* are equal to one *āḍhaka,* which is the volume of the clepsydra of the *Artha-śāstra,* and the measurement of the golden needle is also the same.

Unfortunately, the date of the *Purāṇas* is not exactly known. Winternitz wrote that the original text of the *Vāyu-purāṇa* is "probably not later than the 5th century AD"[41]. Winternitz also wrote that Pargiter may be right in thinking that the *Viṣṇu-purāṇa* "cannot be earlier than the 5th century AD", but it is not much later[42]. As regards the *Brahma-purāṇa,* Winternitz wrote: "surely only a small portion of what has come down to us as the *Brahma-purāṇa* can only claim to be an ancient and genuine *Purāṇa*"[43]. Kane wrote that the *Vāyu-purāṇa* "must be placed between 350 AD and 550 AD"[44], and "it would not be far from the truth to hold that the present *Viṣṇu[-purāṇa]* was composed between 300 to 500 AD"[45].

Let us now see the description of the clepsydra in the *Bhāgavata-purāṇa,* which is somewhat different from the above descriptions. Winternitz wrote that "there are good grounds for assigning it to the 10th century AD"[46], and Kane wrote that "no reliable and cogent evidence has been adduced to prove that current *Bhāgavata[-purāṇa]* can be placed earlier than the 9th century AD"[47]. So, in any case, the following text may not be an early one:

The *Bhāgavata-purāṇa* (3.11.9)[48] reads:

द्वादशार्धपलोन्मानं चतुर्भिश्चतुरङ्गुलैः ।
स्वर्णमाषैः कृतच्छिद्रं यावत् प्रस्थजलप्लुतम् ॥ 9 ॥

"[The clepsydra is] 12 $\frac{1}{2}$ *palas* in measure (volume), and has a hole made by 4-*aṅgula* [needle made of] 4 *māṣas* of gold, in such a way that it is floating [until it is filled with] a *prastha* of water".

Fleet pointed out that "this statement is particularly imperfect, and also mixed: the three *pādas* of the verse belong to the ancient form of the water-clock, but the fourth *pāda* to the later form"[49]. In any case, this is a kind of the floating type of the clepsydra which is mentioned in later *Siddhāntas.*

(f) *The Yavava-jātaka*

The *Yavana-jātaka* (AD 269/70) of Sphujidhvaja also has a description of the clepsydra similar to the ancient type. The *Yavana-jātaka* is basically an astrological text based on a Sanskrit rendering of a Greek text, but its last chapter (Chapter 79) is devoted to mathematical astronomy, and a *Siddhānta* of *Vasiṣṭha* is also referred to.

The *Yavana-jātaka* (LXXIX. 27)[50] reads:

द्वाभ्यामथो काञ्चन [माष] खकाभ्यां समांशका ङ्गुलतुल्यदीर्घाः (र्घा) ।
छिद्रं तदग्रेण समेति यत् स्याद् वारो ऽथ मानस्थिति नाडिकांशा ॥ 27 ॥

"A homogeneous [needle should be made] which is 2-*aṅgula* long and made of 2 *māṣas* of gold. By its tip, a hole [should be made] in such a way that the exact amount of water for one *nāḍikā* will be arrived at".

Here, the measurement of the needle is a half of that of the *Artha-śāstra.* It is difficult to say whether this is an outflow type or a floating bowl type.

(g) *Concluding Remarks*

We have seen several descriptions of the clepsydra in ancient texts. Among them, the earliest clepsydras of the *Jyotiṣa-vedāṅga* and the *Artha-śāstra* seem to be the outflow type if we follow their commentaries. The clepsydra of the *Śārdūlakarṇāvadana* is evidently the outflow type. It is not clear whether a tube was attached to the hole of the clepsydra or not.

We should note that all clepsydras were used to measure one *nāḍikā* (= 24 minutes). One *nāḍikā* is a fixed time-span independent of the season. Perhaps, the clepsydra was repeatedly used to measure longer time. One *nāḍikā* is short enough to measure the seasonal variation of the duration of daytime or nighttime without much error.

Sharma and Lishk conjectured[51] that the maximum and the minimum lengths of daytime in terms of *ghaṭī* or *nāḍikā* in ancient texts actually refer to the amount of water in the clepsydra. So, they thought that the ratio 3:2 of the maximum and minimum lengths of daytime mentioned in ancient texts actually represent the ratio of the amount of water, and, applying Bernoulli's theorem, they argued that the actual ratio of the lengths of daytime was $\sqrt{3}:\sqrt{2}$. I think, however, that their conjecture is untenable, because there is no evidence to show that such a large clepsydra was used in ancient India. If a small clepsydra is repeatedly used, Bernoulli's theorem does not hold. As regards the "ratio 3:2", I have shown that it can be derived from the observations in North India[52].

Pingree[53] seems to have conjectured that the Indian clepsydra had been introduced from Mesopotamia, and he referred to a reference to the use of an outflow type of water-clock in the *mul Apin*. We should note, however, the difference between these clepsydras. As we have seen, the Indian clepsydra was used to measure a definite time-span *nāḍikā*. On the contrary, the clepsydra mentioned in the *mul Apin* seems to have been used to measure a seasonal time by the amount of water. According to Neugebauer[54], the ratio of the longest daylight to the shortest night has been given as 2:1 in the text, and it should be understood as the ratio between weights of water of the clepsydra. Neugebauer supposed that a "watch", which is a variable time-span according to the season, was measured by the clepsydra where different weights of water had to be poured[55]. If so, this Mesopotamian clepsydra must have been quite different from the Indian clepsydra.

(h) *Additional Remarks*

We have seen that the clepsydra is one of the earliest Indian astronomical instruments, and has been described in several early texts. There is one meteorological instrument which is probably as early as the clepsydra – the rain gauge[56]. There is a similarity between the clepsydra and the rain gauge. Both are measured by the same volume-measurement of Magadha[57].

The word *varṣa-pramāna* (measure of rain-fall) already occurred in the *Aṣṭādhyāyī* (III.4.32)[58] of Pāṇini, and this fact suggests that the rain gauge had been used at the time of Pāṇini.

The *Artha-śāstra* (II.5.7)[59] explicitly mentions the rain gauge (*varṣamāna*) as follows:

> "In the magazine, he should place a basin with a mouth one *aratni* (in width) as a rain-gauge". (Translated by Kangle)[60].

And also, *Bṛhat-saṁhitā* (XXIII.2)[61] reads as follows:

> "The quantity of water must be determined by taking a basin, a *hasta* (cubit) in diameter, for hydrometer. Fifty *palas* are equal to one *āḍhaka*, by which standard the water that has fallen is to be measured". (Translated by Kern)[62].

Here, it is seen that one *āḍhaka* is 50 *palas*, just the same as the case of the clepsydra. This system was called *Māghada* measure. On the contrary, one *āḍhaka* is 64 *palas* in *Bṛhat-saṁhitā* (LII. 91)[63], where the condition of soil is discussed. As Ohami[64] has pointed out, water was usually measured by the system where one *āḍhaka* was 64 *palas* at the time of Varāhamihira, but only the clepsydra and the rain gauge were measured by the old *Māgadha* measure where one *āḍhaka* was 50 *palas*. It may be that there was a distinct old tradition of experimental science, i.e. the method of determination of time and rainfall.

Lastly, I would like to notice another meteorological instrument, windvane. The *Kṛṣi-parāśara* (33)[65] reads:

> "By attaching a flag to a post, and making efforts day and night, the monthly rainfall is to be ascertained according to the course of the wind". (Translated by Majumdar and Banerji)[66].

Kṛṣi-parāśara is a Sanskrit work on agriculture, probably written in the middle of the 11th century[67]. It is seen that meteorological phenomena were carefully observed in connection with agricultural activities.

Conclusion

Several astronomical phenomena were noticed and observed by Vedic people, but no astronomical instrument was mentioned in the Vedic *Saṁhitās* and *Brāhmaṇas*.

The earliest astronomical instruments in India are the gnomon and the clepsydra.

The gnomon was used for the determination of cardinal directions, and its method was first described in the *Kātyāyana-śulbasūtra* (I. 2).

The annual variation and diurnal variation of the gnomon-shadow was also mentioned in several works. The annual variation of the gnomon-shadow was considered to follow a linear zig-zag function, and was mentioned in the *Artha-śāstra* (II. 20. 41-42), the *Śārdūlakarṇāvadāna*, the *Sūrya-prajñapti* (X. 10), etc.

The description of the diurnal variation of the gnomon-shadow can be classified into two classes. One type is for the summer solstitial day, and follows a simple algebraic formula. This type is found in the *Artha-śāstra* (II.20.39-40) and the *Sūrya-prajñapti* (IX. 4), etc. This formula was generalized for a whole year, and the generalized formula was recorded in the *Yavana-jātaka* (LXXIX. 32) and the *Pañca-siddhāntikā* (IV. 48-49).

Another type is for the equinoctial day, and does not follow a simple algebraic formula. This type is found in the *Śārdūlakarṇāvadāna* and the *Atharva-jyotiṣa*. This type was mentioned in al-Bīrūnī's *India* also.

The clepsydra was first mentioned in the *Jyotiṣa-vedāṅga*. It was also mentioned in the *Artha-śāstra* (II.20.35), the *Śārdūlakarṇāvadāna*, etc. It seems that the clepsydra mentioned in them were outflow type. On the contrary, it seems that the clepsydra mentioned in the *Jyotiṣa-karaṇḍaka* was floating bowl type, which is common in later *Siddhāntas*. The clepsydra was also mentioned in some *Purāṇas*, and some of them described the clepsydra similar to the *Artha-śāstra*.

There is no reason to assume that these two instruments were introduced from outside of India.

ACKNOWLEDGEMENT

This paper was originally a part of my Ph.D. thesis submitted to Lucknow University. The thesis "A History of Astronomical Instruments in India" consists of four chapters as follows.

Chapter I:	The Beginning of Astronomical Instruments in India.
Chapter II:	Astronomical Instruments in Classical Siddhāntas.
Chapter III:	Works on Astronomical Instruments during Delhi Sultanate and Mughal Periods.
Chapter IV:	Astronomical Observatories of Sawai Jai Singh.

This paper is almost the same as Chapter I of the original thesis.

The thesis is a result of my research on the history of astronomical instruments in India as a research scholar of Department of Mathematics and Astronomy, Lucknow University, India, during the period November 1983-September 1987, and also the subsequent study in Japan. At Lucknow University, I did my research under the guidance of Prof. Kripa Shankar Shukla, and he kindly guided me through correspondence even after I left India. I am grateful to Prof. Shukla for his teaching.

In the thesis, I attempted to investigate the development of astronomical instruments in India by studying various primary sources, and also to explain mathematically the method of observation by those instruments.

I am deeply indebted to the successive Heads of Dept. of Mathemtics and Astronomy, namely, Prof. R.P. Agarwal and Prof. K.D. Singh, and Prof. Sunil Datta (acting head when I submitted the thesis), and also the staff of Dept. of Mathematics and Astronomy, Lucknow University, who helped in my work in various ways.

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The thesis was completed in August 1990, and submitted to Lucknow University in January 1991. The thesis has been declared eligible to receive the degree of Doctor of Philosophy in Mathematics (notice dated 10 April 1992).

Last but not least I am grateful to Dr. A.K. Bag, INSA, for the arranging publication of the present paper.

Notes and References

Section 1. i

1. Wheeler, Motimer, *Civilizations of the Indus Valley and beyond*, London, 1966.
2. See Bag, A.K., Astronomy in Indus Civilization and during Vedic Times, in S.N. Sen and K.S. Shukla (eds.): *History of Astronomy in India*, New Delhi; 1985: pp. 122-130, where hypothesis of H. Heras, A. Parpola, etc. has been summarized.
3. The *Śatapatha-brāhmaṇa* is a *Brāhmaṇa* belonging to the White *Yajur-veda*. It seems that it "marks a phase of transition from the Vedic to the post-Vedic culture". (Sharma, R.S., *Material Culture and Social Formations in Ancient India*, New Delhi, 1983, p. 170).

4. Eggeling, Julius (tr.): *The Śatapatha-brāhmaṇa* according to the text of the Mādhyandina School, Part I (Sacred Books of the East 12), Oxford, 1882, p. 155. Also see Thapar, Romila, *From Lineage to State*, Bombay, 1984, pp. 26-27.

5. Chattopadhyaya, Debiprasad: *Lokayata* – A Study in Ancient Indian Materialism, New Delhi 1981, Chap. 1, passim.

6. Sharma, R.S., op.cit. 1983, p. 169.

7. Sharma, R.S., op.cit., pp. 22-23, and 169. Also see Thapar, R., op.cit., 1984, p. 22.

8. Sharma, R.S., op.cit., p. 22.

9. Sharma, R.S., op.cit., p. 24; and Thapar, R., op.cit., p. 23.

10. Morgan, Louis H., *Ancient Society*, London, 1877, pp. 23-24; and Engels, F., *The Origin of the Family, Private Property and the State*, English tr., Moscow, 1977, p. 26.

11. Wilson, H.H. (tr.): *Ṛg-veda-saṁhitā*, (1850 f.), rep. Delhi, 1977, Vol. IV, pp. 446-448. Cf. Griffith, Ralph T.H. (tr.): *The Hymns of the Ṛgveda*, (1889 f.), rep. Delhi, 1973, pp. 384-385.

12. Keith, Arthur Berriedale, *The Religion and Philosophy of the Veda and Upanishads*, Part I, Harvard, 1925, p. 34.

13. Ibid.

14. Gray, L.H., Calendar (Persian), in J. Hastings (ed.): *Encyclopaedia of Religion and Ethics*, 12 vols., Edinburgh, 1908-1921, Vol. III, pp. 128-131.

15. *Ṛg-veda* (I.164.11), etc.

16. See Kosambi, Damodar Dharmanand: *An Introduction to the Study of Indian History*, 2nd ed., Bombay, 1975, pp. 249-251. Also see Sharma, R.S., op.cit., p. 70; and Thapar, R., op.cit, p. 29.

17. Sharma, R.S., op.cit., p. 36, and 170.

18. Wilson, H.H. (tr.), op.cit., Vol. I, p. 76. Cf. Griffith, R.T.H. (tr.), op.cit., p. 15. Also see Dikshit, Sankar Balakrishna: *Bharatiya Jyotish Sastra*, English tr. by R.V. Vaidya, Part I, Calcutta, 1969, p. 17.

19. Wilson, H.H. (tr.), op.cit., Vol. I, p. 77.

20. Wilson, H.H. (tr.), op.cit., Vol. VI, pp. 276-277. Cf. Griffith, R.T.H. (tr.), op.cit., p. 593. Also see Dikshit, S.B., English tr., op.cit., Part I, p. 44.

21. Wilson, H.H. (tr.), op.cit., Vol. VI, p. 277. Cf. Griffith, R.T.H. (tr.), op.cit., p. 593. Also see Dikshit, S.B., English tr., op.cit., Part I, p. 37.

22. Wilson, H.H. (tr.), op.cit., Vol. VI, p. 279. Cf. Griffith, R.T.H. (tr.), op.cit., p. 594. Also see Dikshit, S.B., English tr., op.cit., Part I, p. 44.

23. Macdonell, A.A. & Keith, A.B., *Vedic Index of Names and Subjects*, Vol. 1, London, 1912, p. 110.

24. Wilson, H.H. (tr.), op.cit., Vol. VI, p. 312. Cf. Griffith, R.T.H. (tr.), op.cit., p. 602.

25. Macdonell, A.A. & Keith A.B., op.cit., Vol. 1, p. 110.

26. Wilson, H.H. (tr.), op.cit., Vol. III, p. 516. Cf. Griffith, R.T.H. (tr.), op. cit., p. 277. Also see Dikshit, S.B., English tr., op. cit., Part I, p. 41.

27. See Dikshit, ibid., and Macdonell, A.A. and Keith, A.B. op.cit., Vol. 1, p. 49.

28. Sharma, R.S., op.cit., p. 56. Also see Thapar, R., op.cit., p. 23.

29. Sharma, R.S., op.cit., pp. 169-170.

30. Ibid, p. 170.

31. Sharma, R.S., op.cit., pp. 70-71. Also see Thapar, R., op.cit., p. 29.

32. Sharma, R.S., op.cit., p. 39 and 61.

33. Sharma, R.S., op.cit., p. 61 and 71.

34. Keith, Arthur Berriedale (tr.), *The Veda of the Black Yajus School entitled Taittiriya Sanhita*, Part II, Harvard, 1914, p. 349. According to Macdonell and Keith, op.cit., Vol. 1, 1912, p. 411, the *Kāṭhaka-saṁhitā* (of the Black *Yajur-veda*) also gives a list of 27 *nakṣatras*.

35. Whitney, William Dwight (tr.), *Atharva Veda Saṁhitā*, Vol. II, Harvard, 1905, pp. 906-909. Cf. Griffith, R.T.H. (tr.): *The Hymns of the Atharvaveda*, Vol. II, (1894), rep. Varanasi, 1968, pp. 265-266. According to Macdonell and Keith, op.cit., Vol. 1, p. 411, the *Maitrāyaṇī-saṁhitā* (of the Black *Yajur-veda*) also gives a list of 28 *nakṣatras*.

36. Keith, A.B. (tr.), op.cit., Part I, p. 157.

37. Keith, A.B. (tr.), op.cit., Part II, p. 607.

38. See Macdonell, A.A. & Keith A.B., op.cit., Vol. 2, p. 161.

39. Keith, A.B. (tr.), op.cit., Part II, pp. 349-350.

40. *Griffith, R.T.H. (tr.): The Texts of the White Yajurveda*, (1899), rep. Varanasi, 1976, p. 117, 124, 126, 127, 129 and 139.

41. Macdonell, A.A. and A.B. Keith, op.cit., Vol. 1, pp. 110-111.

42. Ibid.

43. Dikshit, S.B., English tr., op.cit., Part I, pp. 63-64.

44. Keith, A.B. (tr.), op.cit., Part II, p. 540.

45. Dikshit, S.B., English tr., op.cit., Part I, p. 24. For the *Nārāyaṇa-upaniṣad* (80), see Swāmī Vimalānanda (ed. and tr.): *Mahānārāyaṇopaniṣad*, Madras, 1979, pp. 352-359. Also see the *Praśna-upaniṣad* (I. 9); Hume, Robert Ernst (tr.): *The Thirteen Principal Upanishads*, Oxford, 1931, p. 379, or Max Müller, F. (tr.), *The Upanishads*, Part II (SBE 15), Oxford, 1884, p. 272.

46. Macdonell, A.A. & Keith, A.B., op. cit., Vol. 2, p. 162; Dikshit, S.B., English tr., op. cit., Part I, p. 21.

47. Griffith, R.T.H. (tr.), op.cit., 1899, p. 59 and 209.

48. Keith, A.B. (tr.), op.cit., Part I, p. 57

49. Whitney, W.D. (tr.), op.cit., Vol. I, p. 230. Cf. Griffith, R.T.H. (tr.) op.cit., 1894, Vol. I, p. 197.

50. Whitney, W.D. (tr.), Ibid.

51. Macdonell, A.A. & Keith, A.B., op.cit., Vol. 2, p. 412.

52. Griffith, R.T.H. (tr.), op.cit., 1899, p. 239. Also see Dikshit, S.B., English tr., op.cit., Part I, p. 15.

53. Cf. Keith, A.B. (tr.), op.cit., Part II, p. 446. Unfortunately, this translation does not show the names of the successive years clearly.

54. Dikshit, S.B., English tr., op.cit., Part I, pp. 32-33. Also see Macdonell, A.A. & Keith, A.B., op.cit., Vol. 2, p. 158, where *amānta* is suggested.

55. Keith, A.B. (tr.), op.cit., Part I, p. 274.

56. Sarup, Lakshman: *The Nighaṇṭu and the Nirkuta*, (1920-27), rep. Delhi, 1967, p. 171 (tr.) and p. 191 (text). Also see Dikshit, S.B., English tr., op. cit., Part I, p. 34.

57. Macdonell, A.A. & Keith, A.B., op.cit., Vol. 2, p. 169.

58. See Macdonell and Keith, op.cit., Vol. 2, p. 169; and Dikshit, S.B., English tr., op.cit., Part I, pp. 42-43.

59. Eggeling, J. (tr.), op.cit., Part V (SBE 44), 1900, p. 169.

60. Keith, A.B. (tr.), op.cit., Part I p. 93.

61. Sharma, R.S., op.cit., 1983, pp. 82-83.

62. Sharma, R.S., op.cit., p. 74.

63. Keith, A.B., *The Religion and Philosophy of the Veda and Upanishads*, Part II, Harvard, 1925, p. 316. Sometimes, Śrauta-rituals are also divided into *iṣṭi* (unbloody), *paśu* (animal sacrifice), and *soma*. (See Monier-Williams, *A Sanskrit-English Dictionary*, Oxford, 1899, relevant entries; and Kane, P.V., *History of Dharmaśāstra*, Vol. II, Part II, Poona, 1974, p. 1133).

64. Bühler, Georg (tr.), *The Sacred Laws of the Āryas*, Part I (SBE 2), Oxford, 1879, p. 217.

65. Eggeling, J. (tr.), op.cit., Part I, p. 173. Also see Thapar, R., op.cit., 1984, p. 64.

66. Kane, P.V., op.cit., Vol. II, Part II, p. 1106.

67. Macdonell, A.A. & Keith, A.B, op.cit., Vol. 1, p. 309.

68. Ibid. See Oldenberg, Hermann, *The Gṛihya-sūtras*, Part I (SBE 29), Oxford, 1886, p. 52 and 134; and Part II (SBE 30), 1892, p. 15, 58 and 59.

69. I have used Ānandāśrama ed. (Kāśīnāthaśāstrī Āgāśe ed., *Aitareyabrāhmaṇam*, ASS 32, Part II, Pune, 1977, p. 828). According to the numbering of A.B. Keith, this sentence is included in the Ait. Br., VII. 11 (XXXII. 10). (Keith, A.B., *Rigveda Brahmanas*, Harvard, 1920, p. 297).

70. Dikshit, S.B., English tr., op.cit., Part I, p. 35.

71. Keith, A.B. (tr.), op. cit., 1920, p. 297, foot note. I have checked with Ānandāśrama ed. (Gulāvarāva Vajeśaṁkara Ojhā ed.: *Śāṁkhāyanabrāhmaṇam*, ASS 65, Pune, 1977, p. 7). The *Śāṁkhāyana-brāhmaṇa* is the same as the *Kauṣītaki-brāhmaṇa*, and belongs to the *Ṛg-veda*. The *Aitareya-brāhmaṇa* also belongs to the *Ṛg-veda*.

72. Keith, A.B. (tr.), op.cit., 1920, p. 297.

73. See Dikshit, S.B., English tr., op.cit., Part I, pp. 58-62.

74. See Macdonell, A.A. & Keith, A.B. op.cit. Vol. 1, pp. 243-244; and Keith, A.B., *The Religion and Philosophy of the Veda and Upanishads*, Part I, p. 79.

75. One unnumbered verse in the Yajur-vedic recension (between vs. 4 and 5) of the *Jyotiṣa-vedāṅga* mentions Jupiter (*bṛhaspati*). However, it is considered to be an interpolation, because the *rāśi* (zodiacal sign) is also mentioned in the same verse. The *rāśi* is an imported concept from Grt ce. (See Kuppanna Sastry, T.S., Vedāṅga jyotiṣa of Lagadha, *Indian Journal of History of Science*, Vol. 19, No. 4, 1984, Supplement, p. 50).

76. See, for example, Thibaut, G., *Astronomie, Astrologie und Mathematik*, Strassbourg, 1899, p. 6; Dikshit, S.B., English tr., op.cit., Part I, p. 60; Kane, P.V., op.cit., Vol. V, Part I, 1974, p. 494.

77. Macdonell, A.A. & Keith, A.A., op.cit., Vol. 1, p. 243. See Hume, R.E. (tr.), op.cit., p. 434, or Max Müller (tr.), op.cit., Part II, p. 317.

Section 1. ii

1. Tilak, B.G., *The Orion,* 5th ed., Poona, 1972, p. 169.
2. I have used VVRI ed. (Vishva Bandhu ed., *Ṛgveda with Commentaries,* Part IV, Hoshiarpur, 1964, p. 1759).
3. Wilson, H.H. (tr.), op.cit., Vol. III, p. 421. Cf. Griffith, R.T.H. (tr.), op.cit., p. 255.
4. Tilak, B.G., op.cit., p. 169.
5. See, for example, Das, Sukumar Ranjan, Astronomical Instruments of the Hindus, *Indian Historical Quarterly,* **4**, 256-269, pp. 257-258, 1928, and Gurjar, L.V., *Ancient Indian Mathematics and Vedha,* Poona, 1947, pp. 148-151.
6. Das, Sukumar Ranjan, op.cit., p. 258.
7. Dikshit, S.B., English tr., op.cit., Part I, p. 97, and Part II, p. 227; or Dīkṣit, Śaṁkar Bālakṛṣṇa, Bhāratīya Jyotiṣ, Hindi tr. by Śivanāth Jhārakhaṇḍī, Lucknow 1957, p. 137 and 457.
8. Datta, Bhagavad (ed.): *Ātharvaṇa-jyotiṣam,* Lahore, 1924, Introduction, p. 3.
9. Ibid, pp. 2-3.
10. Sengupta, Prabodh Chandra, *Ancient Indian Chronology,* Calcutta, 1947, p. 157.
11. See, for example, Saha, M.N. *et al.* (ed.), *Report of the Calendar Reform Committee,* New Delhi, 1955, p. 266, and Lishk, S.S. & Sharma, S.D. Season Determination through the Science of Sciatherics in Jaina School of Astronomy, *Indian Journal of History of Science,* **12**(1), 33-44, p. 33, 1977.
12. I have used Ānandāśrama ed., Part I, (1979), p. 486 f. According to the numbering of A.B. Keith, this sentence is included in the Ait. Br. IV. 18 (XVIII. 4). (Keith, A.B. (tr.): *Rigveda Brahmanas,* 1920, p. 210.)
13. Keith, A.B. (tr.), op.cit., 1920, p. 210. In Sengupta, P.C., op. cit., p. 156, the *Divākīrtya* has been translated as "the hymns to be chanted during the day", and the *Virāj* has been interpreted as "a period of ten days".
14. Sengupta, P.C., op.cit., p. 156.

Section 1. iii

1. I have not attempted to survey several *Śrauta-sūtras.* I have used only an English translation of the *Śāṅkhāyana-śrauta-sūtra,* translated by W. Caland and posthumously published by Lokesh Chandra in 1953.
2. Caland, W. (tr.): *Śāṅkhāyana Śrautasūtra,* Nagpur, 1953, p. 5.
3. Caland, W. (tr.): op.cit., p. 60.
4. See Sect. 1, ii, c of this paper.
5. See its *adhyāyas* XIX and XXV.
6. The original translation of A.B. Keith reads "six-month", but it must be a misprint. The original Sanskrit is "*ṣaḷahair*". (Ānandāśrama ed. (1977), p. 67) Also see Sarma, K.V.: Indian Astronomy in the Vedic Age, *Vivekananda Kendra Patrika,* Feb. 1983, 98-108, p. 104; and Shukla, K.S., Main Characteristics and Achievements of Ancient Indian Astronomy in Historical Perspective, in Swarup, G., Bag, A.K. & Shukla, K.S. (eds.): *History of Oriental Astronomy,* Cambridge, 1987, 9-22, p. 11.
7. Keith, A.B. (tr.), op.cit., 1920, p. 452.

8. See Sect. 1, ii, c of this paper.

9. The rationale of the calculation is as follows. Let A be the azimuth of the rising point of the sun, when the azimuth is measured towards east from the north point of the horizon, δ the declination of the sun, and φ the observer's latitude. Then, we have: $\cos A = \sin\delta . \sec\varphi$. (See for example, Smart, W.M., *Textbook on Spherical Astronomy*, 6th ed., Cambridge, 1977, p. 47).
We also know the following equation: $\sin\delta = \sin \epsilon . \sin \lambda$,
where ϵ is the obliquity of the ecliptic, and λ is the solar longitude. I have assumed that $\epsilon = 23°.8$, because it was so in around 500 BC.
Therefore, we can calculate the necessary value from the following equation:
$A = \arccos (\sin \epsilon . \sin \lambda . \sec \varphi)$.

10. Tilak, B.G., *Orion*, 1972, p. 34.

11. I have used Ānandāśrama ed. (Nārāyaṇaśāstrin ed. *Taittirīya-brāhmaṇam* with Sāyana's commentary, ASS 37, Part I, Pune, 1979, p. 245).

12. Sāyana commented that this *"jaghanyam"* is "the place behind", i.e. the east of the *nakṣatra* which is moving towards west. However, I think that it means "lastly", i.e. just before the *nakṣatra* vanishes, in this context.

Section 1. iv

1. Datta, Bhagavad (ed.), *Ātharvaṇa-jyotiṣam*, (The Punjab Sanskrit Series VI), Lahore, 1924; and Śarmā, Choṭelāla and Oṁnārāyana Dvivedī (eds., with Hindi tr.), *Atharvavedīya-jyautiṣam*, Datia (Madhya Pr.), 1965

2. Datta, Bhagavad (ed.), op.cit., Introduction, pp. 2-3.

3. Weber, A., "Über den Vedakalender Namens Jyotisham" (*Abhandlungen der philosophisch-historischen Klasses der Königl. Akademie der Wissenschaften*, 1862, Nr. 1), Berlin, 1862. According to Tilak, B.G., *Vedic Chronology and Vedanga Jyotisha*, Poona (n.d.), p. 45, the Ṛg-vedic recension of the *Jyotiṣa-vedāṅga* had already been published by Captain Jervis at the end of his *Indian Metrology*, 1834.

4. Thibaut, G., Contributions to the explanation of the Jyotisha-Vedāṅga, *Journal of the Asiatic Society of Bengal*, **46**, 411-437, 1877.

5. See Dikshit, S.B., English tr., op.cit., Part I, pp. 66-97; or Dīkṣit, Hindi tr., op.cit., pp. 92-137.

6. I have not seen Lālā Chote Lāl's book, but some of his interpretations can be known from the Appendix (written by Muraḷidhara Jhā) to the S. Dvivedin's edition of the *Jyotiṣa-vedāṅga*.

7. Dvivedin, Sudhākara (ed.), *Yājuṣa-jyautiṣaṁ Somākara-sudhākara-bhāṣya-sahitam, Ārca-jyautiṣaṁ ca*, Benares (1908). (Originally published in the *Pandit* in some instalments in 1907.)

8. Tilak, B.G., Note on the Interpretation of the Vedanga Jyotisha (written in 1914), in his *Vedic Chronology and Vedanga Jyotisha*, Poona (n.d.), pp. 43-104.

9. Shamasastry, R., *Vedangajyautisha*, edited with his own English Translation and Sanskrit commentary, Mysore, 1936.

10. Bhaṭṭācārya, Sīteśacandra (ed. with Bengali tr.): *Vedāṅgajyautiṣam*, Calcutta, 1974.

11. Kuppanna Sastry, T.S., (ed. by K.V. Sarma), Vedāṅga Jyotiṣa of Lagadha with the Translation and Notes, *Indian Journal of History of Science*, **19**, No. 3, (Suppl.) pp. 1-32, 1984, No. 4 (Suppl.) pp. 33-74.

12. The word "*śuci*" can also be interpreted as a proper nown, i.e. "I, Śuci by name".

13. See Ṛg-vedic recension (henceforth R. in this footnote) 22 = Yajur-vedic recension (henceforth Y. in this footnote) 40; and R.7 = Y.8.

14. First published by R. Shamasastry in 1909. And also, first translated into English by R. Shamasastry in 1915 (Shamasastry, R., *Kauṭilya's Arthaśāstra*, Mysore, 1915). Now we have a critical edition with an excellent English translation by R.P. Kangle. (Kangle, R.P., *The Kauṭilīya Arthaśāstra*, Part 1 (text), 2nd ed. Bombay, 1969, Part 2 (translation), 2nd ed. Bombay, 1972, and Part 3 (study), Bombay, 1965). I have also used Venkatanathacharya, N.S. (ed.), *Kauṭalīyārthaśāstra of Śrī Viṣṇugupta*, Mysore, 1960; and Ganapati Sastri (ed.), *The Arthaśāstra of Kauṭalya*, with the Commentary *Śrīmūla* of Ganapati Sastri, Trivandrum, 1924.

15. Kangle, R.P., op.cit., Part 3, p. 106.

16. *Artha-śāstra* (II.20.64 and 66). Kangle, op.cit., Part 1, p. 72 (text); Part 2, p. 141 (tr.).

17. Ibid (II.7.7). Kangle, Part 1, p. 43; Part 2, p. 82.

18. Ibid (II.20.41). Kangle, Part 1, p. 72; Part 2, p. 140.

19. Ibid (II.20.37). Kangle, Part 1, p. 72; Part 2, p. 140.

20. Ibid (II.20.61-62). Kangle, Part 1, p. 72; Part 2, p. 141.

21. Ganapati Sastri's Sanskrit commentary in his ed., op.cit., p. 267.

22. I have used Vaidya, P.L. (ed.) *Divyāvadāna*, (Buddhist Sanskrit Texts 20), Darbhanga, 1959. The *Śārdūlakarṇāvadāna* is the 33rd story of this collection. The *Divyāvadāna* had previously been published by Cowell and Neil, Cambridge, 1886, but the *Śārdūlakarṇāvadāna* which was printed as the appendix of their edition was incomplete. For the general information of the *Śārdūlakarṇāvadāna*, see Zemba, Mokoto: "Matōga-kyō no temmon-rekisū ni tsuite" (On astronomical and calendrical descriptions in the Śārdūlakarṇāvadāna), in Japanese, in Bukkyō-daigaku ed.: *Tōyōgaku-ronsō*, Kyoto, 1952, pp. 171-213.

23. The meaning of this title is the "Mātaṅga-sūtra". I have used Taisho ed. (*Taishō-shinshū-daizōkyō*, Tokyo, 1924-34, Vol. 21, pp. 399-410, (text No. 1300)).

24. He was originally an Indian. Fl. the 3rd century AD.

25. Fl. first half of the 3rd century AD.

26. The meaning of this title is the "Prince Śārdūlakarṇa's twenty-eight-lunar-mansion sūtra". I have used Taisho ed. (Vol. 21, pp. 410-419, (text No. 1301)).

27. Fl. second half of the 3rd century AD.

28. The Tibetan title is the *sTag rna'i rtogs pa brjod pa* (Tiger-ear's knowledge story). I have used Peking ed. (1684-1700), *bKa'-'gyur, mDo-Kè*, 242a-286b (rep. *Pekin-ban Seizō-daizōkyō*, Tokyo, 1954-59, Vol. 40, pp. 353-370).

29. Vaidya, P.L. ed., op.cit., p. 355, lines 22-23.

30. Compare with the *Artha-śāstra* (II.20.66).

31. Peking ed., op.cit., 276a, line 4. The original Tibetan text is as follows: "Lo lnga lnga zhing zla ba re re lhag par 'gyur zhing ..."

32. Vaidya, P.L. ed., op.cit., p. 354, lines 18-20.

33. Vaidya ed., op.cit., p. 354, lines 26-28.

34. Vaidya ed., op.cit., p. 355, lines 4-5.

35. Vaidya ed., op.cit., p.355, lines 12-14.

36. For general information of Jaina astronomy, see Jain, L.C., *Exact Sciences from Jaina Sources*, Vol. 2, *Astronomy and Cosmology*, Jaipur, 1983, and Lishk, S.S., *Jaina Astronomy*, Delhi, 1987.

37. Jain, Jagadīś-candra, *Prākṛt sāhitya kā itihās* (in Hindi), Varanasi, 1985, p. 52.

38. For general information, see Weber, A., Ueber den auf der Kön. Bibl. zu Berlin befindlichen Codex der Sūryaprajñapti, *Indische Studien*, **10**, pp. 254-316, 1868 and Thibaut, G. On the Sūryaprajñapti, *Journal of the Asiatic Society of Bengal*, **49**, No. 3, pp. 107-127, and No. 4, pp. 181-206, 1880. The *Sūryaprajñapti* has been edited by J.F. Kohl (Kohl, Josef Friedlich: *Die Sūryaprajñapti versuch einer textgeschichte*, Stuttgart, 1937).

39. I have used Rajkot ed. (*Śrī Candraprajñaptisūtram*, with the Sanskrit commentary of Ghāsīlālajī Mahārāja, Rajkot, 1973).

40. *Candraprajñapti* (X.20.4). Rajkot ed., op.cit., pp. 403-411.

41. *Candraprajñapti* (X.10). Rajkot ed., op.cit., pp. 316-330. Here, the length of the gnomon-shadow for each month is given.

42. I have used H. Jacobi's English translation (Jacobi, Hermann (tr.): *Jaina sūtras*, Part II, (SBE 45), Oxford, 1895). It is also necessary to see Jacobi, Hermann, Einteilung des Tages und Zeitmessung im alten Indien, *Zeitschrift der Deutschen Morgenländischen Gesellschaft*, **74**, 247-263, 1920. In this paper (p. 257, footnote 1), he corrected a mistake in his aforesaid translation.

43. I have used Ratlam ed. (*Vallabhīyācāryīyaṁ Śrījyotiṣkaraṇḍakaṁ Prakīrṇakam*, with the Sanskrit commentary of Malayagiri, Ratlam, 1928).

44. Jain, Jagadīś-candra, op.cit., p. 566.

45. Thibaut, G. & Dvivedī, Sudhākara (ed. and English tr., with Sanskrit com.), *The Pañcasiddhāntikā*, Benares, 1889, text p. 31, Skt. com. pp. 61-63, and English tr. pp. 67-68. And also, Neugebauer, O. and Pingree, D. (ed. and English tr., with English com.), *The Pañcasiddhāntikā of Varāhamihira*, 2 parts, Copenhagen (1970-71), Part I, pp. 106-107 (text and tr.), and Part II, pp. 80-83 (com.).

Section 1. v

1. See, for example, Dikshit, S.B., English tr., op.cit., Part I, p. 86 f.; or Dīkṣit, Hindi tr., op.cit., p. 121 f.

2. Kuppanna Sastry, T.S., op.cit. (*IJHS*, **19**, No. 3, Suppl.), p. 13 f., 1984.

3. Ibid., pp. 13-14.

4. By the celestial sphere, I mean the *nirayana*-coordinates which are fixed to the fixed stars.

5. Since the positions of the solstices have been expressed as "the first point" and "the middle" of the *nakṣatras* in the text of the *Jyotiṣa-vedāṅga* (R.6, Y.7), their accuracy is not more than a quarter *nakṣatra*. In the following calculations, I have roughly assumed that the annual precession is 50".

6. Dikshit, S.B., English tr., op.cit., Part I, pp. 88-89; or Dīkṣit, Hindi tr., op.cit., pp. 124-125.

7. Vasu, Śriśa Chandra (ed., and English tr.): *The Ashṭādhyāyī of Pāṇini*, Allahabad, 1891, Vol. II, p. 871. In this connection, we should also note that the word "*varṣa-pramāṇa*" (a measure of rain-fall) occurs in the *Aṣṭādhyāyī* (III.4.32). (Vasu, Ś.C. (ed. and tr.), op.cit., Vol. I, p. 569. Also see Srinivasan, T.M., Measurement of Rainfall in Ancient India, *Indian Journal of History of Science*, **11**, No. 2, 148-157, 1976, especially p. 149.) This word pre-supposes the existence of the rain gauge, which again presupposes an idea to observe natural phenomena by an instrument.

8. Kosambi, D.D., *An Introduction to the Study of Indian History*, 2nd ed., Bombay, 1975, p. 282.

9. Sharma, R.S., *Material Culture and Social Formations in Ancient India*, New Delhi, 1983, p. 90.

10. Pingree, David, The Mesopotamian Origin of Early Indian Mathematical Astronomy, *Journal for the History of Astronomy*, **4**, 1-12, p. 3, 1973.

11. *Jyotiṣa-vedāṅga* (Y. 29 (a-b)).

12. See Neugebauer and Pingree (ed. and tr.), *The Pañcasiddhāntikā of Varāhamihira*, Part II, Copenhagen, 1971, p. 81.

13. See the *Āryabhaṭīya* (I.3). (Shukla, K.S. and Sarma, K.V. (ed. and tr.), *Āryabhaṭīya of Āryabhaṭa*, Part I (Critically edited with Translation and notes), New Delhi, 1976, p. 6). Actually, Āryabhaṭa gives the number of the rotation of the earth.

14. Pingree, D., op.cit., 1973, p. 7.

15. See, for example, Dikshit, S.B., English tr., op. cit., Part I, pp. 89-90; or Dīkṣit, Hindi tr., op.cit., pp. 125-126. The rationale of this estimation is as follows. Let D be the angle equivalent to the length of daytime, φ the latitude of the observer, and δ the declination of the sun. Then, we have the following equation:
$\cos D/2 = -\tan \varphi \, . \, \tan \delta.$
(See, for example, Smart, W.M., op.cit., p. 47; or Gorakh Prasad, *Text Book on Spherical Astronomy*, Allahabad, 1981, p. 19.)
At the winter solstice, δ is equal to the obliquity of the ecliptic (ϵ). Therefore:
$\varphi = \arctan(-\cos D/2/\tan \epsilon).$

16. See Pingree, D., op.cit., 1973, p. 4 et passim.

17. The rationale of the estimation is the same as explained in the footnote 15. I have assumed that $\epsilon = 23°.8$. (It was so around 500 BC.) Therefore, $\delta = 11°.6$ one solar month after the vernal equinox. ($\sin \delta = \sin \epsilon \, . \, \sin \lambda$, where λ is the solar longitude.) When the length of daytime is 16 *muhūrtas*, D/2 is 96°. Therefore:
$\varphi = \arctan(-\cos 96°/\tan 11°.6)$
$= 27°.0.$

18. Here, $\delta = 20°.5$, and $D/2 = 102°$. So $\varphi = 29°.1$.

19. I shall examine the shadow-length in the next section.

Section 2. i

1. Apart from independent pubications, these four texts have been published as: Satya Prakash & Jyotishmati, Usha (ed.), *The Śulba Sūtras*, Allahabad, 1979 and Sen, S.N. and Bag, A.K. (ed. and English tr.): *The Śulbasūtras*, New Delhi, 1983.

2. Satya Prakash & Jyotishmati, U. (ed.), op.cit., p. 109; or Sen, S.N. & Bag, A.K. (ed. and tr.), op.cit., p. 54 (text), and p. 120 (tr.); and also Khadilkar, S.D. (ed. and English tr.): *Kātyāyana Śulba Sūtra*, Poona, 1974, pp. 1-2, Also see Yano, Michio, Knowledge of Astronomy in Sanskrit Texts of Architecture, *Indo-Iranian Journal*, **29**, 17-29, p. 18, 1986.

3. Al-Bīrūnī called so in his treatise on shadows. (See Yano, M., op.cit., 1986, p. 18).

4. Some people have written that this experiment should be done on the equinoctial day. (See Sen, S.N. and Bag, A.K. (ed. and tr.), op.cit., p. 264; and Khadilkar, S.D. (ed. and tr.), op.cit., p. 2.) However, the "two points" in this method need not be the east and west cardinal points on the circle. This method gives the correct direction on any day except for a certain period around the winter solstice, when even the shortest length of the shadow exceeds the radius of the circle in North India.

5. Satya Prakash and Ram Swarup Sharma (eds.), *Āpastamba-śulbasūtram*, with the commentaries of Kapardisvāmin, Karavinda, and Sundararāja, with English tr. of Satya Prakash, New Delhi, 1968, p. 246.

6. Ibid., p. 248.

Section 2. ii

1. Kangle, R.P., op.cit., Part I, p. 72 (text), and Part II, p. 140 (tr.); or Venkatanathacharya, N.S. (ed.), op.cit., p. 116; or Ganapati Sastri, T. (ed.), op.cit., p. 267.

2. Kangle, R.P., op.cit., Part II, p. 140. Cf. Shamasastry, R. (tr.), op.cit., p. 122.

3. *Artha-śāstra* (II.20.10). Kangle, op.cit., Part I, p. 71, and Part II, p. 138.

4. Ganapati Sastri, T. (ed.), op.cit., p. 267. The original commentary of Ganapati Sastri is as follows:

आषाढे मासि आषाढमासान्ते, नष्टच्छायो ऽच्छायः, मध्याह्नः भवति। अतः परं, श्रावणादीनां षण्मासानां पौषान्तानां, द्व्यङ्गुलोत्तरा द्विद्व्यङ्गुलक्रमवृद्धा छाया भवति। तद्यथा – श्रावणस्यान्ते द्व्यङ्गुला भाद्रपदस्य चतुरङ्गुला आश्वयुजस्य षडङ्गुला कार्तिकस्याष्टाङ्गुला मार्गशीर्षस्य दशाङ्गुला पौषस्य द्वादशाङ्गुलेति।-------

5. The latitude can be calculated from the following two equations:
$s = 12 \tan \zeta$, and
$\varphi = \zeta + \delta$,
where s is the length of the shadow in terms of *aṅgulas*, ζ the sun's midday zenith distance, φ the observer's latitude, and δ the sun's declination. I have assumed that the obliquity of the ecliptic is 23°.7, as it was so around 300 BC.

6. Vaidya, P.L. (ed.), op.cit., pp. 354-355.

7. Probably, the sun was considered to be in Mṛgaśiras at the beginning of the month, because the moon was in Āṣāḍhā at the full moon in the previous month. The *pūrṇimānta* (end with full moon) system is inferred here.

8. Vaidya, P.L. (ed.), op.cit., p. 318, line 27.

9. **Peking ed., op.cit., 275 a, line 4. The Tibetan original is: "shing bu sor drug pa". It literally means "six-finger wooden stick". The Tibetan word "*sor*" (finger) was used to translate the Sanskrit word "*aṅgula*".**

10. **Peking ed., op.cit., 275 b, line 3**

11. **Peking ed., op.cit., 276 a, line 3.**

12. I have used Rajkot ed., op.cit., (1973), of the *Candra-prajñapti*, where the Sanskrit rendering of Ghāsīlālajī Mahārāja has been given. For the *Jambudvīpa-prajñapti*, see Lishk, S.S. & Sharma, S.D., op.cit., *IJHS* **12**, 33-34, 1977. The text of the *Sūrya-prajñapti* has been given in Kohl, J.F. ed., op.cit., p. 51 f., where the text has been compared with the *Jambudvīpa-prajñapti*.

13. Rajkot ed., op.cit., p. 318.

14. In the Rajkot ed., the data for the 4th month of summer has been supplied from the *Jambudvīpa-prajñapti*. In this month, the sun turns by a circle (*vṛtta*), by standing on a square (***sama-caturasra-saṁsthita***), by being hanging down (*nyagrodha-parimaṇḍala*), and by being attached to its own body (*sva-kāyam anuraṅginī*).

15. Lishk, S.S. and S.D. Sharma, op.cit., 1977, p. 38.

16. Lishk and Sharma themselves mathematically proved that there is no such place if the latitude is higher than 23°.5 N. However, they argued that the maximum shadow-length can be double of the minimum shadow-length at the latitude 7°.1 N. It is true, but these two shadows are in opposite directions there, as Lishk and Sharma themselves admit. Their interpretation cannot be accepted, because, in the given text, the shadow-length gradually decreases from maximum to minimum without crossing the zero point.

17. Jacobi, H., op.cit., *ZDMG* **74**, p. 256, 1920.

18. Jacobi, H. (tr.), op.cit., 1895, p. 143. Also see Jacobi, H., op.cit., 1920, pp. 256-257, where he corrected his mistake in his translation.

19. I have used Ladnun ed. (Ācārya Tulasi (ed.): *Aṅga-suttāni*, 3 parts, Ladnun (Rajasthan) (1974)), Part I.

20. Ibid, p. 861.

21. Ibid, p. 882.

22. Lishk, S.S. & Sharma, S.D. op.cit., 1977, p. 41.

23. Thibaut, G. & Dvivedī, Sudhākara (ed. and tr.), op.cit., p. 5 (text), and pp. 11-12 (tr.); or Neugebauer, O. & Pingree, D., (ed. and tr.), op.cit., Part I, p. 37. Chapter II of the *Panca-siddhāntikā* has been considered to be based on the *Vasiṣṭha-siddhānta*.

24. Pingree, D., op.cit., *JHA* **4**, p. 5, 1973.

25. Ibid.

26. Neugebauer, Otto, *A History of Ancient Mathematical Astronomy*, 3 parts, Berlin, 1975, Part I, pp. 544-545.

27. Ibid, p. 545.

28. Neugebauer, O., op.cit., 1975, Part II, pp. 737-740.

29. Ibid, p. 739.

30. See Yano, Michio, op.cit., *Indo-Iranian Journal*, **29**, 17-29, p. 26, 1986.

31. The graph shows the following value: $\tan(\varphi + \epsilon) - \tan(\varphi - \epsilon)$, where φ is the latitude of the observer, and ϵ the obliquity of the ecliptic.

Section 2. iii

1. Kangle, R.P., op.cit., Part I, p. 72 (text), and Part II, p. 140 (tr.)

2. This word "*tripauruṣyām*" is an emendation of R.P. Kangle, who followed the suggestion of H. Jacobi (Jacobi, H., op.cit., *ZDMG* **74**, 247-263, pp. 253-254, 1920). The reading of the original manuscript was "*caturpauruṣyām*". (Kangle, R.P., op.cit., Part I, p. 72, footnote.) Venkatanathacharya ed., op.cit., p. 116, and Ganapati Sastri ed., op.cit., p. 266 retained the original "*caturpauruṣyām*". According to George Abraham (Abraham, George, The Gnomon in Early Indian Astronomy, *Indian Journal of History of Science*, **16**, No. 2, 215-218, 1981, p. 215), Harihara Sastri's edition of the *Artha-śāstra-vyākhyā* – *Cāṇakya-ṭīkā* of Bhikṣu Prabhāmati retains "*caturpauruṣyām*", but takes 1/10 day instead of 1/8 day. This reading is also theoretically possible.

3. Venkatanathacharya ed., op.cit., p. 116, and Ganapati Sastri ed., op.cit., p. 267 omit this "*trayo*". Ganapati Sastri added in his Sanskrit commentary that "*trayo, ṣṭamāṁśāḥ*" was meant here.

4. Kangle, R.P., op.cit., Part II, p. 140. Cf. Shamasastry R. (tr.), pp. 121-122. Shamasastry's translation reads: ' when 4 paurushas, 1/8th part".

5. Rajkot ed., op.cit., p. 222. Cf. Kohl ed., *Sūrya-prajnapti*, IX (pp. 35-38). According to J.F. Kohl, the *Jambudvīpa-prajñapti* does not have an equivalent portion to the above portion except for short sentences which state that the length of daytime and nightime is 18 and 12 *muhūrtas* respectively, or vice versa, at the solstice. In the quoted text of the *Candra-prajñapti*, I have corrected some minor misprints.

6. The original Prakrit of the Rajkot ed. is: "bābīsa-sahassa-bhāge gaë vā sese vā". Kohl's edition of the *Sūrya-prajñapti* also reads: "bâvīsasahassa bhāe .....".

7. Jacobi, H., op.cit., *ZDMG* **74**, 254, 1920. In this paper, Jacobi has written that the shadow is 59 *pauruṣīs* when 1/120 of day time has passed. It is not clear whether a certain unpublished manuscript gave such reading or not.

8. Abraham, George, op.cit., *IJHS*, **16**, 215-218, 1981. He followed Harihara Sastri's reading of the *Artha-śāstra* with the commentary of Bhikṣuprabhāmati, where t/d is 1/10 for s/g = 4, instead of t/d = 1/8 for s/g = 3 in Kangle's text. Both are mathematically permissible.

9. Pingree, David (ed. and tr.): *The Yavanajātaka of Sphujidhvaja*, 2 vols, Cambridge Mass. (1978), Vol. I, p. 500 (text), and Vol. II, p. 189 (tr.)

10. Thibaut, G. and Sudhākara Dvivedin (ed. and tr.), op.cit., p. 17 (text), and pp. 34-35 (tr.); or Neugebauer, O. and D. Pingree (ed. and tr.), op.cit., Part I, pp. 66-69 (text and tr.), and Part II, pp. 45-46 (English com.).

11. The actual shadow-length can be calculated by the following equations:
$\cos \zeta = \sin \varphi \cdot \sin \delta + \cos \varphi \cdot \cos \delta \cdot \cos H$, and $S = G \tan \zeta$,
where ζ is the sun's zenith distance, φ the observer's latitude, δ the sun's declination, H the hour angle, S the shadow-length, and G the gnomon-length.
(See, for example, Smart, W.M., op.cit., p. 35; or Gorakh Prasad, op cit., p. 21)
I have assumed that $\delta = \varphi = 23°.7$, the value suitable for 300 BC or so.

12. See the data "on the last day" discussed in Sect. 2, ii, c of this paper.

13. Vaidya, P.L. (ed.), op.cit., p. 336, line 29 – p. 337, line 7. For the Tibetan translation, see Peking ed., op.cit., 263 a, line 5 – 263 b, line 4. For the Chinese translation, see Taisho ed., op.cit., pp. 408-409, and p. 416.

14. The Tibetan translation gives the following fifteen data of the shadow-length: 60, 60, 12, 6, 5, 4, 3, not given (midday), 1, 4(?), 5, 6, 12, 8 (sic), and 60. The Tibetan text adds one more *muhūrta* called "*srin po*" ("demon") without any data of the shadow-length soon after the *muhūrta* of midday, and it does not have its counterpart in the Sanskrit text.
Two Chinese translations give basically the same numerical data as the Sanskrit text, although one of them uses a smaller unit

15. The Tibetan translation also uses "*skyes bu*" ("man") as the unit of measurement.

16. Vaidya, P.L. (ed.), op.cit., p. 337, line 2.

17. Datta, Bhagavad (ed.), op.cit, 1924, pp. 1-2, or Śarmā, Ch. & Dvivedī, O (eds.), op. cit., 1965, pp. 11-12. Bhagavad Datta's edition wrongly reads "navatī ṣaḍaṅgulā.. " in 6a.

18. Datta, Bhagavad (ed.), op.cit., p. 1. Śarmā, Ch. and Dvivedī O. (eds.), op.cit., p. 11 reads: "madhyāhne cābhijinnāma yasmin chāyā pratiṣṭhati".

19. Dikshit, S.B., English tr., op.cit., Part I, p. 98. (Or, Dīkṣit, Hindi tr., op.cit., p. 137.) Also see Lishk, Sajjan Singh and S.D. Sharma: "Standardization of Time-unit *Muhūrta* through the Science of Sciatherics in *Atharva Vedāṅga Jyotiṣa*". *Indian Journal of History of Science*, Vol. 15, No. 2, pp. 193-203, 1980.

20. Sachau, Edward C. (tr.): *Alberuni's India*, London (1910), Vol. 1, p. 339. The Arabic original has been published as: *Kitāb al-Birūnī fī taḥqīq mā li-l-Hind*, Hyderabad Deccan (1958), p. 285. The original Arabic caption for the middle column is as follows: "Ziyādatu ẓ-ẓilli 'alā fay'i z-zawāli".

21. Sachau, E.C. (tr.), op.cit., Vol. 1, p. 338; and Hyderabad ed., op.cit., p. 285.

22. I have assumed that the Indian data are meant for the middle point of the *muhūrta*. I have also assumed that the latitude is 26°N, the central latitude of the Plain of the Ganga.

23. If we interpret that the data are the finite differences of the excess shadow, the line in the figure becomes smooth. Then, the excess shadows become: 186, 90, 30, 18, 12, 7, and 3. However, the results are too large for the actual excess shadows.

24. See Sect. 2, ii, d of this paper. The source has been explained in Neugebauer, O., op. cit., 1975, Part II, pp. 737-740.

25. Neugebauer, op.cit., Part II, p. 738.

26. Pingree, D., op.cit., *JHA* **4**, p. 6, 1973.

27. The shadow-table in the *mul Apin* has been explained in Neugehauer, O., op.cit., 1975, Part I, pp. 544-545. In the table of the *mul Apin*, the shadow is 8 cubits when 11°15' (i.e. 45 minutes of an equinoctial hour) has elapsed after sunrise. Weidner and van der Waerden had interpreted that the gnomon-length is 1 cubit, but Sachs and Neugebauer have shown that it is wrong, and the text only means to measure in cubits.

28. Neugebauer, O., op.cit., 1975, Part I, pp. 544-545.

29. Ibid, p. 545.

30. The length of the shadow is given by the following equation:
$S = G \tan \zeta$,
where S is the shadow-length, G the length of the gnomon, and ζ the sun's zenith distance. I have assumed that $G = 1$. The sun's zenith distance is given by the following equation:
$\cos \zeta = \sin \varphi \,.\, \sin \delta + \cos \varphi \,.\, \cos \delta \,.\, \cos H$,
where φ is the latitude of the observer, δ the sun's declination, and H the hour angle. (See, for example, Smart, W.M., op.cit., p. 35, or Gorakh Prasad, op.cit., p. 21.)
The sun's declination is given by:
$\sin \delta = \sin \epsilon \,.\, \sin \lambda$,
where ϵ is the obliquity of the ecliptic, and λ the sun's longitude. In the calculation, I have assumed that the obliquity of the ecliptic is 23°.7.

Section 2. iv

1. For the Indian clepsydra in general, see Fleet, J.F. The Ancient Indian Water-clock, *Journal of the Royal Asiatic Society*, pp. 213-230, 1915, and Ohami, Isao: "Kodai, Chūsei-Indo ni okeru jikan no shakudo ni tsuite" (On the Measurement of Time in Ancient and Medieval India), in Japanese, *Kagakusi Kenkyu*, Series II, **18** (No. 130), pp. 95-104, 1979.

2. Kuppanna Sastry (ed.), op.cit., *IJHS*, **19**, No. 3 Suppl., p. 29, 1984.

3. As regards the system of measurement, see, for example, Dikshit, S.B., English tr., op.cit., Part I, pp. 78-79, or Dīkṣit, Hindi tr., op.cit., pp. 109-110; or Fleet, J.F., op.cit., 1915.

4. Dvivedin, Sudhākara (ed.), op.cit., 1908, p. 39. A German rendering of Somākara's commentary has been given in Weber, A., op.cit., Berlin, 1862, p. 78.

5. Dikshit, S.B., English tr., op.cit., Part I, p. 78; or Dīkṣit, Hindi tr., op.cit., p. 109.

6. Fleet, J.F., op.cit., 1915, p. 214.

7. Ibid, p. 224.

8. Shamasastry, R. (ed. and tr.), op.cit., 1936, p. 25 (tr.).

9. Kupanna Sastry (ed.), op.cit., 1984, p. 24 and 26 (variant readings).

10. Weber, A., op.cit., 1862, p. 79.

11. Dikshit, S.B., English tr., op.cit., Part I, p. 77; or Dīkṣit, Hindi tr., op.cit., p. 108. Cf. Fleet, J.F., op.cit., 1915, pp. 215-216 (footnote 5).

12. I have rendered into English from Hindi translation (by Śivanāth Jhārakhaṇḍī) of Dīkṣit's book.

13. Dvivedin, Sudhākara (ed.), op.cit., 1908, p. 64.

14. I have rendered into English following Dvivedin's Sanskrit commentary.

15. Fleet, J.F., op.cit., 1915, p. 215.

16. Ibid. I have made Sanskrit words into italic, and used square brackets instead of original italic in Fleet's translation.

17. Kangle, R.P., op.cit., Part I, p. 71 (text), and Part II, p. 139 (tr.); or Ganapati Sastri (ed.), op. cit., p. 266; or Venkatanathacharya (ed.), op.cit., p. 115.

18. Kangle, R.P., op.cit., Part II, p. 139. Cf. Shamasastry (tr.), op.cit., p. 121. Shamasastry translated as: " ... water passes out of a pot through an aperture of the same diameter as that of a wire of 4 aṅgulas in length and made of 4 māshas of gold".

19. Ganapati Sastri (ed.), op.cit., p. 266.

20. The original Sanskrit commentary of Ganapati Sastri is as follows:

सुवर्णमाषकाश्चत्वारश्चतुरङ्गुलायामा इति मेयमानयोरभेदोपचारात् सुवर्णमाषकचतुष्टयप्रमाणा चतुरङ्गुलदीर्घा जलनिर्गमनालशलाकोच्यते, कुम्भच्छिद्रं तत्समानं जलधारकघटच्छिद्रम्, आढकमम्भसः तत्कुम्भगतमाढकमितं जलम्, इतिशब्द इहाध्याहार्यः, इति वा नालिका इत्येतत्त्रितययोगेन वा नालिका परिच्छिद्यते। यथोक्तमानशलाकायुक्तच्छिद्रात् कुम्भादाढकपरिमितं जलं छिद्रतो यावता कालेन गलति सा वा नालिकेत्यर्थः।

21. Venkatanathacharya (ed.), op.cit., p. 115, footnote 5.

22. Kangle, R.P., op.cit., Part II, p. 139, footnote 35.

23. Ibid.

24. Ibid.

25. Ibid.

26. Vaidya, P.L. (ed.), op. cit., p. 337, lines 26-30. For Tibetan translation, see Peking ed., op.cit., 264 b, lines 6-7. For Chinese translation *Madengqie-jing*, see Taisho ed., op.cit., p. 409. Cf. Fleet, J.F., op.cit., 1915, p. 218 (English rendering); and Ohami, Isao, op.cit., 1979, p. 96 (Japanese rendering).

27. The text edited by Cowell and Neil (1866) reads: "... suvarna-mātram upari catur-aṅgulā suvarṇa-śalākā kartavyā vṛtta-parimaṇḍalā samantāc catur-asrā āyatā. yataś caiva śīryeta tatas toya-ghaṭasya chidraṁ kartavyam. ..." J.F. Fleet translated it as follows: "There should be made a gold pin, of the quantity of a suvarṇa, four aṅgulas *in length*, drawn out quite round *or* square: and with it there should be made a hole *in the bottom* of a water-jar, through which, indeed, *the water* may fall out." (Fleet, J.F., op.cit., 1915, p. 218).

It should also be noted that Cowell and Neil's edition reads "*nālikā*" for *nāḍikā* (measurement of time) also. So, Fleet seems to have interpreted that the word *nālikā*, which I have translated as "tube", is also the measurement of time (= *nāḍikā*).

28. Taisho ed., op.cit, p. 409, column 1, lines 18-20.

29. The Chinese character "*yuan*" means round, and "*tong* " means tube.

30. Ratlam ed., op. cit., pp. 6-8, and 12.

31. Pomegranate *(Pueraria granatum)* is well known in India by its fruit (*anār* in Hindi). (See *The Wealth of India, Raw Materials*, Vol. VIII, New Delhi, 1969, p. 317 ff).

32. In this translation, I basically followed the Sanskrit commentary of Malayagiri.

33. Fleet, J.F., op. cit., 1915, p. 220.

34. I have used Ānandāśrama ed. (*Vāyupurāṇam*, (ASS 49), Pune, 1983, p. 397).

35. I have used Gita Press ed. (Śrīmunilāla Gupta (ed. and Hindi tr.), *Śrī-śrī-viṣṇupurāṇa*, Gorakhpur, 1984, p. 506). Cf. Dutt, M.N. (English tr.), *Vishnupuranam*, 1896, p. 435.

36. I have used Śrī Veṅkaṭeśvara ed. (*Brahmamahāpurāṇam*, Bombay, 1906, folio 163 b, Ch. 124, vs. 7 (ii)-8). J.F. Fleet (op. cit., p. 220, footnote 1 (3)) refers to Ānandāśrama ed. (Ch. 231, 7-9 a). It may be that the chapter *(adhyāya)* number differs between these two editions.

37. Fleet, J.F., op. cit., 1915, p. 220.

38. Ibid, p. 221.

39. Ibid. I have changed his old system of romanization of Sanskrit words into modern system, and made Sanskrit words italic. I have used square bracket instead of original italic in Fleet's translation.

40. Fleet, J.F., op. cit., 1915, p. 221.

41. Winternitz, M., *A History of Indian Literature*, Vol. 1, Part II (Epics and Purāṇas), Translated into English by S.Ketkar, Calcutta, 1978, p. 485.

42. Ibid, p. 478.

43. Ibid, p. 469.

44. Kane, P.V., *History of Dharmaśastra*, Vol. V, Part II, 2nd ed., Poona, 1977, p. 907.

45. Ibid, p. 909.

46. Winternitz, M., op. cit., Vol.I, Part II, English tr., p. 487.

47. Kane, P.V., op.cit., Vol. V, Part II, 1977, p. 899.

48. I have used Gita Press ed. (*Śrīmad Bhāgavata Mahāpurāṇa* (text with English tr. of C.L. Goswami), Part I, Gorakhpur 1982, p. 200; or *Śrīmadbhāgavata-mahāpurāṇa* (text with Hindi tr.), Part I, Gorakhpur, 1985, p. 246).

49. Fleet, J.F., op.cit., 1915, p. 220, footnote 1(4). Fleet refers to Nirnaya Sagar ed. of the *Bhāgavata* (3.11.9).

50. Pingree, D. (ed.), *The Yavanajātaka of Sphujidhvaja*, Part I, 1978, p. 499. Pingree wrongly emended the text as follows:

    घट्यामयो कांचन (लो)हकार्या स्याद्द्वादशको ऽङ्गुलतुल्यदीर्घः ।
    छिद्रं तदग्रेण समेति यत् स्याद् वारि प्रमेण तिथिनाडिकानाम् ॥

    This emendation does not make sense. Especially, the word "*tithi*" is not understandable, and the 4th *pada* is metrically defective also. I have reconstructed the original manuscript reading from Pingree's "Apparatus", and added only one word "*māṣa*" in 27a in order to make it metrically correct, while retaining the remaining portion almost as it was in the original manuscript.

51. Sharma, S.D. & Lishk, S.S., Length of the Day in Jaina Astronomy, *Centaurus*, Vol. 22, No. 3, pp. 165-176, 1978.

52. See Sect. 1, iv of this paper.

53. Pingree, D., op. cit., *JHA* 4, 1-12, pp. 3-4, 1973.

54. Neugebauer, O., Studies in Ancient Astronomy. VIII. The Water Clock in Babilonian Astronomy, *ISIS*, 37, 37-43, 1947, (reprinted in his *Astronomy and History*, Selected Essays, New York, 1983, 239-245).

55. Ibid, 1947, p. 40; or 1983, p. 242.

56. For Indian rain gauge, see Srinivasan, T.M., Measurement of Rainfall in Ancient India, *Indian Journal of History of Science*, **11**, No. 2, 148-157, 1976.

57. For Indian volume measurement, see Ohami, Isao, Kodai Indo no doryōkō – Taiseki-tan'i ni tsuite (On Volume Measures in Ancient India), in Japanese, *Kagakusi Kenkyu*, Series II, **21** (No. 141), pp. 16-26, 1982.

58. Vasu, Ś.C. (ed. and tr.), *The Ashṭādhyāyī of Pāṇini*, Allahabad, 1891, Vol. I, p. 569. Also see Srinivasan, T.M., op.cit., 1976, p. 149.

59. Kangle, R.P., op. cit., Part I, p. 40 (text), and Part II, p. 73 (tr.); or Shamasastry, R. (tr.), op. cit., p. 56.

60. Kangle, R.P., op.cit., Part II, p. 73.

61. For the text, see Tripathi, A.V. (ed.), *Bṛhat-saṁhitā* with the commentary of Bhaṭṭotpala (based on Sudhākara Dvivedin's ed.), 2 parts, Varanasi, 1968, Part I, p. 335; or Bhat, M.R. (ed. and tr.), *Varāhamihira's Bṛhat Saṁhitā*, 2 parts, Delhi, Part I, p. 245, 1981-82. Also see Shastri, A.M.: *India as seen in the Bṛhatsaṁhitā of Varāhamihira*, Delhi, pp. 495-499, 1969.

62. Kern, H. (tr.), The Bṛhat-Saṅhitā; or Complete System of Natural Astrology of Varâhamihira, *Journal of the Royal Asiatic Society*, (in several instalments), p. 253, 1871.

63. Tripathi, A.V. (ed.) op.cit., (1968), Part II, p. 608; or Bhat, M.R. (ed. and tr.), op. cit., Part I, p. 485. (In Bhat's ed., this verse is LIII. 93) Or, Kern, H. (tr.), op.cit., *JRAS*, p. 295, (1973) (LIII. 93).

64. Ohami, Isao, op. cit., p. 21, 1982.

65. Majumdar, Girija Prasanna and Banerji, Sures Chandra (ed. and tr.), *Kṛṣi-parāśara* (Bibliotheca Indica 285), Calcutta (1960), p.9 (text), and p. 66 (tr.).

66. Ibid, p. 66.

67. Gopal, Lallanji, *Aspects of History of Agriculture in Ancient India*, Varanasi, 1980, p. 30.