VARIABLE RADIUS EPICYCLE MODEL

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The paper investigates mathematically the improvement of the *Manda* correction of Indian epicycle theory effected by varying the epicycle radius.

Key Words : *Epicycle model, Indian theory, Manda correction, Variable radius*

Introduction

The epicycle theory first appears in Greek astronomy, and a detailed exposition is found in Ptolemy's *Almagest*[1] in his models of the sun, moon and planets.

A few hundred years later it appears in Indian *Siddhāntic* astronomy in a distinctly different form. One feature in the Indian Model[2] is the variable radius of the epicycle. Ptolemy's epicycle radius was a constant. This gave the Indian model an extra degree of freedom enabling a better fit to observations.

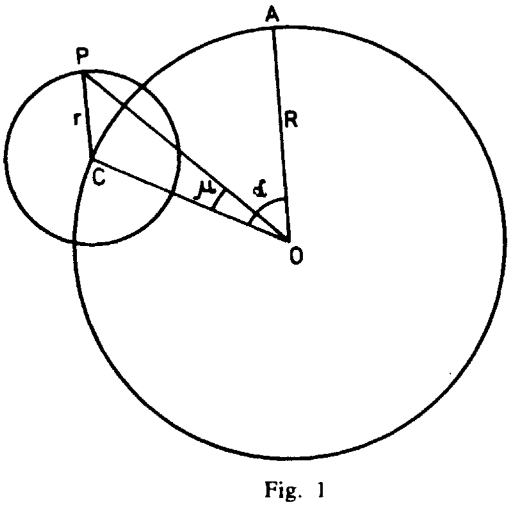

Fig. 1

In the epicycle theory, two corrections are required to find the true position of a planet from the mean motion. In this paper we consider only the first called the *Manda* correction.

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In Fig. 1, the smaller circle is the epicycle. Its centre C moves uniformly on the larger circle called the deferent, centre O. A is the apogee, and CP is parallel to OA; C is the mean position of the planet and P the true position, α is the mean anomaly and the correction μ is called the equation of the centre. r, R are the radii of the circles. The radius r varies according to the formula.

(1) $r = r_o\ (1 + |\sin\alpha|\ \epsilon)$

Where r_o and ϵ are constants.

From Δ OCR, we derive the equation of the centre

(2) $\mu = \sin\mu = ax(1 + 2\ bx + x^2)^{-1/2}$

Where

(3) $x = \frac{r}{R}$, $a = \sin\alpha$, $b = \cos\alpha$

assuming x and hence μ are small quantities.

Calculation of Parameters

The parameters r_0 and ϵ in equation (1) will be determined by minimizing the difference between μ and the corresponding equation of the centre $\mu_0$of the Kepler planetary orbit, which is an ellipse of eccentricty e. We have[3]

(4) $\mu_0 = 2e\ \sin\ \alpha + \frac{5}{4}\ e^2\ \sin\ 2\alpha\ = ae\ (2 + \frac{5}{2}\ b\ e)$

in which e being small, terms of order higher than two are omitted. If we substitute (1) in (2) and omit terms of order higher than two, assuming $x_0 = \frac{r_0}{R}$ and ϵ to be small, we have

(5) $\mu = ax_0\ (1 + a\epsilon - bx_0)$

We define the difference

(6) $\delta\ (\alpha) = \mu\text{-}\mu_0 = a\ \{(x_0\text{-}2e) + ax_0\ \epsilon\ - b\ (\frac{5}{2}\ e^2 + x_0^2)\}$

(7) and measure the deviation by means of the function,

$$S\ (x_{0,}\ \epsilon) = \delta^2(\tfrac{\pi}{4}) + \delta^2(\tfrac{p}{2}) + \delta^2(\tfrac{3\pi}{4})$$

$$= 2(x_0\text{-}2e)^2 + \frac{3}{2}\ x_0^2\ \epsilon^2 + \frac{1}{2}\ (x_0^2 + \frac{5}{2}\ e^2) + (2 + \sqrt{2})\ x_0\epsilon\ (x_0\text{-}2e)$$

δ and S are symmetrical about the line OA, (Fig. 1) and so we need to consider only values of α between 0^0 and 180^0.

The best values of x_0 and $\in$ are found by minimizing S. This gives

(8a) $\frac{dS}{dx_0} = 4(x_0\text{-}2e) + 3x_0\in^2 + 2x_0\ (x_0^2 + \frac{5}{2}e^2) + 2\,(2+\sqrt{2})\ \in(x_0\text{-}e) = O$

(8b) $\frac{dS}{d\in} = 3x_0^2\in + (2+\sqrt{2})\,x_0\,(x_0\text{-}2e) = O$

Substituting

(9) $y = \frac{2e}{x_0}$

in (8a) and (8b), we get

(10 a) $4(1\text{-}y) + 3\in^2 + 8e^2\ (\frac{1}{y^2} + \frac{5}{8}) + (2+\sqrt{2})\in(2\text{-}y) = O$

(10 b) $3e + (2+\sqrt{2})\ (1\text{-}y) = O$

from which we can eliminate $\in$. Then

(11) $e^2 = \frac{0.11467\ (y\text{-}1)}{\frac{8}{y^2} + 5}$

TABLE 1- From different values of y and e for different planets, the following table may be computed :

	y	e	First Approx	Min - Mix Radius	Āryabhaṭa	Sūrya Siddhānta
Venus	1.0052	.0068	5⁰	4⁰52′ - 4⁰-54′	18⁰ - 9⁰	11⁰ - 12⁰
Sun	1.0305	.0167	12⁰	11⁰40′ - 12⁰4′	13⁰ - 30⁰	13⁰40′ - 14⁰
Jupiter	1.214	.0485	35⁰	28⁰45′ - 35⁰45′	31⁰30′ - 36⁰	32⁰ - 33⁰
Moon	1.263	.0549	40⁰	31⁰17′ - 40⁰40′	31⁰30′	31⁰42′ - 32⁰
Saturn	1.269	.0556	40⁰	31⁰33′ - 41⁰14′	40⁰30′ - 58⁰30′	48⁰-49⁰

Results

In Table 1, in Column (1), we have the values of y, calculated by equation (11) which give the observed values, column 2, of the eccentricity e of the Sun, Moon, Venus, Jupiter and Saturn. Equations (9) and (10b) then give the corresponding Values of x_0 and e, from which we can determine the maximum and minimum values of the variable radius r of the epicycle (multiplied by 360^0 in Column 4) Column 3 gives the first order approximation, taking only terms of the first order in e.

Columns 5 and 6 give the corresponding radii of *Āryabhaṭa* and the *Sūrya Siddhānta*. Of special interest are the following :

(a) The calculated radii of the Sun are $11^0 40'$-$12^0 4'$ compared to $13^0 40'$-14^0 of the *Sūrya Siddhānta.*

(b) The calculated radii of Jupiter are $28^0 45'$ - $35^0 45'$ compared to $31^0 30'$-36^0 of Āryabhaṭa.

(c) The minimum - maximum difference of $9^0 41'$ for Saturn is midway between the corresponding 18^0 of Āryabhaṭa and 1^0 of the *Sūrya Siddhānta.*

Planets of larger eccentricity are not considered because the corresponding ϵ are no longer small.

References

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