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EARLY HISTORY OF THE ASTROLABE IN INDIA

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This paper is a study of the early history of the astrolabe in Delhi Sultanate in India. The astrolabe was introduced into India at the time of Fīrūz Shāh Tughluq (reign AD 1351-88), and Mahendra Sūri wrote the first Sanskrit treatise on the astrolabe entitled *Yantra-rāja* (AD 1370). After that, Padmanābha wrote the *Yantra-rāja-adhikāra* (AD 1423), which is a monograph on the astrolabe and is a chapter of his *Yantra-kiraṇāvalī*, and Rāmacandra wrote the *Yantra-prakāśa* (AD 1428), where the astrolabe is described in detail and many other instruments are also described. These texts show that the astrolabe was quite popular in this period, and its principle was well understood by non-Muslim astronomers also.

Special attention is paid to the *Yantra-rāja-adhikāra* of Padmanābha, and its full text with English translation is presented here for the first time.

Key Words : Astrolabe, Fīrūz Shāh Tughluq, Mahendra Sūri, Padmanābha, Rāmacandra, *Yantra-kiraṇāvalī*, *Yantra-prakāśa*, *Yantra-rāja*, *Yantra-rāja-adhikāra*.

1. INTRODUCTION

The whole history of Indian astronomy may roughly be divided into the following periods.

(1) Indus valley civilization period.

(2) Vedic period (c1500 BC—c 500 BC).

(2.a) Ṛg-Vedic period (c1500 BC—c1000 BC)

(2.b) Later Vedic period (c1000 BC—c 500 BC)

(3) Vedāṅga astronomy period.

(3.a) Period of the formation of Vedāṅga astronomy (the 6/5/4th century BC(?)).

(3.b) Period of the continuous use of Vedāṅga astronomy (- the 2/3/4th century AD(?)).

(4) Period of the introduction of Greek astrology and astronomy

(4.a) Period of the introduction of Greek horoscopy

(the 2nd- 3rd century AD).

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(4.b) Period of the introduction of Greek mathematical astronomy

(c 4th century AD(?)).

(5) Classical Siddhānta period (the end of the 5th century AD-the 12th century).

(6) Coexistent period of Hindu astronomy and Islamic astronomy.

(the 13/14th century AD-the 18/19th century AD).

(7) Modern period (coexistent period of modern astronomy and traditional astronomy) (the 18/19th century AD......).

On the observational astronomy in the Vedic and Vedāṅga periods, I published a paper in 1993,[1] and on the astronomical instruments in the Classical Siddhānta period, I published a paper in 1994.[2] Now I am going to publish a series of papers on the astronomical instruments in the coexistent period of Hindu astronomy and Islamic astronomy, which is roughly equivalent to the Delhi Sultanate and Mughal periods.

In the coexistent period of Hindu astronomy and Islamic astronomy, several works on astronomical instruments were composed,[3] and the following Sanskrit works are known to be extant.

(I) Works exclusively devoted to astronomical instruments :

(I.i) Works on single instrument :

(I.i.a) Works on the astrolabe :

(1) *Yantra-rāja* (AD 1370) of Mahendra Sūri.
(Commentators : Malayendu Sūri (c.AD1382), Gopirāja (AD 1540), Yajñeśvara (AD 1842).

(2) *Yantra-rāja-adhikāra* (chapter I of the *Yantra-kiraṇāvalī)* (AD 1423) of Padmanābha.
(Commentator : Padmanābha.)

(3) *Yantra-rāja-vicāra-viṃśādhyāyī* of Nayanasukhopādhyāya (fl.AD 1730).

(4) *Yantra-rāja-racanā* of Savāī Jaya Siṃha (AD 1688-1743).

(5) *Yantra-prabhā* of Śrīnātha.

(6) *Yantra-rāja-kalpa* (AD 1782) of Mathurānātha Śukla.

(7) *Yantra-bhūṣaṇa* (anonymous).

(I.i.b) Works on the quadrant :

(8) *Yantra-cintāmaṇi* (sometime between AD 1150 and AD 1621) of Cakradhara.

(Commentators : Cakradhara, Rāma (AD 1625), Dādābhāī (fl, AD 1720).)

(9) *Turya-yantra-prakāśa* of Bhūdhara (fl.AD 1572).

(10) *Turīya-yantra* of Sevārāma.

(I.i.c) Works on the cylindrical sun-dial :

(11) *Kaśā-yantra* of Hema (the late 15th century AD).

(12) *Pratoda-yantra* of Ganeśa Daivajña (b.AD 1507).

(Commentator : Sakhārāma.)

(I.i.d) Works on other instruments :

(13) *Dhruva-bhramaṇa-yantra-adhikāra* (chapter II of the *Yantra-ratnāvalī*) of Padmanābha (fl.AD 1423).

(Commentator : Padmanābha.)

(14) *Diksādhana-yantra* of Padmanābha (fl.AD 1423)

(15) *Golānanda* (AD 1791) of Cintāmaṇi Dīkṣita.

(Commentator : Yajñeśvara (fl.c.AD1800).)

(16) *Gola-yantra-nirṇaya* of Yallambhaṭṭa

(17) *Koneri-yantra* (of Koneri).

(18) *Valaya-yantra* (anonymous)

(I.ii) Works on several instruments :

(19) *Yantra-prakāśa* (AD 1428) of Rāmacandra.

(20) *Yantra-śiromaṇi* (AD 1615) of Viśrāma.

(21) *Yantra-prakāra* (sometime between AD 1716 and 1724) of Savāī Jaya Siṃha (AD 1688-1743).

(22) *Yantra-sāra* (AD 1772) of Nandarāma Miśra.

(II) Later astronomical *siddhāntas* which have chapter of instruments :

(23) *Sundara-siddhānta* (AD 1503) of Jñānarāja.
(Commentator : Cintāmaṇi (fl.c.1530).)

(24) *Siddhānta-rāja* (AD 1639) of Nityānanda.

(25) *Siddhānta-sārvabhauma* (AD 1646) of Munīśvara (b.AD 1603).
(Commentator : Munīśvara.)

(26) *Siddhānta-samrāṭ* (in or after AD 1732) of Jagannātha.

Among the above works, only (1), (3), (4), (5), (8), (20), (21), and (26), and also the section of the cylindrical sun-dial in (25) have been published,[4] but others remain unpublished.

The above mentioned works are the only works which I have actually seen their publication, manuscripts, or photograph. So, there may be some other works on astronomical instruments in this period.

The earliest Sanskrit work exclusively devoted to an astronomical instrument is the *Yantra-rāja* (AD 1370) of Mahendra Sūri, and the next works are perhaps the works of Padmanābha, one of which is the *Yantra-rāja-adhikāra* (AD 1423). The Sanskrit word "yantra-rāja" means astrolabe, which was introduced into India from Islamic world in the 14th century. The introduction of the astrolabe is the earliest phase of the explicit influence of Islamic astronomy on Indian astronomy, and is the main theme of this paper. Especially, the *Yantra-rāja-adhikāra* of Padmanābha, which is a hitherto unpublished work, is discussed in detail, and its full text is published for the first time.

2. Principle And Early History Of The Astrolabe

a) The astrolabe and the stereographic projection

The astrolabe is a disklike instrument to observe the altitude of a heavenly body, and calculate time, *lagna*, etc. graphically.[5] (see Fig.1.)

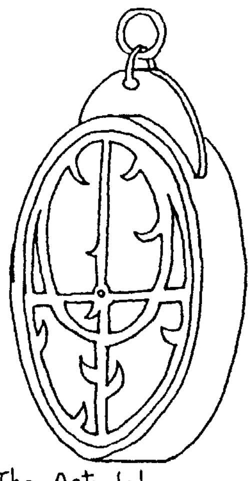
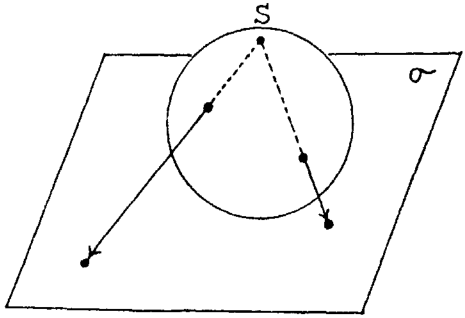


Fig. 1

On the front side of the astrolabe is a disc (or discs changeable for different latitudes), on which the Tropic of Cancer, the equator, the Tropic of Capricorn, the horizon, the parallels of altitude, the vertical circles, the hour circles etc., for the observer's latitude, are drawn by the stereographic projection. Above this disc is put a spider which has the ecliptic circle and some pointers of fixed stars. This spider can be rotated around the centre of the disc which is the projection of the celestial pole.

The back side of the astrolabe is usually divided into four quardrants, each of which is used for graphical calculation etc. On the back side is attached an alidade in order to observe the altitude.

The stereographic projection[6] is a projection of a sphere from one of its points S onto the plane σ which is parallel to the tangent plane of the point S (see. Fig. 2) .The stereographic projection has two important properties, viz :



The Stereographic Projection

Fig. 2

(1) The preservation of circles :

The circles lying on a sphere are projected onto the plane as circles or, if the circles on the sphere pass through the projection centre, as straight lines.

(2) Conformality :

The stereographic projection maps the angles between the curves lying on a sphere as equal-to-them angles between the curves projected onto the plane.

For the convenience of later discussions, let us briefly review the proof of these two properties.[7]

Proof of the first property :

Let AB be a diameter of a circle (see Fig. 3) and CD be a perpendicular dropped from an arbitrary point C on the circle onto the diameter AB. Then, from Euclid (vi.8), we have the following equation :

$$AD \cdot DB = CD^2, \qquad \text{--------} \ (1)$$

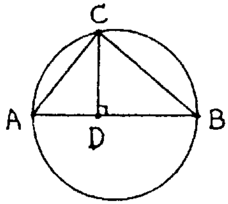


Fig. 3

and conversely, if this equation holds for any pont C on a curve for a segment AB, the curve is a circle.

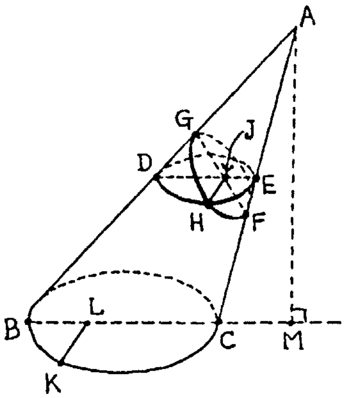


Fig. 4

Now (see Fig. 4), let us consider an oblique circular cone with a vertex A and the base whose diameter is BC, where the straight line BC produced meets the point M which is the foot of the perpendicular dropped from the vertex A onto the plane of the base. Let G be an arbitrary point on the straight line AB. Now, cut the cone by a plane (GHF) which is perpendicular to the plane ABC and passes through the point G in such a way that the angle AGF is equal to the angle ACB, (hence $A\hat{F}G = A\hat{B}C$ also), where F is an intersection of this plane (GHF) and the straight line AC. Then the curve GHF is a circle[8]. Let us prove this fact.

Let H be an arbitrary point on the curve GHF, and J be the foot of the perpendicular dropped from point H onto the straight line GF. Let a segment KL be the perpendicular dropped from an arbitrary point K on the circle BKC onto its diameter BC. Then :

$HJ // KL,$ --------- (2)

because both are perpendicular to the plane ABC.

Now, draw a segment DE, which passes through the point J, in such a way that :

$DE // BC.$ -------- (3)

Then, from (2) and (3), the plane DHE is parallel to the plane BKC. Hence the curve DHE is a circle. Applying the equation(1), we have :

$DJ \cdot JE = HJ^2.$ -------- (4)

Now, from (2),

$A\hat{D}E = A\hat{B}C$, and

$A\hat{E}D = A\hat{C}B$.

And also,

$A\hat{G}F = A\hat{C}B$, and

$A\hat{F}G = A\hat{B}C$.

Therefore,

$G\hat{D}J = E\hat{F}J$, and

$D\hat{G}J = F\hat{E}J$.

Since $D\hat{J}G = F\hat{J}E$, two triangles DJG and FJE are similar. Therefore :

$GJ / DJ = JE / JF,$

or,

$DJ \cdot JE = GJ \cdot JF.$ ------- (5)

From the equations (4) and (5), we have :

$GJ \cdot JF = HJ^2,$

for an arbitrary point H on the curve GHF. Therefore, considering the equation (1), the curve GHF is a circle.

Lastly, let us consider a stereographic projection of a circle (whose diameter is MN) from the point S onto the plane σ, where the plane σ tangents to the point S' which is

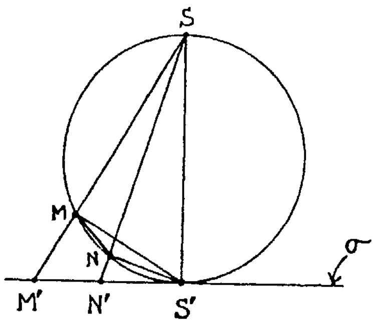


Fig. 5

diametrically opposite to S. (see Fig. 5) Let the segment MN be a diameter of the circle lying on the plane of the great circle which passes through the point S and the centre of the circle. And also, let M' and N' be the projections of the points M and N respectively onto the plane σ. Then, two right-angled triangles SMS' and $SS'M'$ with a common acute angle MSS' are similar, and :

$$SM / SS' = SS' / SM',$$

or,

$$SM \cdot SM' = (SS')^2. \qquad \text{-------} (6)$$

Similarly,

$$SN \cdot SN' = (SS')^2. \qquad \text{--------} (7)$$

From the equations (6) and (7), we have :

$$SM \cdot SM' = SN \cdot SN',$$

or

$$SM / SN = SN' / SM'.$$

So, two triangles SNM and $SM'N'$ with a common acute angle MSN are similar. Hence.

$$\left.\begin{array}{l} S\hat{M}N = S\hat{N}'M', \text{ and} \\ S\hat{N}M = S\hat{M}'N'. \end{array}\right\} \text{------} (8)$$

If we consider an oblique circular cone with the vertex S and the base whose diameter is MN, it is clear from (8) that its section by the plane σ is a circle whose diameter is $M'N'$.

Hence the first property of the stereographic projection has been proved.

Proof of the second property:

The angle between two curves is the angle between their tangents at their intersection. Let a point M be an intersection of two curves on the celestial sphere whose tangents at

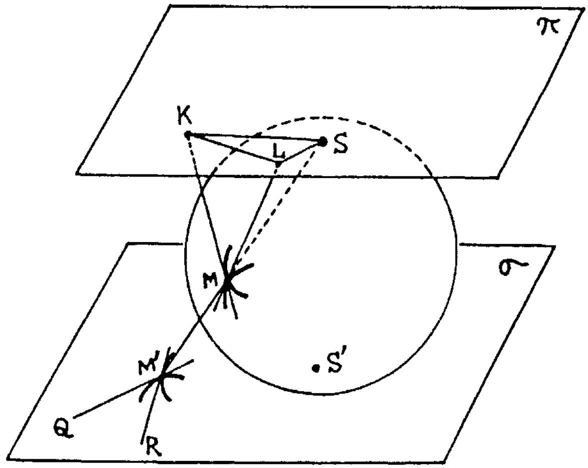


Fig. 6

the point M are MK and ML respectively. (see Fig. 6.) Let the plane σ tangent the point S' which is the opposite of the origin of the projection S. And let M' be the projection of the point M from the point S onto the Plane σ. And also, let $M'Q$ and $M'R$ be the tangents of the two projected curves at the point M'. Let us make a plane π which tangents the point S. And let K and L be the intersections of the plane π with the aforesaid straight lines MK and ML.

Since two tangents to a sphere from the same point are equal, we have:

$KS = KM$, and

$LS = LM$.

Hence, two triangles KSL and KML with the common side KL are mutually equal. Therefore:

$K\hat{S}L = K\hat{M}L.$ ------- (9)

Now, we can consider that the tangents M'Q and M'R are the projections of the tangents MK and ML. So, the straight lines M'Q and M'R are the intersections of the plane σ with the planes KSM and LSM respectively, and we have:

$SK \,//\, M'Q$, and

$SL \,//\, M'R$.

Therefore:

$K\hat{S}L = Q\hat{M}'R$. ------- (10)

From the equations (9) and (10), we have:

$K\hat{M}L = Q\hat{M}'R$.

Hence the second property of the stereographic projection has been proved.

b) The construction of the astrolabe

On the front side of the astrolabe is attached a disc or discs which can be changed according to the observer's latitude. On the disc is attached a spider which can be rotated

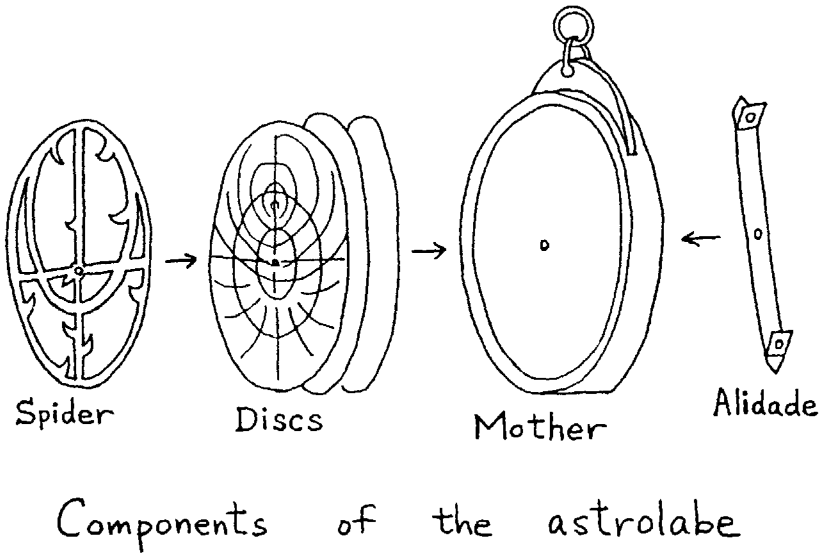


Fig. 7

around the centre of the disc. On the back side of the astrolabe is attached an alidade in order to observe the altitude of a heavenly body. (see Fig. 7.)

The celestial sphere is projected onto the disc from the celestial pole by the stereographic projection. The disc represents horizontal coordinates. The spider, which is a net-like object, has an ecliptic circle and some indicators of fixed stars. The spider represents ecliptic coordinates. Therefore, the rotation of the spider is equivalent to the diurnal rotation of the celestial sphere.

Usually the celestial sphere is projected from the south celestial pole onto the disc. In this case, the centre of the disc is the projection of the north celestial pole. Some authors, such as Padmanābha, described an astrolabe where the celestial sphere is projected from the north celestial pole. In this case, the centre of the disc is the projection of the south celestial pole.

Since the positions of the horizon, parallels of altitude etc. in the equatorial coordinates are different at different latitudes of the observer, one disc with the projection of the horizontal coordinates by the aforesaid method can be used at a particular latitude only. So several discs should be prepared if the astrolabe is to be used at different places.

As regards the practical method of the drawing of the disc, I shall explain under the section of Padmanābha, because he has nicely explained it in his own words.

c) Early history of the astrolabe in Greco-Arabic astronomy

According to O. Neugebauer,[9] the method of the stereographic projection was probably invented by Hipparchus (c 150 BC).

Vitruvius (d. after AD 27) described a clock called "horologium hibernum" or "horologium anaphoricum" which is an application of the stereographic projection in his *De Architectura* (IX. 8). A fragment of this type of clock was really found in Salzburg.[10]

Ptolemy (fl. AD 150) wrote the *Planisphaerium*, and explained the stereographic projection.[11] O. Neugebauer once wrote that the "horoscopic instrument" which has been mentioned in the *Planisphaerium* is the astrolabe in our sense.[12] (The term "astrolabe" used by Ptolemy is the armillary sphere.) However, Neugebauer later wrote that the "horoscopic instrument" might be "an anaphoric clock, but not of the Vitruvian type".[13]

According to O. Neugebauer, the first person who described the astrolabe seems to be Theon of Alexandria (c AD 375), although his work has only been indirectly preserved in later works.[14]

After that, Johannes Philoponus of Alexandria (the 6th century AD) wrote an extensive account on the astrolabe in Greek, and Severus Sebokht of Nisibis (middle of the 7th century AD) wrote a description of the astrolabe in Syriac.[15]

The earliest Arabic treatise on the astrolabe was written by Māshā'allāh before AD 815, but only its Latin version is extant.[16] The well known work *A Treatise on the*

Astrolabe[17] of Geoffrey Chaucer (1340?-1400) is based on the work of Māshā'allāh.

The oldest Arabic work on the astrolabe still extant in the original was written by 'Alī ibn 'Īsā (the 9th century AD).[18]

According to B.A. Rosenfeld and N.D. Sergeeva,[19] the earliest existing presentation of the theory of the stereographic projection with complete proofs of its first property was written by Aḥmad al-Farghānī (the 9th century) in his *Book on Constructing the Astrolabe.*

3. Fīrūz Shāh Tughluq and Mahendra Sūri

i) Fīrūz Shāh Tughluq and the Astrolabe

Fīrūz Shāh Tughluq (reign AD 1351-88) was the third Sultan of the Tughluq dynasty. He is one of the pioneers of the cultural exchange between Hindus and Muslims. A historian Niẓāmuddīn Aḥmad has written in his *Ṭabaqāt-i Akbarī* (c. AD 1593) as follows :

> "......he marched towards Nagarkot.....and there were one thousand and three hundred books of the ancient Brāhmans, in this temple, which is known as Jālāmukhi. And the Sultān sent for the learned men of that tribe; and ordered some of the books to be translated. Among these, 'Izzuddīn Khālid Khānī, who was among the poets of that age, translated into verse a book on natural philosophy and auguries and omens, and called it the Dalāel-i-Firoz Shāhī. This *Faqir* has read it. In truth it is a book containing various philsophical facts both of science and practice." (Translated by B.De)[20]

Like this, Fīrūz Shāh made some Sanskrit works translated into Persian. And also, according to the *Tārīkh-i-Fīrūz Shāhī* written by a contemporary historian Shams Sirāj 'Afif, Fīrūz Shāh constructed a copper clepsydra called *Ṭās-i g'haṛiyāla* in Fīrūzābād (=Delhi), and also had an astrolabe and banners on which astrolabe was painted.[21]

The most important contribution of Fīrūz Shāh in the field of astronomy will be the introduction of the astrolabe into India. An important source material on the astrolabe made by Fīrūz Shāh is an anonymous Persian work entitled *Sīrat-i Fīrūz Shāhī*, (AD 1370), whose manuscript exists in the Khuda Bakhsh Oriental Public Library, Patna.[22]

According to the *Sīrat-i Fīrūz Shāhī*,[23] some "complete astrolabes" *(usṭurlābāt-i tām)*[24] which were considered as Fīrūz Shāh's astrolabe *(Usṭurlāb-i Fīrūz Shāhī)* were put at the top of the roof of a minaret in Fīrūzābād (=Delhi). The reason why Fīrūz Shāh made the astrolabe is as follows. Sometime, some scholars told the Sultan about an ordinary astrolabe which was the "northern astrolabe" *(usṭurlāb-i shamālī)*. By that time, one astrolabe was brought to the Sultan, which was "northern and southern" *(shamālī wa janūbī)*. The Sultan studied and understood it, and ordered to make the "complete astrolabe" which is both "northern" and "southern".

We shall see below that Mahendra Sūri also described the "northern instrument" (an ordinary astrolabe projected from the south celestial pole), the "southern instrument" (an astrolabe projected from the north celestial pole), and the "mixed instrument" (an astrolabe which is a combination of the northern and southern instruments) in his *Yantra-rāja.* This *Yantra-rāja* is the first Sanskrit work on the astrolabe, which was written during the reign of Fīrūz Shāh.

ii) The *Yantra-rāja* of Mahendra Sūri

Mahendra Sūri,[25] a pupil of Madana Sūri of Bhṛgupura, wrote the *Yantra-rāja* in AD 1370. (This date is known from the epoch Śaka 1292 of the star-table in this work.)[26] The word "yantra-rāja" is the Sanskrit word which means "the king of instruments", and has been used to denote the Arabic-Persian word "usṭurlāb". S.B. Dikshit wrote that Mahendra Sūri "must have been a Jain", because "there is at the beginning of the work a salutation to 'sarvajña' (i.e. the Knower of all things)".[27]

There is a commentary (ca.AD 1377-82) written by Malayendu Sūri,[28] a pupil of Mahendra Sūri. There is also another commentary (AD 1540) written by Gopirāja,[29] and yet another commentary (AD 1842) written by Yajñeśvara.[30]

The *Yantra-rāja* with the commentary of Malayendu Sūri was edited by Sudhākara Dvivedin and L.Sarma[31] in AD 1882; and also by K.K. Raikva[32] in AD 1936. The commentaries of Gopirāja[33] and Yajñeśvara[34] have not been published.

According to Malayendu Sūri,[35] Mahendra Sūri wrote the work at the request of Pīroja, i.e. Fīrūz Shāh Tughluq.

The *Yantra-rāja* consists of 182 verses, and is divided into five chapters as follows.

(1) *Gaṇita-adhyāya,*

(2) *Yantra-ghaṭana-adhyāya,*

(3) *Yantra-racana-adhyāya,*

(4) *Yantra-śodhana-adhyāya,*

(5) *Yantra-vicāraṇa-adhyāya.*

As regards the source of his work, Mahendra-Sūri writes in his *Yantra-rāja* (I.3) as follows.[36]

क्लृप्तास्तथा बहुविधा यवनैःस्ववाण्या
यन्त्रागमा निजनिजप्रतिभाविशेषात्।
तान्वारिधीनिव विलोड्य मया सुधावत्
तत्सारभूतमखिलं प्रणिगद्यते ऽत्र॥३॥

"Treatises on instruments have been written in several ways from their own viewpoint in their own language by Yavanas. Having churned them like oceans, I give the nectar-like essence in its entirety."

Here, the Yavanas referred to by Mahendra Sūri must have been Muslims, and he must have consulted Arabic or Persian works.

The first chapter of the *Yantra-rāja* is devoted to the theory of the astrolabe. Here, Mahendra Sūri gave a sine table where the Radius is 3600'. D. Pingree[37] supposed that this sine table was originated in Islamic source. Mahendra Sūri also gave a table of the solar declination whose maximum is 1415′ or 23°35′. This value of obliquity of the ecliptic is the same as the value obtained by al-Battāni (AD 880),[38] and must have been introduced from Islamic source.[39] The value of the obliquity of the ecliptic used in previous Hindu astronomy had been 24°.

Mahendra Sūri also presented a fundamental formula and its numerical table for the drawing of the astrolabe. The *Yantra-rāja* (I. 8) reads:[40]

भुजांशकानां विपरीतमौर्वी मेषप्रमाणेन हता हता च॥
भुजज्यया तु क्रमया ततो यल्लब्धं फलं तच्छकलं दिनस्य॥८॥

"The versed-sine of the angle is multiplied by the measure of Aries, and divided by the sine of the angle; and the result thus obtained is the radius of the diurnal circle."

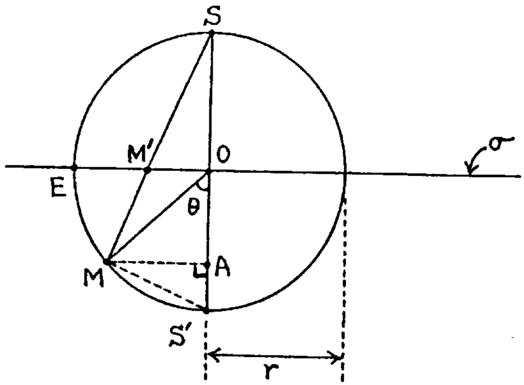


Fig. 8

This verse can be explained as follows. (see Fig 8) I have assumed that the plane of the astrolabe passes through the centre of the celestial sphere.) let O be the center of the celestial sphere $SEMS'$, and the sphere be projected onto the plane σ from the south celestial pole S. The plane σ represents the plane of the astrolabe. Let the angle $S'OM$ be the north polar distance (θ), which is "the angle" *(bhujāṁśa)* in the above text. And also, let the segment OE be the radius *(r)* of the equator on the astrolabe, which has been called "the measure of Aries" *(meṣa-pramāṇa)* in the above text. Then the radius *(OM')* of the projected diurnal circle on the astrolabe, whose north polar distance is θ on the celestial sphere, is expressed as follows.

$$OM' = \frac{\text{r x R}\cdot\text{vers}\theta}{\text{R}\cdot\sin\theta} \qquad \text{------ (1)}$$

Although Mahendra Sūri himself has not given the proof of this formula, I would like to give its proof here, because it is the fundamental formula for the construction of the front disc of the astrolabe. (see Fig.8 again.)

Let A be the foot of the perpendicular drawn from the point M onto the segment SS'. Let us consider two triangles $SS'M$ and $SM'O$. Then, the angle S is common, and $S\hat{M}S' = S\hat{O}M' = 90^{\circ}$. Therefore, two triangles $SS'M$ and $SM'O$ are similar. Similarly, two triangles $SS'M$ and $MS'A$ are similar. Accordingly, two triangles $SM'O$ and $MS'A$ are similar. Therefore:

$$OM' : OS = AS' : AM,$$

or,

$$OM' = \frac{OS \cdot AS'}{AM} \quad . \qquad \text{------ (2)}$$

Now, the segments OS, AS', and AM can be expressed as follows:

$OS = \text{r},$

$AS' = \text{r}\cdot\text{vers}\ \theta$

$AM = \text{r}\cdot\sin\ \theta$

Substituting them in the equation (2), we get:

$$OM' = \frac{\text{r x r}\cdot\text{vers}\ \theta}{\text{r}\cdot\sin\theta},$$

$$= \frac{\text{r x R}\cdot\text{vers}\ \theta}{\text{R}\cdot\sin\theta}.$$

Hence, the equation (1) has been proved.

Basically, all points on the celestial sphere can be projected on the plane of the astrolabe by applying this formula, because the hour circles are projected as straight lines passing through the centre of the disc.

As it is necessary to know the latitude of the observer in order to construct the disc, the commentator Malayendu Sūri has given the latitude of principal cities in India as well as outside of India, such as Samarkand.[41]

Mahendra Sūri has given the longitudes and latitudes of 32 fixed stars.[42] because they are necessary in order to make the spider. As regards the source of these data, the commentator Malayendu Sūri wrote as follows.[43]

शकमते नक्षत्रगोले नक्षत्राणां द्वाविंशत्यधिकं सहस्रमुक्तमस्ति। तन्मध्याद् ग्रन्थकारेण यावन्नक्षत्रगोलं सविस्तरं सम्यगवबुध्यात्र यन्त्रोपयोगीनि द्वात्रिंशन्नक्षत्राणि गृहीतानि।

> "In the doctrine of Śaka, 1022 stars on the celestial sphere have been recorded. Out of them, after knowing the celestial sphere completely and throughly, the author selected 32 stars which are to be employed in the astrolabe."

From this statement, S.B.Dikshit[44] supposed that "the doctrine of Śaka" referred to the star catalogue of Ptolemy, because 1022 stars have been described in the *Almagest* of Ptolemy. G.R.Kaye[45] has shown that the latitudes of Mahendra Sūri's catalogue are exactly the same as Ptolemy's catalogue in most of the cases, and the longitudes differ by exactly 18°53′ in most of cases.

Actually G.R.Kaye missed one phrase "*eṣu dvibāṇā vikalā 'pi syuḥ*"[46] ("there will also be 52 seconds"), and the actual difference between Mahendra Sūri's and Ptolemy's longitudes are 18°53′52″.

Mahendra Sūri's value of the rate of the precession is 720′ per 800 years,[47] or 54″ per year.

Mahendra Sūri also explained equatorial and ecliptic coordinates, the gnomon-shadow etc.

In the second chapter, general construction of the astrolabe is explained.

In the third chapter, graduations and drawing of the astrolabe is explained.

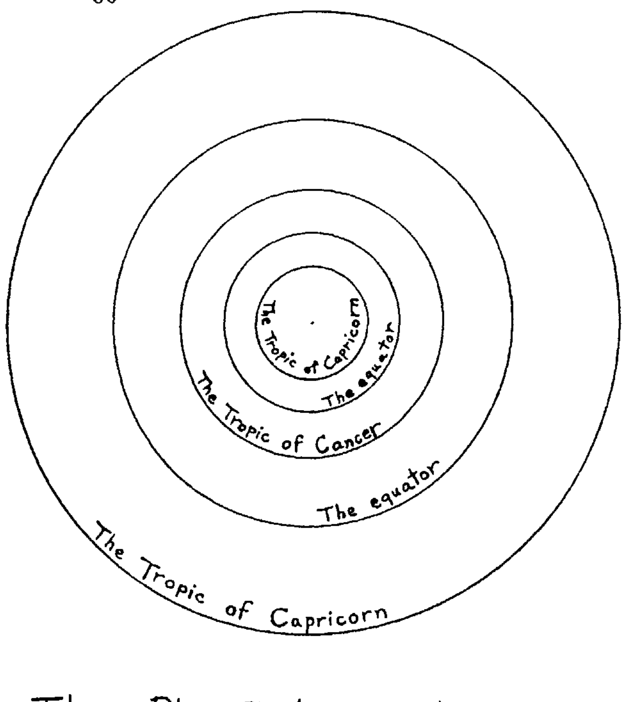
As we have discussed in the section of Fīrūz Shāh Tughluq, an ordinary astrolabe is projected from the south celestial pole, and this type was called "northern instrument" *(saumya-yantra)* by Mahendra Sūri. In this type, the rim of the disc is the Tropic of Capricorn, and its inside is the equator and the Tropic of Cancer. On the contrary, an astrolabe which is projected from the north celestial pole was called "southern instrument" *(yāmya-yantra)* by Mahendra Sūri. In this instrument, the rim of the disc is the Tropic of Cancer, and its inside is the equator etc.

Mahendra Sūri further described a mixed instrument called *Phaṇīndra-yantra* in his *Yantra-rāja* (III.20) as follows.[48]

तन्मिश्रभेदेषु फणीन्द्रयन्त्रे वृत्तत्रयं सौम्यवदेव कार्यम्।
कर्कर्यन्ततः खाग्निलवैर्विभज्य वृत्तद्वयं याम्यवदेव शेषम्॥२०॥

"Among the variations of the mixture of them (northern instrument and southern instrument), in the case of *Phaṇīndra-yantra*, three circles (the Tropic of Capricorn, the equator, and the Tropic of Cancer) should be drawn just like the northern instrument. From the last |circle| of Cancer, dividing by 30, remaining two circles (the equator and the Tropic of Cancer) should be drawn just like the southern instrument."

The meaning of this text is as follows. Firstly, three circles for the northern astrolabe are drawn with radii 30, $19\frac{38}{60}$, and $12\frac{51}{60}$ units, and then the circles for the southern astrolabe are drawn by assuming that the Tropic of Cancer (which has been already constructed with radius $12\frac{51}{60}$ units) contains 30 units. (see Fig. 9)



The Phanīndra-yantra
(reconstructed)

Fig. 9

In the fourth chapter, examination of the astrolabe, such as the ascertainment of straightness of a straight line, is explained.

The fifth chapter, that is the last chapter, is devoted to the methods of the use of the astrolabe.

From the above discussions, it will be clear that Mahendra Sūri was much influenced by Islamic astronomy, and probably obtained much information in connection with the astronomical activities of Fīrūz Shāh Tughluq.

4. YANTRA-RĀJA-ADHIKĀRA OF PADMANĀBHA

i) Introduction

Padmanābha[49] was a son of Nārmada, and the father of Dāmodara. Dāmodara[50] wrote two karaṇa works, the *Bhaṭa-tulya*[51] and the *Sūrya-tulya*[52], in Śaka 1339 (AD 1417). Nārmada was also an astronomer (see vs. 2 of the *Yantra-rāja-adhikāra*).

Padmanābha wrote a work (or works) on astronomical instruments entitled *Yantra-kiraṇāvalī* or *Yantra-ratnāvalī*. David Pingree says that these two titles are of the same work. Pingree writes as follows.

> "*Yantraratnāvalī* = *Yantrakiraṇāvalī* in two adhikāras: *Dikṣādhanayantra* and *Dhruvabhramayantra*. On the latter he wrote his own ṭīkā."[53]

Pingree also wrote as follows.

> "After Mahendra a number of other texts describing traditional Indian instruments were composed in Sanskrit, almost all of them in Gujarāt and Rājasthān, but astrolabe was generally neglected. The earliest of these is the *Yantraratnāvalī* composed in about 1400 by Padmanābha, ... The *Yantraratnāvalī* contains two chapters, each of which describes the construction and use of a single instrument. The first, on the *dikṣādhana*, is not available to me, but I have been able to consult a manuscript of the second, the *dhruvabhramaṇa*, on which Padmanābha himself wrote a commentary. This instrument appears to be an elaboration of Bhāskara's *phalaka*, and is not derived from the Islamic tradition." (David Pingree)[54]

Inspite of Pingree's statement, I shall show that the actual chapter I of the *Yantra-Kiraṇāvalī* is the *Yantra-rāja-adhikāra* (chapter of the astrolabe).

I have seen the unique manuscript of the *Dik-sādhana-yantra*[55] in the Oriental Institute, Baroda, but there was no evidence that it was a chapter of the *Yantra-kiraṇāvalī* or the *Yantra-ratnāvalī*.

The Tagore Library of Lucknow University has two manuscripts of the *Yantra-kiraṇāvalī*.[56] Both are manuscripts of the same work entitled *Yantra-rāja-adhikāra*, or the chapter of the astrolabe, and they clearly states that it is the first *(prathama)* chapter *(adhikāra)* of the *Yantra-kiraṇāvalī* in the colophon. Therefore, it is evident that chapter I of the *Yantra-kiraṇāvalī* is the *Yantra-rāja-adhikāra*, and not the *Dik-sādhana-adhikāra*.[57]

One manuscript (Lucknow 45888) of the *Yantra-rāja-adhikāra* has a commentary also, and the other manuscript (Lucknow 45892) has the text only. No other manuscript of chapter I has been found so far. Probabaly, the author of the commentary of chapter I is Padmanābha himself. Although the name of the commentator has not been given in the colophon of the manuscript (Lucknow 45888), this manuscript has a cancelled colophon in the middle,[58] which states that it is the autocommentary *(sva-vivṛtti)*. In this commentary, he wrote that six instruments would be described ("*atra ṣaḍ yantrāṇi vakṣyati*").[59] It is not known whether all of them were actually described or not.

In this *Yantra-rāja-adhikāra*, Padmanābha mentions the year 1345 Śaka (= 1423 AD), which may be the year of composition. (vs. 39 of Lucknow 45888 or vs. 40 of Lucknow 45892.)

David Pingree[60] found a quotation from the *Gola-yantra-adhyāya* of the *Yantra-kiraṇāvalī* in Padmanābha's commentary *Nārmadī* on the *Karaṇa-kutūhala* (II.17) of Bhāskara II, and some related passages in the same commentary. It may be a fragment of one chapter of the *Yantra-kiraṇāvalī*, which may have been lost.

Now I shall discuss the relationship between the *Yantra-kiraṇāvalī* and the *Yantra-ratnāvalī*. The two Lucknow manuscripts of the *Yantra-rāja-adhikāra* mention the name of the *Yantra-kiraṇāvalī* only. On the contrary, manuscripts of the *Dhruva-bhramaṇa-adhikāra*, which is undoubtedly the chapter II of the *Yantra-ratnāvalī*, usually mention the name of the *Yantra-ratnāvalī*[61] and not the *Yantra-kiranāvalī*. This contrast leads us to a doubt of the identification of these two titles. This problem will have to be investigated further by the examination of other manuscripts of the *Dhruva-bhramaṇa-adhikāra*.

Concludingly, we can say that Padmanābha wrote the following works on astronomical instruments.

(1) *Yantra-rāja-adhikāra* (chapter I of the *Yantra-kiraṇāvalī*) (AD 1423) with his own commentary.

(2) *Dhruva-bhramaṇa-adhikāra* (chapter II of the *Yantra-ratnāvalī*) with his own commentary.

(3) *Dik-sādhana-yantra.*

(4) *Gola-yantra-adhyāya* (fragment) (a chapter of the *Yantra-kiraṇāvalī* (?)).

Padmanābha also wrote the *Nārmadī*,[62] a commentary on the *Karaṇa-kutūhala* (AD 1183) of Bhāskara II.

ii) Text and translation of the *Yanta-rāja-adhikāra*

Now I shall present the full text and English translation of *theYantra-rāja-adhikāra*, chapter I of the *Yantra-kiraṇāvalī* of Padmanābha. I shall basically follow the reading of the manuscript Lucknow 45888 (henceforth Ms. "A"), and also comment the variants of Lucknow 45892 (henceforth Ms. "B"). Some trivial variants, such as *reṣā* for *rekhā*, are silently standardized. There is no figure in the manuscript. Figures in this paper are newly constructed by me.

Roughly, the *Yantra-rāja-adhikāra* can be divided into three parts, viz. the construction of the astrolabe, the star table, and use of the astrolabe.

a) The construction of the astrolabe

Firstly, Padmanābha salutes the God Gaṇeśa and his own father Nārmada.

बिभ्रत्पाशाङ्कुशौ यो वरदभयहर: स्वर्णकान्तिर्गजास्यो
विख्यातो नार्मदीयाभिध इति ऋषिभि: कीर्त्यते वक्रतुण्ड:।
भृग्वाद्यै: स्वीयरूपप्रणवरसलसद्बीजकोशप्रबोध:
स्वान्ते जागर्ति यस्यानिशमिव जनकं तं गणेशं नमामि ॥१॥[1]

Apparatus: (1) Ms. B omits this verse.

1: "I pay homage to my Gaṇeśa-like father as well as fatherlike Gaṇeśa, who bears noose and goad, grants wishes and removes fear, and bears beauty of gold, whose face is like elephant, who is known as Nārmadīya, and is called Vakratuṇḍa ('having a curved trunk') by the Sage Bhṛgu etc., and in whose heart nightlessly awakes the wisdom of the store of essentials such as his own form, the sacred syllable "om", and shining essence."

After this invocation, Padmanābha tells the purpose of this work as follows.

यन्त्रं गोलविदां मुदे प्रकुरुते श्रीपद्मनाभ: सुधी:[1]
सद्वृत्तं विमलं परं त्रिसमयज्ञानप्रदं सप्रभम्।
नि: शेषाज्ञतमो [ऽ] पहं तु परमज्ञानाम्बुजो[2]द्बोधनं
नत्वार्कं च तथा गुणैस्तदपरं श्रीनार्मदं सद्गुरुम् ॥२॥[3]

Apparatus: (1) Ms. B lacks visarga after *-dhī*. (2) A: *jau-* for *jo-* (3) This verse is vs. 1 in Ms. B.

2: "Paying homage to the sun which is well-rounded pure and complete, etc. as well as to the eminent teacher Śrī Nārmada who is superior by his virtue, well-learned Śrī Padmanābha makes an instrument for the sake of pleasure of astronomers, which is well-rounded, pure and complete, which gives the knowledge of three-fold time (past, present and future), which is possessing splendour, which removes the darkness of ignorance wholly and awakes the lotus of the best knowledge."

From this verse, it appears that Padmanābha learned astronomy from his own father Nārmada. According to S.B.Dikshit, Nārmada must have written a commentary on the *Sūrya-siddhānta* or some work based on it, but it is not at present appreciated.[63]

Now Padmanābha explains the construction of the main body of the astrolabe as follows.

यन्त्रं धातुजमिष्टकर्कटयुतं(1) नेमिश्लथाधारकं
कुर्यात्तिर्यगधोर्ध्वगामपि समां रेखां तु केन्द्रोर्ध्वतः।
ऊर्ध्वार्धे परिधौ तु भत्रयलवा ९० नंक्योभयत्रानभो(2)
[ऽ]ग्न्यंशोनेन्दुयुगेषु ४०/४० चार्कलवयुक्तत्त्वेषु(3) २५/५ सूत्रे न्यसेत्॥३॥(4)

Apparatus: (1) A: *vṛttaṃ* for *yutaṃ*. (2) B: *-bhā* for *-bho*. (3) B: *yut* for *yuk*. (4) This verse is vs. 2 in Ms. B.

तत्स्पृग्वृत्तयुगे द्युरात्रवलये(1) नक्रक्रियाद्योस्तयो-
र्मध्ये क्रान्तिलवैस्ततो(2) ऽपि परिधिं चाप्यूर्ध्वरेखां भजेत्।
केन्द्रादङ्गरसैस्तु ६६ नक्रवलयं(3) चैवान्त्यवृतं तु त-
त्कर्काख्यस्य तदादिगः(4) स्ववलये सूर्यो भ्रमत्येव हि॥४॥(5)

Apparatus: (1) B: *rātri* for *rātra*. (2) B: *tatas tu* for *tato 'pi*. (3) A: *natra* for *nakra*. (4) Ms. A lacks visarga after *ga*. (5) This verse is vs. 3 in Ms. B.

3: "A metallic circular instrument, which is made by a pair of compasses with a desired [radius] and has a loose supporter at its rim, should be made. Then a horizontal and a vertical straight line passing through the centre should be drawn. The upper half of the circumference, upto the top, should be graduated with 90 degrees on both sides. Two [horizontal] lines should be drawn at one-third-less forty one (=40°40′) and one-twelfth-added twenty five (=25°5′)."

4: "Two diurnal circles of [the first point of] Capricorn and Aries [should be drawn] in such a way that they tangent them (respective horizontal lienes). In their middle, and [from the equator] upto the rim, the vertical line should be graduated with the degrees of the obliquity of the ecliptic. Then from the centre upto the circle of Capricorn, that is the lowest circle, [the vertical line] should be graduated with 66 degrees. When the sun is at the first point of Capricorn, the sun indeed revolves its own circle."

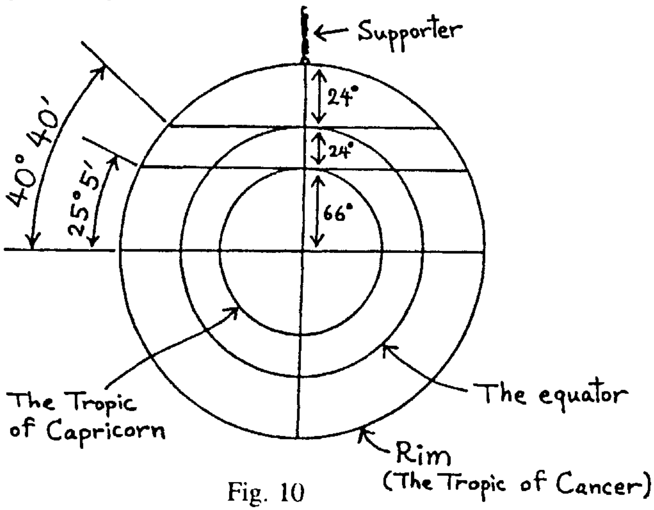


Fig. 10

Here, the construction of the main body of the astrolabe has been explained. The circumference of this circular instrument is assumed to be the Tropic of Cancer, and the equator (=the diurnal circle of the first point of Aries) and the Tropic of capricorn(the diurnal circle of the first point of Capricorn) are drawn inside. So, this is the so called "southern astrolabe", or an astrolabe projected from the north celestial pole. The "loose supporter" is probably attached at the top of the instrument in order to hang it.(see.Fig.10.)

Since Padmanābha takes 66^0 as the supplementary angle of the obliquity of the ecliptic, it is seen that he takes 24^0 as the obliquity of ecliptic, following Hindu traditional value. We shall discuss about two numerical data ($40^040'$ and $25^05'$) in vs. 3 after examining vss.6-7.

The width of a degree of the declination on the vertical line passing through the centre of the astrolabe is not uniform. Therefore, the method of the graduation in vs. 4 is only a rough method. In order to graduate exactly, one should calculate the radius of the diurnal circle for each degree of the declination.

After these verses, Padmanābha gave an approximate formula of R. sine as follows.

इष्टांशा नवतेर्विशोध्य दशहृत्तत्स्वीयघात: पृथक्
तेनोना: शशिकुञ्जरा ८१ स्तु गुणिता इष्टत्रिमौर्व्या हृता:।
भूनागै:(1) ८१ (2)कृतितुर्यभागसहितैर्जीवा भवेदीप्सिता
दो: कोटयो: क्रममौर्विकाविरहिता (3)त्रिज्यान्ययोरुत्क्रमा ॥५॥(4)

Aparatus: (1) Ms. B lacks visarga after-*gai.* (2) B: *kri* -for *kṛ-*. (3) A: *jya* for *jyā.* (4) This verse is vs. 4 in Ms. B.

5: "Desired degrees [of an arc of which R-sine is to be calculated] should be subtracted from ninety, divided by ten, multiplied by itself (i.e. squared), and the result [should be written at two places] separately. It should be subtracted from eighty one, and multiplied by desired Radius. It should be divided by the whole quantity eighty one plus a fourth of "the square" (the value which was writen separately). This is the R-sine (jīvā). The R-sine of an arc or of its complement should be subtracted from the Radius. This is the R-versed-sine *(utkrama-jyā)* of the other".

This approximate formula can be expressed as follows :

$$R\cdot\sin\theta = \frac{\{81-(\frac{90-\theta}{10})^2\}\times R}{81+\frac{1}{4}(\frac{90-\theta}{10})^2}\,. \qquad \text{-------(1)}$$

This is mathematically equivalent to the well known approximate formula which can be traced back to the *Mahābhāskarīya* (VII.17-19(i))[64] of Bhāskara I.

The relationship between R.sine and R.versed-sine is as follows.

R versed-sine θ = R-R.sine $(90^{0}-\theta)$,

R versed-sine $(90^{0}-\theta)$ = R-R.sine θ.

Padmanābha also gives an approximate formula of arcsine as follows.

इष्टज्यात्रिभजीवयोर्विवरत: पञ्चांशको [ऽ]सौ पृथग्
(1)भूनागैर्गुणित(2):पृथक्स्थरहितत्रिज्योधृ (द्धृ) तस्तत्पदम्।
द्विघ्नं(3) नन्दपरिच्युतं दशगुणं चापांशका[:]स्यु[:]स्फुटा(4)
यद्वा ज्याधनुषी तु खण्डकवशात्साध्ये ऽन्यशास्त्रादपि ॥५॥(5)

Apparatus : (1) *gaṇi*-for *guṇi*-. (2) Ms. A lacks visarga after-*ṇita*. (3) A: *vighnaṃ* for *dvighnaṁ*. (4) Ms. B adds visarga after *sphuṭā*. (5) This verse is unnumbered in Ms.A, and is found in folio 6b which has been wrongly incorporated in Ms.B. (See note verse 56) This is vs. 5 in Ms.B.

5':"One fifth of the difference between the R.sine and the Radius [should be written at two places] separately. It is multiplied by eighty one, and divided by the whole quantity the Radius minus the value written separately, [and obtain] its square root. It is multiplied by two, subtracted from nine, multiplied by ten. This is the exact degrees of arc. Or, one may obtain the R.sine and the arc by the finite differences [of the sine table] in some other literature".

This approximate formula of arcsine can be expressed as follows :

$$\theta = (9-2x\sqrt{\frac{81x\frac{1}{5}\,(\text{R-R}\cdot\sin\,\theta)}{\text{R-}\frac{1}{5}\,(\text{R-R}\cdot\sin\,\theta)}}\ )\ x\ 10 \qquad \text{-------}(2)$$

This formula can be derived from formula(1).

After giving these formulae, Padanābha wrote the method to calculate the radius of the equator and the Tropic of Capricorn on the astrolabe as follows.

क्रमोत्क्रमज्ये [ऽ]ङ्गरसांशकानां(1)६६
यन्त्रप्रमाणार्धगुणे(2) पृथक् स्थे।
त्रिज्यापमज्यायुतिभाजिते च(3)
द्युवृत्तखण्डे(4)ऽजमृगाख्ययो(5)[:] स्त: ॥६॥

Apparatus : (1) -*sa*-for-*śa*-. (2) B: -*ṇai* for -*ṇe*. (3) B: *vā* for *ca*. (4) Ms.A lacks *avagraha* before *ja*. (5) B:-*kṣa*-for-*khya*-.

6:"R.sine and R.versed-sine of 66^{0} should be multiplied by the radius of the instrument severally, and put [these results] separately. And [each] should be divided by the sum of the Radius and the R sine of the obliquity of the ecliptic. [The results] are the radii of the diurnal circles of [the first point of] Aries and Capricorn [respectively]".

This verse can be explained as follows. Let a be the radius of the equator, b the radius of the Tropic of Capricorn, and r the radius of the instrument (=the radius of the Tropic of Cancer). Then :

$$a = \frac{r \times R\cdot\sin 66^{0}}{R + R\cdot\sin 24^{0}}, \qquad \text{-------- (3)}$$

$$b = \frac{r \times R\cdot\text{vers } 66^{0}}{R + R\cdot\sin 24^{0}}. \qquad \text{-------- (4)}$$

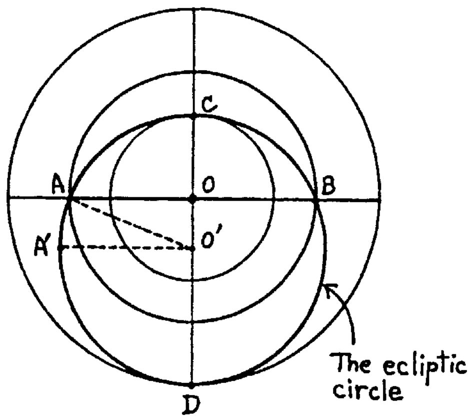


Fig. 11

Let us see the derivation of these formulae explained by Padmanābha in his commentary. (see Fig. 11).

Padmanābha explains by using the ecliptic circle. (Actually the ecliptic circle is introduced later in vs.21.) Let the ecliptic circle tangent the Tropic of Cancer at the lowest point D. Then the ecliptic circle tangents the Tropic of Capricorn at the highest point C. Now, the degrees corresponding to the arc AA', i.e. the angle $AO'A'$ which is equal to the angle OAO', is equal to the obliquity of the ecliptic. (Padmanābha uses this fact without giving its proof, but this is certainly true. Due to the conformality of the stereographic projection, the angle between the normals (AO and AO') of the equator and the ecliptic at the point A is equal to the obliquity of the ecliptic). Let us suppose that the radius ($O'A$) of the ecliptic circle is the Radius of the trigonometric function. Then, from the figure, the following equations can easily be obtained.

$$OA = O'A \cos(O\hat{A}O') = R\cdot\cos\varepsilon \ ,$$

$$OC = O'C - O'A \sin(O\hat{A}O') = R\text{-}R\cdot\sin\varepsilon \ ,$$

$$OD = O'D + O'A \sin(O\hat{A}O') = R + R\cdot\sin\varepsilon \ ,$$

where ε is the obliquity of the ecliptic. If we substitute *a, b,* and *r* for *OA, OC,* and *OD* respectively, we get :

$$a = R\cdot\cos\varepsilon = R\cdot\sin(90^{0}\text{-}\varepsilon) \ ,$$

$$b = R - R\cdot\sin\varepsilon \ ,$$

$$r = R + R\cdot\sin\varepsilon \ .$$

Therefore, if we express the values *a* and *b* in terms of the radius of the instrument *r*, we get:

$$a = r\,\frac{a}{r} = \frac{r \times R\cdot\sin(90^{0}\text{-}\ \varepsilon)}{R + R\cdot\sin\varepsilon},$$

$$b = r\,\frac{b}{r} = \frac{r\,(R\text{-}R\cdot\sin\varepsilon)}{R + R\cdot\sin\varepsilon} = \frac{r \times R\cdot\mathrm{vers}\,(90^{0}\text{-}\ \varepsilon)}{R + R\cdot\sin\varepsilon} \ .$$

Hence proved.

Let us proceed to the next verse.

द्युवृत्तखण्डे त्रिभजीवया घ्ने
यन्त्रप्रमाणार्धहृते ज्यके स्तः।
नक्राजयोः(1) कर्कटके त्रिभज्या
चापान्यथैषां द्युनिशोर्लवा [:] स्युः ॥७॥

Apparatus : (1) Ms. B lacks visarga after-*yo.*

7: "The radii of the diurnal circles of [the first point of] Capricorn and Aries should be multiplied by the Radius and divided by the radius of the instrument severally, [and the results] are the respective R.sine. For [the diurnal circle of the first point of] Cancer, [R.sine is] the Radius. Their corresponding arcs are the degrees of the respective diurnal circles".

This verse can be explained as follows. (see Fig. 12.) Let the radii of the Tropic of Cancer, the equator, and the Tropic of Capricorn be *r*, *a* and *b* respectively, and the "degrees of the diurnal circle" of the Tropic of Cancer, the equator, and the Tropic of

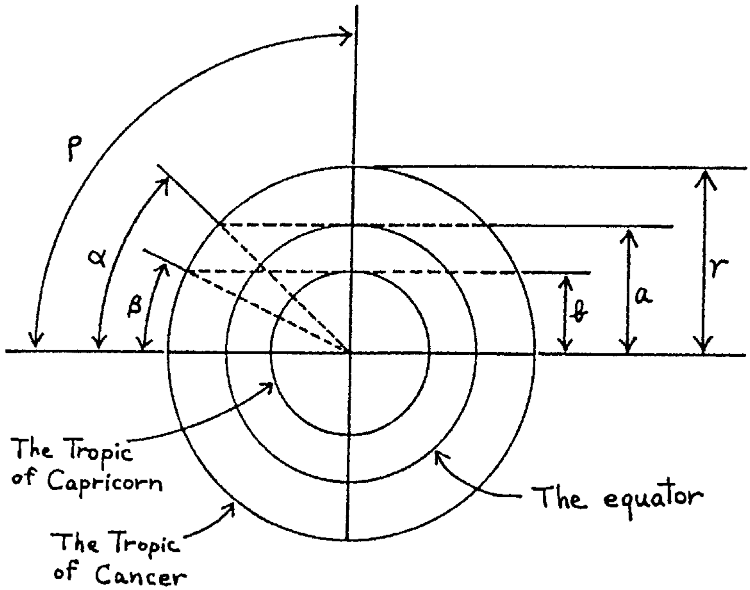


Fig. 12

Capricorn be ρ, α, and β respectively. And also, let the obliquity of the ecliptic be ε. Then, the above verse can be expressed as follows. (In order to avoid confusions, I shall use modern definition of sine here instead of Hindu definition of R.sine.)

$$\rho = \arcsin \frac{r}{r} \quad (=90^{\circ}),$$

$$\alpha = \arcsin \frac{a}{r} \ ,$$

$$\beta = \arcsin \frac{b}{r} \ .$$

These relations will be clear from the figure.

Now, substituting *a* and *b* of equations (3) and (4) in the above equations, we get the following equations :

$$\alpha = \arcsin \left(\frac{\cos \varepsilon}{1 + \sin \varepsilon} \right) , \qquad \text{---------} (5)$$

$$\beta = \arcsin \left(\frac{1 - \sin \varepsilon}{1 + \sin \varepsilon} \right) . \qquad \text{---------} (6)$$

The numerical values of α and β have been given as $40^{\circ}40'$, and $25^{\circ}5'$ respectively in vs.3. However, if we assume that $\varepsilon = 24^{\circ}$, the value used by Padmanābha, the equations (5) and (6) give[65] the values $\alpha \doteq 40^{\circ}30'$ and $\beta \doteq 24^{\circ}57'$. Let us discuss this difference.

Let us estimate the value ε which gives $\alpha = 40^{0}40'$ and $\beta = 25^{0}5'$ by the equations (5) and (6). Then from the equation (5), we get $\varepsilon \doteqdot 23^{0}49'8''$, and from the equation (6), we get $\varepsilon \doteqdot 23^{0}51'48''$.

It is quite interesting that these values are close to Ptolemy's value $\varepsilon = 23^{0}51'20''$ which has been given in his Almagest (I.12). Therefore, I suppose that Padmanābha has borrowed the values $\alpha = 40^{0}40'$ and $\beta = 25^{0}5'$, from certain Islamic source.[66] Padmanābha's source must have been different from Mahendra Sūri's source, because Mahendra Sūri used $\varepsilon = 23^{0}35'$ which is the same as al-Battānī value.

Now, let us see the construction of the horizon and the six o'clock line explained by Padmanābha.

यन्त्रव्यासदलाहतो ऽक्षगुणहृन्मेषांशकानां(1) गुणः
(2)केन्द्राक्षाजकुजाङ्कसङ्गिवलयं लब्धाङ्गुलैः (3)संलिखेत्।
आनेमिं क्षितिजं हि(4) (5)तत्स्वविषये रेखा तथा तिर्यगा
व्यक्षे सा क्षितिजाह्वयान्यविषये तून्मण्डलाख्योच्यते॥८॥

Apparatus:(1) B: *mekhadyuvṛttaḥ śravaḥ* for *meṣāṃśakānāṃ guṇaḥ*. (2) B:*-tthā* for-*kṣā-*. (3) Ms. B. lacks *saṃli*. (4) B:*tu* for *hi*. (5) *sve* for *sva*.

> 8: "The R.sine of the degrees of [the diurnal circle of the first point of] Aries should be multiplied by the radius of the instrument, and divided by the R.sine of the [observer's] latitude. A circle, which passes through the point, whose distance from the centre is equivalent to the latitude, and the intersections of [the diurnal circle of the first point of] Aries and the horizon, should be drawn upto the rim with [the radius of] the obtained *aṅgulas*. [This is] the horizon at one's own place. The horizontal line [passing through the centre] is the horizon at the equator, and the six o'clock line at other places."

Here, "the R.sine of the degrees of Aries" is the R.sine of the angle α which has been given by vs. 7. Let A be the R.sin α, r the radius of the instrument, and ϕ the observer's latitude. Then the radius of the horizon h can be expressed as follows according to the above verse.

$$h = \frac{\mathrm{A \times r}}{\mathrm{R \cdot sin}\phi} \quad \text{--------- (7)}$$

In the commentary, Padmanābha takes R=120, and gives[67] A=78. And also, he uses $\phi = 20^{0}$ in the example of numerical calculations.

Let us see the derivation of the equation (7) explained by Padmanābha in his commentary. Padmanābha refers to the *Līlāvatī* (204) of Bhāskara II in order to derive the equation. The *Līlāvatī* (204)[68] may be rendered into English as follows.

> The diameter, decreased by the square root of the product of the sum and the difference of the chord and the diameter, and halved, will be the arrow *(śara)*.

The diameter is increased by the arrow, and multiplied by the arrow. The double of its square root is the chord (*jīvā*).

The square of half of the chord, divided by the arrow, and increased by the arrow, is the diameter (*vyāsa*). Thus they say regarding the circle.

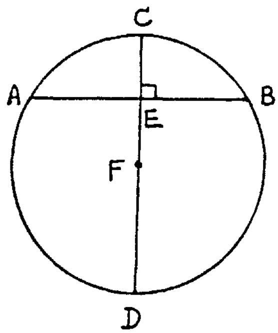


Fig. 13

These rules can be expressed as follows. (see Fig. 13.) Let *s* be the arrow (CE), *g* the chord (AB), and *d* the diameter (CD). Then :

$$\left.\begin{aligned} s &= \frac{1}{2}\left\{d-\sqrt{(d+g)(d-g)}\right\}, \\ g &= 2\sqrt{s(d-s)}, \\ d &= \left(\frac{g}{2}\right)^2 \times \frac{1}{s} + s. \end{aligned}\right\} \text{-------- (8)}$$

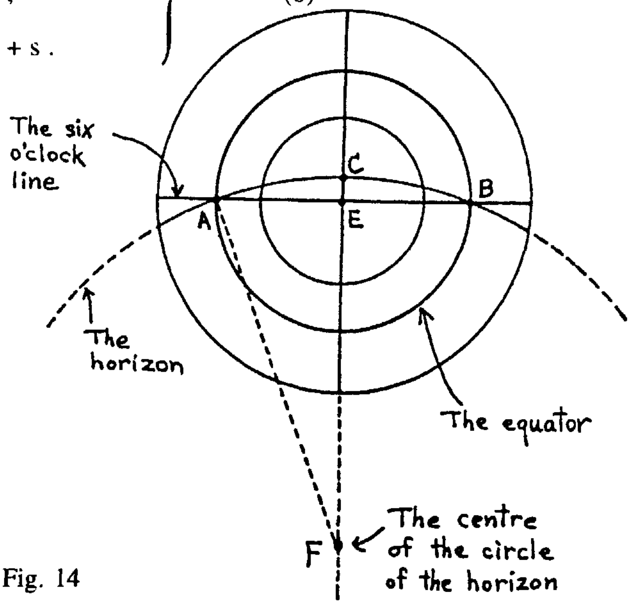


Fig. 14

Now, let us proceed to the derivation of the equation (7). (see Fig. 14.) The points *A* and *B* are the intersections of the equator and the horizon, i.e. the east and west cardinal points. The point *C* is the south cardinal point. Since the angular distance between the south celestial pole and the south cardinal point is equal to the observer's latitude, the distance between the points *E* and *C* corresponds to the observer's latitude.

If we suppose that the radius of the horizon is the Radius of the trigonometric function, the segment *AE* is R sin ϕ, and the segment *CE* is (R-R cosϕ). Therefore :

$$\text{R} \sin \phi : (\text{R-R} \cos \phi) = \text{AE} : \text{CE} . \qquad \text{--------} \ (9)$$

(This fact will be clear from the conformality of the stereographic projection. The angle between the normals (*AF* and *AE*) of the horizon and the equator at the point *A* is equal to (90°-ϕ). Therefore, the angle *EFA* is ϕ. It should also be remembered that the value of the Radius need not be specified in the proportion (9).)

Now, if we substitute *a* for *AE* and *s* for *CE* in the proportion (9), we get :

$$s = \frac{a\ (\text{R-R}\cdot\cos \phi)}{\text{R}\cdot\sin \phi} ,$$

and, substituting this *s* and *2a* for *g* in the third equation of (8)

we get :

$$d = \frac{2a \times \text{R}}{\text{R}\cdot\sin \phi} ,$$

or,

$$\frac{d}{2} = \frac{a\cdot\text{R}}{\text{R}\cdot\sin \phi} . \qquad \text{--------} \ (10)$$

Now, the relation between *A* (= R·sin α) and *a* is :

$$a = \frac{r}{\text{R}} \text{A} . \qquad \text{--------} \ (11)$$

Substituting this for *a* in the equation (10), we get :

$$\frac{d}{2} = \frac{\text{A}\cdot r}{\text{R}\cdot\sin \phi} .$$

Hence proved.

Since the six o'clock line passes through the celestial poles and the east and west cardinal points, it will be clear that it is projected as a horizontal straight line.

Now let us see the construction of the prime vertical and *koṇa* circles explained by Padmanābha.

निघ्नो यन्त्रदलेन लम्बगुणहृन्मेषुद्युरात्रश्रव-
स्तत्सूत्रेण लिखेत्समाख्यवलयं मेषोदयास्तावधि।
तन्मध्यस्थितमत्स्यसूत्रपरिधिस्पृक्चिह्नकाभ्यां[1] पृथक्
तद्व्यासस्वदलघ्नमूलजनिते वृत्ते तु कोणाभिधे॥९॥[2]

Apparatus : (1) B:*sphak* for *spṛk.* (2) This verse is vs.16 in Ms. B.

9 : "The normal radius of the diurnal circle of [the first point of] Aries is multiplied by the radius of the instrument, and divided by the R.sine of the co-latitude. By a string of this length, one should draw the prime vertical in such a way that it passes through the rising and setting points of the [first point of] Aries (i.e. the east and west cardinal points). From the two ends of the perpendicular bisector [of the vertical radius of the prime vertical], which passes through its (prime vertical's) centre, [one should draw] two circles called *koṇa* severally, whose radius is the square root of the product of [the prime vertical's] diameter and its half."

Here, the "normal radius of the diurnal circle of Aries" means R.sin α which already appeared in vs. 8.

The radius p of the prime vertical can be expressed as follows according to the text :

$$p = \frac{A \cdot r}{R \cdot \sin (90^\circ - \phi)} , \qquad \text{-------- (12)}$$

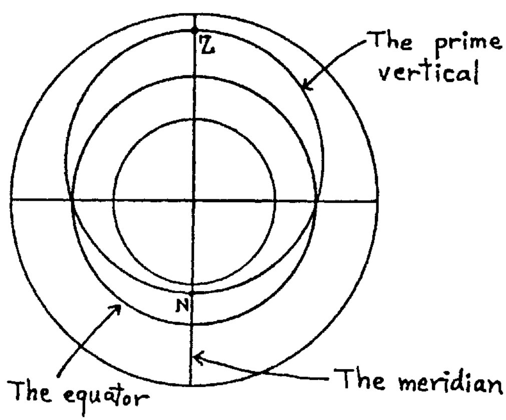


Fig. 15

where A is R·sin α, r the radius of the instrument, and ϕ the observer's latitude. This equation can easily be understood on the analogy of vs.8. (For the construction, see Fig. 15.)

The *koṇa* circles are two great circles on the celestial sphere which pass through the zenith and nadir and the south-east and north-west points or the south-west and north-east points.

According to the text, the radius k of the *koṇa* circle is :

$$k = \sqrt{2p \times p}\ , \qquad \text{--------} \ (13)$$

or,

$k = \sqrt{2}p$. (For the construction, see Fig. 16)

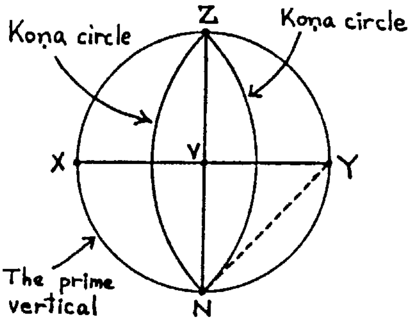


Fig. 16

The equation (13) can easily be understood by considering the triangle *VNY* in Fig. 16, where YN = $\sqrt{2}$ VY.

Now, let us see the cónstruction of the parallels of altitude. Firstly, Padmanābha gives a rough method as follows.

(1) तुल्यान् खाङ्कलवान् समाख्यवलये (ऽ)जक्ष्माजतश्चाङ्कये-
दाव्योमोभयतस्तदूर्ध्वगमिति स्युस्तुल्यचिह्नानि च।
तत्तुल्यत्रयवृत्तमत्स्ययुगलप्रान्तास्यरेखायुते-
स्तेनैवापरिधिं तु (2) वृत्तसकलान्यूर्ध्वे तु साङ्कानि च॥१०॥(3)

Apparatus : (1) B:*rāśisponnatabhāgakān svavalaye svakṣmājataś* for *tulyān---jakṣmājataś.* (2) B: *-pha-*for-*ka-*. (3) This verse is vs.9 in Ms.B.

> 10: "One should graduate the prime vertical circle with equal divisions of 90 degrees [of altitude] from the point of intersection of the horizon and the diurnal circle of Aries on both sides upto the zenith. The vertical line (meridian) should also be graduated with equal divisions. [Taking three points corresponding to the same altitude as obtained above] three circles of an equal radius are drawn and a couple of fish-figures are formed, then the extremities of each figure are joined (i.e. perpendicular bisectors are drawn), and with the point of their intersection [as the centre], all circles [of altitude], marked with the values [of the altitude], are drawn upwards upto the circumference."

This is a rough method to draw the parallels of altitude. For the exact construction of the parallels of altitude, Padmanābha gives the following formula.

यद्वाजद्युनिशाख्यवृत्तगुणिते(1) त्रिज्येष्टकोटिज्यके
भक्ते (ऽ)भीष्टभुजज्यया(2) पृथगिति प्राप्ते फले ये तयोः।
त्रिज्योत्थं(3) मकरादिगे विरहितं युक्तं च(4) कर्कादिगे
कोटदयुत्थेन च(5) केद्र(न्द्र) कर्णदलयोर्युत्यन्तरं स्यात्स्फुटम्(6) ॥१०॥(7)

Apparatus : (1) Ms. B lacks *jye.* (2) A:*pra-*for *pr.*-(3) B:*tvaṁ* for *-tthaṁ* (4) B: tu for *ca.* (5) Ms. A lacks *ca* after-*tthena.* (6) Ms.B lacks *t* after *syā.* (7) In Ms. A, both this verse and its previous verse are numbered as vs. 10. Let us tentatively call this verse vs.10′. In Ms.B, this verse is vs.10.

> 10′ : "Alternatively, the Radius and the R.cosine of the desired angle should be multiplied by [the radius of] the diurnal circle of [the first point of] Aries, and divided by the R.sine of the very desired angle severally. Out of these two results, the value originated in the R.cosine is subtracted from the value originated in the Radius in the case of the first point of Capricorn, and they are added in the case of the first point of Cancer. [These two results are] the exact sum and difference of the position of the centre and the radius."

This verse can be explained as follows. Let *a* be the radius of the equator on the astrolabe, θ the "desired angle", *c* the distance of the centre of the proposed circle from the centre of the astrolabe, and *q* the radius of the proposed circle. Then :

$$\left.\begin{aligned} q + c &= \frac{a\cdot R}{R\cdot\sin\theta} + \frac{a\cdot R\cdot\cos\theta}{R\cdot\sin\theta}, \\ q - c &= \frac{a\cdot R}{R\cdot\sin\theta} - \frac{a\cdot R\cdot\cos\theta}{R\cdot\sin\theta}. \end{aligned}\right\} \text{--------- (14)}$$

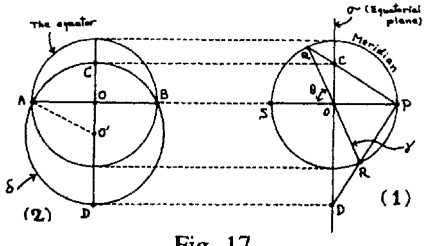


Fig. 17

Although Padmanābha did not explain the derivation of these equations in his commentary, it is easily understood on the analogy of the previous discussions. (see. Fig. 17. In the figure, (1) is the celestial sphere cut by the meridian, and it is projected to the plane σ, and (2) is the projected figure on the plane σ, or on the astrolabe.)

Let the great circle γ, which intersects the six o'clock circle with the angle θ, be projected onto the plane $\acute{o}$ as the circle δ. Then the radius q of the circle δ can be easily obtained on the analogy of vs.8 as :

$$q = \frac{a \cdot R}{R \cdot \sin\,\theta} \quad . \qquad \text{----------} \; (15)$$

Since $O\hat{A}O' = (90^{0}\text{-}\theta)$ in the figure by the conformality,

$OO' : OA = R \cos\,\theta : R \sin\,\theta$.

Therefore, the distance c of the centre $0'$ of the circle δ from the centre O of the projected equator on the plane σ is :

$$c = \frac{a \cdot R \cdot \cos\,\theta}{R \cdot \sin\,\theta} \quad . \qquad \text{----------} \; (16)$$

From the equations (15) and (16), the equations (14) are obtained.

The "first point of Capricorn" in the text refers to the point Q in the figure, or the southernmost point of the circle γ, and the "first point of Cancer" in the text refers to the point R, or the northernmost point of the circle γ. Probably, they were called so on the analogy of the winter solstice and the summer solstice of the ecliptic.

The reason why Padmanābha has given the equations(14) instead of the equations (15) and (16) will be understood by the consideration of the subsequent verse, where the parallels of altitude, which are the small circles on the celestial sphere, are discussed as follows.

(1)अक्षांशोनितभार्धसंभवफलं युत्या द्वयं तद्भवे-
दक्षोत्थं विवरं च संक्रमणतस्ताभ्यां तु (2)केन्द्रश्रुती।
भूजाख्यस्य ततो (ऽ)भिवाञ्छितलवान् संयोज्य तत्रोभयोः
शेषे चेत्षडभाधिके(3) (4)भगणतस्त्यक्त्वा तु साध्ये (ऽ)थवा॥११॥

Apparatus : (1) B:-*chāṃ*-for -*kṣāṃ*-. (2) B:*keṃdra* for *kendra*. (3) Gramatically, *ṣaḍabhā* will have to be emended as *ṣaḍ-bhā*, but this emendation disturbs metric. (4) What I have given here *(bhagaṇatas------'thavā)* is the reading of Ms.B. Ms.A reads *"bhujaguṇaṃ koṭi-prakalpyānayet"*, which is not intelligible to me in this context.

11 : "The [observer's] latitude is subtracted from 180°, and the sum of the two results (the values originated in the R.cosine and in the Radius as directed in vs.10′) [should be obtained]. And the difference of the two results (the values as directed in vs.10′) which are originated in the [observer's] latitude [should also be obtained]. From them, [the position of] the centre and the radius [should be calculated] by the method of concurrence *(saṃkramaṇa)*. As regards the parallels of altitude, the desired angle (altitude) should be added (to 180° minus latitude, and the latitude) severally. If the angle exceeds 180°, one may calculate after subtracting it from 360°."

In this verse, we should remember that the *bhuja* is used as the variable for Hindu trigonometric function, and the definition of the *bhuja* is as follows.

$$\left.\begin{aligned} bhuja &= \theta && (0 \leqq \theta \leqq 90^0) \\ &= 180^0 - \theta && (90^0 \leqq \theta \leqq 180^0) \\ &= \theta - 180^0 && (180^0 \leqq \theta \leqq 270^0) \\ &= 360^0 - \theta && (270^0 \leqq \theta \leqq 360^0) \end{aligned}\right\} \text{---------} (17)$$

And also, the complementary angle of the *bhuja* is the *koṭi*.

In other words, Hindu trigonometric functions can be expressed as follows for any value of θ:

jyā = $| R \cdot \sin \theta |$

koṭi-jyā = $| R \cdot \cos \theta | = | R \cdot \sin (90^0 - \theta) |$

The rule in vs.11 can be expressed as follows. Let ϕ be the observer's latitude, a the radius of the equator on the astrolabe, c the distance of the centre of proposed circle from the centre of the equator, and q the radius of the proposed circle. Then, as regards the horizon, the rule is :

$$\left.\begin{aligned} q + c &= \frac{a \cdot R}{| R \cdot \sin (180^0 - \phi) |} + \frac{a \, | R \cdot \cos (180^0 - \phi) |}{| R \cdot \sin (180^0 - \phi) |} \\ q - c &= \frac{a \cdot R}{| R \cdot \sin \phi |} - \frac{a \, | R \cdot \cos \phi |}{| R \cdot \sin \phi |} \end{aligned}\right\} \text{--------} (18)$$

The equations(18) are application of the rule of vs. 10′. We should note that the *bhuja* of $(180^0 - \phi)$ is the same as the *bhuja* of ϕ. The reason why $(180^0 - \phi)$ is used in the first equation of (18) is understood by considering the calculation for the parallels of altitude.

Let χ be the altitude. Then the rule for the parallel of altitude in the text is :

$$\left.\begin{aligned} q+c &= \frac{a\cdot R}{\mid R\cdot \sin(180^\circ - \phi + \chi)\mid} + \frac{a \mid R\cdot\cos(180^\circ - \phi + \chi)\mid}{\mid R\cdot\sin(180^\circ - \phi + \chi)\mid} \\ \mid q-c\mid &= \frac{a\cdot R}{\mid R\cdot\sin(\phi+\chi)\mid} - \frac{a\mid R\cdot\cos(\phi+\chi)\mid}{\mid R\cdot\sin(\phi+\chi)\mid} \end{aligned}\right\} \quad \text{--------} \ (19)$$

Actually, the second equation of (19) can be used only if ($\phi + \chi$) is less than 90°. If ($\phi + \chi$) is greater than 90°, the second equation should be replaced by the following equation.

$$\mid q-c\mid = \frac{a\cdot R}{\mid R\cdot\sin(\phi+\chi)\mid} + \frac{a\mid R\cdot\cos(\phi+\chi)\mid}{\mid R\cdot\sin(\phi+\chi)\mid} \quad \text{--------} \ (20)$$

This is important, although Padmanābha does not mention this fact in the text.

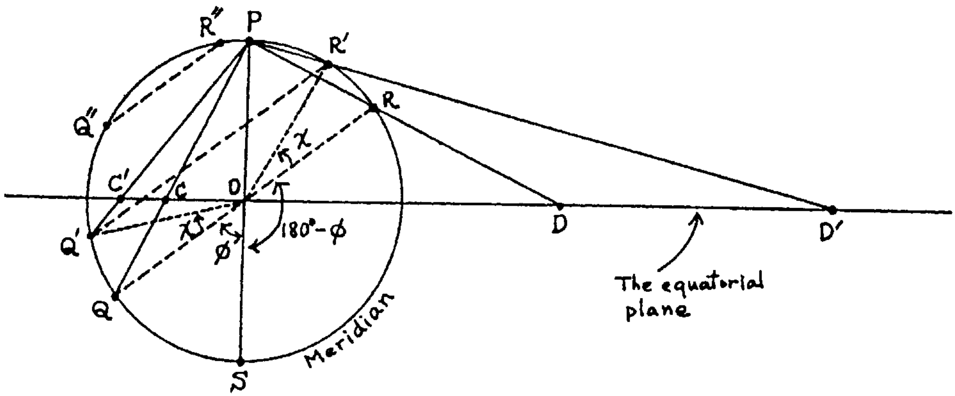


Fig. 18

I shall briefly explain the equations (19) geometrically. (see Fig. 18, which shows the celestial sphere cut by the meridian. Let the celestial sphere be projected onto the equatorial plane.)

In the figure, the segment *QOR* is the diameter of the horizon on the celestial sphere, and it intersects the axis *POS* with the angle ϕ (the observer's latitude). And also, $Q'R'$ is the diameter of the parallel of altitude, where the altitude is χ. Its southernmost point Q' is projected as C', and the northernmost point R' is projected as D'. From the figure, it is clear that the segment OC' can be obtained by substituting ($\phi + \chi$) for ϕ in the second equation of (18), and OD' can be obtained by substituting ($180°-\phi+\chi$) for ($180°-\phi$) in the first equation of (18). Thus, the equations (19) are derived. For the parallels of altitude like $Q''R''$, the equation (20) should be used instead of the second equation of (19).

From the above discussions, the reason why $(q+c)$ and $|q-c|$ have to be used instead of q and c themselves will be clear. The formulae for q and c themselves cannot be utilized for small circles. And also the reason why $(180^0-\phi)$ is used for the first equation of (18) is that by this device the altitude χ can mechanically be added in both cases of equations (19) as $(180^0-\phi+\chi)$ and $(\phi+\chi)$ respectively.

Regarding the "concurrence" *(saṃkramaṇa)*, Padmanābha refers to the *Līlāvatī* (56) of Bhāskara II in the commentary. The *Līlāvatī* (56)[69] may be rendered into English as follows.

> "The sum is decreased and increased [separately] by the difference, and halved. Those [obtained] are said to be original numbers. This is called *saṃkramaṇa*."

This rule can be expressed as follows. Let x and y be the original numbers, and a and b be their sum and difference respectively. That is :

$$a = x + y,$$

$$b = x - y.$$

Then, x and y can be obtained by the following equations.

$$\left.\begin{aligned} x &= \frac{a+b}{2} \\ y &= \frac{a-b}{2} \end{aligned}\right\} \text{-------- (21)}$$

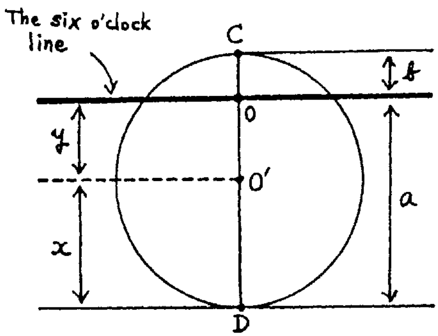


Fig. 19

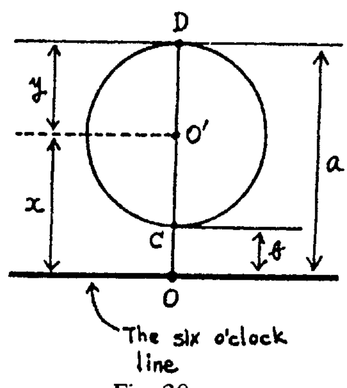


Fig. 20

In vs.11, the values of q and c can be obtained by substituting $(q + c)$ for a, and $|q-c|$ for b in the equations (21). If $(180^0- \phi+\chi)$ is less than 180^0, x and y in the equations (21) are q (the radius) and c (the position of the centre) respectively. (see Fig. 19.) If $(180^0 - \phi + \chi)$ is more than 180^0, x and y are c and q respectively. (see Fig. 20).

Padmanābha gives another method to calculate the (q+c) and | q-c | of the horizon and the parallels of altitude as follows.

दो:कोटिजीवे प्रविधाय ताभ्यां
स्वर्णं कुलीरेण(1) (2)गते त्रिमौर्व्याम् ।(?)(3)
(4)कोटिज्यमां (-कां?) (5)मेषदलेन हन्याद्
दोर्ज्योद्धरेद्वान्तरसंयुती स्त: ॥१२॥

Apparatus (1) A:*-raiṇa* for *-reṇa*. (2) B:*gete* for *gate*. (3) This 12b is difficult to understand. Should it be emended as follows? *"nakre kulīre dhanarṇaṃ trimaurvyām"* (4) B:*koṭyaṃ ca tan* for *koṭijyamāṃ*. (5) A: *keṣa* for *meṣa*.

12: "Writing down the R.sine and the R.cosine [of $(180^0-\phi)$ and ϕ], the R.cosine is added to or subtracted from the Radius in accordance with the case of [the first point of] Cancer and [the first point of] Capricorn respectively(?), multiplied by the diurnal circle of [the first point of] Aries, and divided by the R.sine. They are the sum and difference [of the position of the centre and the radius]".

What this verse tells is that the following equations may be used instead of the equations (18) given in vs.11.

$$\left.\begin{aligned} q + c &= \frac{a \times (R + |R\cdot\cos(180^0 - \phi)|)}{|R\cdot\sin(180^0 - \phi)|} \\ q - c &= \frac{a \times (R - |R\cdot\cos\phi|)}{|R\cdot\sin\phi|} \end{aligned}\right\} \text{--------- (22)}$$

This is mathematically equivalent to the equations (18).

Padmanābha gives another method to calculate the (q+c) and | q-c | as follows.

त्रिज्योद्धृतो दोर्गुणयन्त्रघात [:]
स दोस्ततश्चान्तरमेव साध्यम् ।
तदुद्धृता दो:(1)कृतिरत्र योग-
स्ताभ्यां पुरावद्भवतो(2) (3)ऽथवा तौ(4) ॥१३॥

Apparatus :(1) B:*ttha* for *kṛ-*. (2) Ms.A lacks *dbhava*. (3) Ms.A lacks *avagraha* before *thavā*. (4) B: *te* for *tau*.

13:"The product of the "arm-chord" (*dorguṇa*) and [the radius of] the instrument, divided by the Radius, is the arm (*dos*). Then the difference [between the position of the centre and the radius] should also be determined. The square of the arm divided by it (the difference) is the sum [of the position of the centre and the radius]. From them, two values (the position of the centre and the radius) are obtained as before."

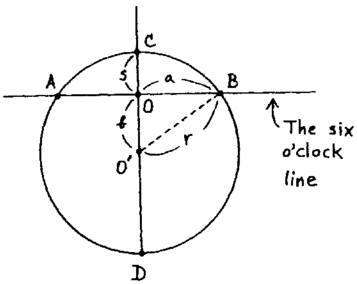


Fig. 21

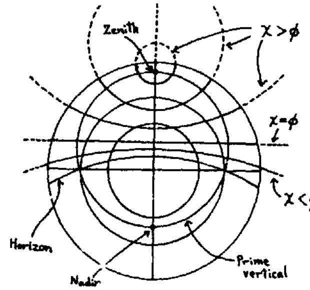


Fig. 22

The "arm-chord" (*dorguṇa*) is the distance of the point of intersection of the desired circle and the horizontal line passing through the centre of the instrument (i.e. the six o'clock line) from the centre of the instrument whose radius is equal to the Radius. In the case of the horizon, the arm-chord is equal to the "normal radius of the diurnal circle of Aries" (see vs. 9.). The arm-chord multiplied by the radius of the instrument and divided by the Radius is of course the corresponding value in the actual instrument. This is called arm (*dos*). Let this value be *a*. (This is *a* in Fig. 21.) The "difference" may be obtained by the method given in vs. 10'. Let this difference be *s*. (This is *s* in Fig. 21.) The "sum" is given as a^2/s in the text. (This is *OD* in Fig. 21.) As regards the rationale of the calculation of the "sum", Padmanābha refers to the *Līlāvatī* (153(a-c)) of Bhāskara II in the commentary. The *Līlāvatī* (153(a-c))[70] may be rendered into English as follows.

> "The quotient of the square of the base (arm, *bhuja*) divided by the difference between the upright (*koṭi*) and the hypotenuse (*karṇa*) [is written down] at two places, and [severally] decreased and increased by the difference between the upright and the hypotenuse. Their halves are the upright and the hypotenuse respectively."

In the Fig. 21, the base is *a*, the upright *b*, and the hypotenuse *r*. The difference between the hypotenuse and the upright, that is *r-b*, is *s*. Then the rules given in the *Līlāvatī* (153(a-c)) may be expressed as follows.

$$\left.\begin{aligned} b &= \frac{1}{2}\left(\frac{a^2}{s} - s\right) \\ r &= \frac{1}{2}\left(\frac{a^2}{s} + s\right) \end{aligned}\right\} \text{-------- (23)}$$

From these equations, the following equation is derived.

$$b + r = \frac{1}{2}\left(\frac{a^2}{s} - s\right) + \frac{1}{2}\left(\frac{a^2}{s} + s\right) = \frac{a^2}{s}$$

This is the "sum" given in vs. 13.

Regarding the parallels of altitude, Padmanābha adds as follows.

प्रागवत्तत्संख्यकोष्टाभ्यां
गृहीत्वा च फले पृथक्।
हत्वा यन्त्रदलेनाप्ते
त्रिंशद्भि[:] स्तो [ऽ]थवा स्फुटौ ॥१४॥

14: "As before, after obtaining two results by two tables of the values, they are multiplied by the radius of the instrument and divided by thirty. They are also exact."

Two tables of the values are the tables of $\dfrac{a\,x(R+|R\cdot\cos\theta|)}{|R\cdot\sin\theta|}$ and $\dfrac{a\,x(R-|R\cdot\cos\theta|)}{|R\cdot\sin\theta|}$ where *a* is the radius of the equator in the instrument whose radius is 30.

Padmanābha does not explain the reason why the radius 30 is used, but we can recall that Mahendra Sūri also mentioned an astrolabe whose radius is 30 in his *Yantra-rāja*, and Bhāskara II also used a circle whose radius is 30 in his *phalaka-yantra* mentioned in his *Siddhānta-śiromaṇi*. It may be that the radius 30 was a kind of standard radius of an instrument.

Padmanābha concludes the explanation of the parallels of altitude as follows.

मध्यादध:संस्थितकेन्द्रचिह्नाद्
व्यासार्धसूत्रेण पृथग् वृता (त्ता) नि।
कुर्यादभीष्टानि च हीयमाने
व्यासे [ऽ]न्यथात्वेन खतस्तदा स्यु: ॥१५॥

15: "[If the altitude is less than the observer's latitude], one should draw circles (parallels of altitude) with desired [interval] by a string of the length of the radius with the mark of centre which is situated below the centre [of the instrument]. Otherwise, [if the altitude is more than observer's latitude], there will be [circles] whose diameter becomes shorter [according as the altitude becomes higher], above [the centre of the instrument]."

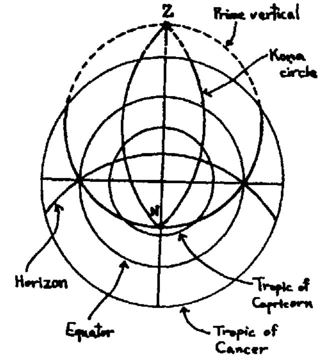
The meaning of this verse is clear. If the altitude is higher than the observer's latitude, $(180^{0}-\phi+\chi)$ is more than 180^{0}. (see Fig. 18 again.) I have drawn an example of the parallels of altitude where the altitude (χ) is less, equal, and more than the observer's latitude (ϕ). (see Fig. 22.).

After that, Padmanābha explained the zenith and nadir as follows.

यस्मिन् देशे सिद्धभागाधिकाक्षा
यन्त्रादूर्ध्वं तत्र खस्वस्तिकं स्यात्।
पातालाख्यं नक्रवृत्तान्तरे(1) त-
त्संपाते स्त:(2) कोणदृग्मण्डलानाम् ॥१६॥(3)

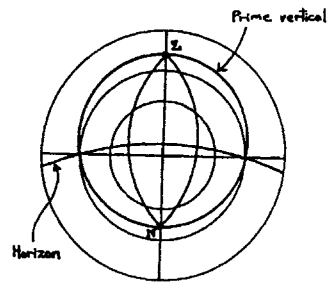
Apparatus : (1) A:*-tta-*for *-ttā-*. (2) Ms. B. lacks visarga after *sta.* (3) A:*dig* for *dṛg.* (4) In Ms.A, this is wrongly numbered as vs. 17, but this should be vs.16. This verse is vs.17 in Ms. B.

> 16: "At the place where the latitude is higher than 24°, the zenith will be above the instrument, and the nadir will be inside of the circle of Capricorn. [The zenith and nadir are] two intersections of the *koṇa* circles and vertical circles."



Example for high latitude
(Latitude > ε)

Fig. 23



Example for low latitude
(Latitude < ε)

Fig. 24

The meaning of this verse will be clear, if we remember that the celestial sphere is projected from the north celestial pole in this text, and the northern part of the celestial sphere from the Tropic of Cancer corresponds to the outside of the rim of the astrolabe. I have drawn an example of the construction for high latitude (Fig. 23), and an example for low latitude (Fig. 24). The point Z is the zenith, and N is the nadir in the figures.

Padmanābha further tells to graduate *nāḍīs* as follows.

यन्त्राद्बाह्ये [ऽ]प्यर्धपर्वान्तराले(1)
वृत्ते नाडीश्चोभयत्रोद्धृतायाः।
केन्द्रादूर्ध्वक्ष्माजयोगे(2) हि(3) याम्यं
तत्सव्यान्ये प्राक्पराख्ये कुजे स्तः॥१७॥(4)

Apparatus : (1) B:*parddhā* for *parvā-*. (2) B: *-rdhvaṃ dṛmā-* for *-rdhvakṣmā-*. (3) B: *tu* for *hi.* (4) This verse is vs. 18 in Ms. B.

> 17: "At the exterior of the instrument, on the circle at the interval of half a *parvan* (or *aṅgula*), *nāḍīs* [should be marked] on both sides from the [horizontal central] line [upwards]. The intersection of the upward line (meridian) from the centre and the horizon is the south

[cardinal point]. From the south [cardinal point], there are east and west horizons [leftwards and rightwards respectively]."

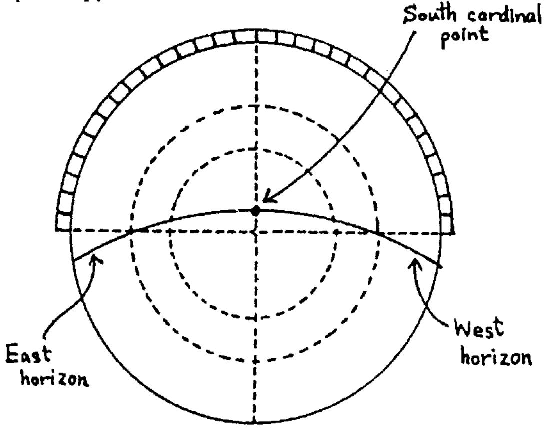


Fig. 25

The meaning of this verse is clear. I have drawn an example. (see Fig. 25.)

Padmanābha now explains a rough method to draw the parallels of angle of depression as follows.

एवं खदृग्गोलदलं हि दृश्यं
(1)संसाधयेदन्यदलं तथेति।
(2)ऊर्ध्वस्थरेखावदध[:]स्थरेखा(3)
केन्द्राद्विभज्याथ समाख्यवृत्ते ॥१८॥(4)

Apparatus : (1) B: *saṃsādhya tatrānyadalaṃ tathaiva* for *saṃsādha----tatheti.* (2) B:*sta* for *stha.* (3) B: *sta* for *stha.* (4) This verse is vs. 19 in Ms. B.

दृश्यार्धभागाङ्कजकेन्द्रसूत्र-
स्पृक्चिह्नकान्येवमथोद्धृतानि(1)।
तच्चिह्नसंगीनि लिखेदधस्ता-
त्साङ्कानि नाडीश्चायथोक्तवत्स्यात् ॥१९॥(2)

Apparatus:(1) A: *-thau-* for *-tho-.* (2) This verse is vs.20 in Ms.B.

18: "Thus the visible half of the celestial sphere is established. Now the other half (invisible half below the horizon) [should also be established]. Just like upward line (visible meridian), the downward line is also graduated from the centre. Regarding the prime vertical also"

19: "the graduation of degrees of the visible half [of the prime vertical] and the centre [of the instrument] are joined by a straight line, and the points of intersection of the line [and the invisible half of the prime vertical circle] are marked. Lines passing through these points (points corresponding to the same angle of depression on the invisible meridian and prime vertical) with marks [of the angle of depression] should be drawn. The *nāḍīs* should also be graduated [at the lower half of the instrument] as before."

This is a rough method to draw the parallels of angle of depression just like the parallels of altitude explained in vs. 10.

After the above rough method, Padmanābha explains the exact method of the construction of the parallels of angle of depressions as follows.

कुजस्य केन्द्रश्रवणौ विधाय
प्राग्वत्ततो [ऽ]क्षे [ऽ]भिमतांशकांश्च।
संशोध्य शेषेष्वपि शोध्यसाध्ये
[1]ताभ्यामधस्थाद्वलयानि वा स्युः॥२०॥[2]

Apparatus:(1) Ms. B adds *tv* after *tābhyām*. (2) This verse is vs.21 in Ms.B.

20: "After obtaining the centre and the radius of the horizon as before, and subtracting desired angle (angle of depression) from the [observer's] latitude and the remainder (180° minus latitude) severally, [one should calculate] two values such as minuend [as directed in vs.10'[. From them, [one should calculate the centre and radius, and] there will be circles [of the parallels of angle of depression which are drawn] below [the horizon]."

The method to draw parallels of angle of depression is the same as the method for parallels of altitude, and only the value of the altitude is replaced by the negative value of the angle of depression.

The above is the method of the construction of the surface of the front side of the astrolabe. After this, Padmanābha proceeds to explain the movable ecliptic circle as follows.

गोलादृश्यदलं चाथ चलद्राशिवृतं (त्तं) त्विह।
वक्ष्ये गोलविदां तुष्टयै प्रत्यक्षं दृश्यते यत:॥२१॥[1]

Apparatus: (1) This verse is vs.22 in Ms.B.

21: "I shall explain the movable ecliptic circle [which rotates on the visible half and] invisible half of the celestial sphere, for the satisfaction of astronomers, as if it is presented before the eyes."

Firstly, Padmanābha gives the radius of the ecliptic circle as follows:

यन्त्रार्धदन्ता ३२ हतिरक्षवेदै [र्]४५
दृता च तद्व्यासदलान्यपत्रम्।
द्युवृत्तयो [र्] व्यासदलप्रमाणं
कुर्यात्सुखार्थं मृगकीटयोर्वा॥२२॥[1]

Apparatus: (1) This verse is vs. 23 in Ms.B.

22 : "[One should] multiply the radius of the instrument with 32, and divide by 45, and [make] another plate [of the ecliptic circle] with the radius [thus obtained]. Or, for the sake of ease, the radius can be obtained from the diurnal circles of [the first point of] Capricorn and Cancer."

This verse can be explained as follows. Let *r* be the radius of the instrument, *E* the radius of the ecliptic on the astrolabe, and ε the obliquity of the ecliptic. If we recall our discussion for vs.6, and Fig. 11, the following equation will be clear.

$$r = E + E \cdot \sin \varepsilon \cdot$$

That is :

$$r = \frac{E}{R} \ (R + R \cdot \sin \varepsilon) \ ,$$

or,

$$E = \frac{r \cdot R}{R + R \cdot \sin \varepsilon)} \ . \qquad \text{--------} \ (24)$$

Padmanābha gives the value of $R \cdot \sin \varepsilon$ as follows in the commentary:

$$R \cdot \sin \varepsilon = 48 \frac{45}{60}$$

for the Radius 120. This value is less than the exact value of $R \cdot \sin 24^{\circ}$. $(\doteqdot 48 \frac{49}{60})$ It may be that he obtained this value from a sine table with interpolation. Substituting this value for $R \cdot \sin \varepsilon$ and 120 for the Radius in the equation (24), we get :

$$E = \frac{r \times 120}{120 + 48 \frac{45}{60}}$$

$$= \frac{32r}{45}$$

This is the method to obtain the radius of the ecliptic circle explained in the first half of vs.22.

The radius of the ecliptic circle can also be obtained as follows. Let *r* be the radius of the Tropic of Cancer, and *b* the radius of the Tropic of Capricorn on the astrolabe. Then :

$$E = \frac{r + b}{2}$$

This is so, because the ecliptic tangents the Tropic of Cancer and the Tropic of Capricorn at diametrically opposite points. (see Fig. 11 again.) This is the method explained in the second half of vs. 22.

Padmanābha continues to explain the ecliptic circle as follows.

तन्मध्यरेखागतकीटकर्ण-
(1)सूत्राग्ररन्ध्रं खलु केन्द्रकीले।
धार्यं दृढं नक्रखजोर्ध्वरेखा(2)
(3)युतिर्यथा स्यात् (4)तथोर्ध्वचिह्नात्॥२३॥(5)

Apparatus : (1) *-rddha* for *-gra.* (2) *B:-rdva* for *-rdhva.* (3) B: *yukti-* for *yuti-.* (4) B: *tathairgha* (?) for *tatordhva.* (5) This verse is vs. 24 in Ms.B.

व्यक्षा मृगाद्या:(1) परिकल्प्य यन्त्र-
नेम्यां च तत्कीलकसूत्रयुत्या।
ते यत्र(2) नेम्यामपसव्यगत्या
तेषां विभागा: स्वधिया ततोऽङ्क्या:॥२४॥(3)

Apparatus: (1) Ms, B lacks visarga after *-dyā.* (2) B: *tatra* for *yatra.* (3) This verse is vs. 25 in Ms. B.

23: "The hole, which is [made] at the point of the tip of the string of the length of the radius of [the Tropic of] Cancer [stretched] along its (ecliptic circle's) [vertical] central line, [should be inserted by] the central pin [of the astrolabe]. One should fix it in such a way that the top of [the Tropic of] Capricorn and [the top of] the vertical line [of ecliptic circle] coincide, and then from the uppermost mark [on the rim of the astrolabe],..."

24: "...one should mark the right ascension [of the first point of signs] beginning with Capricorn [anticlockwise], and join a string with the pin [at the centre of the astrolabe], and stretch towards the rim, and move from right to left, and mark [the first point of signs] there (ecliptic circle) by one's own intelligence,"

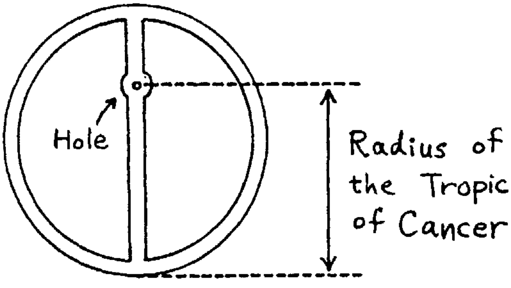


Fig. 26

The design of the ecliptic circle explained in vs. 23 is clear. (see Fig. 26.)

Since the hour circle is projected on the astrolabe as a straight line passing through the centre, the first point of signs can be marked by using right ascensional differences of the first point of signs from the first point of Capricorn as directed in the above verses, when the ecliptic circle is adjusted in such a way that its uppermost point coincides the Tropic of Capricorn. (see Fig. 27.)

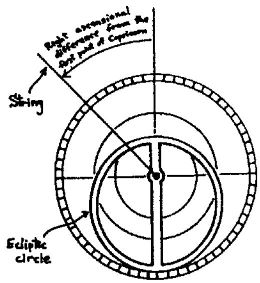


Fig. 27

After the ecliptic circle, Padmanābha tells to make the indicator of the sun as follows.

यन्त्रव्यासार्धाङ्गुलार्धप्रमाणां
पट्टीं कुर्याद्बुध्नरन्ध्रैकपार्श्वां।
एवं शू (सू) च्यग्रां भचक्रोपरिष्टा-
दन्येषां स्युः (1)स्वोन्नतांश प्रमाणाः(2) ॥२५॥(3)

Apparatus: (1) B: *tvo*-for *svo*-. (2) B: *-śu* for *-śa*. (3) This verse is vs. 26 in Ms. B.

25: "One should make an indicator *(paṭṭī)* which is measured by the radius of the instrument and half an *aṅgula*. It has a hole at the bottom of one side, and pointed [at the other end], and is [put] above the ecliptic circle. For other [stars], [one may make their indicators] according to their own altitude."

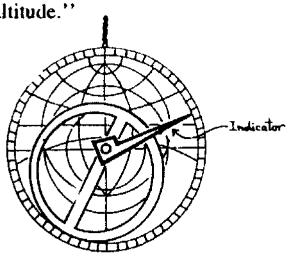


Fig. 28

The indicator of the sun described above is perhaps a radial bar of the length of the radius of the instrument, and of the width at the bottom of half an *aṅgula*, like Fig. 28.

b) The star table

After the above explanation of the method of the construction of the front side of the astrolabe, Padmanābha gives the celestial longitude and celestial latitude as well as the polar longitude and polar latitude of *nakṣatras* and some other stars for the sake of the construction of their indicators. Since their value is much different from Ptolemy's value, Padmanābha's star table must have been based on observations made in India.[71]

(1) Celestial longitude of *nakṣatras*

The celestial longitude of *nakṣatras* is given in vss. 26-30 as follows.

सूर्या: [12] खाम्बुधयो [40] [ऽ]श्विभे(1) ऽंशकलिकास्तत्वा (त्त्वा) नि [25] नागा [8] ध्रुवो
याम्ये हौतभुजे [ऽ]शका नवगुणा [39] लिप्ताद्वयं [2] ब्रह्मभे।
नागाब्धि [48] प्रमिता लवा नव [9] कला सौम्ये (2) कुषटका:(3) [61] शशी [1]
रौद्रे पञ्चरसा [65] गजा[8] स्त्वदितिभे द्व्यङ्का [92] स्त्रिबाणा [53] स्तत:॥२६॥(4)

Apparatus: (1) Ms. B lacks avagraha before *ṃśa*. (2) A: *ṣu* for *ku*. (3) Ms. B lacks visarga after *-ṭkā*. (4) This verse is vs. 27 in Ms. B.

(1)पुष्यस्याङ्गदिशो [106] लवास्तु कथिता:(2) सार्पे(3) (4)नवाभ्रेन्दव: [109]
प्रोक्ता वेद [4] कलोनिताश्च पितृभे नन्दाक्षिचन्द्रा [129] लवा:(5)।
भाग्ये द्व्यब्धिभुजो [142](6) ऽंशका गजयुगा [48] न्यार्यम्णसंज्ञे(7) ध्रुव:
स्त्रयं (त्र्यं) [3] शोना: खशरेन्दवश्च [150] रविभे(8) ऽक्षात्यष्टयो [175] [ऽ]ग्नीन्दव: [13] ॥२७॥(9)

Apparatus: (1) B: *pyṣye for puṣya.* (2) Ms. B lacks visarga after *-tā*. (3) B: *sārpye* for *sārpe*. (4) B: *nava* for *navā*. (5) Ms. B lacks visarga after *-vā*. (6) Ms. A lacks avagraha brfore *ṁśakā*. (7) B: *-rtheṣṇa* for *-ryamṇa*. (8) Ms. B: lacks avagraha before *kṣā-*. (9) This verse is vs. 28 in Ms. B.

त्वाष्ट्रे रामगजेन्दवश्च [183] खशरा [50] स्वातौ द्विनागेन्दव: [182]
सिद्धाश्च(1) [24] द्व्यधिपे ऽर्कलोचन [212] लवा:(2) षड्वर्ग [36] लिप्तान्विता:।
मैत्रे(3) वेदयमाश्विनो(4) [224] (5)[ऽ] ष्टदहना:(6) [38] शाक्रे खरामाश्विनो [230]
भागा बाण [5] कलाधिका नि‌ऋ(ऋ)तिभे द्व्यब्ध्यश्विन:(7) [242] (8) षड्गुणा: [36] ॥२८॥(9)

Apparatus: (1) B: *cā* for *ca*. (2) Ms. A lacks visarga after *-vā*. (3) B: *-trā* for *-tre*. (4) B: *-bhe* for *-no*. (5) B: *ṣu* for *ṣṭa*. (6) Ms. B lacks visarga after *-nā*. (7) B: *-bdhā-* for *bdhya-*. (8) Ms. B lacks *ṣaḍguṇāḥ*, and makes 3 syllables' gap. (9) This verse is vs. 29 in Ms. B.

तोये वेदशराश्विनो [254] युगगुणा [34] वैश्वे खतर्काश्विनो [260]
रूपाक्षीणि(1) [21] तथा [ऽ] भिजि [द्] (2) ध्रुवलवा युग्माङ्गपक्षा [262] दिश: [10]।
खाष्टाक्षीणि [280] गुणा(3) [3] ध्रुवस्तु हरिभे वेदाङ्कपक्षा [294] लवा(4)
मार्तण्डा [12] वसुभे नवेन्दुदहना [319] वेदेषवो [54] वारुणे ॥२९॥(5)

Apparatus: (1) Ms. A. lacks *ṇi* after *rūpākṣī*. (2) A: *hnava* for *dhruva*. (3) Ms. B adds visarga after *guṇā*. (4) Ms. B adds visarga after *lavā*. (5) This verse is vs. 30 in Ms. B.

षड्दैवा(1) [336] ध्रुवकस्त्वजैकचरणे(2) (3)भागा: कलाष्टौ [8] ततो
[ऽ]हिबुध्न्ये(4) गजवारिराशिदहना(5) [348] वेदाब्धयो(6) [44] लिप्तिका:।
पौष्णे शून्य [0] मिति ध्रुवास्तु कथिता भागादिका मध्यमा
दृक्कर्मायनजेन(7) सायनवशात् ते स्यु: स्फुटा:(8) संस्कृता:॥३०॥(9)

Apparatus: (1) B: *deva* for *daiva*. (2) B: *-rathe* for *-raṇe*. (3) B: *nāgāḥ* for *bhāgāḥ* (4) B: *-dhne* for *-dhnye*. (5) Ms. A lacks *ja* after *ga*. (6) A: *-bva-* for *-bdha-*. (7) B: *-yena* for *-jena*. (8) Ms. A lacks visarga after *sphuṭā*. (9) This verse is vs. 31 in Ms. B.

26-30: "[Celestial longitude of *nakṣatras*]

[1]	Aśvibha (=Aśvinī)	$12^{\circ}40'$
[2]	Yāmya (=Bharaṇī)	$25^{\circ}8'$
[3]	Hautabhuja (=Kṛttikā)	$39^{\circ}2'$
[4]	Brahmabha (=Rohiṇī)	$48^{\circ}9'$
[5]	Saumya (=Mṛgaśiras)	$61^{\circ}1'$
[6]	Raudra (=Ārdrā)	$65^{\circ}8'$
[7]	Aditibha (=Punarvasu)	$92^{\circ}53'$
[8]	Puṣya	106°
[9]	Sārpa (=Āśleṣā)	$108^{\circ}56'$
[10]	Pitṛbha (=Maghā)	129°
[11]	Bhāgya (=P. Phālgunī)	$142^{\circ}48'$
[12]	Āryamṇa (=U. Phālgunī)	$149^{\circ}40'$
[13]	Ravibha (=Hasta)	$175^{\circ}13'$
[14]	Tvāṣṭra (=Citrā)	$183^{\circ}50'$
[15]	Svātī	$182^{\circ}24'$
[16]	Dvyadhipa (=Viśākhā)	$212^{\circ}36'$
[17]	Maitra (=Anurādhā)	$224^{\circ}38'$
[18]	Śākra (=Jyeṣṭhā)	$230^{\circ}5'$
[19]	Nirṛtibha (=Mūla)	$242^{\circ}36'$
[20]	Toya (=P. Āṣāḍhā)	$254^{\circ}34'$
[21]	Vaiśva (=U. Āṣāḍhā)	$260^{\circ}21'$

[*]	Abhijit	$262^{\circ}10'$
[22]	Haribha (=Śravaṇa)	$280^{\circ}3'$
[23]	Vasubha (=Dhaniṣṭhā)	$294^{\circ}12'$
[24]	Vāruṇa (=Śatabhiṣaj)	$319^{\circ}54'$
[25]	Ajaikacaraṇa (=P. Bh.ādrapadā)	$336^{\circ}8'$
[26]	Ahirbudhnya (=U.Bh.ādrapadā)	$348^{\circ}44'$
[27]	Pauṣṇa (=Revatī)	0°

Thus the degrees etc. of celestial longitude (*madhyama-dhruva*) have been told. Applying the *ayana-dṛk-karman,* they will be converted into polar longitude *(sphuṭa-dhruva).*"

The *ayana-dṛk-karman* is the method to convert celestial longitude into polar longitude, and explained in vss.80-81.

(2) Polar longitude of *nakṣatras*

अथायनाख्येन(1) च दृक्फलेन
कृतस्फुटान् (2)भध्रुवकान् लवाद्यान्।
वक्ष्ये (3)यथा तद्विशिखान् द्विधोक्तान्
पृथक्कदम्बध्रुवसूत्रसक्तान्(4) ॥३१॥(5)

Apparatus: (1) B: *-kṣena* for *-khyena.* (2) Ms. B lacks *bha* before *dhru-*. (3) B: *tathā* for *yathā.* (4) B: *yātān for saktān.* (5) This verse is vs. 32 in Ms. B.

31: "The degrees etc. of polar longitude (*sphuṭa-dhruvaka*) converted by the *ayana-dṛk-phala* will be told just like the twofold latitudes (*viśikha)* (i.e. celestial and polar latitude) which are measured along the lines passing through the pole of the ecliptic (*kadamba*) and the pole of the celestial equator (*dhruva*) respectively."

The *ayana-dṛk-phala* is the difference between the celestial longitude and the polar longitude. The celestial latitude and the polar latitude of *nakṣatras* are given in vss. 40-47 and vss.48-54 respectively.

Now Padmanābha gives the polar longitude of *nakṣatras* in vss. 32-38.

अष्टौ [8] लवा रामयमाः(1) [23] (2)कलाश्च
दास्रे [ऽ]थ(3) याम्ये (4) कुयमा [21] महीं (ही) ध्राः [7]।
सप्ताग्नयः [37] षट्श्रुतयो [46] [ऽ]ग्निदैवे
ब्राहये नवाम्भोनिधयश्च [49] भागाः॥३२॥(5)

Apparatus; (1) Ms. B lacks *yamāḥ* after *rāma.* (2) A: *lavā-* for *kalā-*. (3) Ms.B lacks *thā.* (4) Ms. B adds *śāliptā* between *yāmye* and *kuyamā.* (5) This verse is vs. 33 in Ms. B.

सौम्ये द्विषट्का(1) [62] यमलाश्च [2] (2)रौद्रे
शराङ्ग [65] भागाः शरबाण [55] लिप्ताः।
(3)रामाङ्क [93] भागा गजराम [38] लिप्ता [:]
पुनर्वसोस्तर्कदिशो (श)स्तु(4) [106] पुष्ये ॥३३॥(5)

Apparatus: (1) Ms. B adds visarga after *-ṭkā*. (2) B: *rodre* for *raudre*. (3) A: *ramā-* for *rāmā-*. (4) B: *-śosu* for *-śastu*. (5) This verse is vs. 34 in Ms. B.

सप्ताभ्रचन्द्रा [107] मानवः [14] फणीन्द्रा (न्द्रे)(1)
(2)नवाश्विचन्द्रा [129] पितृभे(3) ध्रुव [:] स्यात्।
भाग्यस्य सप्ताब्धिभुवो [147] गजाक्षाः(4) [58]
सार्धाश्च बाणेषुभुवो [155] [ऽ] र्यमर्क्षे(5) ॥३४॥(6)

Apparatus: (1) Ms. A adds visarga after *-drā*. (2) B: *tanvā-* for *navā-*. (3) A: *bha* for *bhe*. (4) B: *-khyāḥ* for *kṣāḥ*. (5) Ms. B lacks *r* before *kṣe*. (6) This verse is vs. 35 in Ms. B.

(1)हस्तस्य शून्यागभुव [170] स्त्रयं [3] च
त्वाष्ट्रे [ऽ]ग्निनागेन्दव(2) (वो) [183] लोचने [2] च।
स्वातौ गजाङ्केन्दु [198] लवास्तु सिद्धा [24]
द्विदैवते द्वीन्दुयमाश्च [212] रुद्राः [11] ॥३५॥(3)

Apparatus: (1) B: *karasya* for *hastasya*. (2) According to the Sandhi rule, *-ndava* should become *-ndavo*, but it disturbs metric. (3) This verse is vs. 36 in Ms. B.

मैत्रस्य वेदद्वियमास्तु [224] शक्राः (क्रा) [14]
नन्दाश्विपक्षाश्च [229] जिना [24] स्तथैन्द्रे।
मूले (1)कुसिद्धा [241] कुशरा(2) [51] जलर्क्षे
समुद्रतत्वा (त्वा) [254] न्यमृताशनाश्च [33 ?] ॥३६॥(3)

Apparatus: (1) A: *ṣusiddhā* for *kusiddhā*. (2) B: *kuśalā* for *kuśarā*. (3) This verse is vs.37 in Ms.B.

वैश्वस्य शून्योत्कृतयो [260] दिशश्च [10]
नन्देषुपक्षा [259] विधिभे [ऽ] क्षरामाः [35]।
पञ्चाद्रिपक्षाः [275] श्रवणस्य काष्ठा [10]
वस्व (स्वा)र्क्षके(1) [ऽ] द्व्यष्टयमाश्च [287] सार्धाः ॥३७॥(2)

Apparatus: (1) A: *-bhe* for *-ke*. (2) This verse is vs. 38 in Ms.B.

खबाहुरामा [320] वरुणर्क्षकस्य
तत्वा (त्वा)ग्नयो [325] [ऽ] जैकपदे(1) खपक्षाः [20]।
रसामरा [336] शून्यशरा [50] उपान्त्ये
पौष्णे(2) ध्रुवः(3) शून्य [0] मिति स्फुटाश्च ॥३८॥(4)

Apparatus: (1) Ms. A adds visarga after *pade*. (2) A: *-ṣṇa* for *-ṣṇe*. (3) Ms. A lacks visarga after *dhruva*. (4) This verse is vs. 39 in Ms. B.

32-38: [Polar longitude of *nakṣatras*]

[1]	Dāsra (=Aśvinī)	8° 23′
[2]	Yāmya (=Bharaṇī)	21° 7′
[3]	Agnidaiva (= Kṛttikā)	37° 46′
[4]	Brāhmya (=Rohiṇī)	49°
[5]	Saumya (=Mṛgaśiras)	62° 2′
[6]	Raudra (=Ārdrā)	65° 55′
[7]	Punarvasu	93° 38′
[8]	Puṣya	106°
[9]	Phaṇīndra (=Āśleṣā)	107° 14′
[10]	Pitṛbha (=Maghā)	129°
[11]	Bhāgya (=P.Phālgunī)	147° 58′
[12]	Aryamarkṣa (=U. Phālgunī)	155° 30′
[13]	Hasta	170° 3′
[14]	Tvāṣṭra (=Citrā)	183° 2′
[15]	Svāti	198° 24′
[16]	Dvidaivata (=Viśākhā)	212° 11′
[17]	Maitra (=Anurādhā)	224° 14′
[18]	Aindra (=Jyeṣṭhā)	229° 24′
[19]	Mūla	241° 51′
[20]	Jalarkṣa (=P. Āṣāḍhā)	254° 33′(?)
[21]	Vaiśva (= U. Āṣāḍhā)	260° 10′
[*]	Vidhibha (=Abhijit)	259° 35′
[22]	Śravaṇa	275° 10′
[23]	Vasvārkṣaka (=Dhaniṣṭhā)	287° 30′
[24]	Varuṇarkṣaka (=Śata bhiṣaj)	320°
[25]	Ajaikapada (=P.Bhādrapadā)	325° 20
[26]	Upāntya (U.Bhādrapadā)	336°50′
[27]	Pauṣṇa (=Revatī)	0°

Thus the polar longitude."

After the above table, Padmanābha adds as follows.

(1)पञ्चाब्धिविश्वं(2) [1345] शकवत्सरा(3) गता:(4)
पञ्चेन्दु [15] भागा अयनाख्यकास्तदा(5)।
कृता:(6) सुखार्थं च हिताय तद्विदां
तथा [ऽ] क्षजेनाप्युदयास्तयो [:](7) स्फुटा:॥३९॥(8)

Apparatus: (1) A: *paṃca* for *pañcā.* (2) B: *-śa-* for *bdhi-*. (3) Ms- A adds *ka* between *-tsa* and *rā.* (4) Ms. A lacks visarga after *gatā.* (5) B: *-kṣa-* for *-khya-*. (6) Ms. A lacks visarga after *kṛtā.* (7) A: *-naḥ* for *-nā-*. (8) This verse is vs. 40 in Ms.B.

39: "In Śaka era, 1345 years have passed, and the *ayana-aṃśa* is 15 degrees. For the sake of pleasure and benefit of knowers, the correct rising and setting [points of the ecliptic] are obtained by applying the *akṣa-dṛk-karman.*

This verse shows that this work was composed in around 1345 Śaka (=1423 AD). The *ayana-aṃśa* is the amount of precession, that is the arc of the ecliptic lying between the zero point (junction star of the *nakṣatra* Revatī or the star ζ Piscium) and the current vernal equinox. The *akṣa-dṛk-karman* is the method to find the point of ecliptic which rises or sets simultaneously with the star by applying to the polar longitude, and is explained in vs. 82.

(3) Celestial latitude of *nakṣatras*

The celestial latitude of *nakṣatras* is given in vss. 40-46 as follows.

(1)वक्ष्ये [ऽ] थनक्षत्रशरान् लवाद्यान्
मध्यांश्च दास्रे (2) ऽक्ष [5] कलोनरुद्रा: [11]।
अर्का [12] रसाक्षा [56] यमभे [ऽ]थ वेदा [4]
(3)वेदाध्यां(4) [44]हौतभुजस्य सौम्या:(5)॥४०॥(6)

Apparatus: (1) A: *athocyate dhijyaśarā lavādyā madhyāś ca* for *vakṣye-madhyāṃś ca.* (2) Ms. B lacks avagraha before *kṣa.* (3) A: *vādā-*for *vedā-*. (4) B: *ṣja* (?) for *bdha.* (5) Ms. A lacks visarga after *-myā.* (6) This verse is vs. 41 in Ms.B.

ब्राहयस्य याम्यो [ऽ] ब्धि[4] लवा:(1) खवेदा:(2) [40]
सौम्यस्य काष्ठा(3) [10] स्त्रिभुव(4) [13] स्तु याम्य:(5)।
रौद्रस्य रुद्रा [11] अचला [7] हि याम्य:
सौम्यो [ऽ] दितेर्भस्य रसांशकाश्च [6] ॥४१॥(6)

Apparatus: (1) Ms. B lacks visarga after *lavā.* (2) Ms. B lacks visarga after *vedā.* (3) Ms. A lacks *sṭhā.* (4) Ms. A adds *na* after *va.* (5) A: *-myāḥ* for *-myaḥ.* (6) This verse is vs. 42 in Ms.B.

पुष्यस्य शून्यं [0] फणिभे नगां [7] शा
युगाश्च [4] याम्यः पितृभे वियच्च [0] ।
भाग्ये भगा [12] द्वयम्बुधयो [42] [ऽ]र्यमर्क्षे
विश्वे [13] [ऽ]क्षबाणा [55] उभयोस्तु(1) सौम्यौ(2) ॥४२॥(3)

Apparatus : (1) A: *ta* for *tu.* (2) B: *-myaḥ* for *-myau.* (3) This verse is vs. 43 in Ms.B.

याम्यः करे (1)ऽर्का [12] श्रुतयो [4] [ऽ]थ रूपं[1]
यमेषवो [52] वर्धकिभे च याम्यः।
स्वातौ कुवेदा [41] विशिखाश्च [5]सौम्यो(2)
याम्यो(3) द्विदैवे विधु [1] रक्ष(4) [5] वर्गाः॥ ४३॥(5)

Apparatus: (1) Ms.B lacks avagraha before *rkā.* (2) B:*saumyā* for *saumyo.* (3) B: *yāmye* for *yāmyo.* (4) Ms. B adds *r* between *ra* and *kṣa* (5) This verse is vs. 44 in Ms. B.

मैत्रे शशी [1] शून्यशरा [50] स्तथैन्द्रे
रामा [3] महीध्राज्योभुज [37] ऽथ मूले।
त्र्यं [3] शोननन्दा [9] जलदैवते [ऽ]क्षा [5]
द्विबाहवो [22] [ऽ]थो (1)ऽक्ष [5] लवास्तु (2)वैश्वे॥४४॥(3)

Apparatus : (1) Ms.B lacks avagraha before *kṣa.* (2) B: vi - for *vai* - . (3) This verse is vs. 45 in Ms.B.

अथोत्तरो(1) बाहुरसा [62] लवाः कला(2)
युगेन्दवः(3) [14] स्याद्विधिभे (4)ऽथ वैष्णवे।
शून्याग्नयो [30] [ऽ]क्षाः [5] (5)शशिदिक् च वासवे
तर्काश्विनौ (नो)(6) [26] (7)बाणयमाश्च [25] वारुणे ॥४५॥(8)

Apparatus : (1) B: *rā* for *-ro.* (2) Ms. B gives *kalā* twice. (3) B: *-duva* for *davaḥ.* (4) Ms. A lacks avagraha before *tha.* (5) Ms. A lacks *śi* after *śa.* (6) B: *-nā* for *-no.* (7) B: *paṃca* for *bāṇa.* (8) This verse is vs. 46 in Ms. B.

याम्ये त्रि [3] भागो [ऽ]थ कुवेरदिक् प्राग्-
(1)भद्राख्यकस्योत्कृतयो(2) [26] [ऽ]ग्नयश्च [3]।
गजाश्विनो [28] नागयमा [28] द्वितीय-
भद्रे तु(3) पौष्णे ख [0] मिति(4) प्रदिष्टाः॥४६॥(5)

Apparatus : (1) Ms. A lacks *ka* after *-khya.* (2) A:*-taya* for *-tayo.* (3) A: *ti* for *tu.* (4) B: *maṃhi madhyamāḥ* for *miti pradiṣṭāḥ.* (5) This verse is vs. 47 in Ms.B.

40-46: "Now I shall tell the degrees etc. of the celestial latitude *(madhya-śara)* of nakṣatras.

[1]	Dāsra (=Aśvinī)	10°55′ N
[2]	Yamabha (=Bharaṇī)	12°56′ N
[3]	Hautabhuja (=Kṛttikā)	4°44′ N
[4]	Brāhmya (=Rohiṇī)	4°40′S
[5]	Saumya (=Mṛgaśiras)	10°13′ S
[6]	Raudra (=Ārdrā)	11°7′ S
[7]	Aditer-bha (=Punarvasu)	6° S
[8]	Puṣya	0°
[9]	Phaṇibha (=Āśleṣā)	7°4′ S
[10]	Pitṛbha (=Maghā)	0°
[11]	Bhāgya (=P.Phālgunī)	12°42′ N
[12]	Aryamarkṣa (=U.Phālgunī)	13°55′ N
[13]	Kara (=Hasta)	12°4′ S
[14]	Vardhakibha (=Citrā)	1°52′ S
[15]	Svāti	41°5′ N
[16]	Dvidaiva (=Viśākhā)	1°25′ S
[17]	Maitra (=Anurādhā)	1°50′ S
[18]	Aindra (=Jyeṣṭhā)	3°37′ S
[19]	Mūla	8°40′ S
[20]	Jaladaivata (=P.Āṣāḍhā)	5°22′ S
[21]	Vaiśva (=U.Āṣāḍhā)	5° S
[*]	Vidhibha (=Abhijit)	62°14′ N
[22]	Vaiṣṇava (=Śravaṇa)	30°5′ N
[23]	Vāsava (=Dhaniṣṭhā)	26° 25′N
[24]	Vāruna (=Śatabhiṣaj)	0°20′ S
[25]	Prāg-bhadra (=P.Bhādrapadā)	26°3′ N
[26]	Dvitīya-bhadra (=U.Bhādrapadā)	28°28′N
[27]	Pauṣṇa (=Revatī)	0°

Thus pointed out."

After the above table, Padmanābha adds as follows.

एते शरांशा: कथिता हि भानां
    कदम्बसूत्राश्रयिण[:] स्फुटाख्या:।
ध्रुवाख्यसूत्रापमवर्तिनस्ते(1)
    न स्युस्ततो(2) मध्यमसंज्ञकाश्च ॥४७॥(3)

Apparatus : (1) B: *-kṣa-* for *-khya-*. (2) *B: tamo* for *tato.* (3) This verse is vs. 48 in Ms.B.

47: "The degrees of the latitude (*śara*) of stars measured along the line passing through the pole of the ecliptic have been told. They are not polar latitude (*sphuṭa-śara*) which is connected with declination (*apama*) measured along the line passing through the pole of the celestial equator. They are celestial latitude (*madhya-śara*)."

(4) Polar latitude of *nakṣatras*

The polar latitude of *nakṣatras* is given in vss. 48-54 as follows.

दास्रे दिशो [10] [ऽ]ङ्का(1) [9] यमभे भगा [12] नृपा[:][16]
    कृशानुभे(2) [ऽ]म्भोनिधयो [4] [ऽ]क्षपावका: [35]
ब्राहये [ऽ]ब्धय:(3) [4]षट् [6] कृतयश्च (4)सोमभे
    खभूमयो [10] [ऽ]ङ्का:[9](5) शिवभे (6)भवाश्च [11] षट् [6] ॥४८॥(7)

Apparatus: (1) Ms. B lacks *ṅ* before *kā.* (2) A:*-na*-for-*nu*-. (3) Ms. B lacks *m* before *bho.* (4) B: *saumya* for *soma.* (5) Ms.B lacks *ṅ* before *kāh.* (6) A: *-gā* -vor *-vā* -. (7) This verse is vs.49 in Ms. B.

कलात्रयो [3] नाङ्ग[6] लवा: पुनर्वसो:
    पुष्ये ख [0] माह्ये [ऽ]ङ्ग [6] लवास्त्रिसायका: [53]।
पैत्रे ख [0] मर्का [12] वियद(त)श्च [0] भाग्यभे
    [ऽ]थार्यम्णभे ऽर्का [12] क्षितिभृद्युगा(1) [47] स्तत: ॥४९॥(2)

Apparatus: (1) A: *-bhi*-for-*bhṛ*-. (2) This verse is vs. 50 in Ms.B.

हस्ते भवा [11] स्तुर्यलवोनपक्षा [2]-
    (1)स्त्वाष्ट्रे [ऽ]निले(2) त्र्यं[3] शयुता [ऽ]द्रिरामा: [37]।
द्विदैवते द्वौ [2] द्विशराश्च[52] मैत्रे
    रूपं[1] गजाम्भोनिधय [48] स्तथैन्द्रे(3) ॥५०॥(4)

Apparatus: (1) B: *-t tvā-* for *-s tvā-*. (2) B: *-la* for *-le.* (3) B: *-thedre* for *-thaindre.* (4) This verse is vs.51 in Ms.B.

वेदां [4] शकास्तत्व (त्त्व)(1) [25] कलोनिताश्च
    मूलस्य(2) नागा [8] अचलाग्नयश्च [37]।
आप्यस्य पञ्चा(3) [5] द्वियमाश्च [22] वैश्वे
    बाणा [5] विधेर्भस्य यमाङ्ग [62] भागा:॥५१॥(4)

Apparatus : (1) Ms. A gives *kā* twice. (2) A: *-ś ca* for *-sya.* (3) A: *-ṁca* for *-ñcā.* (4) This verse is vs. 52 in Ms. B.

श्रुतेर्नवाक्षीणि [29] रसाग्नयश्च [36]
    तत्वा (त्त्वा) [25] निधिक्ष्मानि [19] च वासवस्य(1)।
शून्यं [0] खपक्षा [20] वरुणर्क्षकस्य
    वेदां [4] शकोनाब्धियमा [24] स्त्वजांघ्रे:(2)॥५२॥(3)

Apparatus : (1) B: *-śca* for *-sya.* (2) Ms.A lacks visarga after *-ghre.* (3) This verse is vs. 53 in Ms.B.

उपान्त्यभस्याङ्ग [6] कलोनितोत्कृति:(1) [26]
    पौष्णे ख [0] मंशादिशरा[:] परिस्फुटा:।
ब्राहयात्(2) त्रयं (3)सर्पभमर्कभा [द्] द्वयं
    द्विदैवषट्कं वरुणर्क्षकं तथा॥५३॥(4)

Apparatus: (1) Ms. B lacks visarga after - *ti.* (2) Ms.B lacks *t* after *-hmyā.* (3) B: *sārpya* for sarpa. (4) This verse is vs. 54 in Ms.B.

एषां हि याम्या:(1) परिशेषकानां
    सौम्याश्च वेद्या (2)अपमस्य योग्या:।
यतो(3) [ऽ]पमाग्राद् ध्रुवसूत्रगास्त-
    न्मध्योन्नतांशस्थितिदर्शिनस्ते(4)॥५४॥(5)

Apparatus : (1) B: *-yoh* for *-yāh.* (2) *apayaśvayojyāh* for *apamasya yogyāḥ.* (3) B: *yamo* for *yato.* (4) *-dhye na-* for *-dhyonna-.* (5) This verse is vs. 55 in Ms. B.

48-54: "[Polar latitude of *nakṣatrāś*]

[1]	Dāsra (=Aśvinī)	$10^\circ 9'$ (N)
[2]	Yamabha (=Bharaṇī)	$12^\circ 16'$ (N)
[3]	Kṛśānubha (=Kṛttikā)	$4^\circ 35'$ (N)
[4]	Brāhmya (=Rohiṇī)	$4^\circ 36'$ (S)
[5]	Somabha (=Mṛgaśiras)	$10^\circ 9'$ (S)
[6]	Śivabha (=Ārdrā)	$11^\circ 6'$ (S)
[7]	Punarvasu	$5^\circ 57'$ (N)
[8]	Puṣya	0°

[9]	Āhya (=Āśleṣā)	6°53′ (S)
[10]	Paitra (=Maghā)	0°
[11]	Bhāgyabha (=P. Phālgunī)	12°0′ (N)
[12]	Aryamṇabha (=U.Phālguni)	12°47′ (N)
[13]	Hasta	11°(S)
[14]	Tvāṣṭra (=Citrā)	1°45′ (N)
[15]	Anila (=Svātī)	37°20 (N)
[16]	Dvidaivata (=Viśākhā)	2°52′ (S)
[17]	Maitra (= Anurādhā)	1°48′(S)
[18]	Aindra (=Jyeśṭhā)	3°35′ (S)
[19]	Mūla	8°37′ (S)
[20]	Āpya (=P.Āṣāḍhā)	5°22′ (S)
[21]	Vaiśva (=U. Āṣādhā)	5° (S)
[*]	Vidher-bha (=Abhijit)	62° (N)
[22]	Śruti (=Śravaṇa)	29°36′ (N)
[23]	Vāsava (=Dhaniṣṭhā)	25°19′ (N)
[24]	Varuṇarkṣaka (=Śatabhiṣaj)	0°20′ (S)
[25]	Ajāṃghri (=P.Bhādrapadā)	23°45′(N)
[26]	Upāntyabha (=U.Bhādrapadā)	25°54′ (N)
[27]	Pauṣṇa (=Revatī)	0°

Three [*nakṣatras*] from Brāhmya (no.4), Sarpabha (no.9), two from Arkabha (no.13), six from Dvidaiva (no.16), and Varuṇarkṣaka (no.24) are south. The rest are north. They are known to be connected with declination, and from the tip of the delcination (i.e. the position of the star) the meridian altitude is measured along the line passing through the pole of the celestial equator."

(5) Celestial longitude and latitude of other stars

The celestial longitude and latitude of some stars other than *nakṣatras* are given in vss.55-61 as follows.

अग्निभुग्ध्रुवको मध्यो
वेदाक्षां [54] शाश्वतुः[1] [4] कलाः।
उत्तरे [ऽ]ष्टौ[8] लवा[:] क्षेपो[2]
मनु [14] लिप्ताधिकास्तथा ॥५५॥[3]

Apparatus : (1) B: *-ḥ kalābdhayaḥ* for *-ś catuḥ kalāḥ*. (2) B: *-pau* for *-po*. (3) This verse is vs. 56 in Ms. B.

गजाक्षां(1) [58] शाश्च षड् [6] वर्ग-
कला स्याद्(2) ब्रह्महड [द्] ध्रुवः।
शरः सौम्यो वियद्रामा [30]
लवा नन्दाब्धयः [49] कलाः॥५६॥(3)

Apparatus : (1) B: *-tthā-* for *-kṣā-*. (2) A: *khyā* for *syād*. (3) This verse is vs. 57 in Ms. B.

प्रजापतेः(1) शराङ्गां [65] शा-
(2)स्त्रिबाण(3) [53] कलिका ध्रुवः।
गजाग्नयो [38] लवा लिप्ता-
स्तावन्त्यश्च(4) शरो ह्युदक्॥५७॥(5)

Apparatus : (1) Ms. B lacks visarga after *-te*. (2) Ms. B lacks *s* after *-śā*. (3) B: *vrāṇā* for *bāṇa*. (4) B: *-sya* for *-ś ca*. (5) This verse is vs. 58 in Ms. B.

लुब्धकस्य युगाष्टां [84] शा(1)
लिप्ता षड् [6] व र्गसंमिताः(2)।
विक्षेपस्तु वियद्वेदा [40]
लवा (3)याम्यो [ऽ]ब्धयः [4]कलाः॥५८॥(4)

Apparatus : (1) Ms. A adds visarga after *-śā*. (2) Ms. B lacks visarga after *-tā*. (3) B: *dya* and two syllables' gap for *yāmyo 'bdha-*. (4) This verse is vs. 59 in Ms. B.

अगस्त्यस्य शराष्टौ [85] च
भागा(1) लिप्तास्त्रयो [3] ध्रुवः।
सप्ताद्रयो [77] लवा याम्यः
(2)शरो लिप्ता [ऽ]ष्टि [16] संयुताः(3)॥५९॥

Apparatus : (1) B: *gābhā* for *bhāgā*. (2) B: *śiro* for *śaro*. (3) Ms. B lacks visarga after *-tā*. (4) This verse is vs. 60 in Ms. B.

(1)अपांवत्सा पयोर्मध्यौ(2) (3)
चित्रावद्ध्रुवकावुदक्।
शरो भागास्त्रयो(4) [3]लिप्ता
गजा [8] (5)श्चाङ्का [9]श्च दिक्क [10]लाः॥६०॥(6)

Apparatus : (1) Ms. B lacks anusvāra after *apā*. (2) B: *-tsa-* for *-tsā-*. (3) B: *-dhye* for *-dhyau*. (4) B: *triyo* for *trayo*. (5) A: *-jāḥś* for *-jāś*. (6) This verse is vs. 61 in Ms. B.

सौरोक्तरुद्रभस्यांशा-[1]
स्त्र्यद्र्यो [73] [S]गाब्धय: [47] कला:।
ध्रुवकस्तु[2] शरो याम्यो
नन्दां [9] शा:[3] श्रुतय: [4] कला:॥६१॥[4]

Apparatus : (1) A: *śāḥs* for *-śās*. (2) B: *-ś ca* for *-s tu*. (3) Ms. B lacks visarga after *-śā*. (4) This verse is vs. 62 in Ms. B.

55-61: "[Celestial longitude and latitude of some stars:]

[Name of the star :]	[longitude :]	[latitude :]
Agnibhuj	54°4′	8°14′ N
Brahmahṛd	58°36′	30°49′N
Prajāpati	65°53′	38°38′N
Lubdhaka	84°36′	40° 4′S
Agastya	85°3′	77°16′ S
Apāṁvatsa	183°50′	3°8′ N
Āpa	183°50′	9°10′ N
Saurokta-rudra	73°47′	9°4′ S"

Among the above names, the "Saurokta-rudra" is the same as the "Sūryoktārdrā", where the "Rudra" is a synonym of the *nakṣatra* Ārdrā.

After the above table, Padmanābha adds as follows.

मध्यमा ध्रुवका: प्रोक्ता:[1]
शराश्चापममण्डलात्।
दृक्कर्मणायनेनैव
ते स्यु: शरवशात्[2] स्फुटा:॥६२॥[3]

Apparatus : (1) Ms. A lacks visarga after *-ktā*. (2) Ms. B lacks *t* after *-śā*. (3) This verse is vs. 63 in Ms. B.

62 : "The celestial longitude and latitude, which are measured from the ecliptic, have been told. Applying the *ayana-dṛk-karman*, they are converted into polar coordinates, in accordance with their latitude."

(6) Polar longitude and latitude of other stars

The polar longitude and latitude of the above mentioned stars are given in vss. 63-70 as follows.

त्र्यक्षां [53] शा नृप [16] लिप्ताढ्या
ध्रुवको(1) [ऽ]ग्निभुज[:](2) स्फुटः।
सौम्यः(3) शरो गजां [8]शाश्च
नव [9]लिप्ताधिका(4) स्फुटः॥६३॥(5)

Apparatus : (1) B: *dhruve* for *dhruva.* (2) Ms. B adds anusvāra after *gni.* (3) Ms. B lacks visarga after *-mya.* (4) B: *bhujā* for *dhikā.* (5) This verse is vs. 64 in Ms.B.

पञ्चपञ्चां [55] शकाश्चाति -
धृति [19] लिप्ताधिका ध्रुवः।
सद्धृदः सौम्यविशिषः
खाग्नयो [30] [ऽ] ब्ध्यग्नयः [34] कलाः(1) ॥६४॥(2)

Apparatus : (1) Ms. B lacks visarga after *-lā.* (2) This verse is vs. 65 in Ms. B.

प्रजापतिध्रुवस्त्र्यङ्ग [63]-
लवा वेदाग्नयः(1) [34] कलाः।
सौम्येषु(2) नागरामां [38] शा
लिप्ताश्चाश्विहुताशनाः [32] ॥६५॥(3)

Apparatus : (1) B: *-sayaḥ* for *-gnayaḥ.* (2) Ms. B adds *r* between *-ṣu* and *nāga.* (3) This verse is vs. 66 in Ms. B.

सूर्योक्तार्द्राध्रुवस्त्र्यश्वा [73]
लवा नेत्रेषवः [52] कलाः।
दक्षिणो विशिषो नन्द [9]-
लवास्त्रि [3] कलिकाधिकाः(1) ॥६६॥(2)

Apparatus : (1) B: *-kaḥ* for *-kāḥ.* (2) This verse is vs.67 in Ms. B.

(1)लुब्धकस्य द्विनागां [82] शा (2)
राम[3] लिप्ताधिका ध्रुवः।
खवेदां[40]शाः(3) शरो (4)याम्यो
(5)हुताशनकलोनितः॥६७॥(6)

Apparatus : (1) B: *la*-for *lu*-. (2) Ms. B adds visarga after *-śā.* (3) Ms.B lacks visarga after *-śā.* (4) B:*māṃmye* for *yāmyo.* (5) B:*-śīna* for *-śana.* (6) This verse is vs. 68 in Ms.B.

कवष्टौ[81] (1)भागा ध्रुवो [ऽ]गस्ते(2)
रावणघ्न(3) [3]कलाधिकाः।
याम्येषू(षु) रसशैलां [76]शा
यमलाब्धि[42] कलाधिकाः(4) ॥६८॥(5)

Apparatus : (1) B: *bhya*-for *bhā*. (2) Ms.B adds visarga after-*ste*. (3) Ms.B adds visarga after-*ghna*. (4) Ms.B lacks visarga after -*kā*. (5) This verse is vs.69 in Ms.B.

कलाङ्का[9] (1)ढ्याक्षधृत्यं(2)[185] शा(3)
अपांवत्सध्रुव:(4) स्मृत:।
उदग्राम[3]लवा बाणो
नग[7]लिप्तोनितास्तथा(5) ॥६९॥(6)

Apparatus : (1) B:-*dyā*-for -*ḍhyā*-. (2) B:-*tyāṁ*-for -*tyaṁ*-. (3) Ms.B. adds visarga after -*śā*. (4) Ms.A lacks visarga after -*va*. (5) A:-*tas* for -*tās*. (6) This verse is vs. 70 in Ms.B.

त्र्यं[3] शोना गजधृत्यं [188] शा
(1)नन्दां [9] शा विशिखो (2) ह्युदक् (3)
सार्धाश्चैवापसंज्ञस्य(4)
स्फुटा(5) दृक्कर्मसंस्कृता: ॥७०॥(6)

Apparatus : (1) A: *gajāṃśa* for *nandāṃśā*. (2) B:-*śe*-for -*śi*-. (3) Ms.B adds *va* after *udak*. (4) Ms.B lacks *pa* after -*vā*. (5) Ms.B adds visarga after -*ṭā*. (6) This verse is vs.71 in Ms.B.

63-70: "[Polar longitude and latitude of some stars:]

[Name of the star:]	[longitude:]	[latitude:]
Agnibhuj	53°16′	8° 9′ N
Saddhṛd	55°19′	30°34′ N
Prajāpati	63° 34′	38° 32′ N
Sūryoktārdrā	73°52′	9° 3′ S
Lubdhaka	82° 3′	39° 57′ S
Agasti	81° 3′	76° 42′ S
Apāṃvatsa	185° 9′	2° 53′N
Āpa	187°40′	9°30′ N

These are the polar coordinates converted by the "[*ayana*-] *dṛk-karman*"."

Among the above names, the "Saddhṛd" is the same as the "Brahmahṛd", and the "Agasti" is the same as the "Agastya".

The Ms.B has a table of the above mentioned stars in its folio 5b as follows.

Madhyamā dhruvakāḥ (Celestial longitude [and latitude]) :

Agni	Brahma	Prajā°	Sū°Ārdrā	Lubdhaka	Agastya	Apādha	Āpa
1	1	2	2	2	2	6	6
24	28	5	13	24	25	3	3
4	36	53	47	36	3	50	50
8	30	38	9	40	77	3	9
14	48	38	4	4	16	8	10
N	N	N	S	S	S	N	N

Spaṣṭa-dhruvakāḥ (Polar longitude [and latitude]) :

Agni	Brahma	Prajā°	Sū°Ārdrā	Lubdhaka	Agasti	Apavaṃśa	Āpa
1	1	2	2	2	2	6	6
23	25	3	13	22	21	5	7
16?	19?	34?	52	3	3	9	40
8	30	38	9	39	76	2	9
9	34	31	3	57	48	53	26
N	N	N	?	?	?	?	?

In the above table, the longitude is given in terms of signs, degrees, and minutes, and the latitude is given in terms of degrees and minutes, and its direction (north or south) is added.

Among the above figures, the celestial latitude of Brahmahṛd, and the polar latitude of Prajāpati, Agastya, and Āpa are different from the text.

All these stars except for Sūryoktārdrā are mentioned in the *Sūrya-siddhānta* (VIII.10-12 and 20-21) also. The 'Sūryoktārdrā", which means the *nakṣatra* Ārdrā told by Sūrya, is not clear to me. Its position is different from that of the *nakṣatra* Ārdrā in the *Sūrya-siddhānta*. So, the "Sūrya" meant by Padmanābha may by something different.

(7) Other minor stars

The polar longitude and latitude of some other minor stars are given in vss. 71-78 of Ms.A. These verses do not appear in Ms.B.

ध्रुवो याम्यवसो ज्ञेय:
    पञ्चाङ्कां [95] शा: शारस्तथा।
याम्या: षोडश [16] भागाश्च
    नख [20] लिप्ताधिका [:] स्फुट:॥७१॥
नागेन्दु (न्द्र?) विशिखा [58?] भागा
    ध्रुवो वरुणबन्धुज:।
याम्यदिग्विदिश (शिख?) स्तस्य
    शरनेत्र [25] लवास्तथा॥७२॥
नवेन्दुहुतभुग् [319] भागा
    ध्रुव:[1] कलजस्मृत:।
व्योमाक्षा [50] विशिख: सौम्यो
    भागा [:] सार्धा [:] परिस्फुटा:॥७३॥

Apparatus: (1) One syllable is missing before or after *dhruvaḥ*. A Particle such as *ca* or *tu* may be supplied after *dhruvaḥ*.

शिशुमारस्य तारााया: (-या)
    ध्रुव: सप्ताक्षबाहव: [257]।
उत्तरो विशिखो भागा:
    षट् [6] षष्टि [60] भागवर्झि (र्जि) ता:॥७४॥
ध्रुवकस्तोरणाख्यस्य
    पञ्चलोचनबाहव: [225]।
भागास्तस्यैव विशिख:
    सौम्यस्तत्व (त्त्व) [25] लवा[:] स्मृता:॥७५॥
खाम्भोनिधियमां [240] शाश्च
    ध्रुवको [ऽ]श्वमुखस्य च।
षट्पञ्चाशल् [56] लवा: सौम्यो
    विशिख: सदलास्तथा: (-था)॥७६॥
प्राग्मुनेर्ध्रुवकस्त्यङ्क-प्रुचन्द्रा [193]
स्नाङ्गान्यु [63] दक् शर:।
उ(ऊ-) र्ध्वाध: पश्चिमगयो:
    शशीन्द्रा: (न्द्रा) [141] ध्रुवकस्तयो:॥७७॥
स्वा (खा-?) क्षां [50?] शा ऊर्ध्वगस्योदक्
शरो ऽन्यस्य शरेषव: [55]।
    वशिष्ठस्यां (-स्या-) ग्निधृत्यं [183] शा: (-शा)
ध्रुवको [ऽ]ङ्गरसा: [66] शर:॥७८॥

71-78: "[Polar longitude and latitude of minor stars:]

[Name of the star :]	[longitude:]	[latitude:]
Yāmyavas	95^0	$16^0 20'$ S
Varuṇabandhu	58^0(?)	25^0 S
Kalaja	319^0	$50^0 30'$ N
Śiśumāra	257^0	$5^0 59'$ N
Toraṇa	225^0	25^0 N
Aśvamukha	240^0	$56^0 30'$ N
Prāgmuni	193^0	63^0 N
Ūrhdva-paścimaga	141^0	50^0 (?) N
Adhaḥ-paśemaga	141^0	55^0 N
Vaśiṣṭha	183^0	66^0 N''

Since the above text (vss.71-78) is based on one manuscript only, its reading is highly tentative.

Ms. A, after vs. 78 is a cancelled colophon as follows.

नार्मदः (-द) इदं यन्त्रं चकारः (-र)।इत्यन्वयः॥ ॥ ज्योतिश्रीनर्मदात्मजज्योतिश्री पद्मनाभविरचितायां यन्त्रकिरणावल्यां स्वविवृत्तौ यन्त्रराजाधिकारः प्रथमः॥ ॥ शुभं भवतु ॥ ॥ श्रीरस्तु॥ ॥ ग्रन्थसंख्या ७५० स्वस्ति श्रीसंवत् १६३४ वर्षे मार्गशीर्षमासे शुक्लपक्षे षष्ठीभृगुवासरे लिखितम्॥ ।छ॥ ॥

In Ms. A, after the above cancelled colophon is vs. 80. So, vs. 79 does not exist in this manuscript.

c) The use of the astrolabe

The remaining part (vss.80-115 of Ms.A, and vss. 72-107 of Ms.B) of this work is devoted to the use of the astrolabe and related topics. Let us see the text.

Firstly, the "*ayana-dṛk-karman*", that is the method to convert celestial longitude into polar longitude, is explained as follows.

दोर्ज्या विश्व १३ गुणा द्विराम ३२ विहृता क्रान्तिज्यका(1) सा भवेत्
तद्वर्गोनपरज्यकाकृतिपदं (2)द्युज्याथ(3) कोटेरिति(4)।
क्रान्तिज्या वलनाह्वया(5) भवति तन्मध्येषुजीवा वधाद्(6)
(7)द्युज्यावाप्तधनुः रवाग्नि ३०० गुणितं (8)व्यक्षोदयाप्तं(9) फलम्॥८०॥(10)

Apparatus: (1) B: *pya* for *jya*. (2) A: *dā* for *dyu*. (3) B: *jya* for *tha*. (4) A: *-ṭī-* for *-ṭe-*. (5) A: *-ya-* for *-hva-*. (6) Ms. B lacks *d* after *-dhā*. (7) B: *kva* for *dyu*. (8) A: *vakṣyo* (?) for *vyakṣo*. (9) B: *-stāṃ* for *- ptaṃ* (10) This verse is vs. 72 in Ms.B.

तद्बाणायनयोः (1)समान्यककुभोर्हीनं (2)युतं व्योमगे
कुर्यादायनजं(3) स्फुटापमलवाक्षांशैः(4) स्वमध्योन्नतः।
कृत्योरद्वान्तरतः पदेन वलनज्यात्रिज्ययेराहता
बाणज्या(5) त्रिभजीवया च विहृता(6) चापं शरः स्यात् स्फुटः॥८१॥(7)

Apparatus: (1) *A: sā-* for *sa-*. (2) A: *cyu-* for *yu-*. (3) B: *-da-* for *-dā-*. (4) Ms. B lacks visarga after *-śai*. (5) B: *-ṇā* for *-ṇa*. (6) Ms. A adds *c* between *-tā* and *cā-*. (7) This verse is vs. 73 in Ms.B.

80-81: "R.sine of the longitude *(dorjyā)* [of a star] multiplied by 13 and divided by 32 is the R.sine of the declination (*krāntijyā*) [of a point on the ecliptic whose longitude is equal to the star's longitude].

The square-root of the difference which is the square of the Radius minus the square of it (R.sine of the declination) is the "day-radius" *(dyujyā)*.

The R.sine of the declination of [the point on the ecliptic whose longitude is equal to] the complementary angle (*koṭi*) [of the longitude of the star] is called *valana* (or more properly "*aynana- valana-jyā*").

Multiply it by the R.sine of the [star's] celestial latitude (*madhya-iṣu-jīvā*) and divide by the "day-radius". Its corresponding arc is to be multiplied by 300 and divided by the right ascension [of the width of the sign where the star is situated in terms of *vināḍīs*].

The result, the *ayana-*[*dṛk*] *-phala*, is to be subtracted from or added to [the celestial longitude of] the star when it latitude and the *ayana* (when the longitude is from 270° to 90°, the *ayana* is north, and the rest is south) are in the same direction or in different directions respectively.

Using the star's own declination (*sphuṭa-apama*) and the observer's latitude (*akṣa*), its meridian altitude (is calculated).

Multiply the R·sine of the celestial latitude (*bāṇa-jyā*) by the square root of the difference of the squares of the *valana-jyā* and the Radius, and divide by the Radius. Its corresponding arc is the polar latitude (*sphuṭa-śara*).

Firstly, the declination of the point on the ecliptic whose longitude is the same as that of the star is calculated by the following equation.

$$R{\cdot}\sin\lambda \times \frac{13}{32} \quad R{\cdot}\sin\delta', \qquad \text{-------} \ (1)$$

where λ is the star's celestial longitude, and δ' the declination of a point on the ecliptic [whose longitude is equal to the star's longitude.]

The rationale of this equation is as follows. Regarding the relationship between the longitude and declination of a point on the ecliptic, we know the following equation.

$$\sin \lambda' \cdot \sin \varepsilon = \sin \delta',$$

or, in Hindu notation,

$$R\cdot\sin \lambda' \times \frac{R\cdot\sin \varepsilon}{R} = R\cdot\sin \delta',$$

where ε is the obliquity of the ecliptic, and λ' and δ' the longitude and declination of the point on the ecliptic respectively. According to Padmanābha's auto-commentary, $R = 120$ and $R\cdot\sin \varepsilon = 48 \frac{45}{60}$. So,

$$\frac{R\cdot\sin \varepsilon}{R} = \frac{13}{32}, \text{ approximately.}$$

Hence the equation (1)

Then, the following equation is given.

$$\sqrt{R^2 - (R\cdot\sin \delta')^2} = R\cdot\cos \delta' = \text{“day-radius” } (dyujy\bar{a}). \text{ --------- } (2)$$

The "day-radius" is the radius of the diurnal circle of a point on the ecliptic.

The *ayana-valana* is the angle between the secondaries to the equator and to the ecliptic at the star. It is zero when the star's longitude is 90°, and is approximately ε when its longitude is 0° if the star is near to the ecliptic. So, the declination of the point on the ecliptic whose longitude is the complementary angle of the star's longitude can roughly be considered to be the *ayana-valana*, and its R·sine the *ayana-valana-jyā.*

Then, Padmanābha tells to obtain one value as follows. Let this value be R·sin X.

$$ayana\text{-}valana\text{-}jy\bar{a} \times \frac{R\cdot\sin \beta}{\text{“day-radius”}} = R\cdot\sin X, \qquad \text{-------- } (3)$$

where β is the star's celestial latitude.

The R·sin X is converted into its corresponding arc X, and the following value is calculated. Let this value be Y.

$$X \times \frac{300}{RA} = Y, \qquad \text{--------- } (4)$$

where RA is the right ascensional width of the sign where the star is situated in terms of *vināḍīs*. The number 300 is the right ascensional width of a sign in terms of *vināḍīs* if ε is zero.

This result Y is the *ayana-dṛk-phala* (or *āyana-dṛk-phala*), that is the difference between the celestial longitude and the polar longitude of the star. Applying this value to the celestial longitude, the polar longitude is obtained.

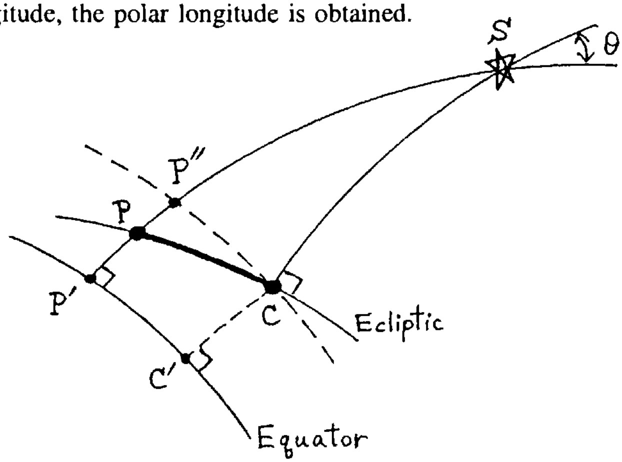


Fig.29

The rationale is as follows. In Fig.29, S is the star, SC the secondary to the ecliptic passing through the star, and SP′ the secondary to the equator passing through the star. Then the point C indicates the star's celestial longitude, and the point P its polar longitude. So, the arc CP is the *ayana-dṛk-phala*. Let CP" be an arc of the diurnal circle of the point C, and CC′ the perpendicular to the equator passing through the point C. The arc CP" can roughly be considered to be the perpendicular to the arc SP′ passing through the point C. Considering the triangle SCP" to be a right-angled spherical triangle, we have the following equation.

$$\text{R·sin (CP")} = \frac{\text{R·sin}\,\beta \;\;\text{x}\;\; \text{R·sin}\,\theta}{\text{R}} ,$$

where θ is the *ayana-valana*. It may be noted here that Padmanābha himself did not use spherical triangle in our sense. His method is as follows. When SC is 90°, R·sin (CP") is R·sin θ , and when SC is 0°, CP" is zero. Hence the above equation by the rule of three. Now, since the arc CP" is an arc of the diurnal circle of the point C whose declination is δ', we have the following equation.

$$\text{R·sin (C′P′)} = \text{R·sin (CP")} \;\text{x}\; \frac{\text{R}}{\text{R·cos}\,\delta'}$$

$$= \frac{\text{R·sin}\,\beta \;\;\text{x}\;\; \text{R·sin}\,\theta}{\text{R·cos}\,\delta'}$$

This is the same as the equation (3), where X corresponds to C'P'. Then, evidently,

$$CP \doteq C'P' \times \frac{300}{RA}$$

Hence the equation (4).

If the star's latitude is north, the value CP is positive when the star's longitude is from 90° to 270°, i.e. in southern *"ayana"*, and the value is negative when the longitude is from 270° to 90°, i.e. in northern *"ayana"*. If the star's latitude is south, the sign is opposite.

Now, let us proceed to the method to obtain polar latitude from the celestial latitude. The reading of the text seems to be expressed as follows.

$$R\cdot\sin B = R\cdot\sin\beta \times \frac{\sqrt{R^2 - V^2}}{R}, \qquad \text{------- (5)}$$

where B is the star's polar latitude, R the Radius, V the *ayana-valana-jyā*, and β the star's celestial latitude.

However, this equation is quite strange. Let us consider the right-angled spherical triangle SPC in Fig. 29. Here, SC is β and SP is B. This triangle can be considered to be a plane triangle if it is small.

Then, approximately,

$$R\cdot\sin(S\hat{P}C) \doteq R\cdot\cos(C\hat{S}P) = \sqrt{R^2 - V^2}.$$

Therefore,

$$R\cdot\sin(SP) \doteq R\cdot\sin(SC) \times \frac{R}{\sqrt{R^2 - V^2}}.$$

Then, the polar latitude should be obtained by the following equation.

$$R\cdot\sin(B) \doteq R\cdot\sin\beta \times \frac{R}{\sqrt{R^2 - V^2}}. \qquad \text{-------- (6)}$$

This equation contradicts the equation (5). This point puzzles me.

Now Padmanābha proceeds to explain the ***akṣa-dṛk-karman***, that is the method to find the point of the ecliptic which rises or sets simultaneously with the star applying to the star's polar longitude.

प्रस्फुटेषुलवसिञ्जीनीहता-
क्षज्यका त्वपहृता(1) विलम्बया।
(2)तद्धनुर्यमदिगुत्तरे शरे
(3)स्वर्णमी दयिकगे(4) [ऽ]स्तगे [ऽ]न्यथा॥८२॥(5)

Apparatus: (1) B: *-hā-* for *-hṛ-*. (2) B: *-dva-* for *-ddha-*. (3) B: *oda-* for *auda-*. (4) A: *-kaye* for *-kage*. (5) This verse is vs.74 in Ms. B.

82: "The R sine of the observer's latitude (*akṣa-jyakā*) is to be multiplied by the R sine of the Star's polar latitude (*prasphuṭa-iṣu-lava-siñjinī*) and divided by |the R sine of | the observer's colatitude (*vilamba*). The corresponding arc of the result is to be added to or subtracted from |the star's polar longitude| according as the star's latitude is south or north when the star is rising. When setting, do oppositely."

The above method can be expressed as the following equation.

$$\text{R·sine of } \textit{akṣa-dṛk-phala} = \frac{\text{B x R·sin } \varphi}{\text{R·cos } \varphi}, \qquad \text{-------- (7)}$$

where B is the star's polar latitude, and φ the observer's latitude.

Applying this value to the star's polar longitude, the longitude of the point of the ecliptic which rises or sets simultaneously with the star is obtained.

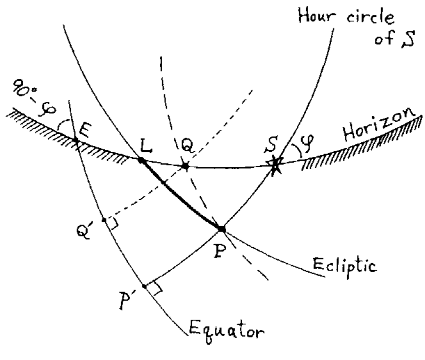


Fig.30

Its rationale is as follows. In Fig. 30, S is the rising star. SP the hour circle passing through the star S, P the point of the ecliptic which meets the hour circle SP, and L the rising point of the ecliptic. Let PQ be the diurnal circle passing through the point P, and Q its intersection with the horizon. The arc PL is the *akṣa-dṛk-phala*. Applying it to the longitude of the point P, that is the polar longitude of the star S, the longitude of the rising point L of the ecliptic is obtained.

Now, the arc PL is approximately the same as the arc PQ, if the star's latitude is small. Let us consider the triangle SQP, which may be considered to be a right-angled spherical triangle, where the angle SPQ is right angle, if it is small. Here,

$SP = B$,

$Q\hat{S}P = \varphi$,

$S\hat{Q}P = 90^{0} - \varphi$

Therefore,

$$R{\cdot}\sin(PL) = R{\cdot}\sin(PQ)$$

$$= \frac{R{\cdot}\sin(SP) \times R{\cdot}\sin(Q\hat{S}P)}{R{\cdot}\sin(S\hat{Q}P)},$$

$$= \frac{B \times R{\cdot}\sin\varphi}{R{\cdot}\cos\varphi}$$

Hence the equation (7).

When the star is rising, the value PL is positive if the star's latitude is south, and is negative if the star's latitude is north. When the star is setting, the sign is opposite.

Regarding the *akṣa-dṛk-karman*, Padmanābha adds as follows. The following verse appears in Ms. A only, where it is numbered as vs.82 again. Let the following verse be called vs.82′.

मार्गणे महति मध्यमस्फुट-
क्रान्तितश्चरलवान्तरैक्यकं ।
तुल्यभिन्नककुभोः पजो (लो?) द्धवं
दृक्प (फ) लं धनमृणं च पूर्ववत्॥८२'॥

82′: "When the star's latitude is large, the difference or the sum of the ascensional differences (*cara*) of the star's "mean declination" (*madhya-krānti*) and of the star's "true declination" (*sphuṭa-krānti*) [is to be calculated], according as the declinations are in the same direction or in different directions. [Otherwise,] the *akṣa-dṛk-phala* (*pala-dṛk-phala*), which is positive or negative, [may be used] as before (i.e. as in vs.82)."

Here, another method to find the point of the ecliptic which rises or sets simultaneously with the star is given. Here, the "mean declination" *[madhya-krānti]* is the declination of the point of the ecliptic whose longitude is equal to the star's polar longitude (P′P in Fig. 30), while the star's own declination (P′S in Fig. 30), is called "true declination" (*sphuṭa-krānti*).

The rationale of this rule is as follows. In Fig.30, the arc PL is the *akṣa-dṛk-phala*. Let QQ′ be the hour circle passing through the point Q, Q′ its intersection with the equator, and E the intersection of the equator and the horizon. The arc PLe is approximately the same as the arc P′Q′. Now, EP′ is the ascensional difference of the star S, and EQ the ascensional difference of the point Q which is equal to that of the point P. Here, the ascensional difference (*cara*) is the arc of the equator lying between the six o'clock circle and the hour circle of the star at rising. Therefore,

$$PL = P'Q'$$

$$= EP' \mp EQ'$$

$$= (\textit{cara} \text{ of S}) \mp (\textit{cara} \text{ of P}).$$

Hence the above rule.

When the star S and the point P are in the same direction with regard to the equator, the difference is taken. If in different directions, the sum is taken, because the ascensional differences of these two points are in the opposite directions.

Now Padmanābha explains the method to represent the *akṣa-dṛk-phala* (= *ākṣa-dṛk-phala)* and the *ayana-dṛk-phala* (=*āyana-dṛk-phala*) on the astrolabe.

पट्टिका ध्रुवगतेष्टवृत्तगा
(1)व्यक्तमा क्षजफलं हि(2) दृश्यते ।
स्वध्रुवादिषुमुखं कदम्बगं
तत्खमध्यगतमायनं(3) तथा ॥८३ ॥(4)

Apparatus : (1) B: *-kṣi* for *-kṣa*. (2) A: *dvi* for *hi*. (3) A: *-gatm* for *-gatam*. (4) This verse is vs.75 in Ms. B.

83 : "Let the indicator (*paṭṭikā*) directed towards the star's [polar] longitude (*dhruva*) meet with the desired circle (horizon) [at the mark of the star], then the "*ākṣa-*[*dṛk*]*-phala*" is manifestly seen.

Let [the indicator of the star which is] directed towards the tip of its latitude (*iṣu*) from the point which indicates its [celestial] longitude (*dhruva*), which points to the pole of the ecliptic, meet with the line which passes through the zenith (meridian line), then the *āyana-*[*dṛk-phala*] [is seen]."

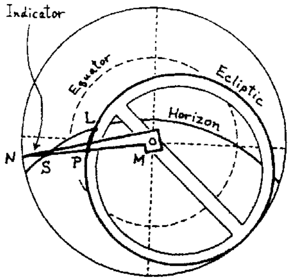


Fig.31

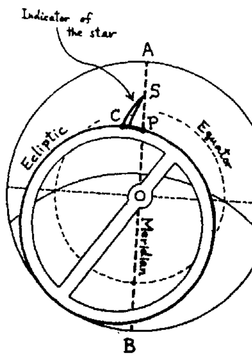


Fig. 32

The method to represent the *akṣa-dṛk-phala* on the astrolabe is as follows. In Fig. 31, MN is the indicator, on which S indicates the position of the star, and P the star's polar longitude. The indicator itself represents the hour circle of the star. Let the point S meet with the horizon. Then the point L indicates the point of the ecliptic which rises simultaneously with the star. Therefore, the arc PL represents the *akṣa-dṛk-phala* . In his auto-commentary, Padmanābha also tells to represent similar values using altitude circles instead of the horizon.

The method to represent the *ayana-dṛk-phala* is as follows. In Fig.32, CS is the indicator of the star, on which S indicates the position of the star, and P the star's celestial longitude. The straight line AB represents the meridian. Let the point S meet with the meridian. Then the intersection P of the meridian and the ecliptic indicates the star's polar longitude, because the meridian AB represents the secondary to the equator passing through the star. Therefore, the arc CP represents the *ayana-dṛk-phala.*

Now Padmanābha explains the construction of the back of the astrolabe. He described two alternative designs of the back. The first design is as follows.

ततो [ऽ]न्यपार्श्वे [ऽ]र्धयवान्तराले(1)
रेखोपरिष्टान्नलिकां तु (2)कुर्यात।
नेम्यामधस्तादुभयत्र तिर्य -
ग्रेखां विदध्यादपि खाङ्कभागान्(3)॥८४॥ (4)

Apparatus: (1) B: *yalā*-for *yavā*-. (2) B: *tiryak* for *kuryāt.* (3) A: *-āt* for *-ān.* (4) This verse is vs.76 in Ms.B.

वेधयेत् (1) सवितृमण्डलं तथा
खेचरं निशि तु वा भतारकम्(2)।
केन्द्रकीलकविलम्बसूत्रगा -
(3)स्ते स्युरिष्टनिजशङ्कुभागका:॥८५॥(4)

Apparatus: (1) Ms. A adds *tu* after *-yet*, but it disturbs metric. (2) B: *-kāṃ* for *-kam.* (3) Ms. B lacks *s* after *-gā.* (4) This verse is vs.77 in Ms. B.

84 : "Then, on the backside, one should make a tube above [the horizontal central] line, at the distance of a half *yava.* On the rim, from the bottom upto the horizontal line on both sides, one should graduate 90 degrees."

85 : "One should observe the sun, or a planet or a star at night [through the tube]. The position of the plumb line which is hung from the central pin is the desired altitude."

A *yava* is an unit of length, and is $\frac{1}{6}$ or $\frac{1}{8}$ *aṅgula.* A tube is fixed to the backside of the astrolabe horizontally, at the distance of a half *yava.* It is clear that the altitude (χ) is obtained by the above method. (see Fig.33.)

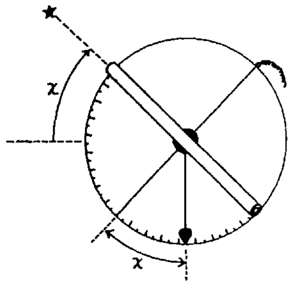


Fig. 33

The second design of the backside of the astrolabe described by Padmanābha is as follows.

पट्टिकामुत सरन्ध्रकीलक-
    द्वंद्वयुकतनुमुखीं नियोजयेत्।
यन्त्रपृष्टगतकेन्द्रकीलके(1)
    वेधयेद्दिविचरं तथानया॥८६॥(2)

Apparatus : (1) A: - *kaṃ* for *-ke*. (2) This verse is vs. 78 in Ms.B.

धारयेल्लम्बवद्यन्त्रं पट्ट्या संविध्यते यदि।
(1)ऊर्ध्वार्धे (2)चोन्नता भागा ज्ञेया:(3) पूर्वापरार्धगा:॥८७॥(4)

Apparatus: (1) B: *ūrddhā* for *ūrdhvā-*. (2) B: *yo-*for *co-*. (3) Ms.B lacks visarga after *-yā*. (4) This verse is vs. 79 in Ms.B.

86: "Or, one should put [a movable] indicator with pointed ends, which has a pair of wedges with a hole, to the central pin on the backside of the instrument.One should observe a heavenly body by it."

87: "One should hang the instrument vertically. If [the heavenly body] is seen along the indicator, the altitude, which is [shown] at the upper half from the east-west line, is known."

This design is also clear. The indicator (*paṭṭī*) is the alidade which is commonly used in the ordinary astrolabe. (see Fig. 34.)

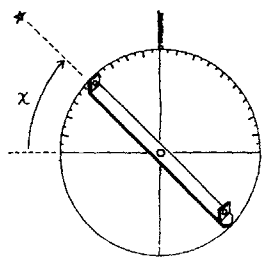


Fig. 34

Now let us see the method of the determination of time from the altitude of the sun thus obtained. Padmanābha writes as follows.

भमण्डलस्थं सचलांशसूर्यं
कुजे [1]निदध्याद्दल [2]ये तु[3] पट्टीम्।
एवं परिभ्राम्य भचक्रमर्कं
निवेशयेदिष्ट नरांशरेखाम्[4] ॥८८॥[5]

Apparatus: (1) B: *-dhyo-* for *-dhyā-*. (2) B: *-jaye* for *-laye*. (3) B: *ta* for *tu*. (4) A: *nirā-* for *narā*. (5) This verse is vs.80 in Ms. B.

पट्टूयग्रके [1]यातविनाडिका:[2] स्यु-
रेवं हि पश्चादगता[3] विनाड्य:।
स्वस्थानत: केन्द्रचतुष्कमेवं
नराद्ययनेहादिकमत्र साध्यम्॥८९॥[4]

Apparatus: (1) B: *yāvṛ* for *yāta*. (2) Ms. B lacks visarga after *-kā*. (3) A: *-dgata* for *-dagatā*. (4) This verse is vs.81 in Ms. B.

88: "One should adjust the position of the tropical longitude of the sun (*sa-calāṃśa-sūrya*) on the ecliptic circle to the horizon, and put the indicator on the six o'clock line. Thus, one should rotate the ecliptic circle [with the indicator] and adjust the position of the sun to the parallel of desired (observed) altitude."

89: "At the end of the indicator is [the graduation of *nāḍikās* and] *vināḍikās* elapsed, or [*nāḍīs* and] *vināḍīs* remaining in the afternoon. The four [*lagnas* which are at four directions] from the centre at their own positions, and the altitude and the hour angle etc. are also obtained here."

The meaning of these verses will be clear. The front side of the astrolabe is now used. If the mark of the sun on the ecliptic circle is rotated from the horizon to the parallel of observed altitude, the rotated angle is equal to the rotation of the celestial sphere, that is the time elapsed. Then, four *lagnas* can directly be read as the intersections of the ecliptic circle with the east and west horizon and the visible and invisible meridian.

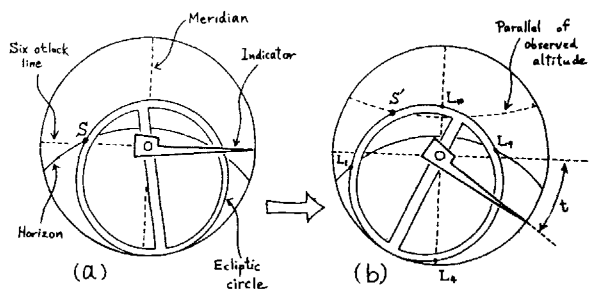


Fig. 35

This is the method of the determination of the time and *lagna* from the sun's altitude. The process of this method is shown in Fig.35. In Fig.35(a), the point S indicates the sun's longitude on the ecliptic circle, and is adjusted to the horizon circle. The indicator is adjusted to the six o'clock line, and fixed to the ecliptic circle. The point S is rotated upto the parallel of observed altitude of the sun. The result is the point S′ in Fig.35(b). Then the angle of rotation (t in Fig. 35(b)) corresponds to the time elapsed since sunrise. Here, four *lagnas* are indicated as L_1 (the 1st *lagna* or the point of the ecliptic which meets eastern horizon), L_4 (the 4th *lagna* or the point of the ecliptic which meets invisible meridian), L_7 (the7th *lagna* or the point of the ecliptic which meets western horizon), and L_{10} (the 10th *lagna* or the point on the ecliptic which meets visible meridian).

Now Padmanābha explains the method of the determination of the time at night from a star's altitude.

(1)भपट्टिका(2) स्वध्रुवके निदध्यात्
तदग्रभं(3) (4)तत्तरणिं प्रकल्प्य(5)।
नृलग्नकालादिकमस्य(6) साध्यं
प्राग्वत्ततो रात्रिगतागतं च॥९०॥(7)

Apparatus : (1) Ms. A lacks *bha* before *paṭṭī-*. (2) Ms. B lacks *kāṁ*. (3) A: *gaṃ* for *bhaṃ*. (4) A: *vaitaraṇiṃ* for *tattaraṇiṃ*. (5) B: *-lya* for *-lpya*. (6) B: *-dhya* for *-sya*. (7) This verse is vs.82 in Ms.B.

दत्वार्कमस्तक्षितिजे तु(1) पट्टी-
मुन्मण्डले (2)भध्रुवके स्वपट्टीम्।
(3)ततः (4)परिभ्रम्य निजांशभागं(5)
तावद्गतं स्याद् दिनपट्टिकाग्रात्॥९१॥(6)

Apparatus : (1) A: *ṃdu* for *tu*. (2) A: *sve* for *bha*. (3) B: *dṛḍhaṃ* for *tataḥ*. (4) B: *bhrā-* for *bhra-*. (5) A: *bhaṃ-* for *bhā-*. (6) This verse is vs. 83 in Ms. B.

इष्टर्क्षपट्ट्यग्रकमात्मशंकौ
धृत्वा ततस्तूद्वलये(1) (2)[ऽ]र्कपट्टीम्।
(3)कुर्याद् दृढं प्रास्य(4) रविं (5)कुजस्थं
प्राग्वद्भवेयुः क्षणदैष्यनाड्यः(6)॥९२॥(7)

Apparatus: (1) Ms. B gives *la* twice. (2) B: *tu* for, *rka*. (3) Ms. B lacks *d* after *kuryā*. (4) B: *-pya* for *-sya*. (5) Ms. B lacks anusvāra after *ravi*. (6) A: *bhājyaṃ* for *nāḍyaḥ*. (7) This verse is vs. 84 in Ms. B.

90) : "Fix the star's indicator (***bha-paṭṭikā***) at its longitude. Providing [the mark of] the star at its tip and that of the sun [on the ecliptic circle], the altitude [of the star], ***lagna***, and time etc. are determined. Then, the time elapsed [since sunset] or remaining [until sunrise] at night [is obtained] as before."

91: "Place [the mark of] the sun [on the ecliptic circle] on the setting (western) horizon, the indicator (radial indicator) on the six o'clock line, and the own indicator (star's indicator) at the star's longitude. After that, rotate [the mark of the star] upto its degrees [of its altitude], then the time elapsed [since sunset] is obtained from the tip of the day-indicator (radial indicator)."

92 : "Place the tip of the star's indicator on the parallel of observed altitude of the star, and the solar indicator (radial indicator) on the six o'clock line, and fix them. Rotate [the mark of] the sun [on the ecliptic circle] upto the [eastern] horizon. The time remaining [until sunrise] in terms of *nāḍīs* at night is obtained as before."

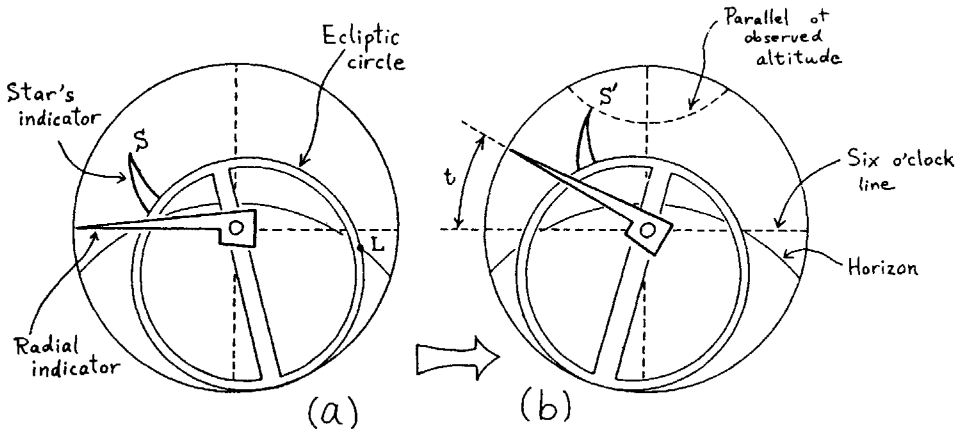


Fig. 36

The process of the determination of time from the star's altitude is shown in Fig.36. Firstly, as in Fig. 36(a), the point L of the ecliptic, which indicates the sun's longitude, is adjusted to western horizon, and the radial indicator is adjusted to the six o'clock line and fixed to the ecliptic circle. Then the tip S of the star's indicator, which indicates the star's position, is rotated upto the parallel of observed altitude of the star. The result is the point S' in Fig.36(b). Then the angle of rotation (*t* in Fig 36(b)) corresponds to the time elapsed since sunset.

The time remaining until sunrise can also be determined by the astrolabe similarly.

Now Padmanābha explains the method of the determination of the longitude of a planet on the astrolabe as follows.

द्वित्राणि विध्वाभिमतानि(1) भानि
तत्संस्थितेः प्राक् स्थिरमृक्षचक्रम्।
कृत्वा ग्रहस्योन्नतभागगे [ऽ] ंशे
चक्रस्य(2) राशिः(3) द्युचरः (4)स्फुटः(5) स्यात्॥९३॥ (6)

Apparatus: (1) A: *-ddhā-* for *-dhvā.* (2) B: *-stha* for *-sya.* (3) B: *-śerkka-* for *-śiḥ dyu-* (4) A: *spu-* for *sphu-.* (5) Ms. B lacks visarga after *-ṭa.* (6) This verse is vs.85 in Ms. B.

93 : "Firstly, after observing desired two or three stars, fix the ecliptic circle in such a way that it corresponds with their position. Then the sign [and degrees etc.] of the [ecliptic] circle which meets the parallel of altitude of the planet is the exact [longitude of the] planet."

The above method is clear. If two or three indicators of stars are adjusted to their observed altitudes, the ecliptic circle of the astrolabe represents the actual position of the ecliptic. Therefore, its intersection with the parallel of altitude of desired planet indicates the longitude of the planet.

This method is not exact if the planet's latitude is not zero. So, Padmanābha adds as follows.

नमनोन्नमने(1) बाणाद्
भवतो [ऽ]त उपस्फुटः।
पूर्वापरसमाने(2) ह
सिद्धयोगदलं स्फुटः॥९४॥(3)

Appartus: (1) B: *-nnā-* for *-nna-.* (2) B: *sā-* for *sa-.* (3) This verse is vs.86 in Ms.B.

94 : "Due to the latitude [of the planet], its zenith distance and altitude are only approximately exact. A half of the sum of the results in the eastern hemisphere and the western hemisphere is exact."

As the direction of the error is usually in the opposite direction in the eastern and western hemisphere, this method gives better result.

Now Padmanābha starts to explain the method to calculate the time from the altitude of the sun etc. and the altitude from the time without using the astrolabe.

यन्त्रवेधविधिना(1) (2)विनाधुना-
वेत्य (3)भामभिमतोन्नतांशकैः(4)।
भास्वतो (5)ऽन्यखचरादिकस्य वा
प्रोच्यते तु समयो(6) द्युरात्रयोः॥९५॥(7)

Apparatus: (1) B: *-dhā-* for *-dhi-.* (2) B: *pinā-* for *vinā-.* (3) Ms. A lacks *ma* after *bhā.* (4) Ms. B adds *ṃ* after *-mato.* (5) Ms.A lacks avagraha before *nya.* (6) A: *-maye* for *-mayo.* (7) This verse is vs.87 in Ms. B.

95: "Without the method of observation by the instrument (astrolabe), now the time in day and night is told using the desired altitude of the sun or other planets etc. after knowing its gnomon shadow."

The method to calculate the R·sine of the altitude from the gnomon-shadow is as follows.

इष्टछायोत्थकर्णेन
(1)भजेत्त्रिज्यानराहतिम्(2)।
लब्धमिष्टोन्नतज्या स्या-
(3)दुत(4) ज्या व्यत्य यद्युते:(5) ॥९६॥(6)

Apparatus: (1) A: *virājyaṁ* for *ttrijyā*. (2) Ms. B lacks *m* after *-ti*. (3) Ms. A lacks *d* after *syā*. (4) B: *unnatā* for *uta*. (5) Ms. B lacks *tya* after *vya*. (6) This verse is vs.88 in Ms. B.

96 : "Divide the product of the Radius and the length of gnomon by the hypotenuse produced by the desired shadow. The result is the desired R·sine of the altitude.

This R·sine also corresponds to the reversed shadow (vertical shadow by a horizontal gonmon)."

The meaning of this verse is clear. The method to calculate the ascensional difference is as follows.

(1)क्रान्त्युत्क्रमज्याभ्रकु(2) १०भागहीन:
सूर्यो१२ (3)द्धृत:(4) (5)क्रान्तिगुणे (6)ऽक्षभाघ्न:।
फलस्य चापं दश १०ताडितं(7) तच्
चरं ततो [ऽ]भीष्टखगद्युरात्रम्॥९७॥(8)

Apparatus: (1) B: *akra-* for *utkra-*. (2) B: *bhṛgu* for *bhraku*. (3) Ms.B lacks *d* before *dhṛ-*. (4) Ms. A lacks visarga after *-ta*. (5) A: *krāṃta* 21 *tiṇo* for *krāntiguṇe*. (6) Ms. A lacks avagraha before *kṣa*. (7) B: *-hi-* for *-ḍi-*. (8) This verse is vs.89 in Ms.B.

97: "The R·sine of the declination [of a planet] if divided by twelve diminished by one tenth of the R·versed-sine of the declination, and multiplied by the equinoctial midday shadow (*akṣabhā*). The corresponding arc of the result multiplied by 10 is the ascensional difference (*cara*) [in terms of *vināḍīs*]. From this, the day and night (i.e. visible time and invisible time) of the planet [is known]."

The above calculation can be expressed as follows.

$$\text{R·sin}\ (cara) = \frac{\text{R·sin}\ \delta\ \times\ \text{akṣabhā}}{12 - \dfrac{\text{R·vers}\ \delta}{10}}\ , \qquad \text{--------}\ (8)$$

where δ is the declination of the planet. If the degrees of *cara* is multiplied by 10, it is converted into *vināḍīs*, because one degree corresponds to 10 *vināḍīs*.

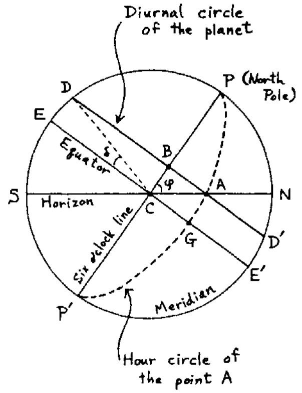


Fig. 37

The rationale of the equation (8) is as follows. Fig.37 is the orthographic projection of the celestial sphere onto the plane of the meridian. Here, CG is the R·sine of *cara* of a planet whose diurnal circle is DD′. Here, evidently,

$$CG = \frac{R}{R\cdot\cos\delta}\ BA = \frac{R}{R\text{-}R\cdot\text{vers}\ \delta}\ BA.$$

In the triangle ABC,

$$\frac{BA}{BC} = \frac{\text{akṣabhā}}{g}$$

$$BC = R\cdot\sin\delta,$$

where *g* is the height of the gnomon, because the *akṣabhā* is equal to g.tan φ, where φ is the observer's latitude. In Hindu astronomy, normal height of the gnomon is 12 *aṅgulas*, and Padmanābha uses the Radius equal to 120. Therefore,

$$CG = \frac{120 \times R.\sin\delta \times \text{akṣabhā}}{(120 - R\cdot\text{vers}\ \delta) \times 12}$$

$$= \frac{R.\sin\delta \times \text{akṣabhā}}{12 - \dfrac{R\cdot\text{vers}\ \delta}{10}}$$

Hence proved.

The method to calculate the meridian altitude of a planet is as follows.

(1)युतायनांशेष्टखगापमांश -
पलैक्यविश्लेषलवा: प्रसाध्या:(2)।
(3)समान्यदिग्धे(4)(-क्षे?)(5) नवते ९० र्विशोध्या -
(6)स्तत्सिञ्जिनी मध्यनरज्यका (7)स्यात् ॥९८॥(8)

Apparatus: (1) B: *surā-* for *yutā-*. (2) Ms. B lacks visarga after *-dhyā*. (3) A: *sāmā-* for *samā-*. (4) B: *-tke* for *-gdhe*. (5) B: *-tai-* for *-te-*. (6) Ms. A lacks *s* after *-dhyā*. (7) B: *sā* for *syāt*. (8) This verse is vs.90 in Ms. B.

98: "Determine the degrees of the sum or difference of the declination of the desired planet, which corresponds to its tropical longitude, and the observer's latitude, according as the same or different directions [of the declination and the intersection of the meridian and equator in the south direction]. Subtract it from 90°, then its R-sine is the R-sine of its meridian altitude."

Meaning of this verse is clear.

Now, the determination of the time from the altitude of the planet is explained as follows.

इष्टोन्नतज्यान्त्यगुणाभिघातान(1)
मध्योन्नतज्याप्तफलस्य चापम्।
दिनार्धनिघ्नं खनवो ९०धृ (द्धृ) तं ता:
पूर्वापरार्धे गतशेषनाड्य: ॥९९॥(2)

Apparatus: (1) B: *-dhā-* for *-ghā-*. (2) This verse is vs.91 in Ms.B.

99: "Multiply the desired altitude [of the planet] by the Radius (*antya-guṇa* or "the last sine"), and divide by the R.sine of its meridian altitude. The arc corresponding to this result is multiplied by a half of the duration of daytime [or visible time of the planet] and divided by 90. This is the elapsed or remaining *nāḍīs* [since the planet's rising or until its setting] in the eastern or western hemisphere respectively."

This method can be expressed as follows.

$$\text{R.sin T} = \frac{\text{R.sin a}' \times \text{R}}{\text{R}\cdot\text{sin a}}\ , \qquad \text{-----------} \ (9)$$

$$t = \frac{\text{T} \times d/2}{90}\ , \qquad \text{----------} \ (10)$$

where a' is the observed altitude of the planet, a its meridian altitude, t the time elapsed since its rising or remaining until its setting, and d the duration of its visible time. Here, T is only a parameter.

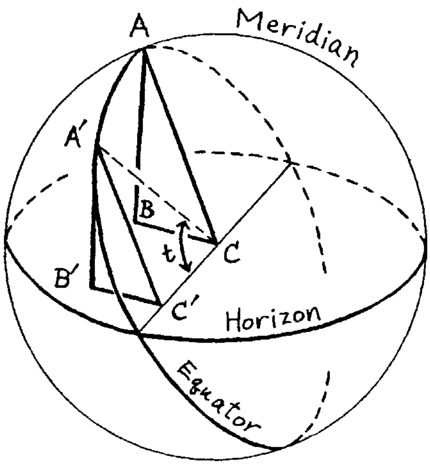


Fig. 38

This method gives correct value on equinoctial days. In Fig.38, the triangle ABC and A'B'C' are similar. Therefore,

$$A'C' = \frac{A'B' \times AC}{AB}$$

Here, AC is the Radius, AB the R.sine of the meridian altitude of the planet, A'B' the R.sine of its observed altitude, and A'C' the R.sine of the time elapsed since its rising. Hence the equation (9), where T is equal to *t* in this case. Except for equinoctial days, this method gives approximate value by applying the equation (10).

The determination of the altitude of the planet from the time is as follows.

अभीष्टनाड्यो गुणिताः खनन्दै९०
दिनार्धभक्ताप्तगुणेन निघ्नी।
मध्योन्नतज्या त्रिभजीवयाप्ता
चापं (1)तदिष्टोन्नतभागकाः(2) स्युः ॥१००॥(3)

Apparatus: (1) *ca* (?) for *ta.* (2) Ms.B lacks visarga after *-kā.* (3) This verse is vs.92 in Ms.B.

100: "The desired *nāḍīs* [since the planet's rising or until its setting] is multiplied by 90, and divided by a half of the duration of daytime (or visible time of the planet). The R.sine of the result is multiplied by its meridian altitude, and divided by the Radius. The arc corresponding to this result is the desired altitude."

This method can be expressed as follows.

$$R\cdot\sin a' = R\cdot\sin\left(\frac{90 \times t}{d/2}\right) \times \frac{a}{R} \qquad \text{---------} (11)$$

This is just the reverse of the previous method.

Padmanābha adds the parallactic correction of the planet's altitude as follows.

त्रिज्याहत: (1)स्वीयगते: शरेन्दु-
भागो [ऽ] ष्टरामाब्धिगुणैर्विभक्त: ।३४३८।
फलेन हीनोन्नतभागजीवा
शङ्कु: (2)स्फुटो [ऽ] स्मादनुपाततो भा ॥१०१॥(3)

Apparatus: (1) A: *-tāḥ* for *-taḥ*. (2) Ms. B lacks visarga after *-ṅku*. (3) This verse is vs.93 in Ms. B.

101 : "One fifteenth of the motion [of the planet in a day in terms of minutes] is multiplied by the Radius, and divided by 3438. The R·sine of the [observed] altitude minus this result is the correct R·sine of the altitude. From this, the shadow (or R·sine of its zenith distance) is calculated by proportion."

In Hindu classical astronomy, the radius of the earth is assumed to be one 15th of the linear mean daily motion of a planet, which is assumed to be uniform among all planets. In his auto-commentary, Padmanābha says that a planet's linear mean daily motion is 11857 *yojanas*, and the earth's radius 791 *yojanas*. So, the observed altitude should be less than the geocentric altitude by one 15th of the planet's mean daily motion.

In order to convert the arc in terms of minutes into its corresponding R·sine, R/3438 is multiplied, where 3438 is the length of the Radius expressed by minutes, that is $360 \times 60/2\pi$.

Padmanābha proceeds to explain the determination of the sun's declination, amplitude (*agrā*), and azimuth using the astrolabe.

उन्मण्डले क्रान्तिलवास्तथैवं
कुजे [ऽ]ग्रकां(1) मेषवृता(त्ता)त्तु(2) केन्द्रम् (3)।
बहिश्च याम्योत्तरगोलकौ(4) (5)तौ
ज्ञात्वा दिगन्तो [ऽ]क्षयुगांशकांश्च(6) ॥१०२॥(7)

Apparatus : (1) Ms. B lacks anusvāra after *-kā*. (2) Ms. B lacks *t* between *-tā* and *tu*. (3) B: *-ṃdre* for *-ndram*. (4) A: *śaila* for *gola*. (5) B: *to* for *tau* (6) Ms. B lacks anusvāra after *-kā*. (7) This verse is vs.96 in Ms.B.

> 102: "[Graduate] the six o'clock line with the degrees of declination, and similarly the horizon with those of amplitude *(agrakā)*. Assuming the inside and outside of the circle of Aries (i.e. equator) to be the southern and northern hemispheres respectively, [graduate the parallels of altitude with] 45 degrees between the directions (i.e. both sides of cardinal azimuthal directions)."

If the astrolabe is graduated as above, the sun's declination, amplitude (deviation from the cardinal point at its rising or setting), and azimuth can be read by adjusting the sun's mark on the ecliptic circle to the six o'clock line, horizon, and parallel of its observed altitude respectively. In his auto-commentary, Padmanābha instructs to determine terrestrial direction by applying the sun's azimuth thus obtained to the direction of the actual sun with the help of the astrolabe kept horizontally.

The method to calculate the sun's amplitude without using astrolabe is as follows.

क्रान्तिज्यामक्षकर्णेन
हत्वा द्वादशभिर्भजेत्।
फलमग्रा भवेत्त्रिज्या-
कर्णवृत्ते कुजानुगा(1) ॥१०३॥(4)

Apparatus : (1) Ms. B adds visarga after *-gā*. (2) This verse is vs.94 in Ms.B.

(1)स्वेष्टकर्णहता(2) छिन्ना
त्रिज्यया शङ्कुशीर्षतः।
कर्णवृत्ते प्रभाग्रस्था
साङ्गुलाद्यापमान्यदिक्(3) ॥१०४॥(4)

Apparatus : (1) A: *se-* for *sve-*. (2) A: *itā* for *hatā*. (3) *-manya* for *-mānya*. (4) This verse is vs.95 in Ms.B.

> 103: "Multiply the R-sine of the [sun's] declination by the equinoctial midday hypotenuse (*akṣa-karṇa*), and divide by 12. The result is the [R-sine of] amplitude (*agrā*) along the horizon, which corresponds to the Radius."

> 104: "This is multiplied by the desired hypotenuse, and divided by the Radius. [The result is the *agrā*] in terms of *aṅgulas* etc. towards the opposite direction of the declination, which is at the tip of the shadow, and corresponds to a circle whose radius is the hypotenuse, and centre the tip of the gnomon."

In Hindu classical astronomy, the sun's *agrā* has two meanings, i.e. (i) the sun's amplitude, that is the sun's deviation from the cardinal point at its rising (or its R-sine), and (ii) the difference between the equinoctial midday shadow and the north-south component of the shadow at the time. The latter corresponds to the R-sine of the former, where it is assumed that the hypotenuse produced by the gnomon and its shadow is the Radius. In the above verses, vs.103 gives the former, and vs.104 the latter.

The vs.103 can be expressed as follows.

$$\text{R}\cdot\sin a = \frac{\text{R}\cdot\sin \delta \ \text{x akṣakarṇa}}{12} \quad , \qquad \text{-----------} (12)$$

where *a* is the sun's amplitude (*agrā* of the first meaning), δ the sun's declination, and the *akṣakarṇa* the equinoctial midday hypotenuse of the 12-*aṅgula* gnomon.

The vs.104 can be expressed as follows.

$$a' = \frac{\text{R}\cdot\sin \text{a x h}}{\text{R}} \quad , \qquad \text{-----------} (13)$$

where a′ is the *agrā* of the second meaning, and *h* the hypotenuse produced by the gnomon and its shadow at the time.

The R·sine of the sun's amplitude correspond to AC in Fig.37, where DD′is the sun's diurnal circle. Here,

BC : AC = R·sin δ : R·sin a = 12 : *akṣakarṇa.*

Hence the equation (12).

Now Padmanābha proceeds to explain various types of the R·sine of altitude of stats.

सूर्यः(1) खेटो (2)[ऽ]थ धिष्ण्यं वा
यत्र यत्रोद्वृता (-त्ता-) दिषु।
(3)गच्छेत्तत्संज्ञकं (4)शङ्कु-
रन्यत्रापि(5) भवेदिह ॥१०५॥(6)

Apparatus : (1) Ms. A lacks visarga after *-rya.* (2) Ms. A lacks avagraha before *tha.* (3) A: *gate* for *gacchet* (4) A : *-kaḥ ketu-* for *-kaṃ śaṅku-*. (5) A: *-tvā-* for *-trā-*. (6) This verse is vs.97 in Ms.B.

105: "Here is some type of the R·sine of altitude or other which is named after the six o'clock line (*ud-vṛtta*) etc. which the sun, planet, or star is passing."

When a heavenly body is on the six o'clock line (*ud-vṛtta or un-maṇḍala*), the R·sine of its altitude is called *ud-vṛtta-śaṅku* or *un-maṇḍala sāṇku.* Similarly, when it is on the prime vertical (*sama-maṇḍala*, the R·sine of its altitude is called, *sama-śaṅku* and when it is on the *koṇa-maṇḍala* (vertical circle passing through north-east and south-west or north-west and south-east cardinal points), the R·sine of its altitude is called *koṇa-śaṅku.*

The method to calculate the *koṇa-śaṅku* and *sama-śaṅku* is as follows.

(1)द्विघ्नत्रिजीवानिहताक्षभानिजा-(2)
ध्यासाढ्यशून्याष्टयमाद्रिभू १७२८० हता(3)।
(4)अग्राकृतेस्त्रिज्यदलांशवर्झि(र्जि)त- (5)
त्रिज्या(6) पृथक्स्‌था च युताक्षभाग्रयोः॥१०६॥(7)

Apparatus: (1) A: *tra* for *tri.* (2) A: *jitā-* for *nijā-* (3) Ms. A adds visarga after *hatā.* (4) B: *athāhate-* for *agrākṛte-*. (5) B: *-te* for *-ta.* (6) Ms. B lacks *jyā.* (7) This verse is vs.98 in Ms.B.

द्विनिघ्नघातस्य(1) पृथक्स्थितस्य
वर्गेण तन्मूलमुदग्विहीनम्।
द्विनिघ्नघातेन युतं च(2) याम्ये(3)
पृथक्स्थिता(4) तेन भजेच्च(5) हत्वा ॥१०७॥(6)

Apparatus : (1) Ms. B lacks *ghā.* (2) B: *ta* for *ca.* (3) B: *-me* for *-mye.* (4) Ms.A lacks anusvāra after *-tā.* (5) A: *bhave-* for *bhaje-*. (6) This verse is vs.99 in Ms. B.

(1)व्योमाब्धिशकै १४४० रिति कोणशङ्कुः(2)
(3)स्यात्तद्धनुस्ते (4)[ऽ]भिमतोन्नतांशाः(5) ।
(6)क्रान्तिज्यकाक्षश्रुतिघाततो(7)(8) यत्
फलं भयाप्तं(9) समशङ्कुरेवम् ॥१०८॥(10)

Apparatus : (1) B: *nāgāgniśakrai* 1438 for *vyomābdhiśakai* 1440. (2) Ms.A lacks visarga after *-ṅku.* (3) B: *-dva-* for *-ddha-*. (4) A: *hato-* for *mato-*. (5) Ms. B lacks visarga after *-śā.* (6) A: *śra-* for *śru-*. (7) A: *dhā-* for *ghā-*(8) B:*-yo* for *-to.* (9) A: *bhaya-* for *bhayā-* (10) This verse is vs.100 in Ms. B.

106-108: " The square of the equinoctial midday shadow (*akṣa-bhā*) multiplied by the double of the Radius is added to 17280, and multiplied by the Radius minus the quotient of the square of [the R-sine of] the sun's amplitude (*agrā*) divided by the half of the Radius, which is put separately, and added to the square of the double of the product of the equinoctial midday shadow and [the R-sine of] the sun's amplitude, which is put separately, then its square root is made. The double of the product [of the equinoctial midday shadow and the R-sine of the sun's amplitude, which was put separately,] is subtracted from the square root, when [the sun's declination is] in north, and added, when in south, then divide it by 1440. Divide the [former] value, which was put separately, by this result. This is the *koṇa-śaṅku* and its corresponding arc is the desired altitude.

The R.sine of the [sun's] declination multipied by the equinoctial midday hypotenuse (*akṣa-śruti*), and divided by the [equinoctial midday] shadow (*bhā* is the *sama-saṅku.*)"

The above method of the calculation of the *koṇa-śaṅku* can be expressed as follows.

$$\textit{koṇa-śaṅku} = \frac{\left(R - \frac{a^2}{R/2}\right)}{\left(\sqrt{(2\cdot s\cdot a)^2 + (2R \times s^2 + 17280) \times \left(R - \frac{a^2}{R/2}\right)} \mp 2\cdot s\cdot a\right)/1440}, \quad \text{-----(14)}$$

where s is the equinoctial midday shadow (*akṣa-bhā*) of the 12-*aṅgula* gnomon, and a the R·sine of the sun's amplitude (*agrā*).

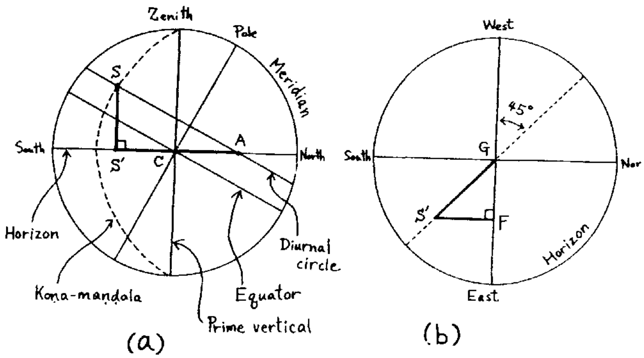


Fig. 39

The rationale of this equation is as follows. Let X be the *koṇa-śaṅku.* In Fig.39(a), which is the orthographic projection of the celestial sphere onto the plane of the meridian, SS′ is the *koṇa-śaṅku*, S′C the *bhuja*, S′A the *śaṅku-tala*, and CA the *agrā*. Here,

$$bhuja = śaṅkutala \mp agrā = X\frac{s}{12} \mp agrā.$$

In Fig. 39(b), which is the orthogrphic projection of the celestial sphere onto the plane

of the horizon, GS′ is the R.sine of the sun's zenith distance or $\sqrt{R^2-X^2}$, and S′F the "*bhuja*". Here,

$$bhuja = \frac{\sqrt{R^2-X^2}}{\sqrt{2}} = \sqrt{(R^2-X^2)/2}$$

From the above two relations,

$$\left(\frac{s}{12}\right)^2 . X^2 \mp \frac{s \cdot a}{6} X + a^2 = \frac{R^2-X^2}{2}$$

or,

$$(S^2 + 72) \cdot X^2 \mp (24 \cdot s \cdot a) X - \left(\frac{R^2}{2} - a^2\right) \cdot 12^2 = 0.$$

This is a quadratic equation of X. Therefore,

$$X = (\pm) \frac{24 \cdot s \cdot a \pm \sqrt{(24 \cdot s \cdot a)^2 + 4(s^2 + 72) \cdot (R^2/2 - a^2) \cdot 12^2}}{2(s^2 + 72)}$$

Now the following value is multiplied to the numerator and denominator.

$$24\cdot s\cdot a \mp \sqrt{(24\cdot s\cdot a)^2 + 4(s^2 + 72)\cdot(R^2/2 - a^2)\cdot 12^2}$$

Then, assuming that R=120, the equation (14) is obtained.

The method to calculate the *sama-śaṅku*, given in the above text, can be expressed as follows.

$$\textit{sama-śaṅku} = \frac{R\cdot\sin\delta \times k}{s} \qquad \text{------- (15)}$$

where k is the equinoctial midday hypotenuse, i.e. the hypotenuse produced by the 12-*aṅgula* gnomon and its equinoctial midday shadow s, and δ the sun's declination.

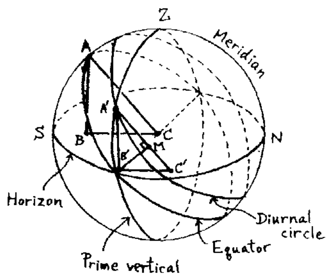


Fig. 40

The rationale of this equation is as follows. In Fig.40, the triangles ABC and A'MB' are similar. So,

A'B' : B'M = AC : CB = k : s,

where A'B' is the *sama-śaṅku*, and B'M the R.sine of the sun's declination. Hence the equation (16).

The method to calculate the *koṇa-karṇa*, or the hypotenuse produced by the 12-*aṅgula* gnomon and its shadow when the sun is on the *koṇa-maṇḍala*, is as follows.

अग्रावर्गात (1) त्रिज्यकावर्गखण्डे-(2)
नाप्तं(3) (4)रूपाच्छुद्धमाद्याह्वय: स्यात्।
द्विघ्नाग्रज्याक्षप्रभाघाततो(5) यत्
(6)त्रिज्यालब्धं मध्यसंज्ञं ततश्च ॥१०९॥(7)

Apparatus : (1) B: *-grā-* for *-gā-*. (2) B: *-gra-* for *-ga-*. (3) B: *-staṃ* for *-ptaṃ*. (4) A: *-śu-* for *-cchu-*. (5) A: *dyā-* for *ghā-*. (6) Ms. B adds *t* between *yat* and *tri*. (7) This verse is vs.101 in Ms.B.

अक्षभाया(1) (2)द्विघ्नवर्गान्(3) नृवर्गे(4)-
णा १४४ द्या (-ढ्या-) दाद्येनाहतान्मध्यकृत्या।
युक्तान्मूलं मध्यमेनोनयुक्तं(5)
(6)त्वाद्येनाप्तं(7) गोलयो: कोणकर्ण:(8) ॥११० ॥(9)

Apparatus: (1) A: *sā* for *yā*. (2) A: *dma* (?) for ***ghna***. (3) **B**: *-grā-* for *-gā-*. (4) B: *-gre-* for *-ge-* (5) B:*nobha* for *-nona*. (6) Ms.A adds *kṛ* between *yuktaṃ* and *tvā-*. (7) B:*-staṃ* for *- ptaṃ*. (8) Ms. A lacks visarga after-*ṇa*. (9) This verse is wrongly numbered as vs.2 in Ms.B. This should be vs.102 there.

109: "The square of |the R-sine of| the sun's amplitude (*agrā*) divided by the half of the square of the Radius, and subtracted from one is called "*ādya*" (first value).

The double of the product of the R-sine of the sun's amplitude (*agra-jyā*) and the equinoctial midday shadow (*akṣa-prabhā*) divided by the Radius is called *madhya*, (middle value)."

110: "The double of the square of the equinoctial midday shadow (*akṣā-bhā*) is added to the square of 12, multiplied by the *ādya*, added to the square of the *madhya*, and its square root is made. Then the *madhya* is subtracted from or added to the result, and divided by the *ādya*. This is the *koṇa-karṇa* in two hemispheres."

The above calculation can be expressed as follows.

$$1 - \frac{a^2}{R^2/2} = \bar{a}dya$$

$$\frac{2 \cdot s \cdot a}{R} = madhya$$

$$\frac{\sqrt{(2 \cdot s^2 + 12^2) \cdot (\bar{a}\text{dya}) + (\text{madhya})^2} \mp (\text{madhya})}{(\bar{a}\text{dya})} = ko\d{n}a\text{-}kar\d{n}a \qquad \text{-------} (16)$$

The rationale is as follows. We have the following proportion.

koṇa-śaṅku : R = 12 : *koṇa-karṇa*

Therefore,

$$ko\d{n}a\text{-}kar\d{n}a = \frac{12\ R}{ko\d{n}a\acute{s}a\dot{n}ku}$$

Applying the equation (14) here, we get the equation (16).

Now Padmanābha proceeds to explain the latitude-triangles (*akṣa-kṣetra*).

क्रान्त्यग्रजं लम्बवदुद्गता (ता)त्[(1)] कुजं[(2)]
सूत्रं तदुन्मण्डलशङ्कुरस्य तु ।
तलात्तु [(3)]याम्योत्तरखण्डके कुजे
ते स्तो [ऽ]ग्रकायाः प्रथमान्त्यसंज्ञके ॥१११॥[(4)]

Apparatus: (1) Ms. B lacks *t* after *-tā*. (2) A: *kujāṃ* for *kujaṃ*. (3) A: *khaṃḍo-* for *yāmyo-*. (4) This verse is vs. 103 in Ms. B.

मध्ये तु[(1)] जात्यद्वितयं [(2)]पलोद्भवं
सर्वेण चेच्छं [(3)] विभवेत्[(4)] तृतीयकम्।
कुजोदयाङ्कात् सममण्डलावधौ[(5)]
[(6)]सूत्रं च तत्तद्ध (द्धृ) तिसंज्ञमुच्यते ॥११२॥[(7)]

Apparatus: (1) B: *ku* for *tu*. (2) *B:pha-* for *pa-*. (3) B: *hita-* for *vibha-*. (4) Ms. A lacks *t* after *-ve*. (5) B: *-yādho* for *-vadhau*. (6) A: *mūlaṃ* for *sūtraṃ*. (7) This verse is vs.104 in Ms.B.

मध्ये तथैषां च तदुद्धृतावधि[(1)]
स्यादग्रकाद्येन दलेन तत्परम्।
एवं [(2)]द्विजात्यानि[(3)]पलोद्भवानि षट्
त्र्यस्राणि[(4)] चास्मिन् प्रकटानि चाष्टसु[(5)] ॥११३॥[(6)]

Apparatus : (1) Ms. A adds anusvāra after *-dhi*. (2) B: *di* (?) for *dvi*. (3) A: *-bhiphalo-* for *-ni palo-*. (4) A: *-sra* for *-srāṇi*. (5) A: *vyaṣṭa-* for *cāṣṭa-*. (6) This verse is vs. 105 in Ms.B.

111: "The vertical line dropped from the tip of the declination (i.e. point on the diurnal circle) on the six o'clock circle upto the horizontal plane is the *unmaṇḍala-śaṅku*.

From its root are two horizontal north-south segments (i.e. bases of the triangles in the eastern and western hemispheres), and from [the tips of the bases on] the rising-setting line are two [hypotenuses] called *prathama-antya*."

112: "Its inside are two latitude [triangles] (i.e. two small right-angled triangles divided by the perpendicular line dropped from the right-angled corner of the original triangle to its hypotenuse). In all cases [of the right-angled triangle], the [second and] third [similar small triangle] are produced.

The segment from the rising-setting line upto the prime vertical is called *taddhṛti*. (Regarding it to be the hypotenuse, a latitude triangle is formed.)"

113: "Their inside are also [latitude triangles], [divided by the line drawn] upto the hypotenuse, formed by the segment *agrakā* etc.

Thus there are six evident latitude triangles, including two fold interior ones, formed by eight segments."

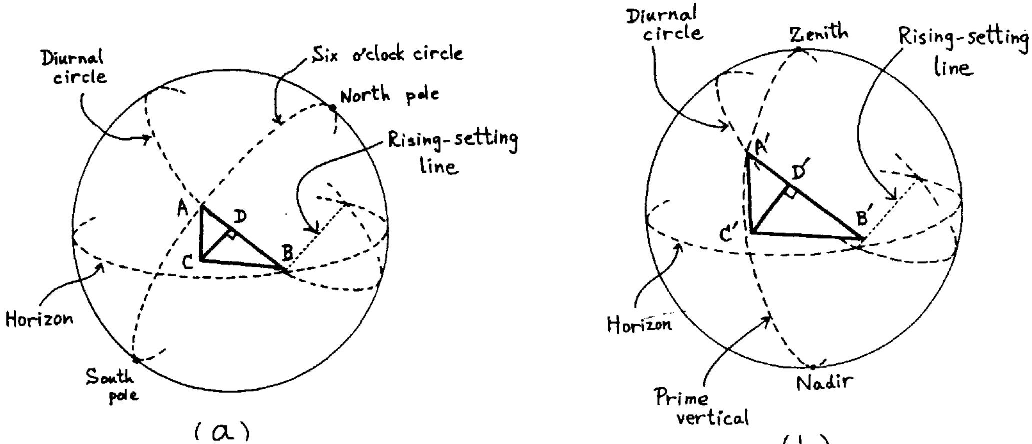


Fig. 41

In Fig.41, the right-angled triangles ABC and A'B'C' are latitude triangles, where the angle CAB or C'A'B' is equal to the latitude of the observer. Here, the segment AC is the *unmaṇḍala-śaṅku*, AB the *prathama-antya*, A'B' the *taddhṛti*, C'B' the *agrakā*, and A'C' is called *sama-śaṅku*. The triangle ABC is divided into two similar small triangles CBD and ACD, and the triangle A'B'C' into C'B'D' and A'C'D'.

Now, the last two verses are conclusion.

कोणोद्वृत्तसमेष्टमध्यजनरः(1) क्रान्त्यग्रकुज्यादयः(2)
क्षेत्राण्यक्षसमुद्भवानि कतिचिच्छायाग्रबाह्वादयः(3) ।
वृत्तानि ग्रहभोदयास्तसमयाद्येवं द्रुतं तद्विदां
प्रत्यक्षं सकलं भवेदिह पृथक् सद्यन्त्रराजाभिधे ॥११४॥(4)

Apparatus: (1) Ms. *A* lacks visarga after *nara*. (2) Ms. A lacks *n* after *krā*. (3) Ms. B. lacks *c* after *-ci*. (4) This verse is vs. 106 in Ms.B.

114: "The altitude of a heavenly body on the *koṇa*-circle, six o'clock circle, prime vertical, any place, or meridian; the declination, amplitude, earth-sine etc.; the latitude triangles; the *agrā* or *bhuja* etc. corresponding to the gnomon-shadow; some circles; or rising or setting time of planets or stars are respectively presented on the *yantra-rāja* for knowers here, quickly, evidently, and wholly."

विनापि दृग्धिष्ण्यखगोलबन्धं
भचक्रसंस्थां विमलामशेषाम् ।
संवेदितुं सन्मनसां सदुक्तिः
श्रीनार्मदो यन्त्रमिदं चकार॥११५॥[1]

Apparatus: (1) This verse is vs. 107 in Ms.B.

115: "In order to know the clear perfect form of zodiac without [armillary sphere which is] the combination of *drg-gola*, *bha-gola*, and *kha-gola*, eloquent Śrī Nārmada made this instrument for good-hearted people."

From this verse, it appears that the astrolabe (*yantra-rāja*) was made by Nārmada, the father of the author Padmanābha.

The above is the full text of the *Yantra-rāja-adhikāra* of Padmanābha. The colophon of Ms.A is as follows.

इति श्रीपद्मनाभविरचितायां यंत्रकिरणावल्यां
यंत्रराजाधिकारः वासनाभाष्यसहितः प्रथमः॥
॥छ॥ ॥छ॥ श्रीकृष्णाय नमः ॥श्रीरस्तु॥ ॥छ॥
..... ॥छ॥ ॥ संवत् १६३४ वर्षे मार्ग्र (र्ग) शीर्षे
मास्य (से) सितपक्षेऽष्टम्यां तिथौ सोमवासरे
लिखितो यंत्रराजाधिकारः ॥रघुनाथेन स्वस्तये॥

The colophon of Ms.B is as follows.

इति श्रीयंत्रकिरणावल्यां यंत्र [रा] जाधिकारः
प्रथमः समाप्तमिति ॥श्रीरस्तु॥ ॥छ॥ ॥
यादृशं पुस्तके दृष्टं तादृशं लिखितं मया।
यदि शुद्धमशुद्धं वा मम दोषो न हीयते ॥९॥
॥श्रीरामाय नमः॥ ॥स्वस्ति श्रीसंवत् १६३४
वर्षे मार्गशीर्षमासे शुक्लपक्षे अष्टमीरवौ
लिखितमिदं॥ ॥श्री॥ ॥छ॥.....

iii) Historical significance of Padmanābha's Yantra-rāja-adhikāra

From the above text, it is clear that the theory of the astrolabe was well understood by Hindu astronomers, Padmanābha and probably his father Nārmada also, as early as c.AD 1423 or still earlier, only about 50 years after Mahendra Sūri's *Yantra-rāja*. Padmanābha explained the astrolabe in his own words using well known methods of Hindu mathematics. This fact shows the ability of astronomers in Delhi Sultanate period as well as the activeness of Hindu-Muslim cultural exchange in the field of astronomy.

Lastly, I should note again that the astrolabe described by Padmanābha is the "southern astrolabe", and the drawings are just opposite to the ordinary "northern astrolabe". And also, I should state that there is no figure in the manuscripts, and the figures in this paper were drawn by me.

5. THE YANTRA-PRAKĀŚA OF RĀMACANDRA

The *Yantra-prakāśa* of Rāmacandra is a voluminous work on several astronomical instruments. Its manuscripts are extant in the Asiatic Society, Calcutta,[72] and the Bhandarkar Oriental Research Institute, Poona.[73] According to the colophon of each chapter in the manuscripts, Rāmacandra's father was Sūryadāsa, mother was Viśālākṣī, grandfather was Śivadāsa, and teacher was Hīrasvāmī. In the Poona manuscript, a star catalogue for Vikrama Saṃ. 1485 (=Śaka 1350 = AD 1428) at Naimiṣāraṇya has been given. So, this work must have been written around AD 1428. Naimiṣāraṇya exists near Sitapur in present Uttar Pradesh.

The *Yantra-prakāśa* is divided into the following six chapters.

(1) *Dyujyā-kendra-vyāsa-gaṇanā-adhyāya,*

(2) *Bhapatra-racanā-adhyāya,*

(3) *Graha-sphuṭīkaraṇa-adhyāya,*

(4) *Tithyādi-gaṇanā-adhyāya,*

(5) *Śaṅku-yaṣṭi-siddhi,*

(6) *Kutūhala-yantra-prakāśana.*

Among these chapters, the first four chapters are devoted to the astrolabe, which Rāmacandra calls *Sulabhā-yantra.* Rāmacandra's description of the astrolabe has some similarities with the *Yantra-rāja* of Mahendra Sūri and its commentary by Malayendu Sūri. For example, the *Yantra-prakāśa* (chapter I) gives a list of the latitudes of several cities, and it is almost identical with the list in the similar to the list of given in the *Yantra-rāja* of Mahendra Sūri and its commentary of Malayendu Sūri on the *Yantra-rāja* of Mahendra Sūri. And also, the *Yantra-prakāśa* (Chapter II) gives a list of the longitudes, latitudes etc. of 32 stars, and it is similar to the list given in the *Yantra-rāja* of Mahendra Sūri and its commentary by Malayendu Sūri. Rāmacandra's value of the longitude is different from the value of Mahendra Sūri by the amount of 52′12″. The *Yantra-prakāśa* (AD 1428) was written 58 years later than the *Yantra-rāja* (AD 1370), and Mahendra Sūri has given the rate of the precession as 54″ per year. Now, 54″ x 58 = 3132″ = 52′12″. This is exactly the amount of the difference of the longitude between Mahendra Sūri's list and Rāmacandra's list. So, Rāmacandra must have been influenced by Mahendra Sūri and Malayendu Sūri's work. This fact shows the popularity of the knowledge of the astrolabe in this period.

Chapter V of the *Yantra-prakāśa* is the description of the gnomon and the staff.

Chapter VI of the *Yantra-prakāśa* is an interesting chapter, and several astronomical instruments have been described there. According to the Poona manuscript, the names of the instruments described there are as follows:

Phalaka, valaya, cūḍā, mudrikā, cāpa, trikoṇa, candra-ardha, turīya-gola, cakra, kaśā, khaṅga-śara-ādi, nāḍī-valaya, cintāmaṇi, bhūja, kapāla, mayūra, brahmacāri, rādhā-vedha, vadhūvara-yoga, meṣa-aja-yuddha, prakāraṇa-śaṅkha-vāda, kāraṇa-ghaṇṭa-paṭaha-ādi-vādana, vānara, ghaṭī, haṃsa, ajasra, apara-ajasra, lagna-udaya, kāhala, narasiṃha-yuddha-bhāṇa-ghāta-ākhu-mārjāra-yuddha-ādi, kāca-ghaṭī, and *kāṣṭha-ghaṭī.*

Unfortunately the description of each instrument in this chapter is mostly very brief, but this is certainly a very important source material. Detailed study of this work will be necessary in the future.

It may be mentioned here that Rāmacandra (=Rāma Vājapeyin) composed several works including three works on the *Śulba-sūtra*. One of them, the *Śulba-vārttika*, was composed in Śaka 1356 (AD 1434).[74]

6. Conclusion

The astrolabe was introduced into India from the Islamic world at the time of Fīrūz Shāh Tughluq (reign AD 1351 - 88), and Mahendra Sūri wrote the first Sanskrit treatise on the astrolabe entitled *Yantra-rāja* (AD 1370). Only after a half century, Padmanābha wrote the *Yantra-rāja-adhikāra* (AD 1423),probably the second Sanskrit monograph on the astrolabe, and Rāmacandra wrote the *Yantra-prakāśa* (AD1428), where the astrolabe has been discussed in detail besides several other instruments. Padmanābha explained the principle of the astrolabe in his own words using traditional Hindu mathematics. Rāmacandra must have been influenced by Mahendra Sūri's *Yantra-rāja* and its commentary written by Malayendu Sūri.

We can conclude that the astrolabe was quite popular in Delhi Sultanate period after Fīruz Shāh's time, and well understood by non-Muslim Indian astronomers, and that there was much exchange of ideas between Muslim astronomy and Hindu astronomy.

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NOTES AND REFERENCES

1 Yukio Ôhashi : "Development of Astronomical Observation in Vedic and Post-Vedic India", *Indian Journal of History of Science*, 28 (3), 1993, pp.185-251.

2 Yukio Ôhashi : "Astronomical Instruments in Classical Siddhāntas", *Indian Journal of History of Science*, 29 (2), 1994, pp. 155-313.

Tentatively, the following paper may be used as a rough sketch of this subject :

3 Yukio Ohashi : "Sanskrit Texts on Astronomical Instruments during the Delhi Sultantate and Mughal Periods", *Studies in History of Medicine and Science*, Vols. X-XI, 1986 -1987 (published in 1991), pp.165-181.

4 The published texts are as follows: Raikva, K.K. (ed.) : *Yantra-rāja* of Mahendra Sūri with the commentary of Malayendu Sūri and the *Yantra-śiromaṇi* of Viśrāma, Nirnaya Sagar Press, Bombay (1936); Bhaṭṭācārya, B.B. (ed.) : *Yantra-rāja-vicāra-viṁśādhyāyī*, Smapurnanand Sanskrit University, Varanasi (1979); Kedāranātha Jyotirvid (ed.): *Yantra-rāja-racanā* of Jaya Siṃha with the *Yantra-prabhā* of Śrīnātha (Rājasthāna Purātana Granthamālā 5), Rājasthāna Purātattva Mandira (now the Rajasthan Oriental Research Institute), Jaipur (1953) ; Bhāgīrathī Prasāda Śarman (ed.) : *Yantra-cintāmaṇi* with the commentary of Rāma; Sarma, S.R. (ed. and tr.) : *Yantra-prakāra* of Sawai Jai Singh, Supplement to *Studies in History of Medicine and Science*, vols.x-xi (1986-87) ; Caturveda, Mularīdhara (ed.) : *Siddhānta-samrāṭ* of Jagannātha, Sagarika Samity, Sagar University, Sagar (1976) ; and Sharma, S.D. (ed. and tr.) : *Pratoda-yantra* (This is actually Munīśvara's version of the pratoda-yantra or cylindrical sundial), Martand Bhavan, P.O. Kurali (Ropar), Punjab (1982).

5 For the astrolabe in general, see Nallino, C.A. : "Asṭurlāb", *E.J. Brill's First Encyclopaedia of Islam, 1913-1936*, reprint: Leiden (1987), vol. 1, pp.501-502; Gunther, Robert T. : *The Astrolabes of the World* 2 vols., Oxford (1932); Hartner, W. : "The Principle and Use of the Astrolabe"(Originally written in 1939), reprinted in his *Oriens-Occidence* (1968) pp. 287-311; Khareghat, M.P. (edited posthumously by D.D. Kapadia) : *Astrolabes* (M.P. Khareghat Memorial Volume II), Bombay (1950); Hartner, W. "Asṭurlāb", *The Encyclopaedia of Islam, New Edition*, vol. I, Leiden (1960), pp.722-728, reprinted in his *Oriens-Occidence* (1968) pp. 312-318; and Miyajima, Kazuhiko: "Asutororābe ni tsuite" (On the Astrolabe, in Japanese), *Kagakusi Kenkyu*, Series II, Vol.14 (No.113), (1975), pp.16-21.

6 For the streographic projection in general, see Rosenfeld, B.A. and N.D. Sergeeva: *Stereographic Projection* (Little Mathematics Library), Translated from Russian by Vitaly Kisin, Moscow (1977). I have borrowed some sentences and some ideas regarding the drawings from this book.

7 See Rosenfeld and Sergeeva, op.cit., and Neugebauer, O.: *A History of Ancient Mathematical Astronomy*, Part Two, Berlin(1975), pp.858-860.

8 Apollonius proved this fact in his *Conic Sections* (I.5). See Neugebauer, O., op.cit., Part Two, p.858 and Heath, Thomas : *A History of Greek Mathematics*, Vol. II, Oxford (1921), reprint : Dover, New York (1981), p.135. Also see Thomas, Ivor (tr.) : *Greek Mathematical Works II* (Loeb Classical Library 362), Cambridge, Mass. (1941), p. 301.

9 Neugebauer, O., op.cit., Part Two, p.869. Also see Hartner, W., op.cit. *(Oriens-Occidence)*, p.288.

10 Benndorf, O., E.Weiβ and A.Rehm: " Zur Salzburger Bronzescheibe mit Sternbildern", *Jahreshefte des österreichischen Archäologischen Institutes in Wien*, Band VI, 1903, pp.32-49. Also see Diels, Hermann : *Antike Technik*, Leipzig (1924), pp.213-219. (I have also consulted its Japanese translation by Hirata, Yutaka : *Kodai Gijutsu*, Tokyo (1970), pp.197-201.) Also see Neugebauer, O., op.cit., Part Two, pp..869-870. I also consulted Morita, Keiichi (tr.) : *Witorūwius Kenchikusho* (Japanese translation of Viturvii *De Architectura*), Tokyo (1979).

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13 Neugebauer, O.: *A History of Ancient Mathematical Astronomy*, Part Two, (1975) p.871.

14 Ibid, pp. 877-879.

15 Drecker, J.: "Des Johannes Philponos Schrift über das Astrolab", *ISIS*, *11* (1928) pp.15-44, and Nau, M.F. : "Le Traité sur l'Astrolabe Plan de Sévère Sabokt", *Journal Asiatique*, Neuvième Sé rie, Tome XIII (1899) pp.56-101 and 238-303. Englis translation of the works of Philoponus and Sabokt (=Sebokht) are op.cit., Vol. 1, pp. 61-81 and pp. 82-103 respectively.

16 Hartner, W., op.cit., p.290.

17 Chaucer, Geoffrey: *The complete works of Geoffrey Chaucer*, Edited by F.N. Robinson, Second ed., Oxford (1974), pp.544-563.

18 Schoy, C.: " Alî ibn 'Îsâ, Das Astrolab und sein Gebrauch", *ISIS*, *9* (1927) pp.239-254.

19 Rosenfeld and Sergeeva, op.cit., p.40.

20 De, B. (tr.): *The Ṭabaqāt-i-Akbarī of Khwājah Nizāmuddīn Aḥmad*, Vol.I, Bibliotheca Indica, Calcutta (1911,1927), rep.1973, pp.248-249.

21 See Elliot, H.M. and John Dowson: *The History of India as told by its own Historians*, vol.III, rep. Allahabad (n.d.), p.338; and Sarma, Sreeramula Rajeswara: "Astronomical Instruments in Mughal Miniatures" in *Studien zur Indologie und Iranistik*, Band 16/17 (1992) 235-276; p.238.

22 Muqtadir, Maulavi Abdul (ed.) : *Catalouge of the Arabic and Persian Manuscripts in the Khuda Bakhsh Oriental Public Library*, Vol. VII, Indian History, Patna (1921), rep. 1977, pp.28-33. The call no. of the manuscript is HL-99.

23 The Patna manuscript (HL-99) f.152a ff. A summary of the account of the astrolabe in this work has been given in Ghori, S.A.K. : "Scientific Exchanges between Soviet Central Asia and India during Medieval Times" in Subbarayappa, B.V. ed.: *Scientific and Technological Exchanges between India and Soviet Central Asia (Medieval Period)*, New Delhi (1985) 78-89; pp.87-89.

24 The word "*tāni*" cannot be seen clearly in my microfilm of the manuscript, and I tentatively surmised this.

25 Dvivedī, Sudhākara: *Gaṇaka-taraṅgiṇī*, Benaras (1892), rep. 1986, pp.45-47; Dikshit, S.B. : *Bharatiya Jyotish Sastra*, English tr. by R.V.Vaidya, Part II, Calcutta (1981) pp. 230-231; and Pingree, David: *Census of the Exact Sciences in Sanskrit*, Ser.A, Vol.4, Philadelphia (1981), pp.393-395.

26 Bombay edition (see note 32 below) p.81.

27 Dikshit, S.B., op.cit., Part II, p.230.

28 Pingree, D., *Census*, op.cit., A-4, pp.363-364.

29 Pingree, D., *Census*, A-2. Philadelphia (1971), p.133.

30 Dikshit, S.B., op.cit., Part II, p.177 and 231.

31 Sudhākara Dvidedin and L.Sarma ed., Benares (1882). (See Pingree, *Census*, A-4, p.395.) The copy of this edition has not been available to me.

32 *Yantra-rājaḥ, Mahendra-guru-viracitaḥ Malayendu-sūri-viracita-ṭīkā-sahitaḥ*, edited by Kṛṣṇaśaṁkara Keśavarāma Raikva, Bombay (1936). The *Yantra-śiromaṇi* of Viśrāma is also included in this edition.

33 I have seen two manuscripts of Gopirāja's commentary at the Anup Sanskrit Library, Bikaner, but taking extensive note was not allowed. (Ms. no.5010, 134 ff; and no.5011, 122 ff.) It is unfortunate that this commentary could not be utilized, because it is a very detailed commentary.

34 I have used BORI 556 of 1899-1915.

35 Bombay ed., p.81.

36 Bombay ed., p.2.

37 Pingree, David: "Islamic Astronomy in Sanskrit", *Journal for the History of Arabic Science*, 2 (1978), 315-330; p.318.

38 See Kaye, G.R. : *The Astronomical Observatories of Jai Singh*, Calcutta (1918), rep. New Delhi (1982), p. 136.

39. See Pingree, D., op.cit. (1978), p. 318.

40. Bombay ed., p.10.

41 Bombay ed., pp.18-19.

42 Bombay ed., (I.22-37), pp.25-28.

43 Bombay ed., p.28.

44 Dikshit, S.B., op.cit., Part II, p.348.

45 Kaye, G.R., op.cit., p.4, and p.116.

46 Bombay ed., (I.38b), p.28.

47 Bombay ed., (I.40), p.29.

48 Bombay ed., p.60. It may be mentioned here that W.H. Morley wrote in his article (included in Gunther, R.T., op.cit., Vol. 1) that there are three kinds of planispheric astrolabes, namely "Shamālī" or *"Northern"*, "Janūbī" or *"Southern"*, and "A third, the Saratání, which takes its name from the position of the sign Saratán, *Cancer*, is by most writers either passed over, or only incidentally mentioned, but receive some attention from Az-Zubair : this kind of Astrolabe was both Northern and Sourthern, and combined the propertes and uses of each. ......" (Gunther, op.cit., Vol. 1, p.7).

49 Dikshit, S.B., op.cit., Part II, p.125, and 231; and Pingree, D., *Census*, A-4, pp. 170-172.

50 Dikshit, S.B., op.cit., Part II, pp.125-127; and Pingree, D., *Census*, A-3, Philadelphia (1976), pp.100-101.

51 I have used a manuscript BORI 346 of 1882/83.

52 I have used a manuscript AS Calcutta IM-5356.

53 Pingree, D., op.cit., A-4, p.170.

54 Pingree, David: *Jyotiḥśāstra*, Astral and Mathematical Literature, Wiesbaden (1981), p.53.

55 Manuscript: Baroda 3160.

56 Manuscripts: Lucknow 45888 (33ff, copied on Monday(sic) 8th tithi, white pakṣa, Mārgaśīrṣa month, Vikrama saṃvat 1634) = AD1577); and Lucknow 45892 (9ff, copied on Sunday (sic) 8th tithi, white pakṣa, Mārgaśīrṣa month, Vikrama Samvat 1634). Lucknow 45888 has a commentary also, and Lucknow 45892 is text only. Their texts are basically identical, and probably these two manuscripts were copied by the same person simultaneously. One folio (f.6) of Lucknow 45888 has mistakingly been incorporated in Lucknow 45892 as its f.6. From the condition of the numbering of these folios, I suppose that this incorporation is originated with the scribe himself.

57 Consulting one manuscript (Lucknow 45888), I reported this fact at the Colloquium No.91, International Astronomical Union, New Delhi, 13-16 Nov.1985. My paper has been published as: Ohashi, Y.: "A Note on some Sanskrit Manuscripts on Astronomical Instruments", in Swarup, G., A.K., Bag, and K.S. Shukla eds.: *History of Oriental Astronomy*, Cambridge (1987) pp.191-195. When I wrote this paper, I was not aware of the existence of another manuscript (Lucknow 45892).

58 Lucknow 45888, f.21b. This cancelled colophon is dated Friday 6th tithi, white pakṣa, Mārgaśīrṣa month, Vikrama saṁvat 1634.

59 Lucknow 45888, f.3a.

60 Pingree, David: "Some little known Commentators on Bhāskara's *Karaṇakutūhala*", *Aligarh Journal of Oriental Studies*, 2 (1985) 157-168; pp. 167-168.

61 I have confirmed that the following manuscripts actually mention the name of the *Yantra-ratnāvalī*: VVRI 469, Baroda 9588 and 3167, BORI 976 of 1886-92, 202 of 1883-84 and 94 of A1882-83, AS Bombay 245-I (BD 298), Ānandāśrama 6673, SOI 9923, 9419 and 7165, and RORI (Jodhpur) 14840. The following manuscripts mention the name of the *Yantra-racanāvalī*: BORI 543 of 1899-1915 and 463 of A1881-82, and Ānandāśrama 3453. One manuscript (GOML (Madras) R 17442) mentions the name of the *Yantra-muktāvalī*, and another manuscript (BORI 329 of 1882-83) mentions the name of the *Yantra-karaṇāvalī*. I have not come across any manuscript of the *Dhruva-bhramaṇa-adhikāra* which exactly mentions the name of the *Yantra-kiraṇāvālī*.

62 I have used the following manuscripts : Baroda 12096, BORI 297 of 1882/83, and Lucknow 47177.

63 Dikshit, S.B., op.cit., Part II, p.125. Also see Pingree, D., *Census*, A-3, p.171. Pingree says that Nārmada wrote a *Nabhogasiddhi* following the Brahmapakṣa.

64 See Shukla, Kripa Shankar (ed. and tr.) : *Mahā-bhāskarīya*, Bhāskara I and his works, part II, Lucknow (1960), pp.207-210.

65 Even if we use Padmanābha's approximate formulae of sine and arcsine, and take four figures, the result is $\alpha = 40°30'$ and $\beta = 24°52'$.

66 I have discussed this problem in Ôhashi, Y., op.cit. (1987).

67 By the equation (3) explained in vs.6, we can calculate as follows.

$$120 \times \frac{\sin 60^\circ}{1 + \sin 24^\circ} = 78.$$

68 Līlāvatī (204) (Ānandāśrama Sanskrit Series no.107, Part 2, Pune (1937), p.205) reads as follows.

ज्याव्यासयोगान्तरघातमूलं व्यासस्तदूनो दलित: शर: स्यात्।

व्यासाच्छरोनाच्छरसंगुणाच्च मूलं द्विनिघ्नं भवतीह जीवा।

जीवार्धवर्गे शरभक्तयुक्ते व्यासप्रमाणं प्रवदन्ति वृत्ते ॥२०४॥

I have also consulted a Japanese translation of the *Līlāvatī* by Hayashi, Takao (in Yano, Michio ed. : *Indo-tenmongaku-sūgaku-shū*,Biblipthca Scientiae 1, Tokyo (1980) pp. 323-324. Also see Colebrooke, H.T.: *Algebra, with Arithmetic and Mensuration, from the Sanscrit of Brahmegupta and Bhāscara.* London (1817), pp.89-90 (no. 206-207).

69 *Līlāvatī* (56) (Ānandāśrama Sanskrit Series no. 107, Part 1, Pune (1937), p.54) reads as follows.

योगोऽन्तरेणोनयुतोऽर्धितस्तौ राशी स्मृतौ संक्रमणाख्यमेतत् ॥५६॥

I have also consulted a Japanese translation by Hayashi, Takao, op.cit., p.233.

Also see Colebrooke, op.cit., p.26 (no.55).

70 *Līlāvatī* (153(a-c)) (Ānandāśrama ed., op.cit., Part 2, p.143) reads as follows.

भुजाद्वर्गितात्कोटिकर्णान्तराप्तं
द्विधा कोटिकर्णान्तरेणोनयुक्तम्।
तदर्धे क्रमात्कोटिकर्णौ भवेताम्

I have also consulted a Japanese translation by Hayashi, Takao, op.cit., p.282-283.

Also see Colebrooke, op.cit., p.60 (no.151).

71 It may be noted here that the values of the longitude and latitude given by Padmanābha are similar to those given by Munīśvara in his *Siddhānta-sārvabhauma* (AD 1646). It is not known whether Munīśvara knew Padmanābha's work directly or not. (For star catalogues of several Indian astronomers, see Dikshit, S.B., op.cit., Part II, pp.338-341.)

72 AS Calcutta G-1363 (Incomplete). See Shāstrī, Haraprasāda, revised and edited by P.C. Sen Gupta: *A Descriptive Catalogue of the Sanskrit Manuscripts in the collection of the Royal Asiatic Society of Bengal*, Volume X, Part I, Astronomy Manuscripts, Calcutta(1945). pp.59-60.

73 BORI 975 of 1886/92 (Complete), 71 ff.

74 See Katre, Sadashiva L.:" Three Works by Rāma Vājapeyin pertaining to Kātyāyana's Śulbasūtra", *Proceedings of the 13th All-India Oriental Conference: Vedic*, (1946), pp.72-78. I am grateful to Dr. Takao Hayashi who provided me with its copy.

For the works of Rāmacandra Vājapeyin, the following work may also be consulted : Pingree, David : *Census of the Exact Sciences in Sanskrit*, Series A, Volume 5, Philadelphia (1994), pp. 467-479. In the section of the *Yantra-prakāśa*, Pingree writes that "S.R. Sarma, who is preparing an edition, has determined that Rāmacandra wrote this in 1428" (Ibid., p. 478). Strangely, he has not mentioned my paper of 1991 (see note 3) where I already determined that Rāmacandra wrote the *Yantra-prakāśa* in 1428 (see p. 168 and note 23 of any paper of 1991).