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# BABYLONIAN MATHEMATICS

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A large number of clay tablets from Mesopotamia, one of earliest civilizations in human beings are available in different Museums. Studies of these tablets have revealed that the Babylonians/Mesopotamians achieved considerable success in the development of Mathematics and were acquainted with :

(A) Reciprocal of Integers in two faces (i) non-recurrent values expressed by decimals of 60 (ii) recurrent values of infinite approximations expressed by inequalities covered with numbers under decimals of 60.

(B) Arithmetical & Geometrical progressions

(C) Squares & Cubes of integers as well as Square-roots & Cube-roots of integers.

(D) Multiplication of integers by another numbers.

(E) A Geometric Number $60^4 = 12{,}960{,}000$ which was later known as Great Platonic Number and was recognised as number governing the Earth and Life on Earth.

(F) Solving Quadratic & Simultaneous equations.

(G) Approximation of values of $\sqrt{2}$, $\sqrt{3}$, and $\pi$ (pi).

(H) Pythagorean numbers to obey the law of Pythagoras on right-angled triangles more than 1000 years before Pythagoras.

(I) Areas of regular Polygons and Identity like $(a + b)^2 = a^2 + 2ab + b^2$

(J) Calculation of Interest and Period of money lending i.e. Annuities.

All the calculations they made were based on sexagesimal system i.e. by multiples or divisors of 60.

***Key-words :*** Abacus, Arithmetical progression, Cube-root, Cuneiform tablets, Geometric progression, Infinite approximation, Metrology, Old Babylonian collection, Pythagorean problem, Sexagesimal scale, Square, Square-root, Surface area.

Babylon (now in *Iraq*) was the capital of Mesopotamia, the land between rivers i.e. the *Tigris* and *Euphrates*. From here come mankind's oldest written historical

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records. The fifth *millenium* (a period of thousand years) BC was time of much migration of people and the establishment of villages and primitive agriculture. It was fully interrelated to Egyptian culture. The culture developed in Mesopotamia was three corner and it was among the bearers of that culture settled in towns, that was through the Nomadic people moving across the desert, the people living in plains of Sumer and Semitic people. Of them Sumerians were dominating Mesopotamia and Mesopotamians were influenced by Egyptians.

As per Sir George Rawlinson, *Sumerians* were civilised before 3000 B.C. Their merchants were familiar with Bills, Notes, Receipts, Accounts, System of measures, Calenders, Countings and Mathematics. We see that Babylonian Mathematics consists of Arithmetics, Algebra, Metrology and Geometry. These informations had been collected from the tablets which were excavated and restored in different Museums mainly from museum of Pennsylvania and British Museum. These tablets were made of clay on which Sumerians and Babylonians or Mesopotamians pressed their writing with a round and pointed stick and the result being circular, semi-circular or wedge shaped (known as Cuneiform) character. Tablets were baked by fire or in the Sun for preservation of their records.

At that time Sumerian city was developed in Mesopotamia and its culture spread in cities *Kish, Erech, Nippur, Larsa, Eridu, Lagash, Umma, Tello* and neighbouring areas.

In the period from 2637 BC to 2582 BC the Semitic people developed their culture in the reign called *Accad* and their king Sharrukin or Sargon conquered Mesopotamia and established a kingdom of *Sumer* and *Accad* and the culture was flourished.

*Mesopotamia* attained its highest position of scientific development in the sixth *Ammuru Dynasty* that is in the dynasty of king Hammurabi (1728-1686 BC). His capital was Babylon. He gave so much importance to his capital that the whole country was called Babylon and its culture became Babylonian culture in place of Sumerian or Mesopotamian culture.

From the discovery of the *Senkret* tablets there had been unearthed some 50,000 tablets at *Nippur* (the modern *Nuffar*). Of them there were many tablets related to Mathematics, Geometry and Astronomy. Probably these tablets were from a large library which seemed to have been destroyed by the *Elamites* about 2150 BC or a little earlier. By that exacavation the discoverers got a most extensive mass of cylinder containing Multiplications, Divisions tablets, tablets of Squares, Square-roots, Geometric Progression, a few computations and some calculations on mathematics. Some other tablets show that Sumerians and Babylonians could solve linear, quadratic, bi-quadratic equations and operations on some negative numbers but in most of the cases they used the scale of sixty.

All tablets we speak of were engraved between 2400 BC to 2000 BC.

Sumerians first gave the strange mixture of decimal and sexagesimal system with additive and multiplicative symbols, (Found in tablet of 28th century from *Breasteds Ancient times*)

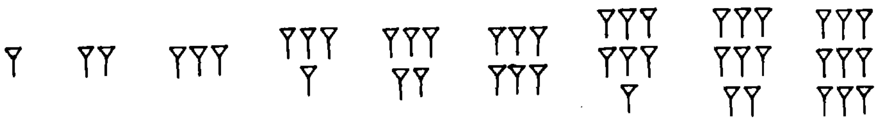
Babylonians used numbers sytems as

(a) the wedge = 1, 60, 3600, 12960000 ...................... = $60^\circ$

(b) the corner = 10, 600, 36000 .................................. = $10.60^\circ$

(c) in sexagesimal positional system

(d) the numerals were 1 to 59 and were constructed additively by symbols expressed in (a) and (b). (b) used zero 0 after Seleucid period (1800-1600 BC)



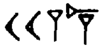
1 2 3 4 5 6 7 8 9

10 11 12 20 30 40 50 59

63=[1]3 132=[2]12 1547=[25]47

$2(30)=2\frac{1}{2}$ $\frac{3}{4}=0(45)$ $\frac{2}{27}=\frac{4}{60}+\frac{26}{60}+\frac{40}{60}=0(4)(26)(40)$

 (*lal* or *la'*) means minus used for operation only.

19 = 20 − 1      20 − 2 = 18

1   10   60   600   3600   36000   216000   2160000   12960000

The sexagesimal scale was first found in the tablet containing the records of magnitude of the illumination of the Moon's portion for everyday from New-Moon to Full-Moon and the whole disk being assumed to consist of 240 parts. The illuminated parts during the first five days are 5, 10, 20, 40, 1.20 = 60+20 = 80 parts which are in geometrical progression with common ratio 2 and from there parts of 11 days counting from 5th day are 1.20, 1.36, 1.52, 2.8, 2.24, 2.40, 2.56, 3.12, 3.28, 3.44, 4 and those are 80, 96, 112, 128, 144, 160, 176, 192, 208, 224, 240, parts and are explained by sexagesimal system as 1.20 = 60+20, 1.36 = 60+36, 1.52 = 60+52. 2.8 = 60 × 2+8, 2.24 = 60 × 2+24, 2.40 = 60 × 2+40, 2.56 = 60 × 2+56, 3.12 = 60 × 3+12, 3.28 = 60 × 3+28, 3.44 = 60 × 3+44, 4 = 60 × 4. So the parts of illumination from 5th day to 15th day after new - moon are in Arithmetical Progression with common difference 16. (Discovered by E. Hinck's in 1854)

Another tablet contained squares and cubes of numbers which being inverted gave square roots and cube roots of same numbers. It contained square of numbers from 1 to 60 and cubes of numbers from 1 to 32

Squares of numbers are 1, 4, 9, 16, ......., 1.4, 1.21, .......2.1, 2.24 .......58.1, 60 and these are explained as

Square of 1 is 1
Square of 2 is 4

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Square of 8 is 1.4 = 60 + 4 = 64
Square of 9 is 1.21 = 60 + 21 = 81

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Square of 11 is 2.1 = 60 × 2 + 1 = 121
Square of 12 is 2.24 = 60 × 2 + 24 = 144

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Square of 30 is 15 = 60 × 15 = 900

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Square of 59 is 58.1 = 60 × 58 + 1 = 3,481
Square of 60 is 60 = 60 × 60 = 3,600

Cubes of numbers are 1, 8, ...................., 56,15, 1[8]16, ..................9[6]8 and are explained as :

Cubes of 1 is 1
Cubes of 2 is 8

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Cubes of 15 is 56.15 = $60 \times 56 + 15 = 3{,}375$
Cubes of 16 is 1[8]16 = $60^2 \times 1 + 60 \times 8 + 16 = 4{,}088$

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Cubes of 32 is 9[6]8 = $60^2 \times 9 + 60 \times 6 + 8 = 32{,}768$

Babylonians used · (dot) for only 60 multiples and [ ] for multiples of 60 when 60 or more multiples of 60 existed in the numbers. They used the *lal* or *la'* meaning minus

They were acquianted with reciprocal of numbers for documentary evidence we find a tablet of reciprocal numbers under the scale of sixty of about 1800 BC.

| Nos. | Reciprocals | Explained as |
|---|---:|---|
| 2 | 30 | 1/2 = 60/2 = 30 |
| 3 | 20 | 1/3 = 60/3 = 20 |
| 4 | 15 | 1/4 = 60/4 = 15 |
| 5 | 12 | 1/5 = 60/5 = 12 |
| 6 | 10 | 1/6 = 60/6 = 10 |
| 8 | 7(30) | 1/8 = 60/8 = 7+30/60 |
| 9 | 6(40) | 1/9 = 60/9 = 6+40/60 |
| 10 | 6 | 1/10 = 60/10 = 6 |
| 12 | 5 | 1/12 = 60/12 = 5 |
| 15 | 4 | 1/15 = 60/15 = 4 |
| 16 | 3(45) | 1/16 = 60/16 = 3+45/60 |
| 18 | 3(20) | 1/18 = 60/18 = 3+20/60 |
| 20 | 3 | 1/20 = 60/2 = 3 |
| 24 | 2(30) | 1/24 = 60/24 = 2+30/60 |
| 25 | 2(24) | 1/25 = 60/25 = 2+24/60 |
| 27 | 2(13)(20) | 1/27 = 60/27 = $2+13/60+20/60^2$ |

| Nos. | Reciprocals | Explained as |
|---|---:|---|
| 30 | 2 | 1/30 = 60/30 = 2 |
| 32 | 1(52)(30) | 1/32 = 60/32 = 1+52/60+30/$60^2$ |
| 36 | 1(40) | 1/36 = 60/36 = 1+40/60 |
| 40 | 1(30) | 1/40 = 60/40 = 1+30/60 |
| 45 | 1(20) | 1/45 = 60/45 = 1+20/60 |
| 48 | 1(15) | 1/48 = 60/48 = 1+15/60 |
| [1] | 1 | 1/[1] = 1/60 = 60/60 = 1 |
| [1]4 | (56)(15) | 1/[1]4 = 1/64 = 56/60+15/$60^2$ |
| [1]12 | (50) | 1/[1]12 = 1/72 = 60/72 = 50/60 |
| [1]15 | (48) | 1/[1]15 = 1/75 = 60/75 = 48/60 |
| [1]20 | (45) | 1/[20] = 1/80 = 60/80 = 45/60 |
| [1]21 | (44)(26)(40) | 1/[1]21 = 1/81 = 60/81 = 44/60+26/$60^2$+40/$60^3$ |

In this chart Babylonians used [ ] as multiple of sixty and ( ) as increasing divisions of sixty. This tablet is not limited but is shown in gaps. There is no reciprocal for 7, for 11. for 13, for 14 etc. The reason is obvious because if we divide 7 into 1 we obtained recurrent sexagesimal fractions 1/7 = 60/7 = 8(34)(17)(8)(34)(17)(8).......... ................. similarly 1/11 = 60/11 = 5(27)(16)(21)(48)......................... appears in finite repetition.

In another tablet invented by A. Sachs we find reciprocals of 7, 11, 13, 14, 17 with approximation by inequalities as follows 8(34)(16)(59)<1/7<8(34)(18), 5(27)(16)< 1/11<5(27)(16)(22), 4(17)(8)<1/14<4(17)(8)(35), 3(31)(45)<1/17<3(31)(45)(53) which shows that they were acquianted with inequalities and infinite approximations.

Babylonians used unit fractions 1/6, 1/3, 1/2, 2/3, 3/4, and so on. As per them these fractions are known as *unit fractions* as its numerators are 1 or 1 less than its denominators.

They also named some fractions whose denominators are 360 or 60 and which are *Sussu* = 10/60 or 1/6 : *Sinipu* = 240/360 = 40/60 = 2/3; *Parab* = 300/360 = 50/60 = 5/6.

Here is a chart (Division and Multiplication Table) containing columns 1 to 12 whose 1st column contains divisors (*igi*) of 60 and other columns contain multiples

| 1st Column | | | Nos. | 2nd Col. | 3rd Col. | 4th Col. | 5th Col. | 6th Col. | 7th Col. | 8th Col. | 9th Col. | 10th Col. | 11th Col. | 12th Col. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| igi | 2 | 30 | 1 | 50 | 48 | 45 | 44(26) (40) | 40 | 36 | 30 | 25 | 25 | 22(30) | 20 |
| igi | 3 | 20 | 2 | [1]40 | [1]36 | [1]30 | [1]28(53)(20) | [1]20 | [1]12 | [1] | 50 | 50 | 45 | 40 |
| igi | 4 | 15 | 3 | [2]30 | [2]24 | [2]15 | [2]13(20) | [2] | [1]48 | [1]30 | [1]15 | [1]12 | [1]7 (30) | [1] |
| igi | 5 | 12 | 4 | [3]20 | [3]12 | [3] | [2]48(56)(40) | [2]40 | [2]24 | [2] | [1]40 | [1]36 | [1]30 | [1]20 |
| igi | 6 | 10 | 5 | [4]10 | [4] | [3]45 | [3]42(13)(20) | [3]20 | [3] | [2]30 | [2]5 | [2] | [1]52 (30) | [1]40 |
| igi | 8 | 7(30) | 6 | [5] | [4]48 | [4]30 | [4]26(40) | [4] | [3]36 | [3] | [2]30 | [2]24 | [2]15 | [2] |
| igi | 9 | 6(40) | 7 | [5]50 | [5]36 | [5]15 | [5]11(6)(40) | [4]40 | [4]12 | [3]30 | [2]55 | [2]48 | [2]37 (30) | [2]20 |
| igi | 10 | 6 | 8 | [6]40 | [6]24 | [6] | [5]55(33)(20) | [5]20 | [4]48 | [4] | [3]20 | [3]12 | [3] | [2]40 |
| igi | 12 | 5 | 9 | [7]30 | [7]12 | [6]45 | [6]40 | [6] | [5]24 | [4]30 | [3]45 | [3]36 | [3]22 (30) | [3] |
| igi | 15 | 4 | 10 | [8]20 | [8] | [7]30 | [7]24(26)(40) | [6]40 | [6] | [5] | [4]10 | [4] | [3]45 | [3]20 |
| igi | 18 | 3(20) | 11 | [9]10 | [8]48 | [8]15 | [8]8(53)(20) | [7]20 | [6]36 | [5]30 | [4]35 | [4]24 | [4]7 (30) | [3]40 |
| igi | 20 | 3 | 12 | [10] | [9]36 | [9] | [8]53(20) | [8] | [7]12 | [6] | [5] | [4]48 | [4]30 | [4] |
| igi | 24 | 2(30) | 13 | [10]50 | [10]24 | [9]45 | [9]37(46)(40) | [8]40 | [7]48 | [6]30 | [5]25 | [5]12 | [4]52 (30) | [4]20 |
| igi | 25 | 2(24) | 14 | [11]40 | [11]12 | [10]30 | [10]22(13)(20) | [9]20 | [8]24 | [7] | [5]50 | [5]36 | [5]15 | [4]40 |
| igi | 28* | 2(13)(20) | 15 | [12]30 | [12] | [11]15 | [11]6(40) | [10] | [9] | [7]30 | [6]15 | [6] | [5]37 (30) | [5] |
| igi | 30 | 2 | 16 | [13]20 | [12]48 | [12] | [11]51(6)(40) | [10]40 | [9]36 | [8] | [6]40 | [6]24 | [6] | [5]20 |
| igi | 35* | 1(52)(30) | 17 | [14]10 | [13]36 | [12]45 | [12]35(33)(20) | [11]20 | [10]12 | [8]30 | [7]5 | [6]48 | [6]22 (30) | [5]40 |
| igi | 36 | 1(40) | 18 | [15] | [14]24 | [13]30 | [13]20 | [12] | [10]48 | [9] | [7]30 | [7]12 | [6]45 | [6] |
| igi | 40 | 1(30) | 19 | [15]50 | [15]12 | [14]15 | [14]4(24)(40) | [12]40 | [11]24 | [9]30 | [7]55 | [7]36 | [7]7 (30) | [6]20 |
| igi | 45 | 1(20) | 20 | [16]40 | [16] | [15] | [14]48(53)(20) | [13]20 | [12] | [10] | [8]20 | [8] | [7]30 | [6]40 |
| igi | 48 | 1(15) | 30 | [25] | [24] | [22]30 | [22]3(20) | [20] | [18] | [15] | [12]30 | [12] | [11]15 | [10] |
| igi | 50 | 1(12) | 40 | [33]20 | [32] | [30] | [29]37(46)(40) | [26]40 | [24] | [20] | [16]40 | [16] | [15] | [13]20 |
| igi | 54 | 1(6)(40 | 50 | [41]40 | [40] | [37]30 | [38]2(13)(20) | [33]20 | [30] | [25] | [20]50 | [20] | [18]45 | [16]40 |
| igi | 60 | 1 | 44(26)(40) | | | | [30]55(18)(31)(6)(40) | | | | | | | |
| igi | 64 | (56)(15) | | | | | | | | | | | | |
| igi | 72 | (50) | | | | | | | | | | | | |
| igi | 80 | (45) | | | | | | | | | | | | |
| igi | 81 | (44)(26)(40) & igi (44)(26)(40) = 81 | | | | | | | | | | | | |

Number those are incorrect are marked by asterick (*)

**Explanation of above chart :-**

1st Column : igi 54 = 60 ÷ 54 = 1 (6) 40 = 1 + 6/60 + 40/60 = 1.111
2nd Column : 50 × 19 = [25] 50 = 15 × 60 + 50 = 950
3rd Column : 48 × 17 = [13] 36 = 13 × 60 + 36 = 816
4th Column : 45 × 11 = [8] 15 = 8 × 60 + 15 = 495
5th Column : 19 × 44 (26)(40) = 19 × (44+26/60 + 40/60) = 19 × 44.4444 = 844.4444
6th Column : 15 × 40 = [10] = 10 × 60 = 600
7th Column : 36 × 13 = [7] 48 = 7 × 60 + 48 = 468
8th Column : 30 × 30 = [15] = 15 × 60 = 900
9th Column : 25 × 16 = [6] 40 = 6 × 60 + 40 = 400
10th Column : 24 × 18 = [7] 12 = 7 × 60 + 12 = 432
11th Column : 22 (30) × 13 = (22 + 30/60) × 13 = 22.5 × 13 = 292.5
12th Column : 20 × 50 = [16] 40 = 16 × 60 + 40 = 1,000

of 50, 48, 44 (26)(40), 40, 36, 30, 25, 24, 22(30), 20 respectively (Mathematical Cuneiform Tablet No CBS 8536 in the Museum of University of Pennsylvania)

There were different symbols or languages used by the Babylonians which were *Soss* = 60, 10 *Sosses* = 600 = *ners*, 6 *ners* = 3600 = *Saru*, *igi - Gal* = denominator, *igi - data* = division. *A - Du* and *Ara* form time such as *Ara* - 1 = 18. *Ara* - 2 = 18 × 2 = 18 × 2 = 36, *Ara* also used for squaring of numbers as 3 *Ara* 3 = 9 = 3×3, *Ba - Di - E* for cubing the numbers as 27*E* 3 *Ba - Di* = $3^3$ = 27, *Ib - Di - E* for squaring of numbers as 9*E* 3 *Ib - Di* = $3^2$ = 9, *A - An* rendered numbers distribution.

Another tablet of Babylon contained geometrical progression of $60^4$ = 12,960,000 (*Babylonian* tablet from *Nippur*, about 2400 B.C. from H.V. Hilprecht excavation). and which is.

| | | | | | |
|---|---|---|---|---|---|
| Line - 1 | 125 | 720 | Line - 2 | IGI - GAL - BI | 1,03,680 |
| Line - 3 | 250 | 360 | Line - 4 | IGI - GAL - BI | 51,840 |
| Line - 5 | 500 | 180 | Line - 6 | IGI - GAL - BI | 25,920 |
| Line - 7 | 1000 | 90 | Line - 8 | IGI - GAL - BI | 12,960 |
| Line - 9 | 2000 | 18 | Line - 10 | IGI - GAL - BI | 6,480 |
| Line - 11 | 4000 | 9 | Line - 12 | IGI - GAL - BI | 3,240 |
| Line - 13 | 8000 | 18 | Line - 14 | IGI - GAL - BI | 1,620 |
| Line - 15 | 16000 | 9 | Line - 16 | IGI - GAL - BI | 810 |

Explanation of Chart :

| | | |
|---|---|---|
| Line - 1 | 125 = 2 × 60 + 5 | 720 = 12 × 60 + 0 |
| Line - 2 | Its denominator | 1,03,680 = 28 × $60^2$ + 48 × 60 + 0 |
| Line - 3 | 250 = 4 × 60 + 10 | 360 = 6 × 60 + 0 |
| Line - 4 | Its denominator | 51,840 = 14 × $60^2$ + 24 × 60 + 0 |
| Line - 5 | 500 = 8 × 60 + 20 | 180 = 3 × 60 + 0 |
| Line - 6 | Its denominator | 25,920 = 7 × $60^2$ + 12 × 60 + 0 |
| Line - 7 | 1,000 = 16 × 60 + 40 | 90 = 1 × 60 + 30 |
| Line - 8 | Its denominator | 12,960 = 3 × $60^2$ + 6 × 60 + 0 |
| Line - 9 | 2,000 = 33 × 60 + 20 | 18 = 10 + 8 |

| | | |
|---|---|---|
| Line - 10 | Its denominator | $6{,}480 = 1 \times 60^2 + 48 \times 60 + 0$ |
| Line - 11 | $4{,}000 = 1 \times 60^2 + 6 \times 60 + 40$ | 9 |
| Line - 12 | Its denominator | $3{,}240 = 54 \times 60 + 0$ |
| Line - 13 | $8{,}000 = 2 \times 60^2 + 13 \times 60 + 20$ | $18 = 10 + 8$ |
| Line - 14 | Its denominator | $1{,}620 = 27 \times 60 + 0$ |
| Line - 15 | $16{,}000 = 4 \times 60^2 + 26 \times 60 + 40$ | 9 |
| Line - 16 | Its denominator | $810 = 13 \times 60 + 30$ |

From the above chart – we also observe that (a) the first numbers of all odd lines (i.e. lines numbered 1, 3, 5, 7, 9, 11, 13, 15) form an increasing geometrical progression with common ratio 2, (b) all the end numbers of the even lines (i.e. lines numbered 2, 4, 6, 8, 10, 12, 14, 16) form a decreasing *Geometrical progression* with common ratio 1/2 (c) both the geometrical progressions mentioned in (a) (b) are divisons of $60^4$ = 12,960,000.

That the first number of every odd lines can be expressed as a fraction which has 12,960,000 as its numerators and closing numbers of the corresponding even lines as its denominators, other words.

$$125 = \frac{12{,}960{,}000}{1{,}03{,}680} \qquad 250 = \frac{12{,}960{,}000}{51{,}840}$$

$$500 = \frac{12{,}960{,}000}{25{,}920} \qquad 1{,}000 = \frac{12{,}960{,}000}{12{,}960}$$

$$2{,}000 = \frac{12{,}960{,}000}{6{,}480} \qquad 4{,}000 = \frac{12{,}960{,}000}{3{,}240}$$

$$8{,}000 = \frac{12{,}960{,}000}{1{,}620} \qquad 16{,}000 = \frac{12{,}960{,}000}{810}$$

but here the closing numbers of all odd lines i.e. 720, 360, 180, 90, 18, 9, 18, 9 are obscure to me.

The question arises in particular is the meaning of the number 12,960,000 ($60^4$ on $3600^2$). This geometric number (12,960,000) which *Plato* (in his Republic VIII 546 - B) called *The lord of better and worse births* is the arithmetical expression of a great law controlling the universe. That is 12,960,000 days = 36,000 years of 360 days known as Great *Platomic year* (the duration of *Babylonian* cycle). A man's life was considered of 100 years = 36,000 days which is as many days as there are years in the *great platonic* year. So the *geometric number* measuring or governing the Earth and life on Earth, was clearly of Babylonian origin.

Babylonians believed that circle of the year consisted of 360 days and also knew that the sides of the regular hexagon inscribed in a circle is equal to the radius of the circle. This property of geometry suggested the division of 360 into six equal parts and 60 being looked upon a kind of symetric number. It might also be probable that 60 had chosen because of its integral divisors 2, 3, 4, 5, 6, 10, 12, 15, 20 and 30. So Babylonians choose sexagesimal system in multiple of fractions.

Now dealing with *metrology* which is the daughter of business. The selling and buying imply the existence of prices and practice of measuring and weighing.

In a *Louvre* tablet no AO - 6770 we see the computation of money to its double at the rate of 20% compound interest. The problem would put into find x in the equation $(1 + 0.12)^x = 2$ where 0.12 = 0 + 12/60 = 1/5 = 20%. The correct result was 3(48) that is 2.8 years = 3 + 48/60 and it was duly obtained by the Sumerian's computer.

For measurement and weighing they used following devices.

a) to measure surface area Babylonians considered

i) *iku* = area of land by which they could produce one Ass - load of food grains i.e. the area of land by which they were able to produce 100 *qa* or 150 *lbs* or 68 *kgs* of foodgrains.

ii) *subtum* = area of land which was large enough to build one house which is 144 square *cubits* of land area or 324 *square feet* of land area. It might be mentioned that they somewhere used *sar* in place of *subtum.*

They also divided *subtum* into 60 equal parts.

1 *Subtum* = 60 *shekels* = 2 (24) *square cubits* = 2 + 24/60 *square cubits* = 5.4 *square feet.*

b) to measure length they used *cubits* whereas

1 *cubits* = 18 *feet*

c) to measure liquids they used following units :

1 *sack* (*naruqqum*) = 120 *qa* (where 1 *qa* was used to measure drinks consumed normally by one primitive man daily)

4 *jars* (*karpatum*) = 1 *sack*

i.e. 1 *Jar* = 30 *qa* and 2 *sarsaranums* = 1 *Jar*, 1 *sarsaranum* = 15 *qa*.

d) to weight the grains or cereal products Babylonians used *talent, mina, shekel, grain* which were in the ratio 3600 : 60 : 1 : 1/180, whereas 1 *talent* = 60 *lbs* (*pounds*) approximately.

At that time food-grains were sent by *Ass-load.*

1 *Ass-load.* = 100 *qa* = 150 *lbs.* = 68 *kgs* (whereas 1 *qa* was considered to be the quantity of food-grains consumed by one primitive man daily)

e) In trading they used *subat kutanu* as unit for measuring textiles and *annukum* for measuring or weighing metal.

1 *subat kutanu* = sufficient size of cloth by which one could make garments for a grown up person.

25 *subat kutanu* = 1 *Ass-load*

They also export lead, silver and copper by *Ass-load.*

1 *Ass-load* of metal (*annukum*) = 2.5 *talent* = 150 *lbs* (*pounds*) = 6.8 *kgs.*

At that time price of Textiles were more than lead and price of silver was from 12 to 16 times of lead.

As they can solve exponential equation (mentioned in *Louvre* tablet no AO 6770) we would not be surprised to learn that they were able to solve linear, quadratic and cubic equations of algebra.

Example of solving linear simultaneous equations of Algebra by Babylonians.

***Problem :***

Sum of length and width of rectangle is 10 and length and one-fourth of width together equal to 7.

| ***Babylonian Method*** | ***Modern Explanation*** |
| --- | --- |
| | In modern notation we could express length $l$ and width w of a rectangle then $l + w = 10$, $l + (1/4)w = 7$ |
| $7 \times 4 = 28$ | Now $4l + w = 28$ <br> $l + w = 10$ |
| $28 - 10 = 18$ <br> $18 \times (1/3) = 6$ (the length) <br> $10 - 6 = 4$ (the width) | Subtracting $3l = 18$ <br> $l = 18 \times (1/3) = 6$ <br> $w = 10 - 6 = 4$ |

This example shows that the Babylonians followed the same sequence of steps that we do but they did not use the letter symbols to represent numbers.

***Problem :***

Through the modern notations we can express a text of *Babylonian Mathematics* that, let us consider a number x and its reciprocal $\bar{x}$ when $x.\bar{x} = 1$, $x + \bar{x} = b$ and $b = 2\ (0)\ (0)\ (33)\ (20)$ and they solved the problem for x.

The detailed solution of the quadratic equations were as follows :

$$b = 2\ (0)\ (0)\ (33)\ (20) = 2 + 0/60 + 0/60^2 + 33/60^3 + 20/60^4$$

$$\text{or, } b/2 = 1\ (0)\ (0)\ (16)\ (40) = 1 + 0/60 + 0/60^2 + 16/60^3 + 40/60^4$$

$$\begin{aligned} \text{or, } (b/2)^2 &= 1\ (0)\ (0)\ (33)\ (20)\ (4)\ (37)\ (46)\ (40) \\ &= 1 + 0/60 + 0/60^2 + 33/60^3 + 20/60^4 + 4/60^5 + 37/60^6 + 46/60^7 + 40/60^8 \end{aligned}$$

Now, subtracting 1 and finding square root we get,

$$\begin{aligned} \sqrt{(b/2)^2 - 1} &= \sqrt{0\ (0)\ (0)\ (33)\ (20)\ (4)\ (37)\ (46)\ (40)} = 0\ (44)\ (43)\ (20) \\ &= 0 + 0/60 + 44/60^2 + 43/60^3 + 20/60^4 \end{aligned}$$

$$\begin{aligned} \text{then, } x &= b/2 + \sqrt{(b/2)^2 - 1} = 1\ (0)\ (0)\ (16)\ (40) + 0\ (44)\ (43)\ (20) \\ &= 1\ (0)\ (45) = 1 + 0/60 + 45/60^2 \\ &= 1.0125 \text{ in decimal} \end{aligned}$$

$$\begin{aligned} \text{and } \bar{x} &= b/2 - \sqrt{(b/2)^2 - 1} = 1\ (0)\ (0)\ (16)\ (40) - 0\ (44)\ (43)\ (20) \\ &= 0\ (59)\ (15)\ (33)\ (20) \\ &= 0 + 59/60 + 15/60^2 + 33/60^3 + 20/60^4 \\ &= 0.9876542 \text{ in decimal} \end{aligned}$$

***Problem :***

In another tablet we find 8th degree special problem which is explained as X and Y are sides of a rectangle, d = diagonal of the rectangle and X.Y = 2 0 (0), $X^3 d$ = 14 (48) (53) (20)

This problem had been expressed as equivalent quadratic equation of $X^4$ as $X^8 + a^2 X^4 = b^2$ Where a = 20 (0) and b = 14 (48) (53) (20)

[As we know that $d = \sqrt{X^2 + Y^2}$ and squaring the second equation we get $X^6 d^2 = b^2$ i.e. $X^6 (X^2 + Y^2) = d^2$; or, $X^8 + X^2Y^2 X^4 = b^2$ or, $X^8 + a^2X^2 = b^2$]

They find $b^2$ = 219 (28) (43) (27) (24) (5) (28) (19) (12)

$= 219 + 28/60 + 43/60^2 + 27/60^3 + 24/60^4 + 5/60^5 + 28/60^6 + 19/60^7 + 12/60^8$

or, $X^4 = \sqrt{a^4 - 4.b^2/2} - a^2/2 = 200$

or, $X^2 = \sqrt{200}$ = 14 (8) (31) (41) (17) (40) .......

or, $X = \sqrt{14\ (8)\ (31)\ (41)\ (17)\ (40)\ .......} = 4$

so $Y = 5$

They also knew algebraic mean of finding successive approximation of square-root of numbers as we find from a small tablet of Old Babylonian Collection.

***Example :***

Babylonian Method of finding square-root of 2 by successive approximation and its value

$$\sqrt{2} = 1\ (25) = 1\frac{25}{60} = 1\frac{5}{12}$$

As first approximation to $\sqrt{2}$ they choose a number such that its square is close to 2 such a number is 1,

so, 1 $\frac{2}{1} = 2.$

Average of 1 and 2 = 1 (30) = $1\frac{1}{2}$

For second approximation we see $\left(1\frac{1}{2}\right)^2 = 2\ (15) = 2\frac{1}{4}$

Which is greater than 2, so $1\frac{1}{2}$ is too large for the value of $\sqrt{2}$

Now $1\frac{1}{2} \cdot \frac{2}{1\frac{1}{2}} = 2$ i.e. $1\frac{1}{2} \cdot 1\frac{1}{3} = 2$

So for third approximation they choose the average of $1\frac{1}{2}$ and $1\frac{1}{3}$

Hence, $\sqrt{2} = \frac{1}{2}\left(1\frac{1}{2} + 1\frac{1}{3}\right) = 1\ (25) = 1.416667$

***Example :***

Finding square root of 7 i.e. $\sqrt{7} = 2\ (38)\ (45) = 2\frac{31}{48}$

As first approximation Babylonians find a number nearest to $\sqrt{7} = 3$

so, $3 \cdot \frac{7}{3} = 7$

for second approximation average of 3 and 7/3 = 1/2 (3 + 7/3) = 8/3;

Now, $(8/3)^2 = 64/9 = 7\frac{1}{9}$

or, $2\frac{2}{3} \cdot 2\frac{5}{8} = 7$

or, $2\frac{2}{3} \cdot \frac{7}{2\frac{2}{3}} = 7$

for third approximation average of $2\frac{2}{3}$ and $2\frac{5}{8} = 1/2\left(2\frac{2}{3} + 2\frac{5}{8}\right) = 2\frac{31}{48}$

So, $\sqrt{7} = 2\frac{31}{48} = 2\ (38)\ (45) = 2.6458333$

Another way to finding square root $\sqrt{2}$ was by application of a geometrical hypothesis from a small tablet of Yale Babylonian Collection.

***Example :***

They found a square of side 30 with diagonal numbers 1 (24) (51) (10) and 42 (25) (35). The relation between the diagonal numbers was 1 (24) (51) (10) $\times$ 30 = 42 (25) (35) = 1 (24) (51) (10) $\div$ 2 as 2 and 30 are reciprocal to one another. From the first part of the relation we see if a = 30 then diagonal = $a\sqrt{2}$ = 42 (25) (35).

Therefore value of $\sqrt{2}$ = 1 (24) (51) (10) was considered then. The accuracy of the approximation of the value of $\sqrt{2}$ can be checked by squaring 1 (24) (51) (10) = 1 (59) (59) (59) (38) (1) (48) which is nearly equal to 2. For estimation of the corresponding error we calculate as error = 2 - 1 (59) (59) (59) (38) (1) (48) = 1.69521 $\times 10^6$.

This is indeed a remarkable good approximation and it was used by *Ptolemy* in computing his table of chords two thousand years later.

The above example of determination of the diagonal of the square from its side is sufficient to prove Pythagorean Theorem was known in *Seleucid* period (1800 - 1600 BC) of Babylon i.e. more than thousand years before Pythagoras. For the purpose we find a chart of Seleucid period or old Babylonian time discovered by Plimpton of Columbia University containing 15 lines and three columns of *Pythagorean* numbers i.e. numbers related to $l^2 + b^2 = h^2$ ($l$ = height, b = base, h = hypothenuse of a right-angled triangle) and fourth column had been deducted.

| Sl. No. | I = h2/l2 | II = b | III = h | $l$ |
|---|---|---|---|---|
| 1. | 1(59)(0)(15) | 1(59) | 2(49) | 2(0) |
| 2. | 1(59)(56)(58)(14)(50)(6)(15) | (56)(7) | 3(12)(1) | (57)(36) |
| 3. | 1(55)(7)(41)(15)(33)(45) | 1(16)(41) | 1(50)(49) | 1(20)(0) |
| 4. | 1(5)(3)(1)(0)(29)(32)(52)(16) | 3(31)(49) | 5(9)(1) | 3(45)(0) |
| 5. | 1(48)(54)(1)(40) | 1(5) | 1(37) | 1(12) |
| 6. | 1(47)(6)(41)(40) | 5(19) | 8(1) | 6(0) |
| 7. | 1(43)(11)(56)(28)(26)(40) | 38(11) | 59(1) | 45(0) |
| 8. | 1(41)(33)(59)(3)(45) | 13(19) | 20(49) | 16(0) |
| 9. | 1(38)(33)(36)(36) | 9(1) | 12(49) | 10(0) |

| Sl. No. | I = h2/l2 | II = b | III = h | *l* |
|---|---|---|---|---|
| 10. | 1(35)(10)(2)(28)(27)(24)(26)(40) | 1(22)(41) | 2(16)(1) | 1(48)(0) |
| 11. | 1(33)(45) | (45) | 1(15) | 1(0) |
| 12. | 1(29)(21)(54)(2)(15) | 27(59) | 48(49) | 40(0) |
| 13. | 1(27)(0)(3)(45) | 7(12)(1) | 4(49) | 4(0) |
| 14. | 1(25)(48)(51)(35)(6)(40) | 29(31) | 53(49) | 45(0) |
| 15. | 1(23)(13)(46)(40) | (56) | (53) | 1(30) |

In this text successive (  ) indicates the increasing divisions by 60 as we have seen before. It contains a few errors as

(a) II-column 9th line we find 9(1) instead of 8(1) which is mere scribal error

(b) II-column 13th line the text has 7(12)(1) instead of 2(41) which shows that they wrote square of 2(41) which is 7(12)(1)

(c) III-column 15th line we find (53) instead of 1(46) which is twice of (53)

(d) Finally there remains an unexplained error in III-column 2nd line where 3(12)(1) should be replaced by 1(20)(25)

In the charts '*l*' has been introduced by computation later but it was not originally in it.

From the chart we can find base angle and vertical angle as from the first line we get b/*l* = 1(59)/2 = 0(59)(30) which is nearly equal to 1 i.e. tan x = 1 = tan 45° = *l*/b. Hence base angle = 45° so vertical angle will be 45° as b/*l* = tan y = 1 = tan 45°

Now from the last line b/*l* = 1(30)/(56) = 1(36)(25)(42)(52) = 1.6074 i.e. $\tan^{-1}$ (b/*l*) = 58° 6'33" i.e. nearly equal to 60° so vertical angle of the triangle is 60°. The ancient Babylonian Mathematicians who composed this text was not only interested in determining triples *l*, b, h, of Phythagorean numbers but also in their ratio h/*l*.

The most interesting fact connected with the Babylonian geometers was their familiarity with the Pythagorean theorem.

In 1916 Weidner gave the translation of what we called *zwei au βerst sinnreiche Methoden* for finding the hypothenuse of a right-angled triangle with known legs; the date was about 2000 BC. The legs being a and b, the first approximation is $c = a + 2ab^2/3600$ and the second $c = a + b^2/2a$

The curius denominator in the first approximation seems to come from juggling with 60, the basis of the number system in Babylonia, the second approximation is better as

$$c = (a^2 + b^2)^{1/2} = a(1 + b^2/a^2)^{1/2} = a + b^2/2a + \ldots\ldots\ldots\ldots$$

by application of binomial theorem which was into the knowledge of *Babylonians* but not in the modern form as we expressed above.

A problem on application of Pythagorean theorem is as follows.

***Problem :***

To find the length of the chord of a circle when we know the radius, and the depth of the sector.

Circumference = 1 depth of the sector = 2, find the chord.

From Struve's translation a cuneiform text we find. *Thou, square 2 and get 4, does thou not see? Take 4 from 20, thou gettest 16. Square 20, thou gettest 6(40). Square 16, thou gettest 4(16). Take 4(16) from 6(40), thou gettest 2(24). find the square-root of 2(24). 12 the square root is the chord.* This is the method.

In modern expression EF = depth of the sector = 2 = b; circumference = 1 = 60
Diameter = d = 60/3 = 20 = AC = BD (Considering $\pi = 3$)

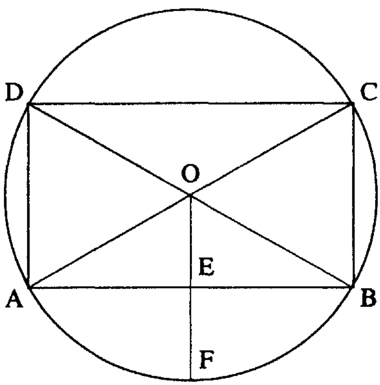


Fig. 1 :

AFBCD is a circle, O is the centre (Fig. 1),

Twice the depth = 2b = 2 × 2 = 4

Subtract 4 from diameter d = d - 2b = 20 - 4 = 16

Square of d = 20 × 20 = 6(40) = 400

i.e. d2 = 6(40) = 400

OE = OF - FE = radius - depth

So, CB = 2 × OE = 2(radius - depth) = d - 2b = 16

Square of d - 2b = 16 × 16 = 256 = 4(16)

Now in right-angled triangle ABC,

$AB^2 = AC^2 - CB^2 = d^2 - (d - 2b)^2$

Therefore, Chord = $\sqrt{d^2 - (d - 2b)^2} = \sqrt{6(40) - 4(16)} = \sqrt{2(24)} = \sqrt{400 - 256} = \sqrt{144} = 12$

In the first line of original translation the phrase was "*Square the depth*" is confusing what we need to do is to double the depth. Here the ratio of the sides of the triangle 12 : 16 : 20 = 3 : 4 : 5 and these figures are almost universal in every *Pythogorean* problem, it seems certain that the *Sumerians* understood the general principle. It is to be noted also that they knew the theorem, usually ascribed to *Thales of Miletus,* that an angle inscribed in a semi-circle is right angle.

Another interesting tablet, dealing with a *Pythogorean* problem, is to be found in *Thureu Dangin* whose translation is given below.

*(Soit) un pala 30", c'est â dire canne. En hant it est descendu de 6" en bas de combein s'est - il e'loigne?*

*'Toi care 30" tu trouveras 15'. Sou (trais) 6" de 30" (tu trouveras 24"). Carre 24", tu trouveras 9'36". Soutrais 9'36" de 15', tu troueras 5'24". 5'24" est combein au carre? c'est 18" au carre. De 18" sur le sol il s'est e'loigne'?*

It is the calculation of the problem that : A straight rod 30 in. stands upright, then it slips so that the upper end slides 6 in. down a vertical wall. How far has the foot come out?

From Babylonian tablet we expressed as follows :

| **Babylonian Solution** | **Modern Solution** |
|---|---|
| Patu 30 (Fig. 2) | Beam of length 30 |
| 30 - 6 = 24 | Now height is 30 - 6 = 24 |
| | Whose hypothenuse = 30 |
| 30 × 30 = 15 | Square of hypothenuse = 900 = 15 |
| 24 × 24 = 9(36) | Square of height = 576 = 9(36) |
| 15 - 9(36) = 5(24) | $(\text{hypothenuse})^2 - (\text{height})^2 = 900 - 576 = 324 = 5(24)$ |
| $\sqrt{5(24)} = 18$ | base = $\sqrt{324} = 18$ |

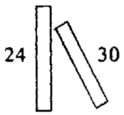


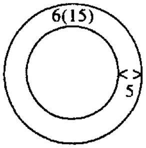

Fig. 2 :

This problem is the direct application of *Pythogoras theorem.*

From books of Moritz Cantor we find the explanation of the following problem extracted from a Babylonian tablet that approximation to the value of $\pi$ (pi) was 3.

***Problem :***

I have drawn boundary of a city (in inner circle in figure). I do not know its length. I have walked 5 from the first circle away from the centre in all directions and I have drawn a second boundary. The area between is [6]15. Diameter of the new and old city was found.

| <u>**Babylonian Solution**</u> | <u>**Modern Solution**</u> |
| --- | --- |
| <br><br>Fig. 3 : | Let the radii of the new and old city be represented by R and r, respectively (Fig. 3). And A = area of the enclosed region. So A = [6]15. $R - r = 5$. |
| Multiply the 5 of the increase with 3; you get 15. | $A = \pi R^2 - \pi r^2 = \pi (R - r)(R + r)$ therefore $[6]15 = 3.5 (R + r)$ Considering approximate value of $\pi$ (pi) = 3. |
| Take the inverse of 15 and multiply this by [6]15, the enclosed area, you get 25 | we find $(R + r)$ by multiplying [6]15 by the inverse of 15, that is by 0(4). Hence we get $(R + r) = 0(4) \times [6]15 = 25$ |
| Write this 25 twice | $R + r = 25$ |
| Add the 5, which you have walked | $(R + r) + (R - r) = 25 + 5 = 30$<br>$2R = 30$ |
| Subtracted the 5, which you have walked | $(R + r) - (R - r) = 25 - 5 = 20$<br>$2r = 20$ |
| Half of the result give you | $R = 15, r = 10$ |

In a temple built by *Solomon* which was round in shape, the diameter from rim to rim being 10 *cubits*; it stood five *cubits* high and it took a line thirty *cubits* long to go round it, which showed that they use value of $\pi$ (pi) = 3.

Babylonians used area of circle A and its circumference is C.

$$A = 0(5)\ C^2 = \frac{1}{12} \times C^2 \text{ using } \pi = 3$$

Explanation : $C = 2 \pi r$ and $A = \pi r^2$ i.e. $A = \pi (C/2\pi)^2$ or, $A = C^2/4\pi$ putting $\pi r = 3$, we get $A = C^2/12 = 0(5)\ C^2$

As per Thureau-Dangin, Babylonian used to divide field into rectangles, right triangles and rectangular trapezoids for survey operation and volume of a frustum of a regular pyramid of square base was calculated by Babylonians in a broken tablet and expressed in modern notation as

$$V = h/3\ [(a + b)^2/4 + (a - b)^2/4] \text{ or, } V = h\left[\frac{1}{3}\left(\frac{a+b}{2}\right)^2 + \frac{1}{3}\left(\frac{a-b}{2}\right)^2\right]$$

which were incorrect and correct formula would be $V = \frac{h}{3}(a^2 + b^2 + ab)$, where h = height, a = side of the base, b = side of top.

Again we find a tablet containing algebraic expression of geometry. I am giving the English version of Clay table No. A0 – 8862 from 0. Neugebauer, (1935-37).

| Lines | Babylonian Expression | Modern Expression |
|---|---|---|
| 1 - 2 | Length, Width | Let the length be x, the width y |
| 3 - 5 | Length and width I have multiplied and thus formed the area. | The area is then equal to xy |
| 6 - 9 | I have further added the excess of the length over the width to the area [3]3 | $x - y + xy = [3]3$ ................ (1) |
| 10 - 11 | Further, I have added the length and the width 27 | $x + y = 27$ ............................ (2) |
| 12 | You follow the method | |
| 13 - 15 | 27 + [3]3 = [3]30 | Add (1) and (2) thus<br>$x + y + x - y + xy = [3]30$<br>$x(2 + y) = [3]30$ ................. (3) |
| 15 - 16 | 2 + 27 = 29 | $2 + x + y = 29$ .................... (4)<br>Here a new variable is introduced<br>$y' = y + 2$<br>(This follows from lines 25 - 27 in the text).<br>We now write (3) and (4) as<br>$xy' = [3]30,\ x + y' = 29$ |
| 16 | Take the half of 29, that is 14(30) | Let $x = 14(30) + a$ and<br>$y = 14(30) - a$<br>$[14(30)]^2 - a^2 = xy' = [3]30$ |
| 17 | 14(30) × 14(30) = [3]30 (15) | $[3]30\ (15) - a^2 = [3]30$ |
| 18 - 20 | [3]30 (15) – [3]30 = 0(15) | $a^2 = [3]30\ (15) - [3]30 = 0(15)$ |
| 20 - 21 | The square-root of 0(15) is 0(30) | $a = \sqrt{0(15)} = 0(30)$ |
| 21 - 22 | 14(30) + 0(30) = 15 length | Hence $14(30) + 0(30) = 15 = x$ |
| 23 - 24 | 14(30) – 0(30) = 14 width | $y' = 14(30) - 0(30) = 14$ |

| | | |
|---|---|---|
| 25 - 26 | Subtract the 2 you added to 27 from 14, the width | $y' = y + 2$<br>$y = y' - 2 = 14 - 2$ |
| 27 | 12, the real width | So $y = 12$ |
| 28 - 29 | 15 length, 12 width I have multiplied<br>15 × 12 = [3]0 area | $x = 15$, $y = 12$<br>so $xy$ = [3]0 |
| 30 - 32 | 15 - 12 = 3 | Now to check $x \dot{-} y = 3$ |
| 32 - 33 | [3]0 + 3 = [3]3 | $xy + x - y$ = [3]3 |

The above tablet is characteristic of Babylonian algebra applied to geometry and they followed the same line of thought as we do, but did not have anything approximating modern notations.

A French archaeologist at *Susu* found 200 tablets and from them we find.

a) a circumcircle of radius 'r' where r = 31(15) = 31 + 15/60 = 31.25 circumscribing an isosceles triangle with sides 50, 50 and 60.

b) Figure of regular hexagon from which they had deducted the value of $\sqrt{3}$ = 1(45) = 1 + 45/60 = 1.75

c) Figure of an equilateral triangle from which deducted the value of $\sqrt{3}$ = 1(45)

d) Figure of a square give the value of $\sqrt{2}$ = 1(25)

e) Area of regular pentagon, hexagon, heptagon and circle estimated. From these we can explain in modern form as $A_n$ = the area, $S_n$ = sides of regular n-gon then (1) Area of regular pentagon $A_5$ = 1(40). $S_5^2$, Area of regular hexagon = $A_6$ = 2(37)(30). $S_6^2$, Area of regular heptagon = $A_7$ = 3(41). $S_7^2$. (2) $C_6$ = Perimeter of the regular hexagon = 0(57)(36). C where C = circumference of the circumscribing circle. Now $C_6$ $3/\pi$. C Shows that $\pi$ = 3(7)(30) = $3\frac{1}{8}$ [$3/\pi$ = 0(57)(36)]

In a tablet we find the following table

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | e | 1 | ib - si | i.e. $\sqrt{1}$ | = | 1 |
| 1, 2, 1 | e | 1, 1 | ib - si | i.e. $\sqrt{121}$ | = | 11 |
| 1, 2, 3, 2, 1 | e | 1, 1, 1 | ib - si | i.e. $\sqrt{12321}$ | = | 111 |
| 1, 2, 3, 4, 3, 2, 1 | e | 1, 1, 1, 1 | ib - si | i.e. $\sqrt{1234321}$ | = | 1111 |

In other way we explain the table as

$1^2 = 1,$

$11^2 = (10 + 1)^2 = 1.100 + 2.10 + 1$

$111^2 = (100 + 10 + 1)^2 = 1.10000 + 2.1000 + 3.100 + 2.10 + 1$

$1111^2 = (1000 + 100 + 10 + 1)^2 = 1.1000000 + 2.100000 + 3.10000 + 4.1000 + 3.100 + 2.10 + 1$

Another broken tablet shows

| | | | | | |
|---|---|---|---|---|---|
| 1 | e | 1 | means | $\sqrt[3]{1} = 1$ | or $1^3 = 1$ |
| 1, 3, 3, 1 | e | 1, 1 | means | $\sqrt[3]{1331} = 11$ | or $11^3 = (10 + 1)^3 = 1.1000 + 3.100 + 3.10 + 1$ |

From the above two tablets we may conclude that they were acquainted with the Binomial Co-efficients. But we get no proof of generalised *Binomial Co-efficients.*

In a tablet we find value of $n^2 + n^3$ where n is an integer :

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| n = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $n^2 + n^3$ = | 1 | 12 | 36 | [1]20 | [2]30 | [4]12 | [6]32 | [9]36 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| [13]30 | [18]20 | [24]12 | [31]12 | [39]26 | [49] | [60] |

which show that they were familiar with polynomials.

**Explanation of above chart** : when n = 13, $n^2 + n^3 = 13^2 + 13^3 = 169 + 2197 = 2366 = 39 \times 60 + 26$ = [39]26.

At that time Babylonians or Sumerians were familiar with the process such as reduction of similar terms, elimination of one unknown quantity by substitution, introduction of an Auxiliary unknown quantity and were aware of identity which we express as $(a + b)^2 = a^2 + 2ab + b^2$ and had an algebraic mean of finding successive approximation of square-root of numbers and for negative numbers they use *lal* or *la'* means minus. In the period 2,200 - 2,000 BC the Babylonians knew to measure the area of rectangle, square, trapezium (trapezoid), right-angled triangle, isosceles triangle, knowledge of Pythagorean theorem, angles of right angled triangle, angles of semi-circle, squares & cubes of numbers as well as multiplication and division. They could measure the volume of rectangular parallelopiped, right circular cylinder, fustrum of a cone & square pyramid. They knew the *Abacus* since it had been suggested that one of the sign (SID) might have been derived from a pictograph of such instrutment. For circular measure Babylonians mostly used to consider value $\pi$ (pi) = 3 which was not correct but the approach was genius.

In conclusion we may say all the Giants of Mathematics of Babylon that is Mesopotamia were inter-related to Egypt, India, China because at that time civilisation and cultivation of science was started in those countries and we find similar type of development of mathematics in those countries then.

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