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ON TWO IMPORTANT PROVISIONS IN VEDĀṄGA-JYOTIṢA

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It is shown that *Ṛgjyotiṣa* verse 12 of the *Vedāṅga-Jyotiṣa* which is identical with *Yajurjyotiṣa* verse 27, gives the rules for correcting the length of the 5-year *yuga* from 1830 to 1831 days and the length of the year from 372 to 371 *tithis*. It is also shown how this was used for modifying the calendar.

Key words : *Vedāṅga-Jyotiṣa*, Corrections for 5-year *yuga*.

Introduction

The *Vedāṅga-Jyotiṣa* was formulated by sage Lagadha when the winter solstice position of the sun was in Dhaniṣṭhā nakṣatra (β Del) around 1400 BC. It replaced the earlier basically observational procedure of making a calendar by a mathematical device which is apparently the first and oldest scheme of its kind in the world. It involves the use of round numbers for describing the positions and motions of the sun and the moon in the first approximation and gives rules for correcting them for better accuracy.

The *Vedāṅga-Jyotiṣa* describes a calendar for a 5-year *yuga* which starts when the sun and the moon together cross into *Vāsava* (*Dhaniṣṭhā*) *nakṣatra* during Māgha month indicating the beginning of *Uttarāyaṇa*, i.e., winter solstice, and *Tapas* (*śiśir*)-*ṛtu*. The *Ṛgjyotiṣa* verse 34 tells how this can be ascertained by observing the *nakṣatra* of the moon on the *kṛṣṇa-caturdaśī* (K 14) preceding the beginning of the *yuga*.

The nominal definition of the *yuga* is given in *Yajurjyotiṣa*, verses 28 to 31. Accordingly one *yuga* contains 5 years, each containing 366 days, 6 *ṛtus* (seasons), 2 *ayanas* (semesters) and 12 solar months of 30½ days. Further

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the *yuga* contains 1830 *sāvana* (civil) days, 1835 sidereal days, 1768 moon-rises, 1809 lunar *nakṣatras*, 135 solar *nakṣatras*, 134 *ayanas* of the moon corresponding to 67 sidereal lunar months, 124 *parvas* (syzygies), 61 *sāvana* months of 30 days each and 62 synodic lunar months containing 1860 *tithis*. These verses do not occur in *Ṛgjyotiṣa*, but they are assumed, because the units of angle and time derived from them are also used in *Ṛgjyotiṣa*. This shows that the two versions of the *Vedāṅga-Jyotiṣa* supplement each other.

Here we wish to point out that the above nominal definition of 5-year *yuga* involves two approximations: (i) 62 synodic lunar months contain 1831 and not 1830 days, but Lagadha has made an approximation for calculational purposes, (ii) 1860 *tithis* in 5 years amount to 372 *tithis* per year. Actually we have close to 371 *tithis* in a year. This is again an approximation introduced for computational purposes, but it makes the *yuga* longer by 5 *tithis*. It is our aim to show that Lagadha does give rules for correcting these approximations without which the whole scheme would be out of use within a short period of time.

Units of angle and time in Vedāṅga-Jyotiṣa

Lagadha defined the units of angle and time by solving for the first time indefinite equations in terms of integers as follows :

(a) The sun is used for measuring angle. As 135 solar *nakṣatras* are traversed by the sun in 1860 *tithis* we write 135 N = 1860 t, where N is the angular measure of a *nakṣatra* and t is the angle covered by the sun in one *tithi*. Then $\left(\frac{N}{t}\right)=\left(\frac{1860}{135}\right)=\left(\frac{124}{9}\right)$. So Lagadha defines 1N = 124 Bh°, Bh° stands for *bhāṃśas*, i.e., parts of a circle. And the sun moves through 9 Bh° in one *tithi*. Accordingly, the 360° circumference of the circle contains $124 \times 27 = 3348$ Bh°.

(b) The moon is used for measuring time. As 1809 *nakṣatras* are covered by the moon in 1830 days we write 1809 n = 1830 d, where n is the time taken by the moon to cover one *nakṣatra* and d is the time length of the day in the same units. Then $\left(\frac{n}{d}\right)=\left(\frac{1830}{1809}\right)=\left(\frac{610}{603}\right)$. So Lagadha defines

1 d = 60̇3 K (*kalās*) and the moon takes 610K to move through one *nakṣatra*.

(c) Further, in order to tie with the 30 *tithis* in a month, or, 30 days in a civil month, and 124 *parvas* in a *yuga* the day is also divided into 30 *muhūrtas* as well as 124 day parts (*divasāṃśas*) which will be denoted by us by D. One day is also divided into 4 quarters (*pādas*) of 31 D. Then We find that one *tithi* contains $\frac{1830}{1860}$ d = 124 D. The *muhūrtas* are also divided into *nāḍikās* and *kāṣṭhās* as *muhūrta* = 2 *nāḍikās*, 1 *nāḍikā* = 124 *kāṣṭhās*. All these time units were measured by a clepsedra.

These units of angle and time are so very different from those used in Babylon that there is no question of any borrowing from Babylonians as suggested by David Pingree[1]. Pingree has drawn attention to the ratio of $\frac{3}{2}$ for the lengths of the longest and shortest days given in the *Vedāṅga-Jyotiṣa* which is the same as that found in Babylonian astronomy. However the ratio holds good for a latitude of 35°N, which is not only true for the northern parts of Babylon but also for northern parts of Afganisthan and Kashmir which were parts of the Indian domain in ancient times. Further Ohasi[2] has demonstrated that the ratio $\frac{3}{2}$ was most probably obtained by extrapolation of the variation in the length of the day found near equinox made at a latitude of 29°N. So it is more probably the case of Babyloanians borrowing from Indians just as they did while using *tithi* as a measure of time.

Consideration of kṣaya tithis

Among the expositions of *Vedāṅga-Jyotiṣa* that of T.S. Kuppanna Sastry[3] is the most authentic and exhaustive. He has described in detail the calculation of the ephemeris of the sun and the moon for the whole period of one *yuga*. Here we shall elaborate only on the determination of *kṣaya tithis*, as our procedure will also be used in the next section on corrections.

As there are 1860 *tithis* in 1830 days we have to drop 30 *tithis* as *kṣaya-tithis* in one *yuga*. It is normally stated that one drops a *tithi* after every 61 *tithis*. But *Vedāṅga-Jyotiṣa* prescribes a different method of dropping a *tithi*

by making some months end on 29th day by reducing the one *ṣaḍaha* of the month from 6 to 5 days. Kuppanna Sastry has referred to *Yajurjyotiṣa* verse 12 which requires that a *parva-tithi* (*pūrṇimā or amāvāsyā*) is to be dropped if it lasts for less than one *pāda* (31 D) of a day, i.e., if it ends before noon. And, in that case, the *Darśa-pūrṇāmāsa-yaṣṭis* are to be performed on the following *pratipada*.

Now, one *parva* is equal to $\left(\frac{1830}{124}\right)$ days, or 14d + 94D. So the *parva* p ends after (14p) days and (94p) D. Let p = $(4x + y)$ where x and y are integers and y is less than 4. Then the excess over full days will be :

$$\begin{aligned}(93 + 1)\,(4x + y)\ \mathrm{D} &= (3x)\ \mathrm{d} + (93y + \mathrm{p})\ \mathrm{D} \\ &= (3x)\ \mathrm{d} + \{(124 - 31)\,y + \mathrm{p}\}\ \mathrm{D} \\ &= (3x + y)\ \mathrm{d} + (\mathrm{p} - 31y)\ \mathrm{D}\end{aligned}$$

So the *parva-tithi* is to be dropped if $(p\text{-}31y)$ is less than 31. Also $(p - 31y)$ should be greater than zero for the *parva* to end in the first *pāda* of the day.

Table 1. Determination of *kṣaya-parvas*

y	Range of p Range of $4x$	Range of x	*Kṣaya-parvas*
0	1 to 30 1 to 30	1 to 7	4, 8, 12, 16, 20, 24 & 28 (7 *amāvāsyās*)
1	32 to 61 31 to 60	8 to 15	33, 37, 41, 45, 49, 53, 57 & 61 (8 *paurṇimās*)
2	63 to 92 61 to 90	16 to 22	66, 70, 74, 78, 82, 86 and 90 (7 *amāvāsyās*)
3.	94 to 123 91 to 120	23 to 30	95, 99, 103, 107, 111, 115, 119, and 123 (8 *paurṇimās*)

Table 1 shows the calculation of *kṣaya-parvas* based on the above criterion. Thus the *amāvāsyās* at the end of 14 even *parvas* and *paurṇimās* at the end of 16 odd *parvas* are *kṣaya-parvas*. We see that we have a total of 30 *kṣaya-tithis* as required. We shall see how this gets modified after correction.

Corrections

(a) *Prescription* : Firstly we have to increase the length of the *yuga* by one day, i.e., 124 D. Secondly, we have to consider the fact that the sun covers the whole zodiac in 371 *tithis* and not 372 *tithis*. This means it ends up 9 Bh° ahead of the starting point during the extra *tithi* in one year. And it ends up 45 Bh° ahead of the starting point at the end of one *yuga*. The required corrections are prescribed in *Ṛgjyotiṣa* Verse 12, or *Yajurjyotiṣa*, verse 27, (vide Text 36, ref. 3, p. 64 for text and Eng. tr. notes) which runs as follows :

trayaṃśo bhaśeṣo divasāṃśa bhāgaś caturdaśaścāpyantya bhinnam /
bhārdhe'dhike cā'pi gate paro'ṃśo dvāvuttame tannava kairavedyam / /

This verse is not properly interpreted by most expositers including Kuppanna Sastry. According to us the first half states that a difference of $\frac{1}{3}$ Bh° in angle and 1 D in time arises at *caturdaśis*. Here *caturdaśi* is figuratively taken as the end of the *parva*, because *paurṇimās* and *amāvāsyās* might be *kṣaya-tithis* quite often. As there are 124 *parvas* in a *yuga* we get a total of $41\frac{1}{3}$ Bh° and 1 day during the *yuga*. The second half specifies that in the case of *navakas*, *i.e.*, *bhāṃśas* (refering to the sun's motion of 9 Bh° in one *tithi*) $\frac{1}{3}$ Bh° also arises at the 10 *bhārdhas*, *i.e.*, *ayanas* and at the end of the *yuga*, thus the difference being twice the usual amount at the end-point. These 11 differences, when added to the earlier 124 differences make up 135 differences which are equal to 45 Bh°.

(b) *Implication for kṣaya-tithis* (see K.D. Abhyankar[4]): With the increase in the length of each *parva* by 1 D the *parva* p will now end after (14p) d + (95p) D. As before we put p = $4x + y$ where x and y are integers and now y is less than 8. Then the excess over full days will become :

$$\begin{aligned}(93 + 2)\,(4x + y) &= (3x)\,\mathrm{d} + (93y + 2\mathrm{p})\,\mathrm{D} \\ &= (3x)\,\mathrm{d} + \{(124 - 31)\,y + 2\mathrm{p}\}\,\mathrm{D} \\ &= (3x + y)\,\mathrm{d} + (2\mathrm{p} - 31y)\,\mathrm{D}\end{aligned}$$

So the *parva-tithi* is to be dropped if $(2p - 31y)$ is less than 31 and greater than zero. Table 2 shows the calculation of *kṣaya parvas* on this criterion. For $y = 3$, $x = 11$ is also possible, which gives $p = 47$. Now, for $p = 47$ we have an excess of ID on 695th day. But that day is already counted as *pratipadā* due to *amāvāsyā* of the 46th *parva* being *kṣaya*. Thus 694th day is *purṇimā* which is not a *kṣayatithi*. Thus the *amāvāsyās* at the end of 15 even *parvas* and the *paurṇimās* at the end of 14 odd *parvas* are to be dropped as *kṣaya-tithis*. The total dropped *tithis* now become 29 which make 1860 *tithis* in 1831 days. In other words, the length of the *yuga* becomes 1831 days instead of 1830 days.

Table 2. Corrected of *kṣaya-parvas*

y	Range of 2p Range of 8x	Range of x	*Kṣaya-parvas*
0	2 to 30 2 to 30	1 to 3	4, 8 & 12 (3 *amāvāsyās*)
1	32 to 60 30 to 58	4 to 7	17, 21, 25 & 29 (4 *paurṇimās*)
2	64 to 92 60 to 88	8 to 11	34, 38, 42 & 46 (4 *amāvāsyās*)
3	94 to 122 88 to 116	12 to 14	51, 55 & 59 (3 *paurṇimās*)
4	126 to 154 118 to 146	15 to 18	64, 68, 72 & 76 (4 *amāvāsyās*)
5	156 to 184 146 to 174	19 to 21	81, 85 & 89 (3 *paurṇimās*)
6	188 to 216 176 to 204	22 to 25	94, 98, 102 & 106 (4 *amāvāsyās*)
7	218 to 246 204 to 232	26 to 29	111, 115, 119, & 123 (4 *paurṇimās*)

(c) *Implications for the 5-year yuga* : We have seen that the sun is ahead of the starting position at 0 Bh° of Dhaniṣṭhā by 45 Bh° at the end of the first *yuga*, but it is still in Dhaniṣṭhā. The difference becomes 90 Bh° at the

end of second *yuga* when also it is in Dhaniṣṭhā. But at the end of the third *yuga* it would be 135 Bh° ahead of the starting point at 11 Bh° Śatabhiṣak. Now 135 Bh° is the distance travelled by the sun in one *pakṣa*. So the sun would have arrived at the starting point one *paḳsa* before the end of the third *yuga*. By implication one should drop one *pakṣa* of the *adhikamāsa* at the end of the 15th year as a *kṣaya-pakṣa* to keep track of the sun. Thus we have $5\frac{1}{2}$ *adhikamāsas* in 15 years and 11 *adhikamāsas* in 30 years. This is the basis of the modified 30-year Dakṣayanīya sacrificial calendar described in *Śatapatha-Brāhmaṇa* (11.1.2.19 and 12).

MODIFICATIONS OF THE 5-YR YUGA CALENDAR

(a) *First approximation* : R. Shamasastry[5] (1936) has pointed out that there was a period when it was the practice to have $7\frac{1}{2}$ *adhikamāsas* instead of 8 in 20 years. He refers to the passage in *Maitrāyaṇīya-Saṃhitā* 1.162 where it is stated that Aditi, the deity of Punarvasu-nakṣatra, got her sons called Ādityas by eating the remnants of cooked rice which formed the embriyos that developed into Ādityas. According to Shamasastry this is an allegory of piling of 12 intercalary days each year into a full intercalary month (*adhikamāsa*) after 3 and 5 years, respectively. It is stated that each dish (*yuga*) produced two ādityas, *i.e.*, *adhikamāsas*. The first dish (*yuga*) gave birth to the twin Ādityas, Dhātā and Aryamā. The second dish (*yuga*) produced the twin Ādityas, Mitra and Varuṇa; and the third dish (*yuga*) gave birth to the twin Ādityas, Aṃśa and *Bhāga*. However the fourth dish (*yuga*) brought forth one full Āditya, Vivasvān and one half born Āditya, Mārtāṇḍa. Thus the fourth *yuga* had one full and one half *adhikamāsa*. This gives $247\frac{1}{2}$ lunar months or 7425 *tithis* in 20 years, which amounts to $371\frac{1}{4}$ *tithis* per year. This is a better approximation than 372 *tithis* per year. It is obvious that in this 20-year cycle the months would be *amānta* in the first cycle and *paurṇimānta* in the second cycle.

(b) *Second approximation* : The Dakṣaṇīya sacrificial calendar makes use of the *Vedāṅga-Jyotiṣa* prescription which makes the length of the year to 371 *tithis* which is a still better approximation. This calendar is made of

360 lunar *ahorātras*, each *ahorātra* being made of *śuklapakṣas* as days and *kṛṣṇapakṣas* as nights. The *pakṣas* occurring in the *adhikamāsas* are not counted for this purpose. The period of 30 years is supposed to be the year of the *pitṛys* (manes) who are considered to reside on the moon. This stands to reason because one day on the moon is in fact equal to one lunar month.

In the first half of the 30-year cycle the year starts on S 1. *Adhikamāsas* are introduced in the 3rd, 5th, 8th, 10th and 13th years of the cycle. But the *kṛṣṇapakṣa* of the *adhikamāsa* coming at the end of the 15th year was dropped as *kṣaya-pakṣa* and the second half of the cycle was started on K 1. Thus the months were *amānta* in the first half and *paurṇimānta* in the second half of the 30 year cycle. Again after introducing *adhikamāsa* in 18th, 20th, 23rd, 25th and 28th years, the *śuklapakṣa* of the *adhikamāsa* coming at the end of the 30th year was dropped as a *kṣayapakṣya* and the calendar was restarted after completing the 30 year sacrifice.

(c) *Third approximation :* However, it was found that three *dakṣāyanīya* sacrificial cycles fell short of 90 tropical years by about 5 *tithis*. So it was decided to use one normal 5-year *yuga* of 1831 days after the third cycle to bring the calendar into unison with the seasons. This was the basis of the 95-years *agnicayana-vidhi* to which attention has been drawn by S. Kak[6] (1993) and the *dakṣāyanīya* sacrificial calendar was again started after the *agnicayana-vidhi*. This makes the year equal to 371.05 *tithis* which is quite accurate.

It is interesting to note that the 95-year cycle includes five 19-year Metonic cycles, but it is made of modified 5-year *yuga* system. Further the Romaka-*yuga* of 2850 years is actually equal to 30 cycles of 95 years. Further it may be pointed out that the 30-year cycle can also be obtained by dropping the *adhikamāsa* at the end of the 30th year as *kṣayamāsa* instead of dropping one *pakṣa* after every 15 years. Lagadha prefers this scheme, because he wanted to start all years in *śukla-pakṣa*.

Conclusions

It is stated by Lagadha that the *Vedāṅga-Jyotiṣa* was compiled for sacrificial purposes. The various sacrifices include *Darśā-pūrṇamāsa-yaṣṭis*,

Cāturmāsya sacrifices, *Ṛtuyajñyas, Dakṣāyanīya* sacrifice, *Agnicayana-vidhi*, etc. It is to be noted that the modified 5-year *yuga* with *Dakṣāyanīya* sacrifice and *Agnicayana-vidhi* can keep track of the seasons for several centuries. But one has to change the starting *nakṣatra* after about 960 years due to precession. This was precisely done by the Jain astronomers when they changed the starting *nakṣatra* from Dhaniṣṭhā to Śravaṇa around 500 BC. As the sacrificial system was frowned upon by the Buddhist philosophers due to its ritualist corruption it lost its astronomical significance. So the purpose and meaning of *Vedāṅga-Jyotiṣa* became obscure, and the Indian astronomers had to rediscover the art of calendar making during the Siddhāntic period.

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