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LUNAR ECLIPSE COMPUTATION IN INDIAN ASTRONOMY WITH SPECIAL REFERENCE TO *GRAHALĀGHAVAM*

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In the present paper, the procedure for the computation of lunar eclipse according to the traditional siddhāntic (Indian astronomical) texts is explained. The remarkable simplifications made by Gaṇeśa Daivajna are presented in relation to the eclipse computation.

Key words: Gaṇeśa Daivajña, *Grahalāghavam*, Indian astronomy, Lunar eclipse, Siddhānta.

INTRODUCTION

In ancient and medieval astronomical texts (*siddhāntas*, *tantras* and *karaṇas*) great importance is given to the phenomenon and computation of eclipses (*grahaṇa*, *uparāga*). The Indian astronomers used to put to test their theories and computations in respect of positions of the heavenly bodies - especially the Sun and the Moon - on the occasions of the eclipses. As and when disagreements occurred between the observed and the computed positions, the great savants of Indian astronomy revised their parameters and when necessary even the computational procedures. Improving the computations of eclipses - based on sustained observations over long periods of time - was an important target of siddhāntic astronomers.

The great Kerala astronomer, Nīlakaṇṭha Somayāji (1444-1545) paying glowing tributes to his *parama guru* (grand-teacher), Parameśvara (1362-1455), remarks " Parameśvara...having observed and carefully examined eclipses and conjunctions

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for 55 years composed his *Samadrgganitam*" [*Pañcapañcaśat varṣakālam nirikṣya grahaṇa grahayogādiṣu/parīkṣya samadrggaṇitam karaṇam cakāra*].

After Bhāskara II (b. 1114 A.D.) there was an apparent decline in mathematical and astronomical output in India; however, after some time there was a tremendous development in Kerala due to stalwarts like Mādhava, Parameśvara and Nīlakaṇṭha Somayāji from about 14th to 17th centuries of the Christian era.

During that period there were quite a few astronomers of great achievement in other parts of India also. In fact, among the astronomical works in common use, especially in *pañcānga*-making, Gaṇeśa's *Grahalāghavam* is the most popular one. Even to this day in most of northern India, Maharashtra, Gujarat and north Karnataka, the *Grahalāghavam* (*GL*) is in vogue.

While the geometrical configurations and mathematical procedure for computations of eclipses presented by the traditional Indian astronomers are quite sound, the relevant parameters, over a period of centuries, need to be upgraded periodically.

Indian Astronomers on Eclipses

The real scientific causes of the lunar and solar eclipses were well known to the ancient Indian astronomers even in the pre-Āryabhaṭan period. The procedure explained in the then extant *pañca siddhāntas* (five systems of astronomy) bear this out.

Āryabhaṭa I (b. 476 A.D.) explains the causes of the two types of eclipses and explains in his characteristic style of brevity:

chādayati śaśī sūryam śaśinām mahatī ca bhucchāyā

— "The Moon covers the Sun and the great shadow of the Earth (eclipses) the Moon".

-*Āryabhaṭiyam*, 4, 37

Varāhamihira (c.505 A.D.) explains at length in his *Bṛhatsamhitā* the real causes of the eclipses and debunks the irrational myths generally entertained by the ignorant masses. He declares:

(i) *bhucchayām svagrahaṇe bhāskaram arkagrahe praviśati induḥ*

— "At a lunar eclipse the Moon enters the shadow of the earth and at a solar eclipse the Moon enters the Sun's disc"

-Bṛ. Saṃ, 5, 8

(ii) Varāhamihira gives all credit to the ancient preceptors for the knowledge of the causes of eclipses in saying :

evam uparāga kāraṇam uktamidam
divyadrgbhiḥ ācāryaiḥ
rāhurakāraṇam asmin niyuktaḥ
śāstra sadbhāvaḥ

—"In this manner, the ancient seers endowed with divine insight have explained the cause eclipse (*uparāga*). Hence the scientific fact is that (the demon) Rāhu is not at all the cause of eclipse",

-Bṛ. Saṃ, 5,13

(iii) Further, demolishing some of the superstitious beliefs regarding eclipses, Varāha remarks:

na kathañcidapi nimittaiḥ grahaṇam vijñāyate nimittāni
anyasminnapi kāle bhavanti atha utpāta rupāṇi

— "An eclipse can by no means be ascertained through omens and other indications. Because the portents such as the fall of meteors and the earth-quakes occur at other times also"

-Bṛ. Saṃ, 5,16

(iv) Pañcagraha samyogānna khila grahaṇasya sambhavo bhavati
tailam ca jale aṣṭamyām na vicintyamidam vipascidbhiḥ

— " Scholars should not believe in the following myth of the ignorant to the effect that an eclipse cannot take place except when there is a combination of five planets in the same zodiacal sign, and that a week before the eclipse i.e., on the preceding 8th lunar day, its characteristics can be inferred from the behaviour or appearanace of a drop of oil poured on the surface of water"

Bṛ. Saṃ, 5,17

Cause of the Lunar Eclipse

On a full moon day the Sun and the Moon are on opposite sides of the earth. The Sun's rays fall on one side of the earth, facing the Sun, and a shadow will be cast on the other side. When the Moon enters the shadow of the earth, a lunar eclipse occurs. This happens when the Sun and the Moon are in *opposition* i.e., the difference between the celestial longitudes of the Sun and the Moon is 180°.

However, a lunar eclipse does not occur on every full-moon day. This is so because the plane of the Moon's orbit is inclined at about 5° with the ecliptic. If the Moon's orbit were in the plane of the ecliptic, then there would have been a lunar eclipse every full-moon day. Generally on a full-moon day the Moon will be either far above or far below the plane of the ecliptic and hence fails to pass through the shadow of the earth. But, on that full-moon day, when the Moon does pass through the earth's shadow, a lunar eclipse occurs.

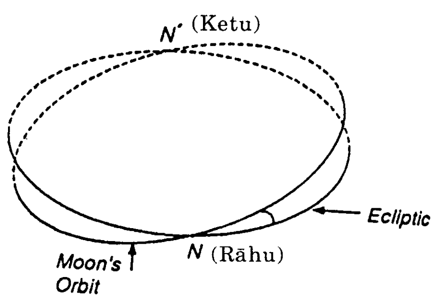

Fig.1 Nodes of the Moon

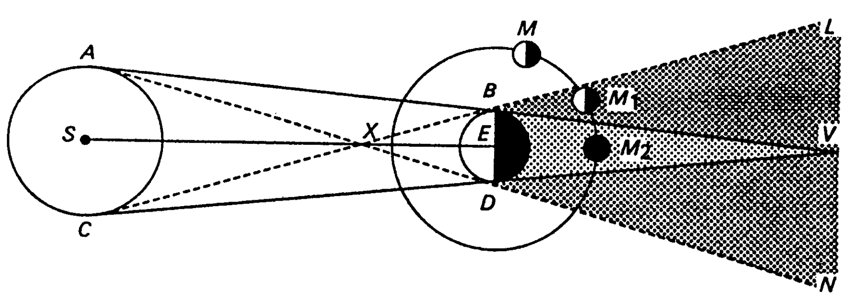

Fig.2. Earth's shadow-cone and the lunar eclipse

In order that an eclipse of the Moon may take place, the Moon must come sufficiently close to the ecliptic. This means that the Moon, on the full-moon day, must be close to one of the *nodes* of the Moon. In Fig. 1 the orbit of the Moon intersects with the ecliptic at two points *N* and *N'*. These two points are referred to as the *ascending* and the *descending* nodes of the Moon. They are called Rahu and Ketu in Indian astronomy.

The lunar eclipse is said to be total when the whole of the Moon passes through the shadow. The eclipse is *partial* when only a part of the Moon enters the shadow.

In Fig. 2, *S* and *E* represent the centres of the Sun and the Earth respectively. Draw a pair of direct tangents *AB* and *CD* to the surfaces of the Sun and the Earth, meeting *SE* in V. If these lines are imagined to revolve round *SE* as axis, they will generate cones. There is, thus, a conical shadow *BVD*, with V as its vertex, across which no direct ray from the Sun can fall. This conical space is called the *umbra.*

The spaces around the umbra, represented by *VBL* and *VDN* bounded by the transverse tangents *AXDN* and *CXBL*, from what is called the *penumbra,* from which only a part of the Sun's light is excluded. It is to be noted that the passage of the Moon through the penumbra does not prompt an eclipse. It results only in diminution of the Moon's brightness. When the Moon is at M_1 (see Fig.2) it receives light from portions of the Sun next to A, but rays from the parts near *C* will not reach the Moon at M_1. Therefore, the brightness is diminished, the diminution growing greater as the Moon approaches the edge of the umbra. An eclipse is considered as just commencing when the Moon enters the umbra or the shadow-cone.

Half-duration of Eclipse and of Maximum Obscuration

The next important step is to determine the instants of the beginning and the end of a lunar eclipse as also of the maximum obscuration. For this, we need to find the duration of the first half and the second half of the total duration of the eclipse. This is explained in Fig. 3.

A half-duration is the time taken by the Moon, relative to the Sun, so that the point *A* in the figure moves through *OA*. We have

$$\begin{aligned} 0A^2 &= OX_1^2 - AX_1^2 = (OE + EX_1)^2 - AX_1^2 \\ &= (d_1 + d_2)^2 - \beta_1^2, \text{ where} \end{aligned}$$

$EX_1 = d_2 =$ semi-diameter of the Moon

$\beta_1 = AX_1 =$ Latitude of the Moon (*Śara*)

when the Moon's centre is at X_1.

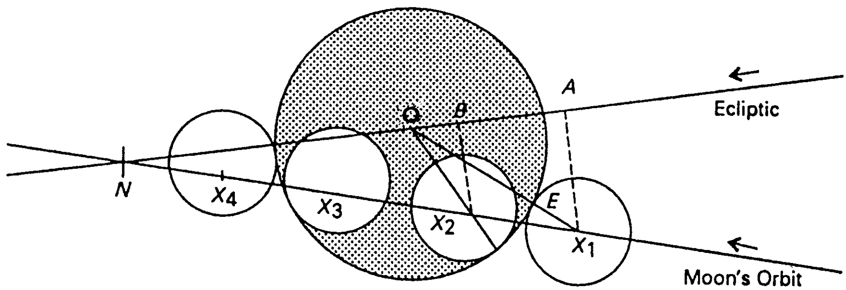

Fig. 3: Half-duration of lunar eclipse

Therefore, we have Half-duration, HDUR $= \dfrac{\sqrt{(d_1 + d_2)^2 - \beta^2}}{(MDM - SDM)}$

where MDM and SDM are respectively the rates of true daily motion of the Moon and the Sun. Since the actual instants of the beginning and the ending of the eclipse and hence the latitudes of the Moon at those instants are not known, the above expression for the half-duration is taken as a first approximation.

By a similar analysis, the half-duration of maximum observation (or totality as the case may be) is given by

$$THDUR = \frac{\sqrt{(d_1 + d_2)^2 - \beta^2}}{(MDM - SDM)}$$

The thus obtained half durations of the eclipse and of maximum obscuration are

(i) *subtracted* from the instant of the opposition to get the beginning moments; and

(ii) *added* to the instant of opposition to obtain the ending moments.

These provide a first approximation to the instants of the beginning and ending of the eclipse and of the totality. For these instants the latitude of the Moon are determined and the same are used in the expressions for *HDUR* and *THDUR*. By the process of successive iteration the precise values for the half-duration as well as for the instants of the beginning and ending of the eclipse and of the totality are obtained. Finally, we have

$$\text{Pramāṇam (Magnitude)} = \frac{\text{Amount of obscuration } (\textit{grāsa})}{\text{Angular diameter of the Moon}}$$

Special Features of Grahalāghavam

Gaṇeśa Daivajña has simplified the methods of computations of planetary positions and eclipses which are otherwise laborious by the traditional methods as explained in the *Sūryasiddhānta*, Brāhmagupta's *Brāhmasputasiddhānta* and *Khaṇḍakhādyaka*, Bhāskara's *Siddhāntaśiromaṇi* etc. The following are some of the special features of Gaṇeśa's simplification:

(i) To avoid handling a large number for the *ahargaṇa* (the number of civil days elapsed since a chosen epoch), Gaṇeśa has introduced a cycle (*cakra*) of 4016 days, approximately 11 solar years. This modified *ahargana* never exceeds 4016 and hence it is very handy and the numerical errors in multiplication etc. can be avoided.

(ii) For the purpose of simplifying *pañcāṅga* making and for the benefit of beginners in astronomy, who may be ignorant of trigonometry, Gaṇeśa has completely avoided the *sine* and *cosine* functions. In fact, the dropping of trigonometric ratios has not seriously affected the accuracy of the results. Gaṇeśa has adopted reasonably very good and justified approximations for the trigonometric functions.

Gaṇeśa Daivajña, highlighting the importance of the special features of his text, declares,
"The earlier preceptors very proudly rose to the peak of their fame even though only in a few places they did calculations without using sines etc. But, I have made the calculations of the entire *siddhānta* (astronomy) simple (*lāghava*) by omitting sine etc. through out. I have my intelligence (and knowledge) enriched only from their works and hence I do not exhibit pride (of my achievement)"

In the following section we describe the procedure given by Gaṇeśa's in his *Grahalāghavam* (*GL*).

LUNAR ECLIPSE PROCEDURE ACCORDING TO *GL*

Despite the approximations introduced in *GL*, doing away with trigonometric ratios, the predictions of eclipses are fairly reliable on account of improved and updated values of the related parameters.

The procedure, as described in *GL,* is presented briefly in what follows. Based on Gaṇeśa procedure we have written a computer program and the same is demonstrated with two examples, one of the medieval period and the other modern one.

(i) Sun's angular diameter (*ravi bimbam*),
SDIA = (SDM - 55)/5 + 10 *aṅgulas*

where SDM is the Sun's true daily motion in minutes of arc (*kalās*).

(ii) Moon's angular diameter (*candra bimbam*)
MDIA = MDM/74 *aṅgulas*

where MDM is the Moon's true daily motion in minutes of arc (*kalās*).

(iii) Angular diameter of the earth's shadow (*bhucchāyā bimbam*).
SHDIA = [3(MDIA)/11 + 3(MDIA) - 8] *aṅgulas*

Note : 1 *aṅgula* = 3 *kalās* ≡ 3'

(iv) Latitude of the Moon (*candra śara*),
MLAT (śara) = 11 (M -R)/7

where *M* and *R* are respectively the true longitudes of the Moon and Rāhu (the ascending node of the Moon) and (M - R) considered here is the *bhuja* (not exceeding 90°) of the difference.

Note: If (M - R), is in III quad. then the argument,
bhuja = (M - R)-180°.

If (M - R) is in IV quadrant, the *bhuja* = 360° - (M - R). On the otherhand, if (M - R) is in II quadrant, the *bhuja* = 180° - (M - R).

Remark : The approximate formula follows from

$$120 \sin \theta \approx 72\, \theta/35 \qquad \text{....(1)}$$

when θ is small, as given by Gaṇeśa (see *GL, Praśnādhikāra,* 22).
According to the traditional texts the Moon's latitude (*śara*) is given by

$$\beta = 270' \sin (M - R) = 270 \times \frac{72}{120 \times 35} (M - R) \textit{ kalās}.$$

$$\beta = \frac{162}{35} (M - R) \textit{ kalās}.$$

Dividing the above result by 3, we get

$$\beta = \frac{54}{35} (M - R) \approx \frac{11}{7} (M - R) \textit{ aṅgulas}$$

The approximations in this case are justified since under the possible circumstances of an eclipse the argument (M - R) is indeed small.

(v) The amount of obscured portion of the full-moon,

Grāsa $= \frac{1}{2}$ [*Chādaka* dia. + *Chādya* dia. - Śara]

where *Chādaka* and *Chādya* are the eclipsing and the eclipsed bodies. The formula is common to both the lunar and solar eclipses. In the case of a lunar eclipse, the Moon is the *chādya* and the earth's shadow is *chādaka*.

(vi) *Mānaikya khaṇḍa* $= \frac{1}{2}$ [*Chādaka* dia. + *Chādya* dia.]

so that we have

Grāsa = *Mānaikya khaṇḍa* - Śara

Therefore,

(a) If *Mānaikya khanda* < *Śara* (i.e., Grāsa <0), there will be no eclipse.
(b) If *Grāsa* > *Chādya* dia., then the eclipse is total (*khagrāsa grahaṇa*).

In that case,

Khagrāsa = *Grāsa* - *Chādya* diameter.

(vii) Half duration of the eclipse and totality :

(a) In Gaṇeśa 's simplified procedure,

let x = $\sqrt{\left[\frac{1}{2}(SHDIA + MDIA) + \acute{S}ara\right] \times 10 \times Gr\bar{a}sa}$

Then, the half-duration of the eclipse,

HDUR = (x -x/6)/MDIA = 5x/6 (MDIA) *gh.*

(where *gh.* = *ghaṭikās; 60 ghaṭikās* = 1 day; 1 *gh.* = 24 minutes).

(b) Similarly,

$$\text{let } y = \sqrt{\left[\frac{1}{2}(SHDIA - MDIA) + \acute{S}ara\right] \times 10 \times Khagr\bar{a}sa}$$

Then, the half-duration of totality,
THDUR = [y - y/6)]/MDIA = (5y/6) / (MDIA) *gh.*

(viii) First and second halves of eclipse and totality :

The difference (true Sun - Rāhu), called *vyagu*, at the instant of opposition is considered and its *bhuja* is determined . The product 2 x *bhuja* in degrees is put in two places as *palas* (i.e., *vighaṭīs*).

(1) If the *vyagu* is in an even quadrant then (2 x *bhuja*) in *palas* is subtracted from and added to the *madhya sthiti* (i.e., mean half-duration HDUR in *gh.* obtained earlier), respectively, to get the corrected first and second half-durations (called *sparśa sthiti* and *mokṣa sthiti*).

(2) If the *vyagu* is in an odd quadrant, then 2 x (*bhuja*) in *palas* is added to and subtracted from the *madhya sthiti* in *gh.*, respectively, to get the corrected *sparśa* and *mokṣa* half-durations.

Similar operations are carried out to get the first and the second half-durations of *totality* by considering the *marda* duration THDUR instead of the *sthiti.*

Example

The lunar eclipse of May 2, 1520 AD (Julian), Wednesday, is considered. This date falls in the epochal year of GL and is taken from the Epighraphia Indica (see Vol. VI, p. 237). The example is worked out using a computer program designed by us.

GRAHALĀGHAVAM **: POSITION OF SUN, MOON AND RAHU**

Date : Year : 1520	:	Month : 5 Date : 2
Time (after sunrise)	:	Hours : 0 Mins : 0
Name of Place	:	Ujjayini
Longitude (-ve for west)	:	Deg. : 75 min : 45
Latitude (-ve for south)	:	Deg : 23 min : 11
Week Day	:	Wednesday
Cakras	:	0
Ahargaṇa : 44	:	Epoch : 19-3-1520 J

True Ravi	:	34° 35' 23"
True Moon	:	205° 56' 7"
Rāhu	:	25° 18' 4"

GRAHALĀGHAVAM: LUNAR ECLIPSE

Sun's true daily motion (SDM)	:	57' 30"
Moon's true daily motion (MDM)	:	736' 15"
Time of opposition after midnight (LMT)	:	$24^h\ 21^m\ 37^s$
True Sun at Oppn.	:	35° 19' 22"
True Moon at Oppn.	:	215° 19' 22"
Node (Rāhu) at Oppn.	:	25° 15' 39"
Moon's diameter (in *aṅgulas*)	:	9.949405
Shadow's diameter (in *aṅgulas*)	:	24.56169

ECLIPSE IS POSSIBLE

(Northern) *Śara* (in *aṅgulas*)	:	15.81192
Grāsa (in *aṅgulas*)	:	1.443631

LUNAR ECLIPSE PARTIAL

Madhya stihiti (in *gh*)	:	1.829996
Sparśa sthiti (in *gh*)	:	2.165401
Mokṣa sthiti (in *gh*)	:	1.494592

SUMMARY OF LUNAR ECLIPSE

		h	m	s
Sparśa (Beginning) Time	:	23	29	39
Madhya (Middle) of Eclipse	:	24	21	37
Mokṣa (Ending) Time	:	24	57	29

MODERN EXAMPLE

The following is a modern example worked out by the *GL* procedure.

Lunar Eclipse for the date 16th July 2000.

For the given date at $5^h 29^m$ a.m. (IST) we have the following:

True Sun= $2^R\ 29^\circ$ 54' 33"
True Moon = $8^R\ 23^\circ$ 36'
End of the *tithi pūrnimā* = $19^h\ 25^m$ (i.e., *parvānta*)
True daily motion of the Sun = 0° 57' 13"

True daily motion of the Moon $= 11^{\circ} 49' 0'' = 709'$
Rāhu $= 3^{R}\, 00^{\circ} 48' 02''$

Duration of *pūrṇimā* from $5^{h} 29^{m}$ IST is
$19^{h} 25^{m} - 5^{h} 29^{m} = 13^{h} 56^{m}$

The motion of the Sun in $13^{h} 56^{m}$ is

$$\frac{57' 13'' \times 13^{h} 56^{m}}{24^{h}} = 33' 13''$$

The motion of the Moon in $13^{h} 56^{m}$ is

$$\frac{709' \times 13^{h}\, 56^{m}}{24^{h}} = 411' 36'' 50''' = 6^{\circ} 51' 36''$$

Therefore, at the *parvānta* we have

True Sun $= 2^{R}\, 29^{\circ} 54' 33'' + 33' 13'' = 3^{R}\, 00^{\circ} 27' 46''$

True Moon $= 8^{R}\, 23^{\circ} 36' + 0^{R}\, 6^{\circ} 51' 36'' = 9^{R}\, 0^{\circ} 27' 36''$

True daily motion of Rāhu $= + 0' 17''$

The motion of Rāhu in $13^{h} 45^{m} = + 0' 10''$

Rāhu at *parvānta* $= 3^{R}\, 00^{\circ} 48' 12''$

Thus we have

Parvānta $= 19^{h} 25^{m}$ IST

True Sun at the *parvānta* $= 3^{R}\, 00^{\circ} 27' 46''$

True Moon at the *parvānta* $= 9^{R}\, 00^{\circ} 27' 36''$

Rāhu at the *parvānta* $= 3^{R}\, 00^{\circ} 48' 12''$

(1) Possibility of Lunar Eclipse :

virāḥvarka = Sun - Rāhu
$= 3^{R}\, 00^{\circ} 27' 46'' - 3^{R}\, 00^{\circ} 48' 12''$
$= 11^{R}\, 29^{\circ} 39' 34'' = 359^{\circ} 39' 34''$

Bhuja of *virāḥvarka* $= 360^{\circ} - 359^{\circ} 39' 34''$
$= 0^{\circ} 20' 26'' < 14^{\circ}$

Since the *bhuja* of *virāḥvarka* is less than 14° there occurs an eclipse.

(2) To find śara (latitude of the moon):

$$\acute{S}ara = \frac{(Bhuja\ of\ Vir\bar{a}\d{h}varka)\times 11}{7}$$

$$= \frac{0^{\circ}20'26''\times 11}{7} = 0.5351587 \quad = +\,0|32\ a\dot{n}gulas,$$ since

varāḥvarka < 180°, *śara* is positive.

(3) Diameters of the Sun, the Moon and the earth's shadow:

(i) Sun's Diameter $= \left[\dfrac{Sun's\ true\ daily\ motion - 55}{5}\right] + 10\ a\dot{n}gulas$

$$= \left(\frac{57'13'' - 55}{5}\right) + 10\ a\dot{n}g. = 10|26\ a\dot{n}g$$

(ii) Moon's diameter $= \left[\dfrac{Moon's\ true\ daily\ motion}{74}\right]\ a\dot{n}g.$

$$= \frac{709}{74}\ a\dot{n}g. = 9|34\ a\dot{n}g$$

(iii) Diameter of earth's shadow =

$$\left(\frac{3}{11}\times Moon's\ diameter + 3\times Moon's\ diameter\right) - 8\ a\dot{n}g$$

$$= \left(\frac{3}{11}\times 9|34 + 3\times 9|34\right) - 8\ a\dot{n}g. = 23|21\ a\dot{n}g$$

(4) To Find *Mānaikyakhaṇḍa* and *Grāsa* :

(i) *Mānaikyakhaṇḍa* $= \dfrac{Shadow\ diameter + Moon's\ diameter}{2}$

$$= \frac{23|21 + 9|34}{2}\ a\dot{n}g \quad = 16|27\ a\dot{n}g$$

(ii) *Grāsa* = *Mānaikyakhaṇḍa* - *Śara*

$$= 16|27 - 0|32 = 15|55\ a\dot{n}g$$

(iii) *Khagrāsa* = *Grāsa* - *Moon's diameter*

= 15|55 - 9|34 = 6|21 *aṅg*

Since *Grāsa* > Moon's diameter, the eclipse is total.

(5) To find *Sthiti* and *Marda* :

Let, x = $[(Sara + Mānaikyakhaṇḍa) \times 10 \times Grāsa)]^{1/2}$

$$= [(0|32 + 16|27) \times 10 \times 15|55]^{1/2}$$

$$= (2703.1806)^{1/2} = 51|59$$

Spaṣṭa sthiti = $\left(x - \dfrac{x}{6}\right) \Big/ Moon's\ diameter$

$$= \left[51|59 - \frac{51|59}{6}\right] \Big/ 9/34\ gh.$$

$$= 4^{gh.}\ 31^{vg}$$

Let y = $\left[\left\{\dfrac{(Shadow\ dia - Moon's\ dia)}{2} + śara\right\} \times 10 \times khagrāsa\right]^{\frac{1}{2}}$

$$= \left[\left\{\frac{(23|21 - 9|34)}{2} + 0|32\right\} \times 10 \times 6|21\right]^{\frac{1}{2}}$$

$$= [471.4875]^{1/2} = 21|42$$

Now, *Marda* = $\left(y - \dfrac{y}{6}\right) \Big/ Moon's\ diameter$

$$= \left(21|42 - \frac{21|42}{6}\right) \Big/ (9|34)\ ghaṭis \quad = 1^{gh}\ 53^{vg}$$

(6) *To find sparśa and mokṣa sthiti:*

We have

Sthiti = $4^{gh}\ 31^{vg}$ and *marda* = $1^{gh}\ 53^{vg}$

Bhuja of *vyagvarka* = 0° 20' 26"

Now, 2 x *bhuja* = 2 x $0^{\circ} 20' 26''$ = 0|40|52 *palas* (or *vighatis*)

Since *vyagu* is in the even quadrant (*vyagu* = $359^{\circ} 39' 34''$), (2 x *bhuja*) in *vighaṭī* is subtracted from and added to the *madhya sthiti* respectively to get the corrected *sparśa* and *mokṣa sthitis.*

Therefore, we have

(i) *Sparśa sthiti* $= 4^{gh}\, 31^{vg} - 0^{vg}\, 40^{pr.\, vg}$
$= 4^{gh}\, 30^{vg}\, 20^{pr.\, vg} = 1^{h}\, 48^{m}\, 07^{s}$

(ii) *Mokṣa sthiti* $= 4^{gh}\, 31^{vg} + 0^{vg}\, 40^{pr.\, vg}$
$= 4^{gh}\, 31^{vg}\, 40^{pr.\, vg}$
$\equiv 1^{h}\, 48^{m}\, 40^{s}$

(iii) *Sparśa marda* $= 1^{gh}\, 53^{vg} - 0^{gh}\, 0^{vg}\, 40^{pr.\, vg}$
$\equiv 0^{h}\, 44^{m}\, 56^{s}$

(iv) *Mokṣa marda* $= 1^{gh}\, 53^{vg} + 0^{gh}\, 0^{vg}\, 40^{pr.\, vg} \equiv 0^{h}\, 45^{m}\, 28^{s}$

(**Note** : 1 *vighaṭi* = 60 *prati vighaṭīs*)

(7) To find *sparśa* and *mokṣa kālas*:

We have

Parvānta = $19^{h} 25^{m}$ (IST)

(i) *Sparśa kāla* = *Parvānta* – *sparśa sthiti*

$= 19^{h}25^{m} - 1^{h}\, 48^{m}\, 7^{s} = 17^{h}\, 36^{m}\, 53^{s}$

(ii) *Mokṣa kāla* = *Parvānta* + *Mokṣa sthiti*

$= 19^{h}25^{m} + 1^{h}\, 48^{m}\, 40^{s} = 21^{h}\, 13^{m}\, 40^{s}$

(8) To find *sammīlana* and *unmilana kālas* (in the case of total eclipse) :

(i) *Sammilana kāla* = *Parvānta* – First half duration of totality

$= 19^{h}25^{m} - 0^{h}\, 44^{m}\, 56^{s} = 18^{h}\, 40^{m}\, 04^{s}$

(ii) *Unmilana kāla* = *Parvānta* + Second half duration of totality

$$= 19^h 25^m + 0^h 45^m 28^s = 20^h 10^m 28^s$$

Summary of the eclipse :		**IST**
(i) Beginning of the eclipse	=	$17^h\ 36^m\ 53^s$
(ii) Beginning of the totality	=	$18^h\ 40^m\ 04^s$
(iii) Middle of the eclipse	=	$19^h\ 25^m$
(iv) End of the totality	=	$20^h\ 10^m\ 28^s$
(v) End of the eclipse	=	$21^h\ 13^m\ 40^s$

Note : According to the *Indian Astronomical Ephemeris* the beginning of the eclipse is at $17^h 27^m .2$ (IST) and the end is at $21^h\ 23^m .8$ IST.

The beginning and the end of totality are respectively at $18^h 32^m$ (IST) and $20^h\ 19^m$ (IST).

There is a difference of about 9^m to 10^m between the *GL* and the modern values which is negligible in view of the fact that the *GL* is about five centuries older.

CONCLUSION

We have presented the procedure for the computation of lunar eclipse according to Gaṇeśa Daivajña 's *Grahalāghavam* (*GL*) and demonstrated the same with a medieval example and a modern one.

The expressions used by *GL* are much simpler than the ones given in the standard traditional texts. This explains the popularity of *GL*.

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