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OBSERVATIONAL ASTRONOMY

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Following up the work of Roger Billard, the mean daily motions of the sun, moon and planets in the theories of Ptolemy, Āryabhaṭa, Lalla and Nīlakaṇṭha Somayaji are calculated, and compared with the results of modern formulae.

Key words: Longitude error, Mean daily motion, Mean longitude, Observational astronomy.

INTRODUCTION

In a very important contribution to the understanding of Indian Astronomy, Roger Billard [1-4] has studied the mean longitudes of the sun, moon's apogee and node, and the planets, as given by Indian astronomers. He has used modern formulae to find the errors of these longitudes, and the methods of mathematical statistics to fix the most probable dates of observations used. His graphs, in which the errors are plotted against time, from BC 500 to AD 1600, reveal the most interesting information. For example, the Āryabhaṭa, graph, reproduced in Fig. 1, shows clearly that he made accurate observations at about AD 510, but there are large errors before and after that date. Consequently the rate of change is far from correct. In the graph of the synodic longitudes the lines of the mean longitudes slope away from the vertical zero-error line. A careful examination of the synodic graphs show that two astronomers, Lalla and Nīlakaṇṭha Somayaji, stand out conspicuously compared to the other Indian astronomers. In these graphs (Figs 2,3) as many as six lines are close to the zero-line from most of the time span, indicating that their mean daily motions are accurate.

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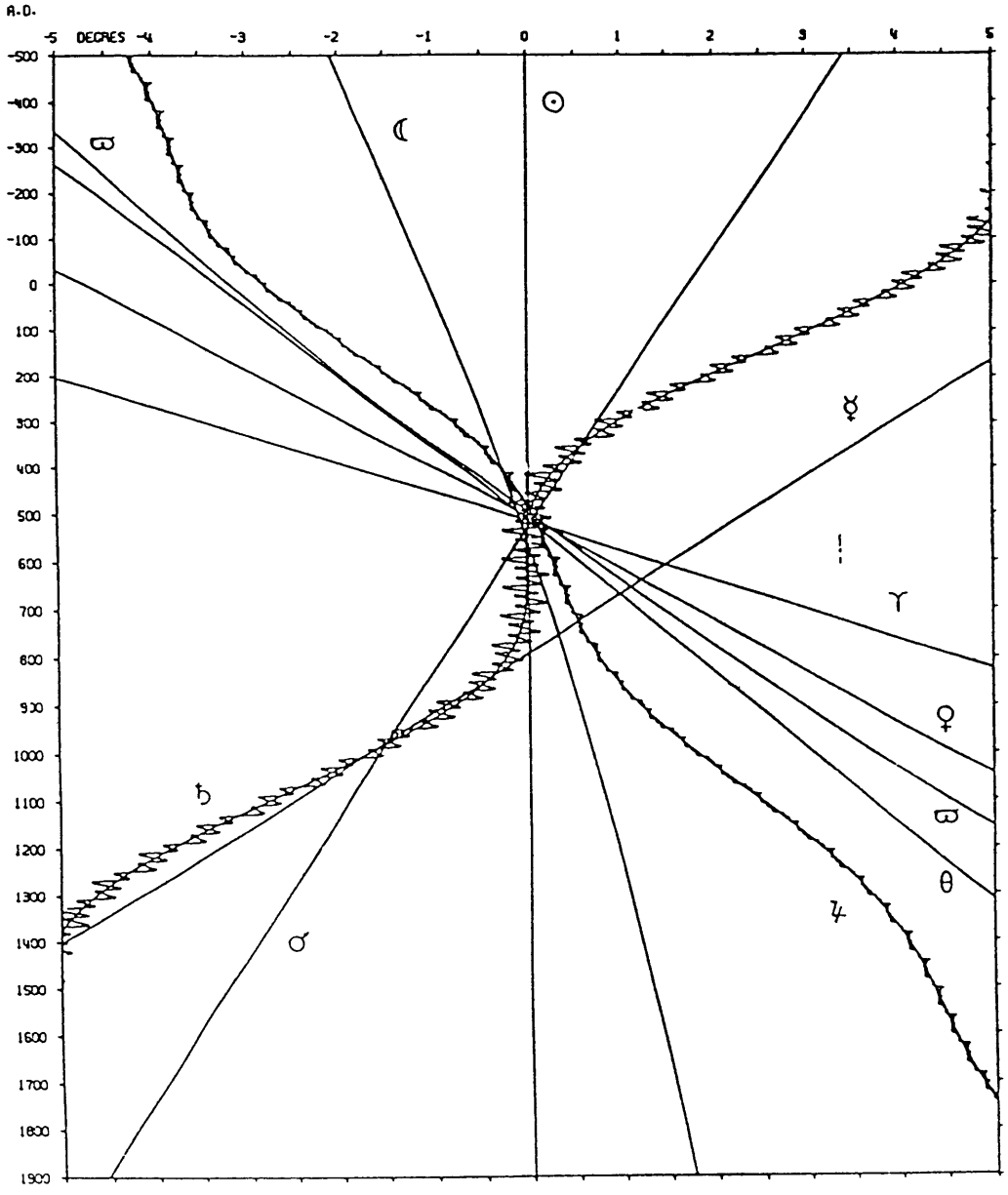

Fig. 1 Errors in mean longitude of Sun (☉), Moon (☾), Mercury (☿), Venus (♀), Mars (♂), Jupiter (♃), Saturn (♄), Moon's line of nodes (θ) between 500 BC to 1900 AD.

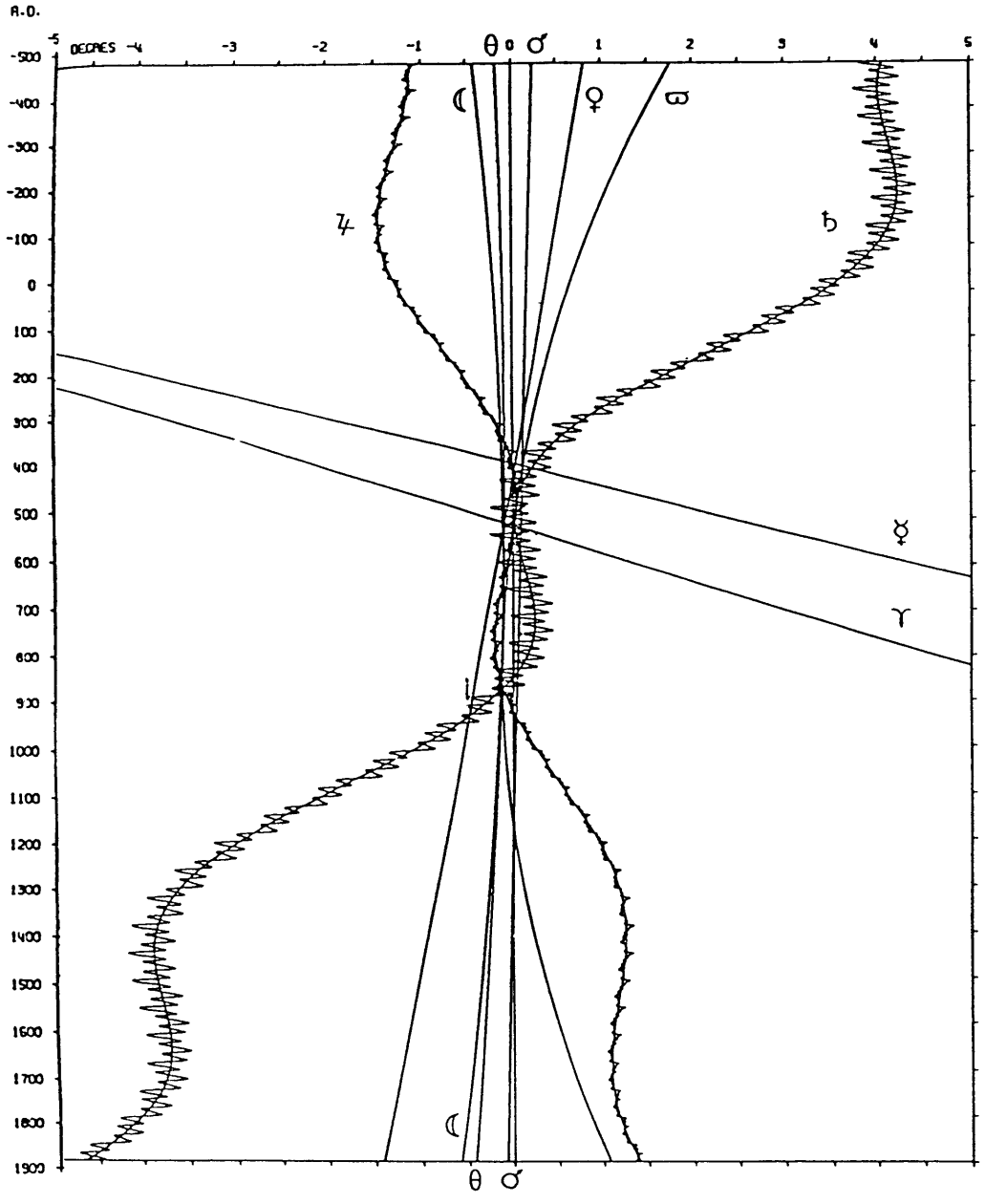

Fig. 2: Errors in mean longitude of planets from 500 BC to 1900 AD

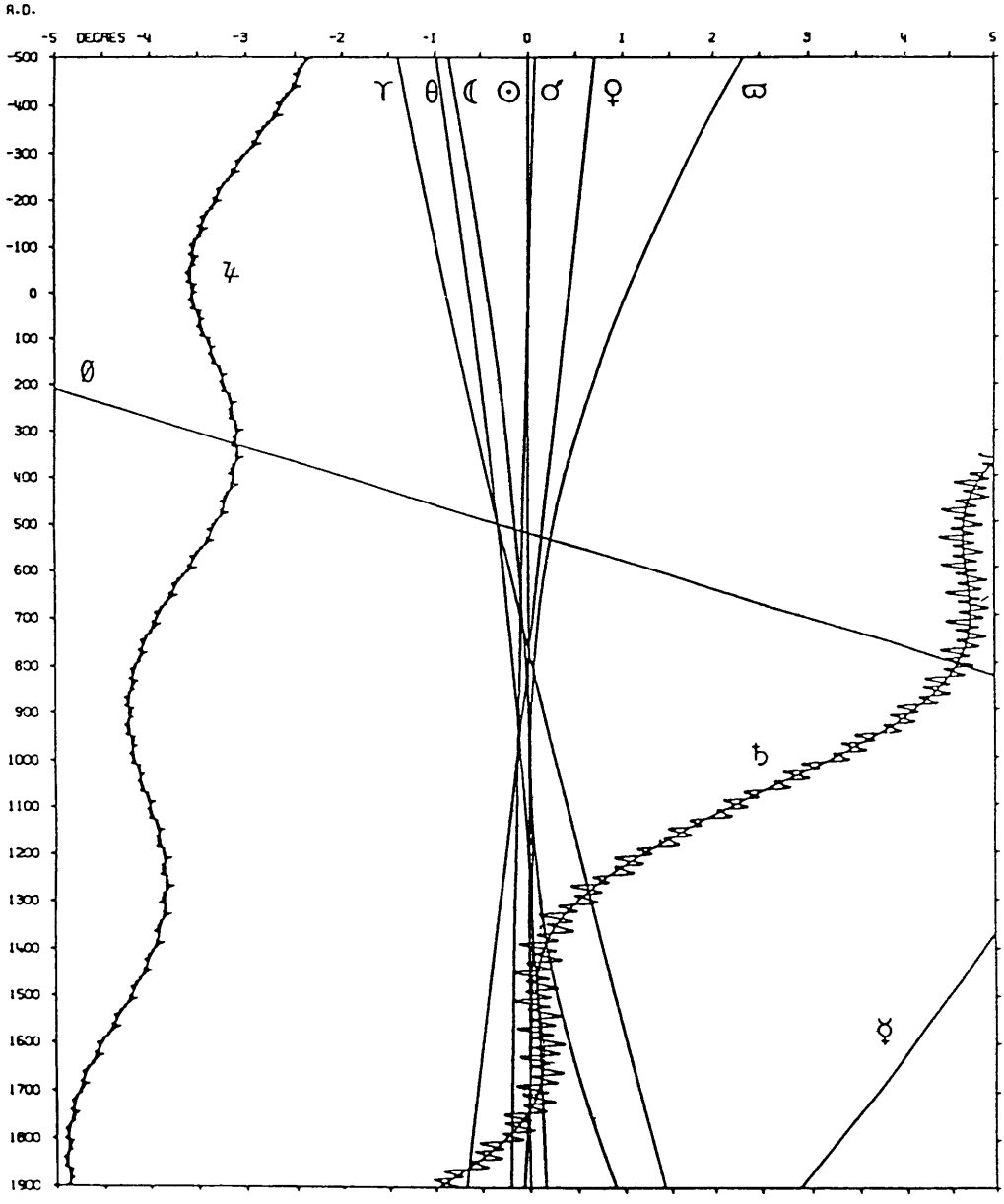

Fig. 3: Errors in mean longitude of planets from 500 BC to 1900 AD

In this paper, the mean daily motions of the moon and planets of Ptolemy and four Indian astronomers, Āryabhaṭa, Brahmagupta, Lalla and Nīlakaṇṭha Somayaji are calculated and their accuracy determined by comparison with those determined by modern theory. The data for the astronomers are given in Roger Billard's *L'Astronomie Indienne* on pp. 53, 78, 143 and 160 and the motion of the moon's apogee according to Āryabhaṭa in the *Āryabhaṭīya* [5], page 7. The source fro the modern theory is the 1994 paper of Simon et al.[6], p. 670 for the moon (Delaunay arguments, 1992 values) and pp. 677-679 for the planets.

The Calculation

We have chosen to calculate the mean daily motions, in longitude, of the moon and the planets relative to the sun, and the moon with respect to its apogee and node. These longitudes are independent of the origin of longitude.

The formula for the daily motions in degrees, based on the mean longitudes of Simon et al. and the conversion formula from ephemeris time to universal time, given in Billard's book on page 44, are of the form

$$(1) \qquad \delta\lambda = a + bt + ct^2$$

The constants a, b, and c are listed in Table 1, the first three rows for the moon relative to the Sun, the Moon's apogee and node, and the remaining five for the planets, Mercury, Venus, Mars, Jupiter and Saturn relative to the Sun.

Table 1 Modern Formula for Daily Motion in the form:
$\delta\lambda = a + bt + ct^2$

S.No.	a	b $\times 10^6$	c $\times 10^9$
1.	12.1907 4962 635	0.1344 9642	0.1504 20
2.	13.0649 9349 908	0.7328 8089	1.1780 84
3.	13.2293 5079 268	0.0571 5958	-0.0236 63
4.	3.1067 2983 369	0.0589 8487	0.0031 32
5.	0.6165 2140 030	0.0120 8137	0.0028 69
6.	0.4615 7629 259	0.0083 6145	-0.0029 27
7.	0.9025 1795 639	0.0215 0704	-0.0046 85
8.	0.9521 4949 047	0.0062 5302	0.0008 07

Table 2: Errors of Daily Motion

S.No.	Ptolemy	Āryabhaṭa	Bhahmagupta	Lalla	Nīlakaṇṭha
1.	12.1907 4693 67	12.1907 5202 60	12.1907 5192 48	12.1907 4746 30	12.1907 4856 37
	12.1907 4718 97	12.1907 4764 28	12.1907 4871 20	12.1907 4816 51	12.1907 4895 77
	0.02	0.36	0.34	0.06	0.03
2.	13.0649 8286 01	13.0649 6830 16	13.0649 9458 57	13.0649 8454 58	13.0649 8648 09
	13.0649 8034 40	13.0649 8277 10	13.0649 8367 98	13.0649 8558 00	13.0649 8986 42
	0.19	1.11	0.83	0.08	0.26
3.	13.2293 5099 87	13.2293 3697 10	13.2293 5699 19	13.2293 4992 99	13.2293 4942 57
	13.2293 4942 71	13.2293 4993 00	13.2293 5000 52	13.2293 5016 11	13.2293 5050 63
	0.10	0.98	0.53	0.02	0.08
4.	3.1066 9904 30	3.1067 0690 96	3.1067 0418 21	3.1067 8356 76	3.1067 1872 36
	3.1067 2874 35	3.1067 2894 96	3.1067 2902 62	3.1067 2918 52	3.1067 2953 89
	0.96	0.71	0.80	1.75	0.35
5.	0.6165 0873 39	0.6165 4660 65	0.6165 4735 72	0.6165 1868 11	0.6165 1967 83
	0.6165 2117 79	0.6165 2121 97	0.6165 2123 53	0.6165 2126 78	0.6165 2134 00
	2.02	4.12	4.24	0.42	0.27
6.	0.4615 7556 71	0.4615 8519 69	0.4615 8447 24	0.4615 7643 60	0.4615 7639 67
	0.4615 7613 69	0.4615 7616 65	0.4615 7617 75	0.4615 7620 03	0.4615 7625 07
	0.12	1.96	1.80	0.05	0.03
7.	0.9025 1284 21	0.9025 0558 73	0.9025 0562 77	0.9025 1416 57	0.9025 1985 52
	0.9025 1755 69	0.9025 1763 27	0.9025 1766 09	0.9025 1771 93	0.9025 1784 87
	0.52	1.33	1.33	0.39	0.22
8.	0.9521 4673 83	0.9521 6445 73	0.9521 6433 84	0.9521 6080 69	0.9521 6265 17
	0.9521 4937 51	0.9521 4939 69	0.9521 4940 50	0.9521 4942 17	0.9521 4945 92
	0.28	1.58	1.57	1.20	1.39

In formaula (1), t = 0 corresponds to Julian year 2000, and for every hundred years earlier is reduced by 1. The dates of the mean parameters of Ptolemy and the four Indian astronomers are taken to be 150 for Ptolemy, 500 for Āryabhaṭa, 630 for Brahmagupta, 900 for Lalla and 1500 for Nīlakaṇṭha as suggested in Billard's book. We ignore the difference between the Julian dates and the corresponding calendar dates, because the change in the daily motions in those intervals in time is negligible.

The results of our calculations are given in Table 2. The rows numbered 1 to 3 are for the moon's motion relative to the sun, moon's apogee and node, and rows 4 to 8 are for the planet's motions relative to the sun. The first line in each row gives the daily motion according to the astronomers, O, the second is the motion calculated from modern theory, M, and the third is the error E defined by

$$(2) \qquad E = \frac{|O - M|}{M}$$

multiplied by 10^6 for the lunar motions and by 10^5 for the planets.

The average values E_1 of $E \times 10^6$ for the 3 lunar parameters and E_2 of the $E \times 10^5$ for the 5 planetary parameters are :

$$(3) \qquad E_1 : \quad 0.10, 0.82, 0.57, 0.05, 0.12$$
$$E_2 : \quad 0.78, 1.94, 1.96, 0.76, 0.45$$

for the five astronomers, as illustrated in figs. 4a and 4b.

Figs. 5a and 5b show the errors of (a) the mean daily anomalistic motion of the moon and (b) the mean synodic motion of Jupiter, the curve in each case indicating the theoretical values and the vertical lines the errors of the five astronomers. There are insignificant changes in the errors if we change the dates of the astronomers unlike the longitudes in Billard's calculations. For example if Lalla's date is taken as 800 AD the average lunar and planetary errors are $E_1 = 0.03$ and $E_2 = 0.76$.

Discussion

As seen in Figs. 4a and 4b, the least error for the lunar parameters is that of Lalla and for the planetary parameters is that of Nīlakaṇṭha, emphasizing what can be seen in the corresponding diagrams in Billard's book (Figs. 2,3).

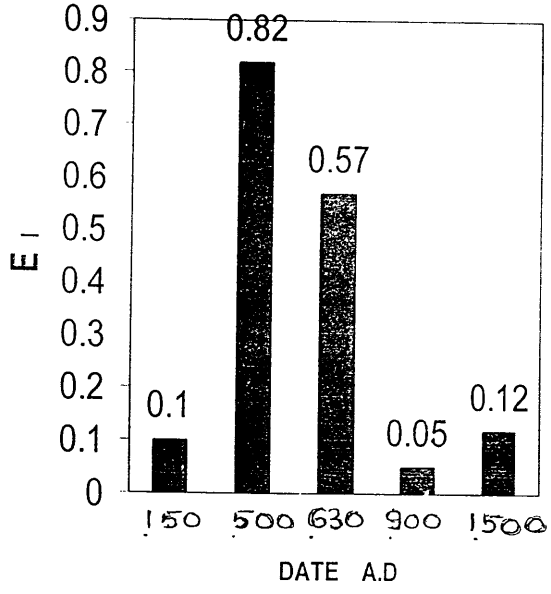

Fig. 4a: Lunar Error

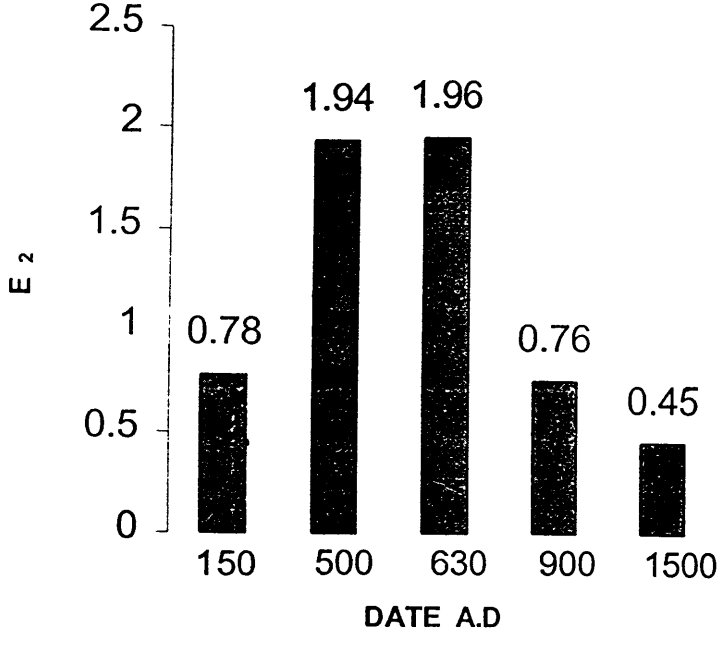

Fig. 4b: Planetary Error

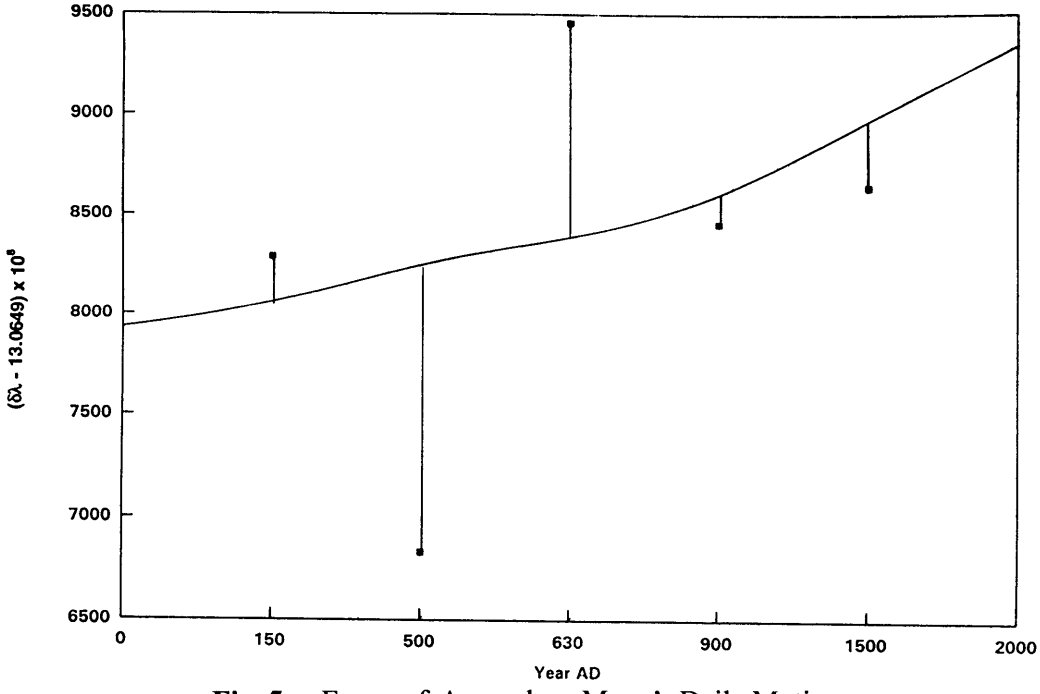

Fig. 5a: Errors of Anomalous Moon's Daily Motion

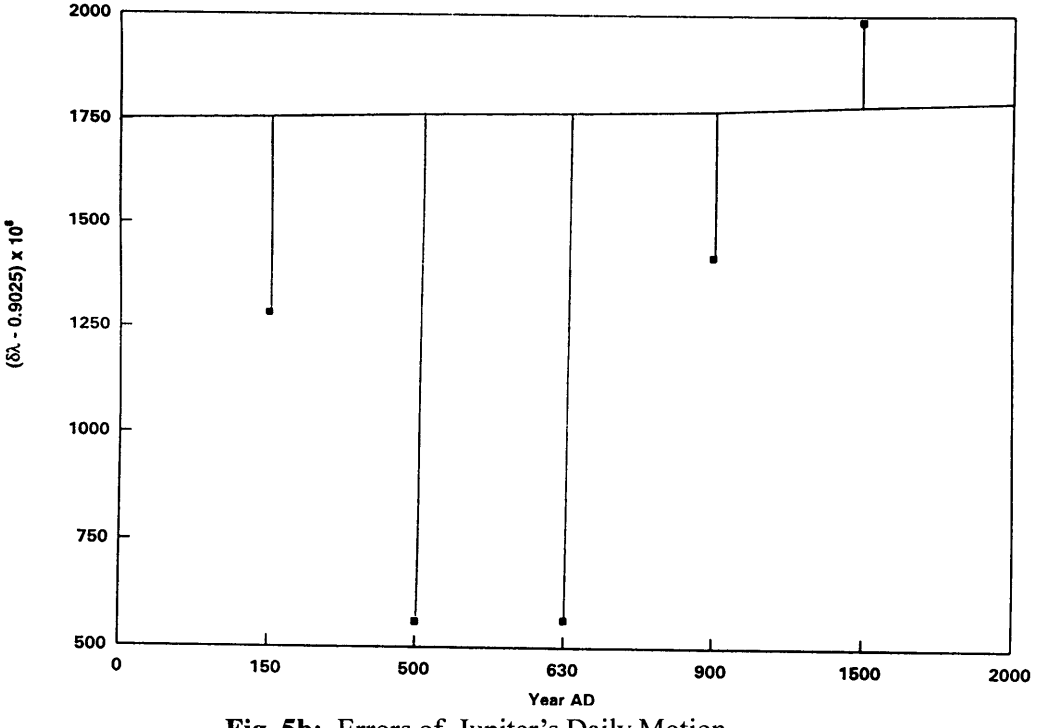

Fig. 5b: Errors of Jupiter's Daily Motion

David Pingree[3] (p. 59) has expressed skepticism of Billard's conclusions partly because of the absence, in the works of Indian astronomers, of accurate star catalogues. In this connection, we must consider the following stanza (Kalakriya, 48) of the *Āryabhaṭīya* quoted by P.C. Sengupta in his introduction to E. Burgess's translation of the *Sūrya Siddhānta*[7].

"The day-maker has been determined from the conjunction of the earth (or the horizon) and the sun; and the moon from her conjunctions with the sun. In the same way the 'star planets' have been determined from their conjunctions with the moon".

This suggests that Āryabhaṭa depended more on the conjunctions of the sun, moon and planets than their conjunctions with stars.

Billiard's analysis and the results in this paper point to the following conclusions:

- The accuracy of Ptolemy, Lalla and Nīlakaṇṭha depended not only on their own observations but probably their using the accurate observations of their predecessors. In the case of Ptolemy, this included Hipparchus and the Babylonians[8]; Lalla and Nīlakaṇṭha would have taken into account the observations of Āryabhaṭa and Brahmagupta.

- For the better assessment of Lalla and Nīlakaṇṭha it would be necessary to compare them with Islamic and European astronomers of the same period.

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