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MEAN PLANETARY POSITIONS ACCORDING TO *GRAHALĀGHAVAM*

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In this paper, the procedure for computations of the mean positions of the Sun, the Moon and the five planets (*tārāgrahas*) according the *Grahalāghavam* (*GL*) of Gaṇeśa Daivajña is presented. The special features of *GL* in simplifying mathematical computations, finding the *ahargaṇa* and determination of mean planetary positions are explained. Illustrative examples from the commentator Viśvanātha are provided.

Key words: Gaṇeśa Daivajña, *Grahalāghavam*, Mean Planets, Indian Astronomy, *Ahargaṇa*.

INTRODUCTION

One of the most popular traditional Indian astronomers is Gaṇeśa Daivajña. He is said to have been born in 1507 A.D. (Śaka 1429) at a place called Nandigrāma on the western sea coast in the Konkan region of Maharastra. His father Keśava Daivajña was also a great astronomer who authored the book *Grahakautukam*. Gaṇeśa's mother was Lakṣmī.[1]

Apart from his most popular text *Grahalāghavam* (*GL*) Gaṇeśa's other astronomical texts are *Tithicintāmaṇi* a commentary on Bhāskara's *Siddhānta śiromaṇi*, a commentary, *Buddhivilāsinī* on the *Līlāvatī* of Bhāskara II. The epoch of *Grahalāghavam* is March 19, 1520 A.D. (Julian), Monday.

In *GL* the mean positions of planets have been given for the instant of the mean sunrise of Monday (at Ujjayinī) March 19, 1520 (Julian) which was a newmoon day.

Gaṇeśa has simplified the method of computations of positions of planets which is otherwise laborious by the traditional method followed by celebrated

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astronomers like Āryabhaṭa I (b.476), Brahmagupta (628) and Bhāskara II (b.1114).

The following are some of the special features of GL:

(i) To avoid a large number for the *ahargaṇa*, the elapsed civil days since the chosen epoch, Gaṇeśa has adopted a cycle (*cakra*) of 4016 days, approximately 11 solar years. His modified *ahargaṇa* never exceeds 4016 days, hence it is very handy. Further, huge numbers for *ahargaṇa* by the conventional method, for multiplication etc., would result in numerical errors. This is avoided greatly by Gaṇeśa's innovation.

(ii) For the benefit of compilers of *pañcāṅga* (astronomical almanac) and beginners in astronomy who may be ignorant of trigonometry, Gaṇeśa has completely avoided the sine and cosine functions.

(iii) In fact dropping of trigonometric ratios has not seriously affected the accuracy of the results; on the contrary it has simplified the procedures greatly. Gaṇeśa has adopted reasonably very good and justifiable mathematical approximations for the trigonometric functions.

AHARGAṆA

For the purpose of finding the mean positions of the planets on any particular day, the total number of civil days elapsed since the beginning of the chosen epoch must first be determined. Then, this duration of time multiplied by the mean daily motion of a planet gives the angular distance covered by the planet during that period. From this motion the completed number of revolutions (multiples of 360°) is removed. The remainder when added to the mean position of the planet at the epoch gives the mean position for the required day.

The number of days elapsed since the chosen fixed epoch, called *ahargaṇa* literally means a "heap of days". The calculation of the *ahargaṇa* depends on the calendar system followed. Since in the traditional Hindu calendar both the luni-solar or lunar calendar and the solar calendar, to which the former is pegged on to, are followed, the intercalary months (*adhikamāsa*) play an important role in calculating the *ahargaṇa*.

The process of determining the *ahargaṇa* essentially consists of the following steps.[2]

(i) Convert the solar years elapsed (since the epoch) into lunar months;

(ii) Add the number of *adhikamāsas* during that period to give the actual number of lunar months that have elapsed upto the beginning of the current lunar year;

(iii) Add the number of lunar months elapsed in the given year;

(iv) Convert these actually elapsed number of lunar months into *tithis* (by multiplying it by 30);

(v) Add the elapsed number of *tithis* in the current lunar month;

(vi) Subtract the *kṣaya dinas*; and finally convert the elapsed number of *tithis* into civil days.

Note: While finding the *adhikamāsas*, if an *adhikamāsa* is due after the lunar month of the current lunar year, then 1 is to be subtracted from the calculated number of *adhikamāsas*. This is because an *adhikamāsa* which is yet to come in the course of the current year will have already been added.

A. K. Bag has explained, in detail, the process of computing the *ahargaṇas* according to different traditional texts in his paper.[3]

The epoch chosen by Gaṇeśa Daivajña is *Śālivāhana śakavarṣa* (year) 1442 *caitra śukla pratipat*, corresponding to March 19, 1520 A.D. (Julian), Monday. The *ahargaṇa* for a given lunar date is determined as follows[4] :

(i) Subtract 1442 from the *Śālivāhana śaka* year (elapsed) of the given date to get the years elapsed (*gatābdi*).

(ii) Divide the remainder by 11. The quotient is called *cakra* (cycles) $= C$.

(iii) Multiply the remainder, obtained in step (ii), by 12 and to the product add the number of lunar months elapsed, counting Caitra as 1 etc. The sum thus obtained is called 'mean lunar months' (*madhyama māsagaṇa*) denoted by M.

(iv) The number of *adhikamāsas* is given by the quotient of $\dfrac{M+2C+10}{33}$

(v) True lunar months (*spaṣṭa māsagaṇa*) = Mean lunar months +

$$adhikamāsas = M + Quotient\ of\ \frac{M+2C+10}{33} \equiv TM$$

(vi) Mean *ahargaṇa* (*madhyama ahargaṇa*) $MAH = (TM \times 30) + TI + \frac{C}{6}$

where TI is the number of *tithis* elapsed in the given lunar month.

(vii) *Kṣayadinas* = Quotient of $\left[\frac{1}{64} \textit{madhyama ahargana} \right] \equiv KD$

(viii) True *ahargaṇa* (*sāvaṇa dinas*) i.e., the number of civil days,

TAH = Mean *ahargaṇa* – *kṣaya dinas*

= $MAH - KD = MAH$ – Quotient of 1/64 (MAH)

(ix) However, since the average values of parameters are considered in the above computations, 1 day may have to be either added to or subtracted from the result of (viii) to get the actual true *ahargaṇa*.

This is done by verifying the weekday as follows:

(a) Multiply the *cakras* by 5 i.e., find $5C$. Add *ahargaṇa* to this i.e., find $(5C + TAH)$.

(b) Divide the result of (a) by 7 and find the remainder. Let

R = Remainder of $\left[\frac{5C + TAH}{7}\right]$. If $R = 0$, then it is Monday; $R = 1$ then it is Tuesday; and so on.

(c) If the calculated weekday is a day next to the actual weekday, then subtract 1 from TAH and if it is one day less than the actual weekday, then add 1 to TAH.

[See Note (1) and (2) appearing later in this section].

Example 1: Commentator Viśvantha cites the following illustration:

Śā. *Śaka* 1534, *Vaiśākha Purṇimā*, Monday, May 14, 1612 A.D. (Gregorian)

(i) Subtract 1442 from 1534 :

Gatābdi = 1534 – 1442 = 92 years (from the epoch)

(ii) Divide the remainder in (i) by 11:
the quotient, *cakras* = 8 ≡ C and the remainder = 4

(iii) Multiplying the remainder from (ii) i.e. 4 by 12 and adding the number of lunar months elapsed in the given year we get
$(4 \times 12) + 1 = 49 \equiv$ M, the *madhyama māsagaṇa*

(iv) No. of *adhikamāsagaṇa* =

$$\frac{M+2C+10}{33}=\frac{49+2(8)+10}{33}=\frac{75}{33};Quotient=2$$

(v) M + No. of *adhikamāsas* $= 49 + 2 = 51 = TM$ is the *spaṣṭa māsagaṇa*

(vi) (a) Mean *ahargaṇa* (*madhyama ahargaṇa*)

$= (TM \times 30)$ + (No. of *tithis* elapsed in the given lunar month)

$= (51 \times 30) + 14 = 1544$

(b) Add INT 1/6 [C] i.e., INT (8/6) = 1 to the result of (vi) (a)

$\therefore \quad 1544 + 1 = 1545 \equiv MAH$

(**Note:** INT stands for the integer value).

(vii) *Kṣaya dinas* $= \text{INT}\left(\frac{MAH}{64}\right) = \text{INT}\left(\frac{1545}{64}\right) = 24 \equiv KD$

(viii) *Sāvaṇa ahargaṇa* (i.e, No. of civil days in the running *cakra*)

$= MAH—KD = 1545\text{-}24 = 1521 \equiv TAH$

(ix) Week day verification

$5C+TAH= 5(8) +1521=1561$

$$\therefore \quad R = \text{Remainder of} \left\{\frac{1561}{7}\right\} = 0$$

That is, the weekday comes out as Monday.

Since the weekday obtained from calculation is the same as the actual weekday (known), nothing needs to be added to or subtracted from TAH. Therefore,

True *ahargaṇa* = 1521 No. of *cakras* = 8

Note 1: Sometimes when (Śaka year 1442) is divided by 11, to get *cakras* the remainder could be 0. In that case even 2 may have to be added to or subtracted from the obtained *Sāvana dinas* to get the true *ahargaṇa* for the weekday.

Note 2: Sometimes there could be an *adhikamāsa* in a particular given lunar year.

(i) If the given date is before the *adhikamāsa* of that lunar year, then subtract 1 from the no. of *adhikamāsas* obtained in the calculation.

(ii) If the given date is after the *adhikamāsa* of that lunar year, then add 1 to the no. of *adhikamāsas* obtained in the calculation.

These two cases are demonstrated in the following examples of Viśvanātha.

Example 2:

Śaka 1555 *Caitra Śukla Pratipat*, Friday [March 11, 1633]. In this year *Vaiśākha* is the *adhikamāsa* which comes after the given date.

We shall find the *cakra* and *ahargaṇa* for the given date:

(i) *Gatābdi* = 1555 – 1442 = 113

(ii) *Cakras*, C = INT (113/11) = 10, Remainder = 3

(iii) Mean *māsagaṇa* = (Remainder × 12) + No. of elapsed lunar months

$= (3 \times 12) + 0 = 36 \equiv M$

(iv) No. of *adhikamāsas* $= \text{INT}\left[\dfrac{M+2C+10}{33}\right] = \text{INT}\left[\dfrac{36+2(10)+10}{33}\right]$

$= \text{INT}\,[66/33] = 2$ (Note : Remainder = 0)

Since the given date falls before the *adhika Vaiśākhamāsa*, subtract 1 from the number obtained above. Therefore, the *adhikamāsas* elapsed = 2 – 1 = 1.

(v) True lunar months = M + No. of *adhikamāsas* = 36 + 1 = 37 ≡ TM

(vi) Mean *ahargaṇa*, MAH = (37 × 30) + *Tithis* elapsed + INT (C/6)

= 1110 +0 + INT (10/6) = 1110 +1 = 1111.

(vii) *Kṣayadinas*, KD = INT[MAH/64] = INT [1111/64] = 17

(viii) *Sāvaṇa ahargaṇa* = $MAH - KD$ = 1111 – 17 = 1094 ≡ TAH

(ix) Weekday verification:

$5C + TAH$ = 5(10) + 1094 = 1144; Remainder of [1144/7] = 3

i.e., Thursday; but the actual weekday : Friday.

Therefore, the true *ahargaṇa* = TAH + 1 = 1095

(x) Christian date:

No.of civil days since epoch = 10 (4016) + 1095 = 41,255

Kali *ahargaṇa* of the *GL* epoch : 16,87,850

∴ Kali *ahargaṇa* of the date : 17,29,105

From Table 1 to 3 under Kali *ahargaṇa* we have

1600 A.D. (G)	:	17,16,982
year 33	:	12,053
March 11	:	70
Total	:	17,29,105

corresponding to March 11, 1633 AD (Gregorian).

Example 3: *Śaka* 1530 (Bhādrapada is *adhikamāsa*) *Kārtika Śukla Pratipat*, Saturday. We have

(i) *Gatābdi* = 1530 – 1442 = 88

(ii) *Cakra*, C = INT (88/11) = 8, Remainder = 0

(iii) Mean lunar months $M = (0 \times 12) + 7 = 7$.

(iv) No.of *Adhikamāsas* = $\text{INT}\left[\dfrac{M+2C+10}{35}\right] = \text{INT}\left[\dfrac{7+16+10}{33}\right] = 1;$

Remainder = 0.

Since *Kārtika* month (i.e., the given month) occurs after the *adhika* Bhādrapada *māsa*, add 1 to the calculated no. of *adhikamāsas*. Therefore, no. of *adhikamāsas* = 1 + 1 = 2.

(v) True lunar months $TM = M$ + No. of *adhikamāsas* = 7 + 2 = 9

(vi) Mean *ahargaṇa* :

$MAH = (TM \times 30)$ + (*tithis* elapsed in the given month) + INT (C/6) = $(9 \times 30) + 0 +$ INT (8/6) = 271

(vii) *Kṣayadinas* = INT (271/64) = 4 ≡ KD

(viii) *Sāvaṇa ahargaṇa* $TAH = MAH - KD = 271 - 4 = 267$

(ix) Weekday verification : $5C + TAH = 40 + 267 = 307$

Remainder of (307/7) = 6 i.e., Sunday

But the given weekday is Saturday

∴ True *ahargaṇa* = $TAH - 1 = 266$

(x) Christian date:

No. of civil days since epoch = 8 (4016) + 266 = 32,394

Kali ahargaṇa of *GL* epoch : 16,87,850.

∴ *Kali ahargaṇa* of the given date : 17,20,244

From Tables 1 to 3 we have

1600 AD	:	17,16,982
year 08	:	2,922
December 6	:	340
Total	:	17,20,244

corresponding to December 6, 1608 AD (Gregorian).

Finding the Christian date from the *ahargaṇa* and vice versa:

In the above examples we saw how to get the *cakras* and the *ahargaṇa* from the given *Candramāna*(i.e., lunar) date. Now, we shall see how from the thus obtained *cakras* and *ahargaṇa* the corresponding Christian date can be obtained.

In Table 1 the Julian days and the *ahargaṇas* for the epochs of the *Kaliyuga* and the *Grahalāghavam* are given for the beginnings of the Christian centuries from 1500 AD (Julian) to 2100 AD (Gregorian).

Table 1: Julian Day Number (JDN) and *Ahargaṇa*

Chris.	Julian Day	Kali	*Graha Lāhavam*	
Year	(JDN)	*Ahargaṇa*	*Ca.*	*Ahargaṇa*
1500 (J)	2268933	1680467	-2	649
1500 (G)	2268923	1680457	-2	639
1600 (G)	2305448	1716982	7	1020
1700 (G)	2341972	1753506	16	1400
1800 (G)	2378496	1790030	25	1780
1900 (G)	2415020	1826554	34	2160
2000 (G)	2451545	1863079	43	2541
2100 (G)	2488069	1899603	52	2921

Note: In Table 1, the JDN refers to the noon (GMT) and the *Kali* and GL *ahargaṇas* refer to the Ujjayin mean sunrise of the *day preceding* January 1 for the *non-leap* century years. In the case of *leap* century years (e.g. 1600 and 2000 Gregorian), the numbers refer to Jan. 1 itself.

Epochs chosen:

(i) The epoch adopted for the Kali era is the mean sunrise of February 18th of 3102 BC (i.e., the year – 3101) in the *audayika* system.For any year before Christ (BC), for mathematical convenience, the negative sign is prefixed to 1 less than the numerical value of the Christian year. For example, 46 BC is considered as – 45, and 3102 BC as --3101.

This convention is adopted since 1 BC is taken as the '0' year of the Christian era.

Table 2: Ahargaṇa for Year Beginnings

Year	Days	Graha	Lāghavam	Year	Days	Graha	Lāghavam
		Ca.	*Ahar.*			*Ca.*	*Ahar.*
0	0	0	0	50	18262	4	2198
1	365	0	365	51	18627	4	2563
2	730	0	730	52	18993	4	2929
3	1095	0	1095	53	19358	4	3294
4	1461	0	1461	54	19723	4	3659
5	1826	0	1826	55	20088	5	8
6	2191	0	2191	56	20454	5	374
7	2556	0	2556	57	20819	5	739
8	2922	0	2922	58	21184	5	1104
9	3287	0	3287	59	21549	5	1469
10	3652	0	3652	60	21915	5	1835
11	4017	1	1	61	22280	5	2200
12	4383	1	367	62	22645	5	2565
13	4748	1	732	63	23010	5	2930
14	5113	1	1097	64	23376	5	3296
15	5478	1	1462	65	23741	5	3661
16	5844	1	1828	66	24106	6	10
17	6209	1	2193	67	24471	6	375
18	6574	1	2558	68	24837	6	741
19	6939	1	2923	69	25202	6	1106
20	7305	1	3289	70	25567	6	1471
21	7670	1	3654	71	25932	6	1836
22	8035	2	3	72	26298	6	2202
23	8400	2	368	73	26663	6	2567
24	8766	2	734	74	27028	6	2932
25	9131	2	1099	75	27393	6	3297
26	9496	2	1464	76	27759	6	3663
27	9861	2	1829	77	28124	7	12
28	10227	2	2195	78	28489	7	377
29	10592	2	2560	79	28854	7	742
30	10957	2	2925	80	29220	7	1108
31	11322	2	3290	81	29585	7	1473
32	11688	3	3656	82	29950	7	1838
33	12053	3	5	83	30315	7	2203
34	12418	3	370	84	30681	7	2569
35	12783	3	735	85	31046	7	2934
36	13149	3	1101	86	31411	7	3299
37	13514	3	1466	87	31776	7	3664
38	13879	3	1831	88	32142	8	14
39	14244	3	2196	89	32507	8	379
40	14610	3	2562	90	32872	8	744
41	14975	3	2927	91	33237	8	1109
42	15340	3	3292	92	33603	8	1475
43	15705	3	3657	93	33968	8	1840
44	16071	4	7	94	34333	8	2205
45	16436	4	372	95	34698	8	2570
46	16801	4	737	96	35064	8	2936
47	17166	4	1102	97	35429	8	3301
48	17532	4	1468	98	35794	8	3666
49	17897	4	1833	99	36159	9	15

Notes: (1) In Table 1, the letters J and G in brackets represent respectively the *Julian* and the *Gregorian* calendars. The *Gregorian* calendar came into effect from October 15, 1582, Friday.
(2) In Table 3, the first two columns are headed by C and B which stand respectively for a *common* (non-leap) year and *bissextile* (leap) year.
For a given date in a leap year, only for January and February, the column headed by B must be used. For other months even in a leap year and for all months in a common year the first column under C must be used.

(ii) The epoch of the *Grahalāghavam* (*GL*) :Gaṇeśa Daivajña in his *GL* has adopted the mean sunrise (at Ujjayinī) of March 19, 1520(Julian) AD, Mon day, as the epoch.(ii) The epoch of the *Grahalāghavam* (*GL*) : *Gaṇeśa Daivajña* in his *GL* has adopted the mean sunrise (at Ujjayinī) of March 19, 1520(Julian) AD, Monday, as the epoch.

(iii) Julian Day Number (JDN) : The reckoning of the Julian days starts from the mean noon (GMT) on January 1, 4713 BC, Monday. On that day, at the mean noon (GMT), JDN = 0

The procedure for finding the Christian date from the *cakras* and the *ahargaṇa*:

(i) Multiply the number of *cakras* C by 4016 (the number of days in a *cakra*) i.e., find 4016 C. To this 4016 C add the *ahargaṇa* A i.e., find $(4016C + A)$. The Kali *ahargaṇa* for the *GL* epoch is 16,87,850).Add this constant to $(4016\,C + A)$ i.e., find $(4016\,C + A$ + 16,87,850). This gives the Kali *ahargaṇa* for the required date.

From Tables 1 to 3, for the thus obtained Kali *ahargaṇa* the corresponding Christian date can be obtained as shown in the following example.[5]

Finding the weekday from the *ahargaṇa*:

Let C and A respectively be the *cakras* and the *ahargaṇa* according to *GL*. Multiply C by 5 and to this product add A i.e., find $(5C + A)$.

Dividing $(5C + A)$ by 7, let the remainder be R. If $R = 0$, then the given date falls on a Monday; if $R = 1$, Tuesday etc.

Example: In the example considered above, $C = 8$, and $A = 1521$. Therefore, $5C + A = 5\,(8) + 1521 = 1561$. When 1561 is divided by 7, the remainder $R = 0$. Therefore, the given date is a Monday.

Table 3: Ahargaṇa for Days of a Year

Dates C	Dates B	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
0	1	0	31	-	-	-	-	-	-	-	-	-	-
1	2	1	32	60	91	121	152	182	213	244	274	305	335
2	3	2	33	61	92	122	153	183	214	245	275	306	336
3	4	3	34	62	93	123	154	184	215	246	276	307	337
4	5	4	35	63	94	124	155	185	216	247	277	308	338
5	6	5	36	64	95	125	156	186	217	248	278	309	339
6	7	6	37	65	96	126	157	187	218	249	279	310	340
7	8	7	38	66	97	127	158	188	219	250	280	311	341
8	9	8	39	67	98	128	159	189	220	251	281	312	342
9	10	9	40	68	99	129	160	190	221	252	282	313	343
10	11	10	41	69	100	130	161	191	222	253	283	314	344
11	12	11	42	70	101	131	162	192	223	254	284	315	345
12	13	12	43	71	102	132	163	193	224	255	285	316	346
13	14	13	44	72	103	133	164	194	225	256	286	317	347
14	15	14	45	73	104	134	165	195	226	257	287	318	348
15	16	15	46	74	105	135	166	196	227	258	288	319	349
16	17	16	47	75	106	136	167	197	228	259	289	320	350
17	18	17	48	76	107	137	168	198	229	260	290	321	351
18	19	18	49	77	108	138	169	199	230	261	291	322	352
19	20	19	50	78	109	139	170	200	231	262	292	323	353
20	21	20	51	79	110	140	171	201	232	263	293	324	354
21	22	21	52	80	111	141	172	202	233	264	294	325	355
22	23	22	53	81	112	142	173	203	234	265	295	326	356
23	24	23	54	82	113	143	174	204	235	266	296	327	357
24	25	24	55	83	114	144	175	205	236	267	297	328	358
25	26	25	56	84	115	145	176	206	237	268	298	329	359
26	27	26	57	85	116	146	177	207	238	269	299	330	360
27	28	27	58	86	117	147	178	208	239	270	300	331	361
28	29	28	59	87	118	148	179	209	240	271	301	332	362
29	30	29	-	88	119	149	180	210	241	272	302	333	363
30	31	30	-	89	120	150	181	211	242	273	303	334	364
31	-	31	-	90	-	151	-	212	243	-	304	-	365

Finding the *ahargaṇa* from the Christian date:

In Table 1, for the beginning of the Christian century (column 1)— in which the given date lies – Kali *ahargaṇa* (column 3) and the *cakras* and the (balance) *ahargaṇa* according to *GL* are given in column 4 and 5. In Table 2 the days elapsed at the beginning of each year are given. Table 3 provides the cumulative days corresponding to each day of a year (see Note at the end of Table 2). For example, consider October 7 of the year 2001 AD. For this year the century beginning year is 2000 (G). In Table 1, against 2000 (G), we have

	Kali *ahargaṇa*	*GL ahargaṇa* Ca.	*ahar.*	Julian Day Number
2000 (G)	18,63,079	43	2541	24,51,545
Year 1	365	0	365	365
October 7	280	0	280	280
Total	18,63,724	43	3186	24,52,190

Weekday from *GL ahargaṇa* : Here $C = 43$ and $A = 3186$.

$\therefore \quad 5C + A = 215 + 3186 = 3401$

Dividing $(5C + A)$ by 7, remainder $R = 6$, we get Sunday.

Note: In Table 1, column 2 gives the Juian day number (JDN). To find the weekday from JD of the given date, divide JDN by 7 and let R be the remainder. If $R = 0$, it is Monday if $R = 1$, Tuesday etc.

MEAN POSITIONS OF THE SUN, MOON AND PLANETS

Revolutions of bodies and daily motions

The following is the usual procedure for finding the mean position of any planet. Let λ_0 be the mean position at an epoch. If d is the daily mean motion of the planet in degrees and A is the *ahargaṇ a* from the epoch, then the mean motion from the epoch till the given day is $\lambda_1 = (A \times d)$. Then, the mean position on the given day is

$$\lambda = \lambda_0 + \lambda_1 = \lambda_0 + (A \times d)$$

In the texts where the beginning of *Kaliyuga* is used as the epoch, the *ahargaṇa* is calculated from the mean midnight preceding 18th of February and the mean sunrise of 3102 BC respectively in the *ardharātrika* and the *audayika* systems. Therefore, in this method, the *ahargaṇa* A is obtained till the midnight preceding the given day. Further, the midnight or the sunrise is as at the Ujjayani (23°11′ N latitude and 75° 46′ E longitude) meridian passing through *Laṇkā* on the equator.

Therefore, while calculating the positions of the planets, corrections will have to be applied to account for:

(i) the time interval between the midnight at Ujjayinī and the midnight at the given place;

(ii) the difference in the duration from the local midnight to the given time.

In addition to these, some more important corrections will also have to be applied. These will be discussed later.

Some astronomical texts consider the mean sunrise at *Laṇkā* on February 18,3102 BC as the starting time of the Kali era.

The mean motion of the Sun, the Moon and other planets are given in terms of the number of revolutions (each of 360° extent) completed in the course of a *Kalpa* of 432 × 10^7 years. Table 4 and 5give the necessary details for determining the mean positions of the Sun and the Moon according to different texts.

Looking at Table 4, we notice that while the revolutions of the Sun in a *Kalpa* (432 × 10^7 years) are the same according to the different texts, those of the Moon and its apogee and node (*Rāhu*) are different. Further, the number of civil days in a *Mahāyuga* (or multiplied by 1000 for a Kalpa) are slightly different according

Table 4: Revolutions of the Sun and the Moon in a Kalpa

(1 *Kalpa* = 432 × 10^7 years = 1000 *Mahāyugas*)

Bodies and points	Ravi	Candra	Candra's Apogee	Candra's (Asc.) Node	Civil days in a *Mahāyuga*
Ārybhaṭa I	432,00,00,000	5775,33,36,000	48,82,19,000	23,21,16,000	1,57,79,17,500
Brahmagupta (*Khnd·akhādyaka*)	432,00,00,000	5775,33,36,999	48,82,19,000	23,21,16,000	1,57,79,17,800
Sūryasiddhānta	432,00,00,000	5775,33,36,000	48,82,03,000	23,22,38,000	1,57,79,17,828
Ārybhaṭa II (*Mahā sıddhhānta*)	432,00,00,000	5775,33,34,000	48,82,08,674	23,23,13,354	
Bhāskara II (*Siddhāntaśiromaṇ i*)	432,00,00,000	5775,33,00,000	48,82,05,858	23,23,11,168	1,57,79,16,450

Table 5: Mean daily motions of the Sun, Moon, etc.

Bodies and points	Modern Astronomy	*Sūrya Siddhānta*	*Siddhānta śiromaṇi*	*Graha Lāghavam*	*Khaṇḍa Khādyaka*
Ravi	0° 59 08.2	0°59 08 10 09.7	0°59 08 10 21	0°59 08	0°59 08
Candra	13° 10 34.9	13° 10 34 52 02	13° 10 34 53 00	13° 10 35	13° 10 31
Kuja	0° 31 26.5	0° 31 26 28 10	0° 31 26 28 07	0° 31 26	0° 31 26
Budha's śīghrocca	4° 05 32.4	4° 05 32 20 42	4° 05 32 18 28	*3° 06 24	4° 05 32
Śukra's śīghrocca	1° 36 07.7	1° 36 07 43 37	1° 36 07 44 35	*0° 37 00	1° 36 07
Guru	0° 04 59.1	0° 04 59 08 48	0° 04 59 09 09	0° 05 00	0° 04 59
Śani	0° 02 00.5	0° 02 00 22 53	0° 02 00 22 51	0° 02 00	0° 02 00
Candra's mandocca	0° 06 40.92	0° 06 40 58 42	0° 06 40 53 56	0° 06 41	0° 06 40
Candra's *pāta* (Rāhu)	−0° 03 10.77	−0° 03 10 44 43	−0° 03 10 48 20	−0° 03 11	−0° 03 10

* In *GL* the *śighra kendras* of Buddha and Śukra are given directly unlike other texts in which the *śīghroccas* are given.

to Āryabhaṭa I and the *Sūrya-siddhānta*, for example. These differences have resulted from the corrections made periodically, and give rise to slightly different mean daily motions. The mean daily motions are given in Table 5 according to the *Grahalāghavam*[6], the *Siddhāntaśiromaṇi*[7] of Bhāskara II, as compared to the modern values and those of the *Khaṇḍakhādyaka*[8] of Brahmagupta and *Sūryasiddhānta*[9].

It is clear that the daily motion of a celestial body is given, in revolutions per day, by

$$d = \frac{\textit{Number of revolution in a Mahāyuga}}{\textit{Number of civil days in a Mahāyuga}}$$

For example, according to the *Siddhāntaśiromaṇi*, the number of revolutions completed by the Moon is 5,77,53,300 in a Mahāyuga that has 1,57,79,16,450 civil days. Therefore, the Moon's daily motion is

$$d_{Moon} = (5{,}77{,}53{,}300/\ 1{,}57{,}79{,}16{,}450) \times 360 \text{ (in degrees)}$$
$$= 13^{\circ}10'34''.8796$$

Similarly, for the Sun, the mean daily motion is

$$d_{Sun} = (43{,}20{,}000/157{,}79{,}16{,}450) \times 360 \text{ (in degrees)} = 0^{\circ}59'08''.1726$$

Gaṇeśa Daivajña compares his values of the mean positions of the bodies with those according to other popular astronomical systems like the *Saurapakṣa*, *Āryapakṣa* and *Brahmapakṣa*. He says:

sauro'rko'pi vidhūccamaṅkalikonābjo gurustvāryajo asṛgrāhūca kajam jñakendrakam atha ārye seṣubhāgaḥ śaṇiḥ / śaukram kendramajāryamadhyagamitīme yānti dṛktulyatām siddhaistairiha parvadharmanaya satkāryādikam tvādiśet // -GL, 16.

"The values of the Sun and the Moon's *mandocca* (*Candrocca*) obtained from this text are equivalent to those according to the *Sūrya-siddhānta* (SS). By subtracting 9′ from the Moon (as per this text) we get as per SS. The positions of Kuja, Guru, Rāhu are in accordance with the *Āryapakṣa*. Budha *kendra* (Budha's *śīghra* anomaly) is in accordance with the *Brāhma pakṣa.* By adding 5° to Śaṇi (obtained from this text), we get that according to the *Ārya Siddhānta*. The *Śukra* (*śīghra*) *kendra* (of this text) is half of the sum of those obtained from the *Ārya* and *Brahma pakṣas* (i.e., the average of the latter two)….."

Mean position of bodies according to *GL*:

For determining the mean position of a body we need its *kṣepaka* and *dhruvaka* as explained hereafter.

Kṣepaka is the mean position of a body at the chosen time of the epoch and *dhruvaka* is the residual motion of a body in a *cakra* (after removing the completed revolutions of 360° each). The *dhruvakas* and *kṣepakas* of the different bodies (and points) are given in Tables 6 and 7 respectively.

Table 6: Dhruvakas of bodies

	Ravi	Candra	Candra *Ucca.*	Rāhu	Kuja	Budha *Kendra*	Guru	Śukra *Kendra*	Śani
Rāśi	0	0	9	7	1	4	0	1	7
Amśa (°)	1	3	2	2	25	3	26	14	15
Kalā (°)	49	46	45	50	32	27	18	2	42
Vikalā (°)	11	11	0	0	0	0	0	0	0

Table 7: Kṣepakas (epochal positions) of bodies

	Ravi	Candra	Candra *Ucca*	Rāhu	Kuja	Budha *Kendra*	Guru	Śukra *Kendra*	Śani
Rāśi	11	11	5	0	10	8	7	7	9
Amśa (°)	19	19	17	27	7	29	2	20	15
Kalā (′)	41	6	33	38	8	33	16	9	21

The use of *dhruvaka* and *kṣepaka* is explained below:

From the motion of a body obtained from the *ahargaṇa* subract the product of the *dhruvaka D* and the *cakra C* and to it add the *kṣepaka K*. This gives the mean position of the body for the mean sunrise (at *Laṇkā* and Ujjayinī).

i.e. Mean longitude = (*Ahargaṇa* derived motion) – $(C \times D) + K$

In the case of the Moon, the distance (in *yojanas*) between one's place and the central meridian (*rekhā*), chosen as the meridian passing through *Laṇkā* and Ujjayini is divided by 6 to get the correction in *kalās* (minutes of arc). This is added to or subtracted from the earlier obtained position of the Moon according as one's place is to the west or to the east of the central meridian (*rekhā*).

Explanation: The above correction described for the Moon is referred to as *deśāntara saṃskāra* due to the difference in the sunrise timings at the given place and at *Laṇkā*. The *deśāntara* correction for the Moon

$$= \frac{\text{The distance in } yojanas}{\text{Circumference of earth in } yojanas} \times \text{Moon's true daily motion}$$

Consider the mean daily motion = 790′35″ and *paridhi* (circumference) of the earth = 4967 yojanas. We get the *deśāntara* correction for the moon

$$= \frac{790'35''}{4967} \times \left(\text{distance in } yojanas\right) = \frac{1}{6.2827} \times \left(\text{distance in } yojanas\right)$$

$$\approx \frac{1}{6} \times (\text{distance in } yojanas)$$

where the distance in *yojanas* is the distance of the given place from the meridian passing through *Laṇkā* and Ujjayinī. The modern known values of the earth's equatorial and polar radii are respectively 6378.16 km and 6356.775 km. Considering the mean radius of the earth in miles and the circumference of the earth given as 4967 *yojanas* in the GL commentary, we get 1 *yojana* ≈ 5 miles. The circumference of the earth as 4967 *yojanas* is taken by Bhāskara II in his *Siddhānta Śiromaṇi*.

Expressions for the mean positions of the bodies

According to *GL* we have the following expressions for the mean positions in which *A* = *ahargaṇa*, *C* = *cakra*, *D* = *dhruvaka* and *K* = *kṣepaka*. In what follows the expression in the first pair of brackets is the *ahargaṇa*-derived mean longitude.

(1) Mean Sun, Budha (Mercury) and Śukra (Venus)

$$\lambda = \left(A - \frac{A}{70} - \frac{A}{150 \times 60}\right) - C \times D + K$$

Note: In Indian astronomy, the mean longitudes of Budha and Śukra are taken the same as that of the Sun.

(2) Moon: $\lambda = \left(A \times 14 - A \times \frac{14}{17}\right) - \frac{A}{140 \times 60} - C \times D + K$

(3) Moon's mandocca (candrocca): $\lambda = \left(\frac{A}{9} + \frac{A}{70 \times 60}\right) - C \times D + K$

(4) Rāhu (Moon's ascending node):

$$\lambda = \left[360^o - \left(\frac{A}{19} + \frac{A}{45 \times 60}\right)\right] - C \times D + K$$

(5) Kuja (Mars): $\lambda = \left(\frac{10 \times \mathrm{A}}{19} - \frac{10 \times \mathrm{A}}{73 \times 60}\right) - C \times D + K$

(6) Budha's *śīghrakendra* : SK $= \left(\frac{3A}{28} + 3A - \frac{A}{38 \times 60}\right) - C \times D + K$

(7) Guru (Jupiter): $\lambda = \left(\frac{A}{12} - \frac{A}{70 \times 60}\right) - C \times D + K$

(8) Śukra's *śīghrakendra* : SK $= \left(\frac{3A}{5} + \frac{3A}{181}\right) - C \times D + K$

(9) Śani (Saturn): $\lambda = \left(\frac{A}{30} + \frac{A}{156 \times 60}\right) - C \times D + K$

Derivations of expressions for the mean longitudes

The expressions for the mean longitudes of the Sun, the Moon, the Moon's apogee and node, given above, are now derived using the revolutions of these bodies and the numbers of civil days in a *Mahāyuga* of 432×10^4 years.

(i) **Mean longitude of the Sun**

The mean longitude of the Sun is given by

$$\lambda = \left(A - \frac{\mathrm{A}}{70} + \frac{\mathrm{A}}{150 \times 60}\right)^{\circ} - Cakra \times 1^{\circ}.8192 + 349^{\circ}.683$$

where A is the *ahargaṇa* according to GL.
Mean motion of the Sun in a *Cakra*:
Number of civil days in a *Mahāyuga* = 1577917828 according to *Sūrya Siddhānta.*
Number of revolutions of the Sun in a *Mahāyuga* = 4320000

$$\therefore \text{ Mean daily motion of the Sun} = \frac{4320000\, rev.}{1577917828} = 59'8''.17$$

Therefore, in a *Cakra* of 4016 days, we have

Sun's motion $= 4016 \times 59'8''.17 = 3958^\circ.1808 = 10.994947$ revolutions
Subtracting 11 ^{rev}, we get *dhruvaka* $= -1^\circ.81972 = -1^\circ 49' 11''$ (approx).
The *ahargaṇa*-derived mean motion of the Sun is given by $A \times 59'8''.17$
Multiplying and dividing by 70, we get

$$\frac{70 \times A \times 59'8''.17}{70} = \frac{4139'32'' \times A}{70} = \frac{68^\circ 59'32'' \times A}{70}$$

By adding and subtracting 28″ to the constant factor in the numerator, the mean motion of the Sun

$$= \frac{A \times \left(68^o 59'32'' + 28''\right)}{70} = A\left(\frac{69^o - 28''}{70}\right)$$

$$= \frac{A \times 69^\circ}{70} - \frac{A \times 28''}{70} = \frac{A(69^\circ + 1^\circ - 1^\circ)}{70} - \frac{A \times 28''}{70}$$

$$= \frac{A \times 70}{70} - \frac{A}{70} - \frac{A \times 28''}{70} = A - \frac{A}{70} - \frac{A}{\frac{70 \times 60 \times 60}{28}} \text{ degrees}$$

$$= A - \frac{A}{70} - \frac{A}{150 \times 60} \text{ in degrees as given in } GL.$$

(ii) Mean longitude of the Moon

The mean longitude of the Moon is given by

$$\lambda = A \times 14 - 14 \times \frac{A}{17} - \frac{A}{140 \times 60} - Cakra \times 3^\circ.76972 + 349.1 \text{ in degrees}$$

where A is the *ahargaṇa* according to *GL*.

Mean daily motion of the Moon $= 790'34''.9 = 13^\circ 10'34''.9$

Mean motion of the Moon in a *Cakra* $= 4016 \times 13^\circ.176361 = 52916^\circ.266$ $= 146^{rev}.98963$.

Subtracting 147 rev., we get

Dhruvaka $= -3^\circ.73400$ (taken as $-3^\circ.76972$ by *GL*).

The mean motion of the Moon $= A \times 13^\circ 10'34''.9$

Multiplying and dividing by 17 we get

$$\frac{A\times17\times13°10'34''.9}{17}=\frac{A\times223°59'53''}{17}$$

$$=\frac{A\times\left(223°59'53''+7''-7''\right)}{17}=\frac{A\times\left(224°-7''\right)}{17}=\frac{A\times224°}{17}-\frac{A\times7''}{17}$$

$$=\frac{A\times(224°+14°+14°)}{17}-\frac{A\times7''}{17}=\frac{A\times238°}{17}-\frac{A\times14°}{17}-\frac{A\times7''}{17}$$

$$=A\times14°-\frac{A\times14°}{17}-\frac{A\times7''}{17\times60\times60}\text{ degrees}$$

$$=A\times14°-\frac{A\times14°}{17}-\frac{A°}{\frac{1020\times60}{7}}\doteq A\times14°-\frac{A\times14°}{17}-\frac{A°}{145°.71429\times60}$$

Note: In the last term *GL* has made an approximation viz. $\frac{A°}{140\times60}$

(iii) Mean longitude of the Moon's apogee (*mandocca*)

The mean longitude of the Moon's apogee is given by

$$M=\frac{A}{9}+\frac{A}{70\times60}-272.75\times Cakra+167.55\text{ in degrees}$$

where A is the *ahargaṇa* according to *GL*.

Number of civil days in a *Mahāyuga* = 1577917828

Number of revolutions of the Moon's apogee = 488203

According to the *Sūrya Siddhānta*

Mean daily motion of the Moon's apogee $=\frac{488203\,revns.}{1577917828}=6'40''.98$

Mean motion of the Moon's apogee in a *Cakra*:

The mean motion of the Moon's apogee in 4016 days

$$=4016\times6'\,40''.98=447°.31547=1.242543\text{ revns.}$$

Subtracting 2 revolutions (i.e., 720°) we get

Dhruvaka = – 272°.68452 (taken as 272°.75 subtractive by *GL*)

The mean motion of the Moon's apogee is given by $A \times 6' 40'' .98$

By multiplying and dividing by 9 we have

$$\frac{A \times 6'40''.98 \times 9}{9} = \frac{A \times 1^\circ 0' 8''}{9} = \frac{A^\circ}{9} + \left[\frac{A \times 8}{9 \times 60}\right]' = \frac{A^\circ}{9} + \frac{A'}{67.5}$$

In the last term, *GL* takes 70 instead of 67.5 in the denominator. This approximation results in a maximum error of $2'$.

(iv) Mean longitude of Rāhu (Moon's node)

The mean longitude of Rāhu is given by

$$\text{Rāhu} = \left[360 - \left(\frac{A}{19} + \frac{A}{45 \times 60}\right) - (212.83 \times Cakra) + 27.63\right] \text{degrees}$$

where A is the *ahargaṇa* according to *GL*.

Mean daily motion of Rāhu $= -3'10.8$

Mean motion of Rāhu in a *Cakra*

$$= -4016 \times 3' 10''.8 = -212^\circ.848 = -212^\circ 50' \equiv \textit{Dhruvaka}$$

which is the same as given in *GL*.

The mean motion of the Moon's node (Rāhu) is given by $A \times 3' 10'' .8$ (subtractive) for the *ahargaṇa* A.

Multiplying and dividing by 19 we have

$$\frac{A \times 3'10''.8 \times 19}{19} = \frac{A \times 1^\circ 0' 25''}{19} = \frac{A^\circ}{19} + \frac{A \times 25''}{19} = \frac{A^\circ}{19} + \left[\frac{A \times 25}{19 \times 60}\right]' = \frac{A^\circ}{19} + \frac{A^\circ}{45.6 \times 60}$$

GL has taken 45 instead of 45.6 in the denominator of the last term.

(v) Mean longitude of Kuja (Mars):

$$\text{Mean Kuja} = \frac{10}{19} A^\circ - \frac{10A}{73} \min - (C \times D) \deg + K \deg$$

where A is the *ahargaṇa* according to *GL*.

Number of revolutions of Kuja in a *Mahāyuga* = 2296824

Number of civil days in a *Mahāyuga* = 1577917828

$\therefore$ Mean daily motion of Kuja = $\dfrac{2296824 \, revns.}{1577917828} = 31'26''.4$

Mean motion of Kuja in a *Cakra* = $4016 \times 31'\,26''.4$

$= 304°23'\,2'' = -55°36'58'' \equiv$ *Dhruvaka* (by substracting 360°).

Multiplying and dividing the *ahargaṇa* derived motion by 19 we have

$$\frac{A \times 31'26''.4 \times 19}{19} = \frac{A \times 597'21''}{19}$$

$$= \frac{A \times (597'21'' + 2'39'' - 2'39'')}{19} = \frac{A \times 600'}{19} - \frac{A \times 2'39''}{19}$$

$$= \frac{A \times 600'}{19} - \frac{A \times 2'39'' \times 10}{19 \times 10} = \frac{A \times 10°}{19} - \frac{A \times 10}{\frac{190}{2'39''}}$$

$$= \frac{A \times 10°}{19} - \frac{A \times 10'}{71.698113} \approx \frac{A \times 10°}{19} - \frac{A \times 10'}{72}$$

(vi) Mean *śighra* kendra of Budha

Mean *śighra kendra* (anomaly of conjunction) of Budha is given by

$$3A° + \frac{3A°}{28} - \frac{A}{38}\text{ min} - (C \times D)\text{deg.} + K\text{ deg.}$$

where A is the *ahargaṇa* according to *GL*.

Number of revolutions of Budha's *śighrocca* in a *Mahāyuga* = 17937020

Number of civil days in a *Mahāyuga* = 1577917828

$\therefore$ Mean daily motion of Budha's *śighrocca* = $\dfrac{17937020 \, revens.}{1577917828} = 4°5'\,32''.31$

śighra anomaly = Budha's *śighrocca* – Mean Budha

Daily motion of *śīghra* anomaly

= Daily motion of Budha *śīghrocca* – Daily motion of mean Ravi

$= 4°5'\,32''.31 - 59'\,8''.17 = 3°6'\,24''.14$

Multiplying and dividing the *ahargaṇa*-derived motion by 28 we have

$$\frac{A\times 3°6'24''.14\times 28}{28} = \frac{A\times 86°59'15''.92}{28} = \frac{A\times 87°}{28} - \frac{A\times 44''.08}{28}$$

$$= A^{o}\times\left(3+\frac{3}{28}\right) - \frac{A\times 44''.08}{28} = 3A^{o} + \frac{3A^{o}}{28} - \frac{A'}{\dfrac{28\times 60}{44.08}}$$

$$= 3A^{o} + \frac{3A^{o}}{28} - \frac{A'}{38.11}$$

The denominator of the last term has been taken as 38 in *GL*.

Note: In other texts *śīghrkendra* is obtained by calculating Budha's *śīghrocca* and mean longitude separately and then subtracting the latter from the former. But the *Grahalāghavam* directly gives the formula to calculate the *śīghrakendras* for the inferior planets Budha and Śukra.

The expressions for the mean positions of the remaining bodies can be derived similarly.

Illustrative examples and comparison

The mean positions of the bodies are computed for two dates – one of the early part of the 17th century and the other of our times – using the above expressions given by GL and the same are compared with the corresponding values obtained according to *Karaṇakutūhalam* (*KK*) and modern astronomy.

Example (1): *Śālivāhana śaka* 1534, *Vaiśākha Paurṇimā,* Monday. This corresponds to May 14, 1612 A.D. (Gregorian). For that date, at the mean sunrise at Ujjayinī we get the follwing:

Cakra = 8 and *Ahargaṇa* $A = 1521$.

The mean positions of the Sun, the Moon, the Moon's apogee (*mandocca*), Rāhu and the five planets (*tārāgrahas*) are computed using the algorithm of *GL*.

These results are compared with those obtained according to *KK* of Bhāskara II (epoch : Feb. 24 1183 AD) and the modern astronomical formulae in Table 8.

Table 8: Nirayaṇa Mean Positions for May 14, 1612 AD (G)

Bodies	*Grahalāghavam*			*Karaṇakutūhalam*			Modern		
	D	M	S	D	M	S	D	M	S
Ravi	34	13	41	34	24	15	33	52	28
Candra	200	10	19	200	43	20	199	32	19
Candra's *mandocca*	314	11	16	312	09	54	312	36	23
Rāhu	44	21	06	43	53	22	44	06	09
Kuja	299	55	13	297	43	06	299	01	44
Budha's *śīghrakendra*	47	14	50	47	01	30	*39	29	08
Guru	128	15	16	130	25	42	129	59	52
Śukra's *śīghrakendra*	95	41	36	99	31	21	*94	06	53
Śani	330	36	45	325	51	24	332	34	27

Note: (1) The *Ayanāṃśa* for the year 1612 AD according *GL* and *KK* is 18°10′. The mean *Ayanāṃśa* adopted for that date according to N.C. Lahiri and the Calendar Reform Committee Report works out to be 18°26′51″.

(2) While *GL* gives *śīghrakendras* of Budha and Śukra directly, KK and other siddhantic texts provide the *śīghroccas* of the inferior planets and their mean longitudes (taken same as that of the Sun) separately.

(3) In the last column, based on modern computations, of Table 8, the *śīghrakendras* of Budha and Śukra are obtained as the difference between the mean position of each of them and that of the Sun.

By comparing the mean positions of the bodies according to *GL*, *KK* and the modern computations, we observe that they are reasonably similar. The small differences are ignorable due to the fact that the two traditional texts differ by four centuries between them and the later one (*GL*) is five centuries behind our times. However, in the case of Budha's *śīghrakendra*, in the modern computation, the mean elongation of Mercury (Budha) from the mean Sun is given. This differs from the *GL* value by about 7°46′. Similarly, in the case of the mean anomaly of Venus, the difference is about 1°35′.

Example 2 : August 11, 1998 at the mean sunrise at Ujjayinī. The mean positions of the bodies according to *GL*, *KK* and modern computations are provided in Table 9.

Table 9: Nirayaṇa Mean Positions for Aug. 11, 1998

Bodies	*Grahalāghavam*			*Karaṇakutūhalam*			Modern		
	D	M	S	D	M	S	D	M	S
Ravi	115	09	59	115	28	00	114	42	5
Candra	334	31	31	335	20	50	333	48	36
Candra's *mandocca*	184	42	18	182	07	07	182	07	25
Rāhu	128	02	49	126	47	55	127	23	10
Kuja	64	33	30	62	09	21	64	23	04
Budha's *śīghrakendra*	197	07	48	197	39	00	192	04	37
Guru	330	17	58	333	03	12	325	25	31
Śukra's *śīghrakendra*	310	12	47	316	36	03	308	01	31
Śani	08	14	04	03	19	02	6	23	21

On comparison of the corresponding values of the planetary positions according to GL, KK and the modern computations in Table 9, we notice differences to the extent of even 4 to 5 degrees for the present times.

Necessity of updating parameters

In the absence of instruments like telescope, sophisticated mathematical techniques and, of course, Newton's gravitational theory and Kepler's laws of planetary motion, such differences are to be expected. However, the later Indian astronomers, especially from Kerala, introduced suitable *bījas* (corrections) to the various parameters for the computations of planetary positions and eclipses. Even much before, for example, Lalla (8th Cent.) introduced the *bīja* corrections to Brahmagupta's *Khaṇḍakhādyaka*.

In fact, Indian astronomers have repeatedly pointed out that the *śāstra* (science) does become *ślatha* (inadequate, weak) over a period of time. It is left to great savants in the field to devise corrective measures to establish *dṛggaṇitaikya*, concordance between theory and observation. In fact, the famous Kerala astronomer Parameśvara (1360-1455), in his commentary on the *Mahābhāskarīyam* (*MBh*), states:

kālāntaretu saṃskāraścintyatām gaṇakottamaiḥ

"In course of time, the (necessary) corrections must be decided by expert-mathematicians." - *Siddhāntadīpikā* Com., śl. 93 on *MBh*, v. 77

Bhāskara II (b. 1114), in his polite style of commenting on the mistakes and errors of his predecessors, remarks : "It is necessary to speak out the truth correctly before those who have implicit faith in tradition. It is impossible to believe in whatever is said earlier (by predecessors) unless every erroneous statement is criticized and condemned" (*Siddhānta Śiromaṇi*, Gola, 60):

kartavye sphuṭavāsanāprukathane pūrvokti viśvāsīnam
tattaddūṣaṇamantareṇa nitarām nāsti pratītiryataḥ //

There is a detailed statement in the *Bṛhat-tithicintāmaṇi* of Gaṇeśa Daivajña describing how the *śāstra* which is *tathya* (accurate) at one period of time becomes inaccurate and needs *samsthāpana* (re-establishment) in later period.

The present authors humbly submit that they have worked out important corrected parameters such as the (i) the number of civil days in a *Mahāyuga* or *Kalpa*, (ii) the numbers of revolutions of the heavenly bodies in that period, (iii) *paridhis* (peripheries) of *manda* and *śīghra* epicycles and (iv) the mean epochal positions. These *bījas* (corrections) are introduced within the framework of the traditional siddhāntic algorithms to yield results comparable to the modern ones. These suggested updating of parameters will be presented by the present authors in due course.

Conclusion

In the present paper we have presented the procedures according to Gaṇeśa Daivajña's *Grahalāghavam*, of obtaining the *ahargaṇa* for a given day and then tocompute the mean positions of the heavenly bodies. We have also given the derivations for the GL expressions for the same from the *bhagaṇas* (revolutions) of the bodies in a Mahāyuga (432×10^4 years). For determining the *ahargaṇa* from a Christian date (Julian or Gregorian) tables are provided for the benefit of researchers and students in the field. Computation of lunar eclipse according to *GL* was presented in our paper in *IJHS* [12] showing therein the efficacy of Gaṇeśa's procedure.

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