INTRODUCTION

Gaṇeśa Daivajña and *Grahalāghavam*

In this work, we have provided an English Exposition - with explanatory notes, derivations and worked examples - of the popular Sanskrit *karaṇa* text, *Grahalāghavam* . The author of this handbook of Indian astronomy is Gaṇeśa Daivajña who belonged to the early 16th century.

There is a general belief that after Bhāskara II there was a decline in the development of mathematics and astronomy in India. While there may be some truth in this belief, thanks to historical reasons, it is also true that the post - Bhāskara period saw an intensely creative activity in mathematics in the regions south of the Vindhyas. In fact Kerala became the cradle of tremendous and rich mathematical output - often anticipating developments in the European mathematics. In other parts of India too there shone great luminaries, like Gaṇeśa Daivajña, upholding the great tradition of Indian astronomy and mathematics.

In fact, the astronomical works of no other astronomer are in use among the makers of traditional *pañcāṅgas* (astronomical almanacs) in most parts of India today as those of the great and popular astronomer, Gaṇeśa Daivajña.

1. Date and place of Gaṇeśa and Keśava Daivajña

Gaṇeśa Daivajña's father was the famous astronomer Keśava Daivajña and his mother's name was Lakṣmī. He was born in 1507 AD (*śaka* 1429) at a place called Nandigrāma on the western sea-coast.

S.B. Dikshit points out that Nandigrāma is at present a village called Nandagaon in the Janjeera State in the Konkan region. It lies about 40 miles to the south of Mumbai (Bombay). The family belonged to the *gotra* (patrilineal ancestry) of *Kuśika*. Gaṇeśa's grand-father (Keśava's father) was Kamalākara, also an eminent astronomer. Gaṇeśa's teacher in astronomy was his father himself while Keśava's *guru* was Vaijanātha.

Gaṇeśa's father, Keśava, composed several works and commentaries on astronomy and astrology among which his astronomical work, *Grahakautuka* was highly respected. In fact, Keśava is regarded as one of the best observational astronomers of ancient and medieval India. In his *Mitākṣarā* auto-commentary on the *Graha kautuka,* Keśava observes:

"The figures as calculated from the *Brahmā, Āryabhaṭa* and *Saura siddhāntas* exhibit a vast difference in the positions of Mercury (Budha) and Venus (Śukra). Saturn (Śani) has shown an excess of five degrees when actually observed in the sky at the time of conjunction with stars and planets and while setting and rising... Similarly, a difference is recorded in epochal positions and in the annual rates of motion....".

"Hence the future calculators should calculate planetary positions by adopting the figures of revolutions increased or decreased in conformity with the actual observed phenomena of conjunctions, rising and setting of stars and planets in their own times. The writer (Keśava) has accordingly found, out the mean position of the Moon, instead of its maximum equation of centre, by reversed steps, from the observation of the lunar eclipse at the ending moment of the fullmoon, since the equation of centre is neither positive nor negative. The Moon's apogee was finally fixed by reversing the steps of calculation, after observing the eclipse at the moment of the fullmoon in the celestial globe of observation since the maximum correction is neither additive nor subtractive. The moon's position was found to be 5 minutes (of arc) less as compared to that calculated from the *Sūryasiddhānta.* The apogee agreed with that of the *Brahmapakṣa* Thus, the writer has calculated the positions of planets by a short method after observing their actual positions at the present time".

2. Works of Gaṇeśa Daivajña

Gaṇeśa composed several important works on astronomy, astrology etc among which his astronomical treatise *Grahalāghavam* is the most famous. In fact, the remarkable popularity of the *Graha* *lāghavam* surpassed that of his father's *Grahakautuka* which was truly an important text in its own right.

Gaṇeśa's other works are: *Laghu*- and *Bṛhat-Tithi Cintāmaṇi,* a commentary on Bhāskara's *Siddhānta Śiromaṇi,* a commentary on Bhāskara's *Līlāvaṭī* (called *Buddhivilāsinī). Vivāha vṛndāvana ṭīkā, Muhūrta tattvatīka, Śrāddha nirṇaya* etc. Gaṇeśa himself mentions another work of his, *Parvanirṇaya.*

Among Gaṇeśa's works, the *Grahalāghavam* appears to have been composed first, believed to be when he was just 13 years old. The epoch of the *Grahalāghavam* is March 19, 1520 AD, Monday.

The work, *Laghu tithi cintāmaṇi* was composed in *śaka* 1447 (1525 AD) and the *Buddhivilāsinī* commentary on Bhāskara's *Līlāvatī* in the year 1545 AD. Another work, *Pāta sāraṇī,* was composed some time after 1538 AD.

Gaṇeśa's *Vṛndāvanaṭīkā* gives the date of its composition in a very interesting fashion: "Take 12 *as* the number for the *samvatsara (hāyan).* Add one *(lava)* for *ayana* to it. Add 6 to the sum of these two numbers (i.e, to 12 + 1) so *as* to obtain (19) *as* the number of the *yoga.* Add 4 to the sum which would give 23 *as* the number of the *nakṣatra* and one *(lava)* for the *pakṣa.* If *a pakṣa* is added to one more *pakṣa* (i.e, 1 + 2), it would give 3 *as* the number denoting the week day. Take 1 *as* the *tithi* number and 11 as the month number. Multiply the sum of all these numbers by 21 and increase the product by 9 (*nanda*).The result is the *śaka* year number". This gives us the following:

Samvat	*Ayana*	*Yoga*	*Nakṣ*	*Pakṣa*	*Vāra*	*Tithi*	*Māsa*	*Śaka*
(12	+ 1	+ 19	+ 23	+ 1	+ 3	+ 1	+ 11) x 21 + 9	= 1500

i.e,(71 x 21) + 9 = 1491 + 9 = 1500.

This means that the text, *Vivāha vṛndāvana tikā* was composed on *Māgha* (11th lunar month) *Śukla* (1st *pakṣa) Pratipat* (1st *tithi)* of *śaka* 1500 (i.e., 1578 AD), *Bahudhānya* (12th) *samvatsara,* during *Uttarāyaṇa* (1st *ayana), Dhaniṣṭhā* (23rd) *nakṣatra* and *parigha* (19th) *yoga.*

The date of the said text suggests that Gaṇeśa was *about* 71 years old when he composed it. However, Dikshit refers to a manuscript copy of the *Vṛndāvanaṭīkā* which gives the year of its composition as *śaka* 1476 (i.e. 1554 AD), *Ananda samvatsara,* which is 24 years earlier than the above-cited date (1578 AD)

In the *Grahalāghavam,* the positions of planets have been given for the moment of sunrise of Monday, the newmoon day of *Phālguṇa* of *śaka* 1441 corresponding to March 19, 1520 AD (Julian).

3. Special features of *Grahalāghavam*

Gaṇeśa has simplified the method of computations of the positions of planets which is otherwise laborious by the traditional method.

To avoid a huge number for the *ahargaṇa* (number of civil days since the epoch), Gaṇeśa has adopted an *ahargaṇa* cycle of 4016 days, approximately constituting 11 solar years. Therefore, the modified *ahargaṇa,* being the remainder exceeding a completed number of cycles (of 4016 days each), never exceeds 4016 days and is hence handy.

From the point of view of a *pañcāṇga*-maker or a beginner who is ignorant of trigonometry, *Grahalāghavam* is easy to use since Gaṇeśa has completely dispensed with the trigonometric functions.

In fact, the dropping of trigonometric ratios has by no means seriously affected the accuracy of results. Even the other *karaṇa* texts, using the sine tables, generally give the values of sines of angles only in intervals of 15^0 or $3\frac{3}{4}^0$.

Therefore, the sine-values for intermediate angles, by linear interpolation, are only approximate.

4. Popularity of *Grahalāghavam* and its commentators

As mentioned earlier, the *Grahalāghavam* of Gaṇeśa Daivajña is the most popular astronomical text, among the ancient and medieval texts, currently used in most parts of India. Further, among the *karaṇa* works (hand-books) on Indian astronomy, the *Grahalāghavam* is considered as the most comprehensive, exhaustive and easy to use text.

The *Grahalāghavam* carries with it very useful and authoritative commentaries by reputed astronomers like Gaṅgādhara (1586 A.D.), Mallāri (1602 A.D.) and Viśvanātha (around 1612 A.D.)

Gaṅgādhara's commentary on the *Grahalāghavam* is called *Manoramā*. His father was Nārāyaṇa who authored *Muhūrta mārtāṇḍa*. Gaṅgādhara was a Vājasaneyī Brāhmin belonging to the *Kauśikagotra,* the same as that of Gaṇeśa Daivajña. Gaṅgādhara lived in a village called Tapar, lying to the north of *Ghṛṣṇeśvara* (Lord Śiva) temple which is to the north of Devagiri (Daulatabad).

Viśvanātha was brother of the highly accomplished astronomer Viṣṇu who composed a *karaṇa* with the epochal year 1608 AD This *karaṇa* text is based on the *Sūrya siddhānta.* Viṣṇu also wrote a commentary on Gaṇeśa Daivajña's *Bṛhat tithi cintām aṇi* in which he explains the theory also. Viśvanātha has written an *udaharaṇa* on his brother's *karaṇa.*

The two popular commentators of *Grahalāghavam* are Viśvanātha and Mallāri. They were born in illustrious Mahārāstrian Brāhmin families of astronomers. Mallāri's father was Divākara, a pupil of Gaṇeśa. Kamalākara, author of the *Siddhānta tattva viveka* and Raṅgaṇātha who wrote a commentary on the *Sūryasiddhānta* were descendants of Viśvanātha and Mallāri. Nṛsiṃha, nephew and pupil of Gaṇeśa, wrote his commentary *Harṣakaumudī* in 1548. The other commentators are Gaṅgādhara (1586), Nārāyaṇa (Kaśī, before 1635) and Kamalākara (before 1662).

Viśvanātha has given a large number of examples in his commentary to illustrate the methods of the *Grahalāghavam*. The *Graha lāghavam* is extensively used by the *pañcāṅga*-makers particularly in Maharashtra, Gujarat, northern parts of Karnataka, the Hyderabad Deccan region of Andhra Pradesh and by the Deccanis of Varanasi, Gwalior and Indore. It is pointed out that even the government almanacs published at Indore and Gwalior used the *Grahalāghavam* and the *Tithicintāmaṇi* of Gaṇeśa Daivajña.

5. Some special results of Gaṇeśa Daivajña

(i) Phase of the Moon

From the newmoon to the fullmoon, the *phase* of the Moon increases, given by

$$\text{phase} = (1 + \cos d)/2$$

where d is the elongation $S\hat{M}E$ of the earth from the Sun as seen from the Moon. In fact, we have d = 180° - E, so that

$$phase = (1 - \cos E) / 2$$

where E is the elongation of the Moon from the Sun as seen from the earth. On a newmoon day, E = 0° so that the phase of the Moon is zero. On a fullmoon day, E = 180° so that the phase of the Moon is one.

In Indian astronomy, the phase of the Moon is measured by the width of the illuminated part of the Moon which is called *sita* (or *śukla)*. The width of the unilluminated part, equal to the difference between the Moon's diameter and the *sita* is called *asita*. In fact,

$$sita = \frac{(M - S) \times (Moon's\ \ ang.\ diameter)}{180}$$

where M and S denote the celestial longitudes of the Moon and the Sun respectively in degrees.

Gaṇeśa Daivajña gives the formula

$$sita = \left(1 - \frac{1}{5}\right)T$$

aṅgulas where T is the number of *tithis* elapsed in the bright fortnight *(śukla pakṣa)* and the Moon's diameter is taken as 12 *aṅgulas*. Of course, this formula is approximate and a similar formula is given by Brahmagupta.

(ii) Rationale for four sides to form trapezium

Bhāskara II investigates the possibility of four sides forming a trapezium. He gives the condition: "in a trapezium the sum of the other flank side and the face is smaller than the sum of the smaller flank side and the base". *(Līlāvatī*, 185)

Gaṇeśa Daivajña has provided a rationale for this statement of Bhāskara II.

(iii) Proof of the *Śulva theorem* (Pythagoras' theorem)

Gaṇeśa Daivajña in his *Buddhivilāsinī* commentary on the *Līlāvatī* provides a fully geometrical proof for the geometrico-algebraic rationale provided by Bhāskara II for the so-called Pythagoras' theorem on a right-angled triangle.

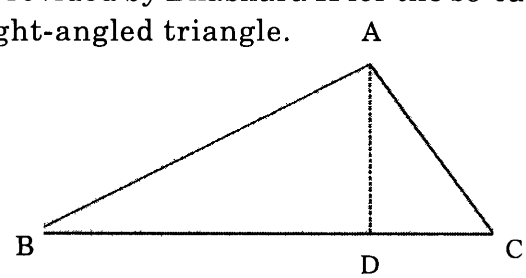

Fig.1: Proof of the Śulva theorem

Let ABC be a triangle right-angled at A (Fig 1). Let AD be the perpendicular to BC. Then the triangles ADB, CDA and CAB (the vertices are in the order of correspondence of equal angles) are *similar*. Therefore, from triangles ADB and CAB, we have

$$\frac{AB}{BC} = \frac{BD}{AB} \text{ or } BD = \frac{AB^2}{BC} \qquad .....(1)$$

Similarly, from triangles *CDA* and *CAB*,

$$DC = \frac{AC^2}{BC} \qquad .....(2)$$

From (1) and (2), we get

$$BD + DC = BC = \frac{AB^2 + AC^2}{BC}$$

or

$$BC^2 = AB^2 + AC^2$$

(iv) Construction of rational quadrilaterals

Gaṇeśa Daivajña, commenting on the method of Brahmagupta to obtain a quadrilateral with rational sides, says that four rational right-angled traingles *(jātyas)* are to be formed out of two basic rational right triangles as follows:

If $m^2 - n^2$, $2mn$, $m^2 + n^2$ and $p^2 - q^2$, $2pq$, $p^2 + q^2$

are the sides of two rational right triangles, then according to Gaṇeśa, the following are the traingles out of which the quadrilaterals are built up :

(i) $(m^2 - n^2)(p^2 - q^2)$, $2mn\,(p^2 - q^2)$, $(p^2 - q^2)(m^2 + n^2)$

(ii) $(m^2 - n^2)(2pq)$, $4mnpq$, $2pq\,(m^2 + n^2)$

(iii) $(p^2 - q^2)(m^2 - n^2)$, $2pq\,(m^2 - n^2)$, $(p^2 + q^2)(m^2 - n^2)$

(iv) $(p^2 - q^2)\,2mn$, $4pqmn$, $(p^2 + q^2)(2mn)$

However, T.A. Sarasvati Amma wonders whether this is the procedure intended by Brahmagupta and Bhāskara II.

(v) Evaluation of π

Āryabhaṭa (b. 476 AD) has given the value of π as

$$\pi = \frac{62832}{20000} = 3.1416$$

and he points out that this value is approximate *(asanna).*

Bhāskara II also gives the value

$$\pi = \frac{3927}{1250} = 3.1416 \text{ and } \pi = \frac{355}{113} = 3.1415929$$

the former value being the same as the one given by Āryabhaṭa (obtained by removing the common factor 16 in the ratio).

The mode of arriving at this value of π is by considering the perimeter of a regular polygon inscribed in a circle. The ratio of the perimeter of the polygon to the diameter of the circle approximates the constant π. Of course, the more the sides of the polygon, the better is the approximation.

Gaṇeśa Daivajña suggests that the number of sides of the inscribed polygon, starting from 12, is successively doubled as 24, 48 until it is 384. The diameter of the circle is taken as 100 units. Then the ratio of the perimeter of the 384-sided polygon inscribed in a circle of diameter 100 would give the approximate value of π as 3927/1250.

Gaṇeśa's commentary, *Buddhivilāsinī on* the *Līlāvatī,* is an extremely useful text to understand the rationales for the formulae and methods used by Bhāskara II and his predecessors.

6. *Grahalāghavam (GL)*

In our present work, for each *śloka* we have given a working English translation along with mathematical explanation, derivations, worked examples, tables and diagrams wherever necessary. This work is based on the printed edition of *GL,* with commentaries of Mallāri and Viśvanātha, edited by Pt. Kedaranath Joshi and published by Motilal Banarsidass, Varanasi, 1981.

The text of *Grahalāghavam* consists of 187 *ślokas* distributed into 14 chapters. However, the commentaries of Mallāri and Viśvanātha contain a fifteenth chapter of 15 *ślokas* called *Pañcāṅga candragrahaṇa.* The text seems to have undergone quite a few changes in the hands of descendants and commentators. However, the popularity of the *Grahalāghavam* remains unchallenged. The mean positions of the Sun, the Moon and the five *tārāgrahas* viz. Kuja, Budha, Guru, Śukra and Śani are computed in Chapter 1. The epoch adopted by Gaṇeśa Daivajña is March 19, 1520 AD (J), Monday, at the mean sunrise at Ujjayinī. We have provided derivations for the formulae of the text using the basic data of the numbers of revolutions *(bhagaṇas),* of the heavenly bodies in the course of a *Mahāyuga* of 432 x 10^4 years.

The true positions of the Sun and the Moon using their equations of centre *(Mandaphalam)* are worked out in the *Śpaṣṭādhikāra* (Chapter 2). The approximation for the *Mandaphalam* used by Gaṇeśa avoiding the trigonometric ratios is adequately explained. In fact, this justified device has greatly simplified computations.

In Chapter 3 computations of the true positions of the five star-planets are worked out by applying the two important equations namely *Mandaphalam* and *Śīghraphalam.* While the first equation corresponds to the "equation of centre" due to the eccentricities of orbits, the second one corresponds to converting the heliocentric positions to the geocentric. We have demonstrated the procedures of the text with worked examples - one relevant to the time of the commentator Viśvanātha (early 16th Cent). The second example is for a modern date.

In the *Tripraśnādhikāra* (Chapter 4) the problems of *dik, deśa* and *kāla* i.e., direction, place and time are discussed. While the mathematical expressions involve the trigonometric ratios *jyā* and *koṭijyā,* the greatness of the text lies in providing justified approximations by avoiding these trigonometric ratios. The prupose of this exercise is mainly to simplify the computations for the benefit of the *Pañcāṅga* makers and lay students.

This intended purpose is amply served. The important parameters of *krānti* (declination), *akṣāṃśa* (latitude of a place), *lagna* (ascendant), *palabhā* (shadow of the gnomon, *śaṅku)* etc. are explained with examples.

The Lunar and Solar eclipses are studied in Chapters 5 and 6. In these chapters computations of the circumstances of the two types of eclipses are explained. The determinations of the *sparśa* (beginning), *madhya* (middle) and *mokṣa* (end) of an ordinary eclipse and of the *sammīlanam* and the *unmīlanam* of a total eclipse *(khagrāsa)* are demonstrated with traditional and modern examples. In the case of solar eclipse, the effects of the parallax on the longitude *(lambanam)* and on the latitude *(nati)* of the Moon are explained.

The speciality of *GL* lies in computing the posibility and circumstances of eclipses without using *ahargaṇa,* through *māsagaṇa* (the number of completed lunar months). This procedure is explained in detail in Chapters 7 and 8. We have elucidated the procedures with examples.

In Chapter 9 *(Udayāstādhikāra),* an exhaustive study of the heliacal rising and setting of the planets and their retrograde motion *(vakragati)* etc., using the *śīghrakendra* (the angular distance of a planet from the Sun) is presented.

The shadow-related problems regarding planets and stars and using them to find out directions and elapsed time are explained in Chapters 10 and 11. Finding the *lagna* and the time by observing a star on the meridian is also discussed.

Chapter 12 *(Śṛṅgonnatyadhikāra)* deals with the rising of the Moon's "horns". Finding the *'valanam'* (deflection) is also explained in this chapter.

In Chapter 13, conjunctions of planets *(grahayuti)* is presented for which their angular diameters have to be determined. The method of determining the days elapsed *(gata)* or to be completed *(gamya)* for the occurrence of a conjunction is explained. Similarly, the conjunction of a planet with the Moon (an occultation) can be predicted by determining the *śara* (latitude), which is corrected for its *nati*.

Two important *yogas* between the Sun and the Moon viz., *Vyātīpāta* and called *Vaidhṛtipātas* are discussed in Chapter 14. These occur when the declinations *(krānti)* of the Sun and the Moon have the same magnitude and lie either on the same side or on opposite sides of the celestial equator.

In Chapter 15, obtaining the true positions, diameters etc. of the Sun and the Moon from the count of elapsed lunar months *(māsagaṇa)* is discussed in detail.

In concluding Chapter 16, the author Gaṇeśa explains how to obtain planetary positions etc for the times before his epoch.

At the end of this text, Appendices and a detailed Bibliography are included. The Appendices include tables of *mandaphala* (equation of centre) and the *śīghraphala* of the planets, the list of 27 *yogas,* Bhāskara I's. approximate formula for sine and the *udayamānas* of *rāśis.*

In the course of the text, mathematical derivations, comparison with modern results and illustrative diagrams are provided.

CHAPTER 1

MADHYAMĀDHIKĀRA

(Mean Positions of Planets)

Śloka **1** : The invocatory verse has two interpretations :

(a) *Keśava*'s (i.e, *Bāla Mukunda*'s) words viz, the *Vedas* (i.e, *Śrutis*) are supreme. By observing the practices enjoined in the *Vedas* the mind gets purified leading to the *Supreme Knowledge* and which (*Vedas*) have brief, difficult and immutable words implying many meanings which were made simple and clear through the later explanatory works (i.e, *Smṛtis* and commentaries) by the incarnations of Lord *Viṣṇu*. Victory to such words of the Lord !

(b) The author's father's i.e., Keśava's words are glorious. By studying his works the mind becomes clear and absorbs the knowledge of stars and planets. Though the words are brief these are clear in their meanings and further flawless and which are elucidated further by the commentaries of his disciples.

Remarks : Gaṇeśa Daivajña's preceptor and father was Keśava Daivajña who composed a handbook of astronomy called *Grahakautukam*. This text was very popular before his son's work *Grahalāghavam* outshone it.

Śloka **2** : O mathematician ! Meditate on that form of *Viśṇu (Rāma's)* which rendered *Śiva's* bow into three pieces and threw it to the ground, who is decorated with the garland of *sattva* quality, who gives life to every living being, who grants the wishes, who has taken the human incarnation, who runs the universe and is very handsome.

(b) O mathematician ! You read this handbook (*karaṇa grantha*) which is devoid of sines and cosines (the trigonometric functions), which is decorated with constant multipliers and divisors (i.e., parameters), which contains operations with angles (like anomaly etc.), which provides proper knowledge of the equation of centre etc., which also adopts the knowledge of shadow of the gnomon (*śaṅku*) and which is mind-capturing and error-free.

***Śloka* 3** : Even though very great scholars composed (astronomical) handbooks, their (mathematical) results are not achieved if sines (and arcs) are left out. Therefore, I (Gaṇeśa) set forth to compose a (planetary) handbook of procedures which are very simple and clear.

Now, the author describes the procedure for finding the *Ahargaṇa* (the number of civil days elapsed since the chosen epoch) :

***Ślokas* 4 and 5** : The procedure described in these two *ślokas* is explained below, for convenience, in a simple algorithmic style.

The epoch chosen by Gaṇeśa Daivajña is *Śālivāhana Śaka Varṣa* (year) 1442 *Caitra śukla pratipat*, corresponding to March 19, 1520 AD (Julian), Monday. The *Ahargaṇa* for a given lunar date is determined as follows:

(i) Subtract 1442 from the *Śālivāhana śaka* year (elapsed) of the given date to get the years elapsed (*gatābdi*).

(ii) Divide the remainder by 11. The quotient is called *cakra* (cycles) $\equiv$ C.

(iii) Multiply the remainder, obtained in step (ii), by 12 and to the product add the number of lunar months elapsed, counting *Caitra* as 1 etc. The sum thus obtained is called 'mean lunar months' (*madhyama māsagaṇa*) denoted by M.

(iv) number of *adhikamāsas* is given by the quotient of $\dfrac{M+2C+10}{33}$

(v) True lunar months (*spaṣṭa māsagaṇa*) = Mean lunar months + *adhikamāsas* = M + Quotient of $\dfrac{M+2C+10}{33} \equiv TM$

(vi) Mean *ahargaṇa* (*Madhyama ahargaṇa*)

$$MAH = (TM \times 30) + TI + \frac{C}{6}$$

where *TI* is the number of *tithis elapsed* in the given lunar month.

(vii) *Kṣaya dinas* = Quotient of $\left[\frac{1}{64}\ \textit{Madhyama Ahargaṇa}\right] \equiv KD$

(viii) True *ahargaṇa* (*sāvana dinas*) i.e, the number of civil days,

TAH = Mean *ahargaṇa* – *kṣaya dinas*

$= MAH - KD$

$= MAH -$ Quotient of $\frac{1}{64}\ (MAH)$

(ix) However, since the average values of parameters are considered in the above computations, 1 day may have to be either *added to* or *subtracted from* the result of (viii) to get the actual true *ahargaṇa.*

This is done by verifying the weekday as follows :

(a) Multiply the *cakras* by 5 i.e., find $5C$. Add *sāvana ahargaṇa* to this i.e., find $(5C + TAH)$.

(b) Divide the result of (a) by 7 and find the remainder. Let R = Remainder of $\left\{\frac{5C + TAH}{7}\right\}$. If $R = 0$, then it is Monday; $R = 1$ then it is Tuesday; and so on.

(c) If the calculated weekday is a day next to the actual weekday, then *subtract* 1 from *TAH* and if it is one day less than the actual weekday, then *add* 1 to *TAH.*

[See Note (1) and (2) appearing later].

Example 1 :

Śā.Śaka 1534, *Vaiśākha Pūrṇimā*, Monday ≡ May 16, 1612 A.D. (Gregorian)

(i) Subtract 1442 from 1534 :

Gatābdi = 1534 − 1442 = 92 years (from the epoch)

(ii) Divide the remainder in (i) by 11 :

the quotient, *cakras* = 8 ≡ C and the remainder = 4

(iii) Multiplying the remainder from (ii) i.e, 4 by 12 and adding the number of lunar months elapsed in the given year we get

$(4 \times 12) + 1 = 49 \equiv M$ is the *madhyama māsa gaṇa*

(iv) No. of *Adhikamāsas* = $\frac{M+2C+10}{33} = \frac{49+2(8)+10}{33} = \frac{75}{33}$;

Quotient = 2.

(v) M + No. of *adhikamāsas* = 49 + 2 = 51 = TM is the *spaṣṭa māsagaṇa*

(vi) (a) Mean *ahargaṇa* (*Madhyama ahargaṇa*)

= (TM × 30) + (No. of *tithis* elapsed in the given lunar month)

= (51 × 30) + 14 = 1544

(b) Add INT $\frac{1}{6}[C]$ i.e., INT (8/6) = 1 to the result of (vi) (a)

∴ 1544 + 1 = 1545 ≡ MAH

(**Note** : INT stands for the integer value).

(vii) *Kṣayadinas* $= \text{INT}\left(\frac{MAH}{64}\right) = \text{INT}\left(\frac{1545}{64}\right) = 24 \equiv KD$

(viii) *Sāvana ahargaṇa* (i.e., No. of civil days in the running *cakra*)

$$= MAH - KD = 1545 - 24 = 1521 \equiv TAH$$

(ix) Week day verification :

$$5C + TAH = 5(8) + 1521 = 1561$$

$$\therefore \quad R = \text{Remainder of } \left\{\frac{1561}{7}\right\} = 0$$

That is, the weekday comes out as Monday.

Since the weekday obtained from calculation is the same as the actual weekday (known), nothing needs to be added to or subracted from *TAH*. Therefore,

True *ahargaṇa* = 1521 No. of *cakras* = 8

Note 1 : Sometimès when (*Śaka* year 1442) is divided by 11, to get *cakras* the remainder could be 0. In that case even 2 may have to be added to or subtracted from the obtained *sāvana dinas* to get the true *ahargana* for the weekday. See the following example.

Example 2 : *Śaka* 1574 *Caitra Śukla Pratipat Ravivāra* (i.e, April 7, 1652 A.D., Sunday)

(i) *Gatābdi* = 1574 – 1442 = 132, years since the epoch.

(ii) $C = \text{INT}\left(\frac{132}{11}\right) = 12$ Remainder of $\left(\frac{132}{11}\right)$ is 0.

(iii) $(0 \times 12) + 0 = 0 \equiv M.$

(iv) Number of *adhikamāsas* $= \text{INT}\left[\frac{M+2C+10}{33}\right] = \text{INT}\left[\frac{0+24+10}{33}\right]$

i.e, *adhikamāsas* $= \text{INT}\left[\frac{34}{33}\right] = 1$

(v) $TM = M$ + No. of *adhikamāsas* = 0 + 1 = 1

(vi) *Mean ahargaṇa* : $MAH = (1 \times 30) + 0 + \text{INT}\,[\frac{1}{6}\,(12)] = 30 + 2$ = 32.

(vii) *Kṣaya dinas,* $KD = \text{INT}\left(\frac{MAH}{64}\right) = \text{INT}\left\{\frac{32}{64}\right\} = 0$

(viii) *Sāvana ahargaṇa* = $MAH - KD$ = 32 – 0 = 32

∴ *Cakras* = 12, *Ahargaṇa* = 32 (in the running *cakra*)

(ix) Weekday verification : R = Remainder of $\left(\frac{5C+TAH}{7}\right)$

i.e., remainder of $\left(\frac{92}{7}\right)$ or $R = 1$

i.e., Tuesday. But the actual weedkay is Sunday.

Therefore subtracting 2 from the *sāvana ahargaṇa* we get

Actual *ahargaṇa* = 32 – 2 = 30.

Thus, *cakras* = 12 and *ahargaṇa* = 30.

(x) Christian date :

No. of civil days since epoch = (*cakras* × 4016) + *ahargaṇa*

= 12 (4016) + 30 = 48,222

Kali ahargaṇa of the GL epoch = 16,87,850 (fixed)

Kali Ahargaṇa of the given date : 17,36,072

From Tables 1.1 to 1.3, we have

1600 A.D. (G)	:	17,16,982 (under *kali Ahargaṇa*)
Year 52	:	18,993
April 7	:	97
Total	:	**17,36,072**

corresponding to April 7, 1652 AD (G)

Note 2 : Sometimes there could be an *adhikamāsa* in a particular given lunar year.

(i) If the given date is before the *adhikamāsa* of that lunar year, then subtract 1 from the number of *adhikamāsas* obtained in the calculation.

(ii) If the given date is after the *adhikamāsa* of that lunar year, then add 1 to the number of *adhikamāsas* obtained in the calculation.

This is demonstrated in the following example.

Example 3 :

Śaka 1555 *Caitra Śukla Pratipat*, Friday [March 11, 1633]. In this year *Vaiśākha* is the *adhikamāsa* which comes after the given date.

We shall find the *cakra* and *ahargaṇa* for the given date :

(i) *Gatābdi* : 1555 – 1442 = 113

(ii) *Cakras*, $C = \text{INT}\left(\frac{113}{11}\right) = 10$, Remainder = 3

(iii) Mean *māsagaṇa* = (Remainder × 12) + No. of elapsed lunar months = (3 × 12) + 0 = 36 ≡ *M*

(iv) No. of *adhikamāsas*

$$= \text{INT}\left[\frac{M + 2C + 10}{33}\right] = \text{INT}\left[\frac{36 + 2(10) + 10}{33}\right]$$

$$= \text{INT}\left[\frac{66}{33}\right] = 2 \text{ (Note : Remainder = 0)}$$

Since the given date falls before the *adhika Vaiśākha māsa*, subtract 1 from the number obtained above. Therefore, the actual *adhikamāsas* elapsed = 2 – 1 = 1.

(v) True lunar months = *M* + No. of *adhikamāsas* = 36 + 1 = 37 ≡ *TM*

(vi) Mean *Ahargaṇa, MAH* = (37 × 30) + *Tithis* elapsed

$$+ \text{INT}\left(\frac{C}{6}\right) = 1110 + 0 + \text{INT}\left(\frac{10}{6}\right) = 1110 + 1 = 1111.$$

(vii) *Kṣayadinas*, $KD = \text{INT}\left[\frac{MAH}{64}\right] = \text{INT}\left[\frac{1111}{64}\right] = 17$

(viii) *Sāvana ahargaṇa* = $MAH - KD$ = 1111 – 17 = 1094 ≡ *TAH*

(ix) Weekday verification :

$$5C + TAH = 5(10) + 1094 = 1144 \text{ ; Remainder of } \left[\frac{1144}{7}\right] = 3$$

i.e., Thursday; but the actual weekday : Friday

Therefore, the true *ahargaṇa* = *TAH* + 1 = 1095

(x) Christian date : No. of civil days since epoch

= 10 (4016) + 1095 = 41,255

Kali ahargaṇa of the *GL* epoch : 16,87,850

∴ *Kali ahargaṇa* of the date : 17,29,105

From Table 1.1 to 1.3 under *kali ahargaṇa* we have

1600 AD(G)	:	17,16,982
33	:	12,053
March 11	:	70
Total	:	**17,29,105**

corresponding to March 11, 1633 A.D. (Gregorian).

Example 4 : *Śaka* 1530 (*Bhādrapada* is *adhikamāsa*) *Kārtika Śukla Pratipat*, Saturday. We have

(i) *Gatābdi* = 1530 – 1442 = 88

(ii) *Cakra*, $C = \text{INT}\left(\frac{88}{11}\right) = 8$, Remainder = 0

(iii) Mean lunar months $M = (0 \times 12) + 7 = 7$.

(iv) No. of *Adhikamāsas*

$$= \text{INT}\left[\frac{M+2C+10}{33}\right] = \text{INT}\left[\frac{7+16+10}{33}\right] = 1$$

Remainder = 0

Since *Kārtika* month (i.e, the given month) occurs after the *adhika Bhādrapada māsa*, add 1 to the calculated number of *adhikamāsas*. Therefore, number of *adhikamāsas* = 1 + 1 = 2.

(v) True lunar months $TM = M$ + No. of *adhikamāsas* = 7 + 2 = 9

(vi) Mean *ahargaṇa* :

MAH = ($TM \times 30$) + (*Tithis* elapsed in the given month)

$$+ \text{INT}\left(\frac{C}{6}\right) = (9 \times 30) + 0 + \text{INT}\left(\frac{8}{6}\right) = 271$$

(vii) *Kṣaya dinas* = $\text{INT}\left(\frac{271}{64}\right) = 4 \equiv KD$

(viii) *Sāvana ahargaṇa* $TAH = MAH - KD = 271 - 4 = 267$

(ix) Weekday verification : $5C + TAH = 40 + 267 = 307$

Remainder of $\left(\frac{307}{7}\right) = 6$ i.e., Sunday.

But the given weekday is Saturday

$\therefore$ True *ahargaṇa* = $TAH - 1 = 266$

(x) Christian date :

No. of civil days since epoch = 8 (4016) + 266 = 32,394

Kali ahargaṇa of *GL* epoch : 16,87,850.

$\therefore$ *Kali ahargaṇa* of the given date : 17,20,244

From Tables 1.1 to 1.3 we have

1600 A.D.	:	17,16,982
Year 08	:	2,922
December 6	:	340
Total	:	**17,20,244**

corresponding to December 6,1608 AD (Gregorian).

Finding the Christian date from the *ahargaṇa* and vice versa

In the above examples we saw how to get the *cakras* and the *ahargaṇa* from the given *Cāndramāna* (i.e, lunar) date. Now, we shall see how from the thus obtained *cakras* and *ahargaṇa* the corresponding Christian date can be obtained.

In Table 1.1 the *Julian days* and the *ahargaṇas* for the epochs of the *Kaliyuga* and the *Graha lāghavam* are given for the beginnings of the Christian centuries from – 3200 (Julian) to 2200 AD (Gregorian).

Epochs chosen :

(i) The epoch adopted for the *Kali* era is the mean midnight between February 18th and 19th of 3102 BC (i.e, the year –3101).

For any year before Christ (BC), for mathematical convenience, the negative sign is prefixed to 1 less than the numerical value of the Christian year. For example, 46 BC is considered as –45 and 3102 BC as –3101.

This convention is adopted since 1 BC is taken as the "0" year of the Christian era.

(ii) The epoch of the *Graha lāghavam* (*GL*) : *Ganeśa Daivajña* in his *GL* has adopted the mean sunrise (at *Ujjayinī*) of March 19, 1520 (Julian) AD, Monday as the epoch.

(iii) *Julian days* (*JD*) : The reckoning of the Julian days starts from the mean noon (*GMT*) on January 1, 4713 BC, Monday. On that day, at the mean noon (*GMT*), $JD = 0$.

The procedure for finding the Christian date from the *cakras* and the *ahargaṇa* :

(i) Multiply the number of *cakras* C by 4016 (i.e, the number of days in a *cakra*) i.e., find 4016 C. To this 4016 C add the *ahargaṇa* A i.e, find (4016 $C + A$). The *Kali ahargaṇa* for the *GL* epoch is 16,87,850. Add this constant to (4016 $C + A$) i.e, find (4016 $C + A$ + 16,87,850). This gives the *Kali ahargaṇa* for the required date.

From Tables 1.1 to 1.3, for the thus obtained *Kali ahargaṇa* the corresponding Christian date can be obtained as shown in the following example.

Finding the weekday from the *GL ahargaṇa*

Let C and A respectively be the *cakras* and the *ahargaṇa* according to *GL*. Multiply C by 5 and to this product add A i.e., find $(5C + A)$.

Dividing $(5C + A)$ by 7, let the remainder be R. If $R = 0$, then the given date falls on a Monday; if $R = 1$, Tuesday etc.

Example : In the example considered above, $C = 8$, and $A = 1521$. Therefore, $5C + A = 5\ (8) + 1521 = 1561$. When 1561 is divided by 7, the remainder $R = 0$. Therefore, the given date is a Monday.

Finding the *ahargaṇa* from the Christian date :

In Table 1.1, for the beginning of the Christian century (column 1) in which the given date lies – *Kali ahargaṇa* (column 3) and the *cakras* and the (balance) *ahargaṇa* according to *GL* are given in columns 4 and 5. For example, consider October 7th of the year 2001 AD. For this year the century beginning year 2000 (G). From Table 1.1 to 1.3, we have

	Kali ahar.	*GL ahargaṇa*		Julian days
		Ca.	*ahar.*	
2000 (G)	18,63,079	43	2541	24,51,545
Year 1	365	0	365	365
October 7	280	0	280	280
	18,63,724	43	3,186	24,52,190

Weekday from *GL ahargaṇa* : Here $C = 43$ and $A = 3186$.

$\therefore\ 5C + A = 215 + 3186 = 3401$

Dividing $(5C + A)$ by 7, remainder $R = 6$, we get Sunday

Note : In Table 1.1, column 2 gives the Julian days (*JD*). To find the weekday from *JD* of the given date, divide *JD* by 7 and let R be the remainder. If $R = 0$, it is Monday; if $R = 1$, Tuesday etc.

Table 1.1 : *JD, Kali* and *Graha Lāghavam Ahargaṇas*

Chris. Year	Julian Days	*Kali Ahargaṇa*	*Graha Cakras*	*Lāghavam Ahargaṇa*
–3200 (J)	552258	–36208	–430	2822
–3100 (J)	588783	317	–421	3203
–3000 (J)	625308	36842	–412	3584
–2900 (J)	661833	73367	–403	3965
–2800 (J)	698358	109892	–393	330
–2700 (J)	734883	146417	–384	711
–2600 (J)	771408	182942	–375	1092
–2500 (J)	807933	219467	–366	1473
–2400 (J)	844458	255992	–357	1854
–2300 (J)	880983	292517	–348	2235
–2200(J)	917508	329042	–339	2616
–2100 (J)	954033	365567	–330	2997
–2000 (J)	990558	402092	–321	3378
–1900 (J)	1027083	438617	–312	3759
–1800 (J)	1063608	475142	–302	124
–1700 (J)	1100133	511667	–293	505
–1600 (J)	1136658	548192	–284	886
–1500 (J)	1173183	584717	–275	1267
–1400 (J)	1209708	621242	–266	1648
–1300 (J)	1246233	657767	–257	2029
–1200 (J)	1282758	694292	–248	2410
–1100 (J)	1319283	730817	–239	2791
–1000 (J)	1355808	767342	–230	3172
–900 (J)	1392333	803867	–221	3553
–800 (J)	1428858	840392	–212	3934
–700 (J)	1465383	876917	–202	299
–600 (J)	1501908	913442	–193	680
–500 (J)	1538433	949967	–184	1061

(Contd...)

Table 1.1 (Contd.)

Chris. Year	Julian Days	*Kali Ahargaṇa*	*Graha Ca.*	*Lāghavam Ahargaṇa*
–400 (J)	1574958	986492	–175	1442
–300 (J)	1611483	1023017	–166	1823
–200 (J)	1648008	1059542	–157	2204
–100 (J)	1684533	1096067	–148	2585
0 (J)	1721058	1132592	–139	2966
100 (J)	1757583	1169117	–130	3347
200 (J)	1794108	1205642	–121	3728
300 (J)	1830633	1242167	–111	93
400 (J)	1867158	1278692	–102	474
500 (J)	1903683	1315217	–93	855
600 (J)	1940208	1351742	–84	1236
700 (J)	1976733	1388267	–75	1617
800 (J)	2013258	1424792	–66	1998
900 (J)	2049783	1461317	–57	2379
1000 (J)	2086308	1497842	–48	2760
1100 (J)	2122833	1534367	–39	3141
1200 (J)	2159358	1570892	–30	3522
1300 (J)	2195883	1607417	–21	3903
1400 (J)	2232408	1643942	–11	268
1500 (J)	2268933	1680467	–2	649
1500 (G)	2268923	1680457	–2	639
1600 (G)	2305448	1716982	7	1020
1700 (G)	2341972	1753506	16	1400
1800 (G)	2378496	1790030	25	1780
1900 (G)	2415020	1826554	34	2160
2000 (G)	2451545	1863079	43	2541
2100 (G)	2488069	1899603	52	2921
2200 (G)	2524593	1936127	61	3301

Table 1.2 : *Ahargaṇa* for Year Beginnings

Year	Days	*Graha Ca.*	*Lāghavam Ahar.*	Year	Days	*Graha Ca.*	*Lāghavam Ahar.*
0	0	0	0	28	10227	2	2195
1	365	0	365	29	10592	2	2560
2	730	0	730	30	10957	2	2925
3	1095	0	1095	31	11322	2	3290
4	1461	0	1461	32	11688	3	3656
5	1826	0	1826	33	12053	3	5
6	2191	0	2191	34	12418	3	370
7	2556	0	2556	35	12783	3	735
8	2922	0	2922	36	13149	3	1101
9	3287	0	3287	37	13514	3	1466
10	3652	0	3652	38	13879	3	1831
11	4017	1	1	39	14244	3	2196
12	4383	1	367	40	14610	3	2562
13	4748	1	732	41	14975	3	2927
14	5113	1	1097	42	15340	3	3292
15	5478	1	1462	43	15705	3	3657
16	5844	1	1828	44	16071	4	7
17	6209	1	2193	45	16436	4	372
18	6574	1	2558	46	16801	4	737
19	6939	1	2923	47	17166	4	1102
20	7305	1	3289	48	17532	4	1468
21	7670	1	3654	49	17897	4	1833
22	8035	2	3	50	18262	4	2198
23	8400	2	368	51	18627	4	2563
24	8766	2	734	52	18993	4	2929
25	9131	2	1099	53	19358	4	3294
26	9496	2	1464	54	19723	4	3659
27	9861	2	1829	55	20088	5	8

(Contd...)

Table 1.2 (Contd.)

Year	Days	Graha Ca.	Lāghava Ahar.	Year	Days	Graha Ca.	Laghava Ahar.
56	20454	5	374	78	28489	7	377
57	20819	5	739	79	28854	7	742
58	21184	5	1104	80	29220	7	1108
59	21549	5	1469	81	29585	7	1473
60	21915	5	1835	82	29950	7	1838
61	22280	5	2200	83	30315	7	2203
62	22645	5	2565	84	30681	7	2569
63	23010	5	2930	85	31046	7	2934
64	23376	5	3296	86	31411	7	3299
65	23741	5	3661	87	31776	7	3664
66	24106	6	10	88	32142	8	14
67	24471	6	375	89	32507	8	379
68	24837	6	741	90	32872	8	744
69	25202	6	1106	91	33237	8	1109
70	25567	6	1471	92	33603	8	1475
71	25932	6	1836	93	33968	8	1840
72	26298	6	2202	94	34333	8	2205
73	26663	6	2567	95	34698	8	2570
74	27028	6	2932	96	35064	8	2936
75	27393	6	3297	97	35429	8	3301
76	27759	6	3663	98	35794	8	3666
77	28124	7	12	99	36159	9	15

Note : (1) In Table 1.3, the first two columns are headed by C and B which stand respectively for a *common* (non leap) year and *bissextile* (leap) year.

For a given date in a leap year, only for January and February, the column headed by B must be used. For other months even in a leap year and for all months in a common year the first column under C must be used.

(2) In Table 1.1, the letters J and G in brackets represent respectively the *Julian* and the *Gregorian* calendars.

Table 1.3 : *Ahargaṇa* for Days of a Year

Dates		Jan.	Feb.	Mar.	Apr.	May	Jun.	July	Aug.	Sep.	Oct.	Nov.	Dec.
C	B												
0	1	0	31	-	-	-	-	-	-	-	-	-	-
1	2	1	32	60	91	121	152	182	213	244	274	305	335
2	3	2	33	61	92	122	153	183	214	245	275	306	336
3	4	3	34	62	93	123	154	184	215	246	276	307	337
4	5	4	35	63	94	124	155	185	216	247	277	308	338
5	6	5	36	64	95	125	156	186	217	248	278	309	339
6	7	6	37	65	96	126	157	187	218	249	279	310	340
7	8	7	38	66	97	127	158	188	219	250	280	311	341
8	9	8	39	67	98	128	159	189	220	251	281	312	342
9	10	9	40	68	99	129	160	190	221	252	282	313	343
10	11	10	41	69	100	130	161	191	222	253	283	314	344
11	12	11	42	70	101	131	162	192	223	254	284	315	345
12	13	12	43	71	102	132	163	193	224	255	285	316	346
13	14	13	44	72	103	133	164	194	225	256	286	317	347
14	15	14	45	73	104	134	165	195	226	257	287	318	348
15	16	15	46	74	105	135	166	196	227	258	288	319	349
16	17	16	47	75	106	136	167	197	228	259	289	320	350
17	18	17	48	76	107	137	168	198	229	260	290	321	351
18	19	18	49	77	108	138	169	199	230	261	291	322	352
19	20	19	50	78	109	139	170	200	231	262	292	323	353
20	21	20	51	79	110	140	171	201	232	263	293	324	354
21	22	21	52	80	111	141	172	202	233	264	294	325	355
22	23	22	53	81	112	142	173	203	234	265	295	326	356
23	24	23	54	82	113	143	174	204	235	266	296	327	357
24	25	24	55	83	114	144	175	205	236	267	297	328	358
25	26	25	56	84	115	145	176	206	237	268	298	329	359
26	27	26	57	85	116	146	177	207	238	269	299	330	360
27	28	27	58	86	117	147	178	208	239	270	300	331	361
28	29	28	59	87	118	148	179	209	240	271	301	332	362
29	30	29	-	88	119	149	180	210	241	272	302	333	363
30	31	30	-	89	120	150	181	211	242	273	303	334	364
31	-	31	-	90	-	151	-	212	243	-	304	-	365

Śloksa 6,7,8 : The dhruvas and ksepakas of all planets

Kṣepaka is the mean position of a heavenly body at the time of the epoch *and dhruvaka* is the multiplier of the completed *cakras* for a given date. In fact, *dhruvaka* is the residual motion of a body in a *cakra* after removing the completed revolutions.

The *dhruvakas* and *kṣepakas* of the heavenly bodies are tabulated below.

Table 1.4 *Dhruvakas* of bodies

	Ravi	Candra	*Cand-rocca*	Rāhu	Kuja	Budha *Kendra*	Guru	Śukra *Kendra*	Śani
Rāśi	0	0	9	7	1	4	0	1	7
Amśa (°)	1	3	2	2	25	3	26	14	15
Kalā (′)	49	46	45	50	32	27	18	2	42
Vikalā(″)	11	11	0	0	0	0	0	0	0

Table 1.5 *Kṣepakas* of bodies

	Ravi	Candra	*Cand-rocca*	Rāhu	Kuja	Budha *Kendra*	Guru	Śukra *Kendra*	Śani
Rāśi	11	11	5	0	10	8	7	7	9
Amśa (°)	19	19	17	27	7	29	2	20	15
Kalā (′)	41	6	33	38	8	33	16	9	21

***Śloka* 9** : The use of *dhruvaka* and *kṣepaka* are explained :

From the motion of a body obtained from the *ahargaṇa* subtract the product of the *dhruvaka* and the *cakra* and to it add the *kṣepaka*. This gives the mean position of the body for the mean sunrise (at *Laṅkā* and *Ujjayinī*).

In the case of the moon, the distance (in *yojanas*) between one's place and the central meridian (*rekhā*), chosen as the meridian passing through *Laṅkā* and *Ujjayinī*, is divided by 6 to get the correction in *kalās* (minutes of arc). This is added to or subtracted from the earlier obtained position of the moon according as one's place is to the west or to the east of the central meridian (*rekhā*).

Explanation : The above correction described for the moon is referred to as *deśāntara saṃskāra* due to difference in the sunrise timings at the given place and at *Laṅkā*. The *deśāntara* correction for the moon

$$= \frac{\text{The distance in } yojanas}{\text{Circumference of earth in } yojanas} \times \text{Moon's daily motion}$$

Consider the mean daily motion = 790′35″ and *paridhi* (circumference) of the earth = 4967 *yojanas*. We get the *Deśāntara* correction for the moon

$$= \frac{790'35''}{4967} \times (\text{Distance in } Yojanas) = \frac{1}{6.2827} \times (\text{distance in } yojanas)$$

$$\approx \frac{1}{6} \times (\text{distance in } yojanas)$$

where the distance in *yojanas* is the shortest distance of the given place from the meridian passing through *Laṅkā* and *Ujjayinī*. The modern known values of the earth's equatorial and polar radii are respectively 6378.16 km and 6356.775 km. Considering the mean radius of the earth in miles and the circumference of earth given as 4967 *yojanas* in the *GL* commentary, we get 1 *yojana* ≈ 5 miles. The circumference of the earth as 4967 *yojanas* is taken by Bhāskara II in his *Siddhānta Śiromaṇi*.

***Śloka* 10** : The method of calculating the mean longitudes of Ravi, Budha and Śukra has been given in this *śloka* in a systematic manner. It is as follows :

(i) Divide *ahargaṇa* A by 70 and then subtract the quotient from A to get the resultant in degrees.

(ii) Divide A by 150. (The result will be in *kalās*). Divide the result by 60 in order to convert it into degrees.

(iii) Subtract the result of step (ii) from that of step (i).

(iv) Multiply *cakra* $\equiv C$ by *dhruvaka* D and subtract the product from step (iii)

(v) Add *kṣepaka* K to step (iv). The result gives the mean planet in degrees.

i.e., Mean longitude of the planet = $\left\{\left(A - \frac{A}{70} - \frac{A}{150 \times 60}\right) - C \times D\right\} + K$

Note : In traditional Indian astronomy the mean longitudes of Budha and Śukra are taken the same as that of the sun.

Example : For the given date, *samvat* 2036, *śaka* 1901

Phālguṇa śukla pūrṇimā (i.e., 1st March 1979)

$A \equiv$ *Ahargaṇa* $\equiv 3328$, $C \equiv$ *Cakra* $\equiv 41$

$D \equiv$ *Dhruvaka* $= 0^R\, 1°49'\, 11''$ $\quad K \equiv$ *Kṣepaka* $= 11^R\, 19°\, 41' = 349°41''$

Mean longitude of the sun = $\left\{\left(A - \frac{A}{70} - \frac{A}{150 \times 60}\right) - C \times D\right\} + K$

$$= \left\{\left(3328 - \frac{3328}{70} - \frac{3328}{150 \times 60}\right) - 41 \times 0^R 1°49'11''\right\} + 11^R 19°41'$$

$= 3555°.1621 =$ **$10^R 15°\ 9'\ 43''.6$** [removing completed revolutions].

Note : Step (iii) gives *aharganotpannagraha* (i.e., mean longitude derived from the *ahargana*).

Modern Example :

Given date : 11th August 1998. For the given date, $A = 2033$, $C = 43$.

For the sun, $D = 0^R\ 1°\ 49'\ 11''$ and $K = 349°41''$

$$\text{Mean longitude of the sun} = \left[\left(A - \frac{A}{70} - \frac{A}{150 \times 60}\right) - C \times D\right] + K$$

$$= \left[\left(2033 - \frac{2033}{70} - \frac{2033}{150 \times 60}\right) - 43 \times 1°49'11''\right] + 349°41' = 2275°.1665$$

= **$115°\ 9'\ 59''.5$** [removing completed revolutions].

Another method : Instead of the method explained in *śloka* 10, we can as well adopt the following method in which the *ahargaṇa* is multiplied by the mean daily motion.

In the example considered earlier, $A = 3328$, $C = 41$, $D = 1°49'11''$, $K = 11^R\ 19°\ 41'$ and the mean daily motion of the sun, $m = 59'08''10'''$. [**Note** : Here, $1'' = 60'''$ i.e, one *vikalā* = 60 *prativikalās*]. Therefore, *Ahargaṇa* generated mean longitude = $A \times m$

$= 3328 \times (59'08''10''') = 3328 \times 0°.9856018 = 3280°.083$

$= 40°04'58'' = 1^R\ 10°\ 04'\ 58''$(removing the multiples of revolutions).

Now, $C \times D = 41^\circ \times 1^\circ 49' 11'' = 74^\circ 36' 31'' = 2^R\, 14^\circ\, 36'\, 31''$

$$\therefore \quad (A \times m) - (C \times D) = 1^R\, 10^\circ 04' 58'' - 2^R\, 14^\circ 36' 31''$$

$$= 10^R\, 25^\circ 28' 27''$$

Adding the *Kṣhepaka*, $K = 11^R\ 19^\circ\ 41'$, we get

$$10^R\, 25^\circ 28' 27'' + 11^R 19^\circ 41' = \mathbf{10^R 15^\circ 09' 27''}$$

This is the mean position of the Sun, Budha and Śukra.

Mean position of the Moon

The mean position of the Moon can be determined as follows:

(i) Multiply *ahargaṇa* A by 14 and divide the product by 17.

(ii) Subtract step (i) from $14 \times A$.

(iii) Divide A by 140×60. Subtract the quotient from step (ii); the difference gives the *ahargaṇa*-derived Moon.

(iv) Subtract the product of *dhruvaka* and *cakra* ($C \times D$) from step (iii).

(v) Add *kṣepaka* K to step (iv) which gives the mean position of the Moon.

i.e., Mean longitude of the Moon =

$$\left[\left\{\left(A \times 14 - \frac{A \times 14}{17}\right) - \frac{A}{140 \times 60}\right\} - C \times D\right] + K$$

Example : $A = 3328$, $C = 41$, $D = 3^\circ 46' 11''$, $K = 11^R 19^\circ 6'$

Mean longitude of the Moon = $\left\{\left(A \times 14 - \frac{A \times 14}{17}\right) - \frac{A}{140 \times 60}\right\} - C \times D + K$

$$= \left\{ \left(3328 \times 14 - \frac{3328 \times 14}{17} \right) - \frac{3328}{140 \times 60} \right\} - 41 \times 3°46'11'' + 11^R 19°6'0''$$

$= 44045°.439 = \mathbf{4^R 5°26'20''}$

(removing completed revolutions).

Modern Example :

Given date : 11th August 1998

For the given date, $A = 2033$, $C = 43$

For the Moon : $D = 3°46'11''$, $K = 11^R 19°6'$

$$\text{Mean longitude of the Moon} = \left[\left(A \times 14 - \frac{A \times 14}{17} \right) - \frac{A}{140 \times 60} \right] - C \times D + K$$

$$= \left[\left(2033 \times 14 - \frac{2033 \times 14}{17} \right) - \frac{2033}{140 \times 60} \right] - 43 \times 3°46'11'' + 11^R 19°6'$$

$= 26974°.525 = 11^R 4°31' = \mathbf{334°31'}$

removing completed revolutions.

***Śloka* 11** : Mean positions of *Candrocca* (Moon's apogee) and Rāhu.

The method is explained as below :

(i) Divide *ahargaṇa* A by 9.

(ii) Again divide *ahargaṇa* A by 70 and it is in *kalās*.

Divide the result by 60 to get it in degrees.

(iii) Adding step (i) and step (ii), we get *ahargaṇa* derived *mandocca*.

(iv) Subtract the product of *cakra* and *dhruvaka* ($C \times D$) from step (iii)

(v) Add *kṣepaka* K to step (iv).

This gives the *Candrocca* (*mandocca* of the Moon).

i.e., Mean position of *Mandocca* = $\left[\left(\frac{A}{9}+\frac{A}{70\times 60}\right)^{\circ} - C\times D\right] + K$

Example : $A = 1521$, $C = 8$, $D = 9^R2^\circ45'$, $K = 5^R17^\circ33'$

Mean position of *Mandocca* = $\left[\left(\frac{A}{9}+\frac{A}{70\times 60}\right)^{\circ} - C\times D\right] + K$

$$= \left[\left(\frac{1521}{9}+\frac{1521}{70\times 60}\right)^{\circ} - 8\times 9^R2^\circ45'\right] + 5^R17^\circ33'$$

$= 314^\circ54'43'' =$ **$10^R14^\circ54'43''$**

Modern Example :

Given date : 11th August 1998

For the given date, $A = 2033$, $C = 43$

For *Mandocca*, $D = 9^R2^\circ45'$, $K = 5^R17^\circ33'$

$\therefore$ Mean position of *Mandocca* = $\left[\left(\frac{A}{9}+\frac{A}{70\times 60}\right)^{\circ} - C\times D\right] + K$

$$= \left(\frac{2033}{9}+\frac{2033}{70\times 60}\right)^{\circ} - 43\times 9^R2^\circ45' + 5^R17^\circ33'$$

$= -11334^\circ.327 = 185^\circ.67294$ (adding multiples of 360°)

$= 185^\circ 40' 22'' = 6^R\ 5^\circ\ 40'\ 22''$.

Mean position of *Moon's Pāta* (Rāhu)

The following is the method to determine the mean position of *Pāta* (Rāhu).

(i) Divide *ahargaṇa* A by 19 (the result in degrees)

(ii) Divide *ahargaṇa* A by 45. Divide the result in *kalās* by 60 to get it in degrees

(iii) Add step (i) with step (ii)

(iv) Divide step (iii) by 30 to get the result in *Rāśis* and subtract the quotient from 12. This gives *cakra śuddha* Rāhu.

(v) Subtract the product of D and C from step (iv).

(vi) Add K to step (v). Then we will get

$$\text{Mean Rāhu} = \left\{12 - \frac{\left(\frac{A}{19} + \frac{A}{45 \times 60}\right)}{30}\right\}^R - C \times D + K$$

$$= \left\{360^\circ - \left(\frac{A}{19} + \frac{A}{45 \times 60}\right)\right\}^\circ - C \times D + K$$

Example : $A = 1521$, $C = 8$, $D = 7^R 2^\circ 50'$, $K = 27^\circ 38'$

$$\text{Mean longitude of Rāhu} = \left\{360^\circ - \left(\frac{A}{19} + \frac{A}{45 \times 60}\right)\right\}^\circ - C \times D + K$$

$$= \left\{360^\circ - \left(\frac{1521}{19} + \frac{1521}{45 \times 60}\right)\right\}^\circ - 7^R 2^\circ 50' \times 8 + 27^\circ 38'$$

$= 44^\circ.350702 = 44^\circ 21' 2''.5 = 1^R\, 14^\circ 21' 2.5''$.

Modern example :

Given date : 11th August 1998

For the given date, $A = 2033$, $C = 43$

For Rāhu : $D = 7^R\, 2^\circ 50'$, $K = 27^\circ 38'$

$$\text{Mean longitude of Rāhu} = \left\{360^\circ - \left(\frac{A}{19} + \frac{A}{45 \times 60}\right)\right\}^\circ - C \times D + K$$

$$= \left\{360^\circ - \left(\frac{2033}{19} + \frac{2033}{45 \times 60}\right)\right\}^\circ - 43 \times 7^R 2^\circ 50' + 27^\circ 38'$$

$= -8871^\circ.953 = 128^\circ.04704$ (adding multiples of 360°)

$= 128^\circ 2' 49'' = 4^R\ 8^\circ\ 2'\ 49''$

***Śloka* 12** : (a) Mean position of Kuja

(i) Multiply *ahargaṇa* A by 10

(ii) Divide step (i) by 19 (the result is in degrees)

(iii) Divide step (i) by 73 (the result is in *kalās*)

(iv) Subtract step (iii) from step (ii), the result of which will be the *ahargaṇa* derived mean Kuja.

(v) Take the product of *cakra* and *dhruvaka* and subtract the product ($C \times D$) from step (iv).

(vi) Add *kṣepaka* (K) to step (v). Then

$$\text{Mean Kuja} = \left[\frac{10 \times A^\circ}{19} - \frac{10 \times A'}{73}\right] - C \times D + K$$

Example : Consider the date May 15, 1612 (G), Monday.

$A = 1521$, $C = 8$, $D = 1^R 25^\circ 32'$, $K = 10^R 7^\circ 8'$

$\therefore$ Mean longitude of Kuja

$$\left[\frac{10 \times 1521\ ^\circ}{19} - \frac{10 \times 152\ 1'}{73}\right] - 8 \times 1^R 25^\circ 32' + 10^R 7^\circ 8'$$

$= 800^\circ.52632 - 208'.35616 - 8 \times 55^\circ.53333 + 307^\circ.13333$

$= 659^\circ.92038 = \mathbf{9^R 29^\circ 55' 13''}$

[removing the completed revolutions]

Modern Example : Given date : 11th August 1998

For the given date : $A = 2033$, $C = 43$

For Kuja : $D = 1^R 25^\circ 32'$, $K = 10^R 7^\circ 8'$

$$\therefore \text{Mean longitude of Kuja} = \left[\frac{10A^\circ}{19} - \frac{10A'}{73}\right] - C \times D + K$$

$$= \left[\frac{10 \times 2033\ ^\circ}{19} - \frac{10 \times 2033}{73}\right]' - 43 \times 1^R 25^\circ 32' + 10^R 7^\circ 8'$$

$= -\ 1015^\circ.4416 = 64^\circ.558448$

$= \mathbf{64^\circ 33' 30''} = 2^R\ 4^\circ\ 33'\ 30''$

(by adding multiples of 360°)

The method of finding *śīghrakendra* of Budha is as follows :

Śīghra normaly of Budha

(i) Multiply *ahargaṇa* by 3.

(ii) Divide 3*A* by 28.

(iii) Add step (i) and step (ii); the result will be in degrees.

(iv) Subtract $\frac{A}{38}$ *kalās* (minutes of arc) from step (iii)

the result of which will be the *ahargaṇa*-derived mean Budha *kendra*.

(v) Subtract the product of *cakra* and *dhruvaka* from step (iv)

(vi) Add *kṣepaka* (*K*) to (*v*).

i.e, *Śīghrakendra* of Budha $= \left[\left(\frac{3A}{28}+3A\right)^{\circ}-\left(\frac{A}{38}\right)'\right]-C\times D+K$

$$= \left[\left(\frac{3A}{28}+3A\right)-\left(\frac{A}{38\times 60}\right)'\right]^{\circ} - C\times D+K$$

Example : $A = 1521,\ C = 8,\ D = 4^{R}\ 3^{\circ}\ 27' = 123^{\circ}\ 27'$

and $K = 8^{R}\ 29^{\circ}\ 33' = 269^{\circ}\ 33'$

i.e, Mean *śīghrakendra* of Budha

$$= \left[\left(\frac{3A}{28}+3A\right)^{\circ}-\left(\frac{A}{38\times 60}\right)^{\circ}\right]-C\times D+K$$

$$= \left(\frac{3\times 1521}{28} + 3\times 1521\right)^{\circ} - \left(\frac{1521}{38\times 60}\right)^{\circ} - 8\times 123^{\circ}\ 27' + 269^{\circ}\ 33'$$

$$= 4007^{\circ}\cdot 2472 = 47^{\circ}\ \ 14'\ \ 49'' = 1^{R}\ \ 17^{\circ}\ \ 14'\ \ 49''$$

[removing the completed revolutions].

Modern example :

Given date : 11th August 1998

For the given date $A = 2033$, $C = 43$

We have for the Budha *kendra* : $D = 4^{R}\ 3^{\circ}\ 27'$, $K = 8^{R}\ 29^{\circ}\ 33'$

$$\text{Budha } \acute{S}\bar{\imath}ghra \text{ kendra} = \left[\left(\frac{3A}{28} + 3A\right)^{\circ} - \left(\frac{A}{38\times 60}\right)^{\circ}\right] - C\times D + K$$

$$= \left[\left(\frac{3\times 2033}{28} + 3\times 2033\right) - \left(\frac{2033}{38\times 60}\right)\right]^{\circ} - 43\times 4^{R}\ 3^{\circ}\ 27' + 8^{R}\ 29^{\circ}\ 33'$$

$$= 1277^{\circ}\cdot 12976 = 197^{\circ}\cdot 12976 = \mathbf{197^{\circ}\ 7'\ 47''}$$

(Removing the multiples of 360 °)

Note : Usually the practice, in other *Siddhāntic* texts, is first to find the *Śīghrocca* of Budha and then to subtract the mean Budha (same as the mean Ravi) from it to get the *Śīghrakendra* of Budha. Similar is the practice in the case of Śukra's *Śīghrakendra.*

However, *Grahalāghavam* gives the method of determining the *śīghrakendras* of Budha and Śukra directly without finding the *śīghroccas.*

***Śloka* 13** : Mean positions of Guru and Śukra *kendra* are explained.

(i) Mean position of Guru :

The method is as explained below.

(i) Divide *ahargaṇa* A by 12. The result is in degrees.

(ii) Divide *ahargaṇa* A by 70. This will be in *kalās* (minutes).

(iii) Subtract step (ii) from step (i)

(iv) Subtract the product of *cakra* (C) and *dhṛuvaka* (D) from (iii).

(v) Add *kṣepaka* (K) to (iv).

i.e., Mean longitude of Guru = $\left[\left(\frac{A}{12}\right)^{\circ} - \left(\frac{A}{70}\right)^{\prime}\right] - C \times D + K$

$$= \left[\left(\frac{A}{12}\right) - \left(\frac{A}{70 \times 60}\right)^{\prime}\right]^{\circ} - C \times D + K$$

Example : $A = 1521$, $C = 8$, $D = 26^{\circ}\ 18'$

and $K = 7^{R}\ 2^{\circ}\ 16' = 212^{\circ}\ 16'$

$\therefore$ Mean longitude of Guru = $\left[\left(\frac{A}{12}\right)^{\circ} - \left(\frac{A}{70 \times 60}\right)^{\circ}\right] - C \times D + K$

$$= \left[\left(\frac{1521}{12}\right)^{\circ} - \left(\frac{1521}{70 \times 60}\right)^{\circ}\right] - 8 \times 26^{\circ}\ 18' + 212^{\circ}\ 16'$$

$= 128^\circ{\cdot}25452 = 128^\circ\ 15'\ 16'' = \mathbf{4^R\ 8^\circ\ 15'\ 16''}$

Modern example :

Given date : 11th August 1998

For the given date, $A = 2033$, $C = 43$

and for Guru $D = 0^R\ 26^\circ\ 18'$, $K = 7^R\ 2^\circ\ 16'$

$$\text{Now, Mean longitude of Guru} = \left[\left(\frac{A}{12}\right)^\circ - \left(\frac{A}{70 \times 60}\right)^\circ\right] - C \times D + K$$

$$= \left[\left(\frac{2033}{12}\right)^\circ - \left(\frac{2033}{70 \times 60}\right)^\circ\right] - 43 \times 26^\circ\ 18' + 7^R\ 2^\circ\ 16'$$

$$= -749^\circ{\cdot}70071 = 330^\circ{\cdot}29929 \ \text{(adding } 1080^\circ)$$

$$= \mathbf{330^\circ 17'\ 57''} = 11^R\ 0^\circ\ 17'\ 57''$$

Mean *śīghrakendra* of Śukra :

The method to find out Śukra *kendra* is as follows :

(i) Multiply *ahargaṇa* A by 3.

(ii) Divide step (i) by 5.

(iii) Divide step (i) by 181.

(iv) Add step (ii) and step (iii). The result is in degrees.

(v) Subtract the product of *cakra* (C) and *dhruvaka* (D) from step (iv)

(vi) Add *kṣepaka* K to step (v)

i.e., Mean position of Śukra *Kendra* = $\left[\frac{3A}{5} + \frac{3A}{181}\right]^{\circ} - C \times D + K$

Example : $A = 1521$, $C = 8$, $D = 1^{R}\ 14^{\circ}\ 2' = 44^{\circ}\ 2'$,

$K = 7^{R}\ 20^{\circ}\ 9' = 230^{\circ}\ 9'$

$\therefore$ Mean position of Śukra *Kendra* = $\left[\frac{3A}{5} + \frac{3A}{181}\right]^{\circ} - C \times D + K$

$= \left[\frac{3 \times 1521}{5} + \frac{3 \times 1521}{181}\right] - 8 \times 44^{\circ}\ 2' + 230^{\circ}\ 6'$

$= 815^{\circ}\ 41'\ 35'' = 95^{\circ}\ 41'\ 36'' = 3^{R}\ 5^{\circ}\ 41'\ 35''$

[removing the completed revolutions].

Modern example :

Given date : 11th August 1998

For the given date, $A = 2033$, $C = 43$

For Śukra *Kendra*, $D = 1^{R}\ 14^{\circ}\ 2'$, $K = 7^{R}\ 20^{\circ}\ 9'$

$\therefore$ Mean position of Śukra *kendra* = $\left(\frac{3A}{5} + \frac{3A}{181}\right)^{\circ} - C \times D + K$

$= \left(\frac{3 \times 2033}{5} + \frac{3 \times 2033}{181}\right)^{\circ} - 43 \times 1^{R}\ 14^{\circ}\ 2' + 7^{R}\ 20^{\circ}\ 9'$

$= -409^{\circ}\ 47'\ 14'' = \mathbf{310^{\circ}\ 12'\ 46''}$ (adding 720°)

***Śloka* 14** : Mean longitude of Śani.

The method is as explianed below :

(i) Divide *ahargaṇa A* by 30. The result will be in degrees.

(ii) Divide *ahargaṇa A* by 156. The result will be in minutes of arc (*kalās*). Convert it into degrees by dividing by 60.

(iii) Add step (i) and step (ii).

(iv) Subtract the product of *cakra* (C) and *dhruvaka* (*D*) from step (iii).

(v) Add *kṣepaka K* to (iv).

i.e., Mean longitude of Śani $= \left[\left(\frac{A}{30}\right)^{\circ} + \left(\frac{A}{156}\right)'\right] - C \times D + K$

$$= \left[\left(\frac{A}{30}\right)^{\circ} + \left(\frac{A}{156 \times 60}\right)^{\circ}\right] - C \times D + K$$

Example : $A = 1521,\ C = 8,\ D = 7^R\ 15^{\circ}\ 42' = 225^{\circ}42'$

and *Kṣepaka* $K = 9^R\ 15^{\circ}\ 21' = 285^{\circ}\ 21'$

$\therefore$ Mean longitude of Śani $= \left[\left(\frac{A}{30}\right)^{\circ} + \left(\frac{A}{156 \times 60}\right)^{\circ}\right] - C \times D + K$

$$= \left[\left(\frac{1521}{30}\right)^{\circ} + \left(\frac{1521}{156 \times 60}\right)^{\circ}\right] - 8 \times 225^{\circ}42' + 285^{\circ}\ 21'$$

$= -1469^\circ\cdot 3875 = 330^\circ\cdot 6125$ (adding $5 \times 360^\circ$) = $\mathbf{11^R\ 0^\circ\ 36'\ 45''}$

Modern example :

Given date : 11th August 1998

For the given date, $A = 2033$, $C = 43$

For Śani, $D = 7^R\ 15^\circ\ 42'$ *and* $K = 9^R\ 15^\circ\ 21'$

$$\therefore \text{ Mean longitude of Śani} = \left[\left(\frac{A}{30}\right)^\circ + \left(\frac{A}{156 \times 60}\right)^\circ\right] - C \times D + K$$

$$= \left[\left(\frac{2033}{30}\right)^o + \left(\frac{2033}{156 \times 60}\right)^o\right] - 43 \times 7^7 15^o 42' + 9^R 15^o 21'$$

$$= -9351^o.7661324 = 8.2338676\ (adding\ 26 \times 360^o)$$

$$= 8^o\ 14'\ 2''$$

***Śloka* 15 : Mean daily motions of planets**

Mean daily motions of planets (*madhyama gati*) are given in Table 1.6

Table 1.6 Mean daily motions of bodies

Bodies	Sun	Moon	*Candrocca*	Rāhu	Kuja	Budha *Kendra*	Guru	Śukra *Kendra*	Śani
Kalās (minutes)	59	790	6	3	31	186	5	37	2
Vikalās (seconds)	8	35	41	11	26	24	0	0	0

***Śloka* 16** : The values of the Sun and the Moon's *mandocca* (*Candrocca*) obtained from this text are equivalent to those according to the *Śūrya Siddhānta* (*SS*). By subtracting 9′ from the Moon (as per this text) we get as per *SS*. The positions Kuja, Guru, Rāhu are in accordance with the *Āryapakṣa*. Budha *kendra* (Budha's *śīghra* anomaly) is in accordance with the *Brāhma pakṣa*. By adding 5° to Śani (obtained from this text), we get that according to the *Ārya Siddhānta*. The Śukra (*śīghra*) *kendra* (of this text) is half of the sum of those obtained from the *Ārya* and *Brāhma pakṣas* (i.e., the average of the latter two).

Derivations of expressions for the mean longitudes

The expressions for the mean longitudes of the Sun, the Moon, the Moon's apogee and node, given in Table 1.6 are now derived using the revolutions of these bodies and the numbers of civil days in a *Mahāyuga* of 432×10^4 years.

(i) Mean longitude of the Sun

The mean longitude of the Sun is given by

$$\lambda = \left(A - \frac{A}{70} - \frac{A}{150 \times 60} \right)^{\circ} - Cakra \times 1^{\circ}.81972 + 349^{\circ}.683$$

where A is the *ahargaṇa* according to *GL*.

Mean motion of the sun in a *Cakra*

1 *Cakra* = 4016 days

Number of civil days in a *Mahāyuga* = 1577917828 according to *Surya Siddhānta*.

Number of revolutions of the Sun in a *Mahāyuga* = 4320000

$$\therefore \text{ Mean daily motion of the Sun} = \frac{4320000 \ rev.}{1577917828} = 59'8'' \cdot 17$$

Therefore, in a *Cakra*

Sun's motion $= 4016 \times 59'8''\cdot17$

$= 3958^{\circ}\cdot180756$

$= 10\cdot99494654$ revolutions

Subtracting 11^{rev}, we get *dhruvaka* $= -1^{\circ}\cdot8192444$

$= -1^{\circ}49'9''$ (approx)

The *ahargaṇa* derived mean motion of the Sun is given by $A \times 59'8''\cdot17$

Multiplying and dividing by 70, we get

$$\frac{70 \times A \times 59'8''\cdot17}{70} = \frac{4139'32'' \times A}{70}$$

$$= \frac{68^{\circ}59'32'' \times A}{70}$$

By adding and subtracting $28''$, the mean motion of the Sun

$$= \frac{A \times (68^{\circ}\ 59'\ 32'' + 28'' - 28'')}{70} = A \times \left(\frac{69^{\circ} - 28''}{70}\right)$$

$$= \frac{A \times 69^{\circ}}{70} - \frac{A \times 28''}{70} = \frac{A \times (69^{\circ} + 1^{\circ} - 1^{\circ})}{70} - \frac{A \times 28''}{70}$$

$$= \frac{A\times 70}{70} - \frac{A}{70} - \frac{A\times 28''}{70} = A - \frac{A}{70} - \frac{\dfrac{A}{70\times 60\times 60}}{28} \text{ degrees}$$

$$= A - \frac{A}{70} - \frac{A}{150\times 60} \text{ in degrees.}$$

(ii) Mean longitude of the Moon

The mean longitude of the Moon is given by

$$\lambda = A\times 14 - 14\times\frac{A}{17} - \frac{A}{140\times 60} - Cakra\ \times 3^\circ\cdot 76972\ + 349\cdot 1 \text{ in deg.}$$

where A is the *ahargaṇa* according to *GL*.

Mean daily motion of the Moon $= 790'34''\cdot 9 = 13^\circ\ 10'\ 34''\cdot 9$

Mean motion of the Moon in a *Cakra* $= 4016\ \times 13^\circ\cdot 176361$

$$= 52916^\circ\cdot 26622 = 146^{rev.}\cdot 9896284$$

Subtracting 147 rev., we get

Dhruvaka $= -3^\circ\cdot 7337785$ (taken as $3^\circ\cdot 76972$ by *GL*).

$$= -3^\circ\ 44'01\ .6$$

The mean longitude of the Moon = $A\times 13^\circ 10'34''\cdot 9$

Multiplying and dividing by 17 we get

$$\frac{A\times 17\times 13^\circ 10'34''\cdot 9}{17} = \frac{A\times 223^\circ 59'53''}{17}$$

$$= \frac{A\times(223^\circ 59'53''+7''-7'')}{17} = \frac{A\times(224^\circ-7'')}{17} = \frac{A\times 224^\circ}{17} - \frac{A\times 7''}{17}$$

$$= \frac{A\times(224^\circ+14^\circ-14^\circ)}{17} - \frac{A\times 7''}{17} = \frac{A\times 238^\circ}{17} - \frac{A\times 14^\circ}{17} - \frac{A\times 7''}{17}$$

$$= A\times 14^\circ - \frac{A\times 14^\circ}{17} - \frac{A\times 7''}{17\times 60\times 60} \text{ deg.}$$

$$= A\times 14^\circ - \frac{A\times 14^\circ}{17} - \frac{A}{\frac{1020\times 60}{7}}$$

$$= A\times 14^\circ - \frac{A\times 14^\circ}{17} - \frac{A}{145\cdot 71429\times 60}$$

Note : In the last term the text has made an approximation viz. $\frac{A^\circ}{140\times 60}$

(iii) Mean longitude of the Moon's apogee (*mandocca*)

The mean longitude of the Moon's apogee is given by

$$M = \frac{A}{9} + \frac{A}{70\times 60} - 272.75 \times Cakra + 167.55 \text{ in degrees}$$

where A is the *ahargaṇa* according to *GL*.

Number of civil days in a *Mahāyuga* = 1577917828

Number of revolutions of the Moon's apogee according to the *Sūrya Siddhānta* = 488203

Mean daily motion of the Moon's apogee $= \frac{488203 \text{ revns.}}{1577917828} = 6'40''.98$

Mean motion of the Moon's apogee in a *cakra*

1 *cakra* = 4016 days

Mean daily motion of Moon's apogee = $6'40''.98$. Therefore, in 1 *cakra,*

the mean motion of the Moon's apogee $= 4016 \times 6'40''.98$

$$= 447^{\circ}.3154667 \qquad = 1.242542963 \; revns.$$

Subtracting 2 revolutions we get

Dhruvaka = $-\,272^{\circ}.6845333$ (taken as $272^{\circ}.75$ by *GL*)

$$= -\,272^{\circ}41'04''.32$$

The mean longitude of the Moon's apogee is given by $A \times 6'40''.98$

By multiplying and dividing by 9 we have

$$\frac{A \times 6'40''.98 \times 9}{9} = \frac{A \times 1^{\circ}0'8''}{9} = \frac{A^{\circ}}{9} + \frac{A \times 8'}{9 \times 60} = \frac{A^{\circ}}{9} + \frac{A'}{67.5}$$

In the last term, *GL* takes 70 instead of 67.5 in the denominator. This results in a maximum error of $2'$.

(iv) Mean longitude of *Rāhu*

The mean longitude of Rāhu is given by

$$\textbf{\textit{Rāhu}} = \left[360 - \left(\frac{A}{19} + \frac{A}{45 \times 60}\right) - (212.83 \times cakra) + 27.63\right] \text{degrees.}$$

where A is the *ahargaṇa* A according to *GL*

Mean daily motion of Rāhu $= 3'10''.8$

Mean motion of Rāhu in a *cakra* $= 4016 \times 3'10''.8$

$$= 212^\circ.848 = 212^\circ 50' \equiv \textit{Dhruvaka}$$

which is the same as given in *GL*

The mean motion of the Moon's node (Rāhu) is given by $A \times 3'10''.8$

for the *ahargaṇa* A.

Multiplying and dividing by 19 we have

$$\frac{A \times 3'10''.8 \times 19}{19} = \frac{A \times 1^\circ 0'25''}{19} = \frac{A^\circ}{19} + \frac{A \times 25''}{19} = \frac{A^\circ}{19} + \frac{A \times 25'}{19 \times 60}$$

$$= \frac{A^\circ}{19} + \frac{A^\circ}{45.6 \times 60}$$

GL has taken 45 instead of 45.6

(v) Mean longitude of *Kuja*

Mean Kuja $= \frac{10}{19} A^\circ - \frac{10A}{73}$ min $- (C \times D)$ deg $+ K$ deg

where A is the *ahargaṇa* according to *GL*.

Number of revolutions of Kuja in a *Mahāyuga* $= 2296824$

Number of civil days in a *Mahāyuga* = 1577917828

$$\therefore \text{ Mean daily motion of Kuja} = \frac{2296824 \text{ revns.}}{1577917828} = 31'\ 26''.4$$

Mean motion of Kuja in a *cakra* = $4016 \times 31'26''.4$

$$= 304^\circ 23' 2'' = -55^\circ 36' 57'' = \textit{Dhruvaka}$$

(by subtracting 360°)

Multiplying and dividing the *ahargaṇa* derived motion by 19 we have

$$\frac{A \times 31'26''.4 \times 19}{19} = \frac{A \times 597'}{19}$$

$$= \frac{A \times (597'\ 21'' + 2'\ 39'' - 2'\ 39'')}{19} = \frac{A \times 600'}{19} - \frac{A \times 2'39''}{19}$$

$$= \frac{A \times 600'}{19} - \frac{A \times 2'39'' \times 10}{19 \times 10} = \frac{A \times 10^\circ}{19} - \frac{\frac{A \times 10}{190}}{2'39''}$$

$$= \frac{A^\circ \times 10}{19} - \frac{A' \times 10}{71.698113} = \frac{A \times 10^\circ}{19} - \frac{A' \times 10}{72}$$

(vi) Mean *śīghra kendra* of Budha

Mean *śīghra kendra* of Budha is given by

$$3A^\circ + \frac{3A^\circ}{28} - \frac{A}{38} \text{ min} - (C \times D) \text{ deg.} + K \text{ deg.}$$

where A is the *ahargaṇa* according to *GL.*

In a Mahāyuga, we have

Number of revolutions of Budha's *śīghrocca* = 17937020

Number of civil days = 1577917828

$$\therefore \text{Mean daily motion of Budha's } \textit{śīghrocca} = \frac{17937020 \text{ revns.}}{1577917828} = 4^\circ 5' 32''.31$$

Śīghra anomaly = Budha's *śīghrocca* – Mean Budha

Daily motion of *śīghra* anomaly

= Daily motion of Budha *śīghrocca* – Daily motion of mean Ravi

$$= 4^\circ 5' 32''.31 - 59' 8''.17 = 3^\circ 6' 24''.14$$

Multiplying and dividing the *ahargaṇa* derived motion by 28 we have

$$\frac{A \times 3^\circ 6' 24''.14 \times 28}{28} = \frac{A \times 86^\circ\, 59'\, 15''.92}{28} = \frac{A^\circ \times 87^\circ}{28} - \frac{A \times 44''.08}{28}$$

$$A^\circ \times \left(3^\circ + \frac{3^{\circ''}}{28}\right) - \frac{A \times 44''.08}{28} = 3A^\circ + \frac{3A^\circ}{28} - \frac{A'}{\frac{28 \times 60}{44''.08}}$$

$$= 3A^\circ + \frac{3A^\circ}{28} - \frac{A'}{38.11}$$

The denominator of the last term has been taken as 38 in *GL.*

Note : In other texts *śīghra kendra* is obtained by calculating Budha's *śīghrocca* and mean longitude separately and then subtracting the latter from the former. But the *Graha Lāghavam* directly gives the formula to calculate the *śīghra kendra* for Budha and Śukra (the inferior planets).

(vii) Mean longitude of Guru

The mean longitude of Guru is given by

$$\frac{A^\circ}{12} - \frac{A'}{70} - (C \times D)\,\text{deg} + K\,\text{deg}$$

where A is the *ahargaṇa*, C is the *cakra* and D *dhruvaka* and K the *kṣepaka* respectively. According to *GL*, here $K = 212^\circ 16'$ and $D = 26^\circ 18'$. We have

Number of civil days in a *Mahāyuga* = 1577917828

Number of revolutions of Guru in a *Mahāyuga* = 364220

Mean daily motion of Guru $= \dfrac{364220 \text{ rev.}}{1577917828} = 4'59''.1468$

Mean motion of Guru in a *cakra* $= 4016 \times 4'59''$

$$= \frac{4016 \times 364220}{1577917828} = 333^\circ 42' \Rightarrow 333^\circ 42' - 360^\circ$$

$= -\ 26^\circ 18'$ so that *Dhruvaka* $D = 26^\circ 18'$

Mean motion of Guru for *ahargaṇa* $A = 4'\,59''.1468$

Multiplying and dividing by 12 we get

$$\frac{A \times 4'59''.1468 \times 12}{12} = \frac{A \times 59'\,49''.762}{12} = \left(\frac{60' - 10''.238}{12}\right) A$$

$$= \frac{A^\circ}{12} - \frac{10''.238 \times A}{12} = \frac{A^\circ}{12} - \frac{10.238 \times A}{12 \times 60} = \frac{A^\circ}{12} - \frac{A'}{70.32}$$

Note : The *GL* has taken 70 in the denominator of last term.

(viii) Mean *śīghra kendra* of Śukra

The mean longitude of Śukra's *śīghrakendra* is given by

$$\frac{3A^\circ}{5}+\frac{3A^\circ}{181}-(C\times D)\text{ deg.}+K\text{ deg.}$$

where A is the *ahargaṇa*, C the *cakra* and D *dhruvaka* and K the *kṣepaka* according to *GL.*

We have in Mahayuga

Number of revolutions of Śukra's *śīghrocca* = 7022388

Number of civil days in a *Mahāyuga* = 1577917828

$\therefore$ Mean daily motion of Śukra's *śīghrocca* $= \dfrac{7022388}{1577917828}\times 360^\circ$

$= 1^\circ\ 36'\ 7''.74$

Śīghra anomaly = Śukra's *śīghrocca* – Mean Śukra.

Here the mean Śukra is the same as the mean Sun.

$\therefore$ Daily motion of the *śighra* anomaly

= Daily motion of Śukra's *śīghrocca* – Daily motion of mean Sun

$= 1^\circ\ 36'\ 7''.74\ -\ 59'\ 8''.17\ =\ 36'\ 59''.57$

The mean motion for *ahargaṇa* $A = A\times 36'\ 59''.57$

By multiplying and dividing the *ahargaṇa* by 5 we get

$$\frac{A\times 36'\,59''.57\times 5}{5}=\frac{A\times 184'57''2'''}{5}=\frac{3^\circ 4'57''2'''\times A}{5}$$

$$\frac{A\times 3^\circ}{5}+\frac{A\times 4'57''2'''}{5}=\frac{A\times 3^\circ}{5}+\frac{A(3'+1'57''2''')}{5}$$

$$=\frac{3A^\circ}{5}+\frac{A\,(3'+1'57''2''')^\circ}{5\times 60}=\frac{3A^\circ}{5}+\frac{A\times 3^\circ}{\frac{5\times 60\times 3}{4|57|2}}=\frac{3A^\circ}{5}+\frac{3A^\circ}{181}$$

as taken in the *Grahalāghavam*.

(viii) Mean longitude of Śani

Mean Śani = $\frac{A^\circ}{30}+\frac{A^\circ}{156}-(C\times D)$ deg. + K deg.

where A is the *Ahargaṇa*, C the *cakra* and D *dhruvaka* and K the *kṣepaka* according to *GL*.

Number of civil days in a *Mahāyuga* = 1577917828

Number of revolutions of Śani in a *Mahāyuga* = 146564

Mean daily motion of Śani $=\frac{146564\times 360^\circ}{1577917828}=2'0''.38$

The mean motion for *ahargaṇa* $A = A\times 2'\,0''.38$

Multiplying and dividing the *ahargaṇa* derived motion by 30 we get

$$\frac{A\times 2'0''.38\times 30}{30}=\frac{60'11.6''\times A}{30}=\frac{A^\circ\times 1}{30}+\frac{11'.6\times A}{30\times 60}$$

$$= \frac{A^\circ}{30} + \frac{A'}{\frac{30 \times 60}{11.6}} = \frac{A^\circ}{30} + \frac{A'}{155.172}$$

The second term is considered as $\frac{A'}{156}$ by *GL*

$$\text{i.e., Mean Śani} = \frac{A^\circ}{30} + \frac{A'}{156}$$

CHAPTER 2

Ravi Candra Sphuṭādhikāra

(True Positions of the Sun and the Moon)

2.1. Introduction

In obtaining the mean positions of the Sun and the Moon, it was assumed that these bodies move in circular orbits round the earth with uniform angular velocities. However, by observations it was found that the motions are non-uniform.

The procedure for calculating the major corrections to the mean positions in order to obtain the true positions, is related to the epicyclic theory which is explained in the following section.

2.2. Epicyclic theory and *Mandaphala*

The theory is that while the mean Sun or the Moon moves along a big circular orbit (dotted in Fig.2.1), the actual (or true) Sun or Moon

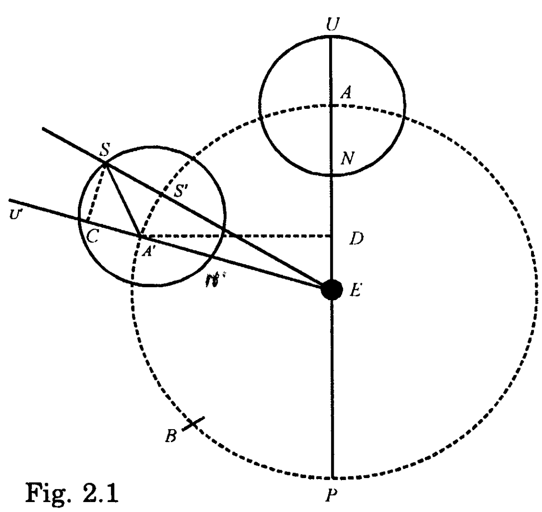

Fig. 2.1
Epicyclic Theory

moves along another smaller circle, called *epicycle*, whose centre is on the bigger circle.

The bigger circle *ABP* with the earth *E* as its centre is called the *kakṣavṛtta.* Let *A* be the position of the mean Sun when the true Sun is farthest from the earth. The line *AEP* is called the apse line (or ***nīcoccarekhā***) and *AE* is the *trijyā* (radius) of this orbit. The epicycle, with *A* as centre and a prescribed radius (smaller than *AE*) is called the *nīcoccavṛtta*. Let the apse line *PEA* cut the epicycle at *U* and *N*. The two points *U* and *N* are respectively called the *mandocca* (apogee)and the *mandanīca* of the Sun. Note that as the Sun moves (as seen from the earth) along the epicycle, he is farthest from the earth when he is at *U* and nearest when at *N*.

The epicyclic theory assumes that as the centre of the epicycle (i.e. mean Sun) moves along the circle *ABP* in the direction of the signs (from west to east) with the velocity of the mean Sun, the true Sun himself moves along the epicycle with the same velocity but in the opposite direction (from east to west). Further, the time taken by the Sun to complete one revolution along the epicycle is the same as that taken by the mean Sun (i.e., centre of the epicycle), to complete a revolution along the orbit.

Now, in Fig 2.1, suppose the mean Sun moves from A to A'. Let $A'E$ be joined cutting the epicycle at U' and N' which are the current positions of the apogee (*mandocca*) and the ***mandanīca.*** While the mean Sun is at A', suppose the true Sun is at S on the epicycle so that $U'\hat{A}'S = U'\hat{E}A$. Join *ES* cutting the orbit (i.e., circle *ABP*) at S'. Then A' is the *madhya* (mean Sun) and S' is ***spaṣṭa*** **(or *sphuṭa*)** Ravi. The difference between the two positions viz, $A'\hat{E}S'$ (or arc $A'S'$) is called the equation of centre (*mandaphala*).

Now, in order to obtain the true position of the Sun, it is necesary to get an expression for the equation of centre which will have to be applied to the mean position.

In Fig. 2.1, SC and $A'D$ are drawn perpendicular to $U'N'E$ and UNE respectively. The arc AA' (or $A\hat{E}A'$), the angle between the mean Sun and the apogee is called the mean anomaly (*mandakendra*) of the Sun.

We have, in the right-angled triangle $A'DE$,

$$\sin A\hat{E}A' = \sin D\hat{E}A' = A'D / A'E$$

so that

$$A'D = R \sin AA' = R \sin m$$

(where $R = A'E$ and m = arc AA') is called *mandakendrajyā*. From the similar right-angled triangles SCA' and $A'DE$, we have

$$SC / SA' = A'D / A'E$$

so that

$$SC = A'D \times SA' / A'E$$

Since SA' and $A'E$ are respectively the radii of the epicycle and the orbit, these are proportional to the circumferences of the two circles; that is,

$SA' / A'E =$ circumference of the epicycle / circumference of the orbit

$\therefore$ SC = (circumference of epicycle / circumference of orbit) $\times A'D$

Taking the circumference of the orbit as 360°, we have

SC = (circumference of the epicycle) $\times$ (***mandakendrajyā***) / 360°.

Now, taking SC approximately the same as $A'S'$, we have

Equation of centre (***mandaphala***)

= (circumference of the epicycle) $\times$ (***mandakendrajyā***) / 360°

$= (r/R)(R \sin m)$

where *R* sin (*m*) the "Indian sine" of the anomaly *m* of the Sun. The maximum value of the equation of centre is *r*, the radius of the epicyle. By observation this can be obtained as the maximum deviation of the Sun's position from the calculated mean position. Note that when the Sun is at his apogee or perigee, the mean and true positions coincide since sin (*m*) is 0 when $m = 0^\circ$ or 180°.

The maximum equation of centre for the sun was observed by Bhāskara II to be $2^\circ 11' 30'' (i.e. 131'.5)$ which is the value of *r*. Therefore,

circumference of the epicycle of the Sun

$$= (131.5 / 3438) \times 360^\circ = 13^\circ.66$$

This value is given by Bhāskara II.

Note : The same epicycle theory is applied to the Moon also. In the case of the Moon, Bhāskara II has given the maximum equation of centre as $302'$. Most texts have taken the epicycles as of varying radii and not fixed.

Table 2.1 : Peripheries of Epicycles of Apsis

Bodies	*Āryabhaṭīyam*	*Khaṇḍa Khādyaka*	*Saura siddhānta* (Varāhamihira)	*Sūrya Siddhānta*
Ravi	13°30′	14°	14°	13°40′ *to* 14°
Candra	31°30′	31°	31°	31°40′ *to* 32°
Kuja	63.0° *to* 81.0°	70°	70°	72° *to* 75°
Budha	22.5° *to* 31.5°	28°	28°	28° *to* 30°
Guru	31.5° *to* 36.5°	32°	32°	32° *to* 33°
Śukra	9.0° *to* 18.0°	14°	14°	11° *to* 12°
Śani	40°.5 *to* 58.5°	60°	60°	48° *to* 49°

From Table 2.1 we notice that the *Khaṇḍa Khādyaka* of Brahmagupta and the *Saurasiddhānta* (as given by *Varāhamihira*) take the epicycles as of constant periphery (and hence radius). The *Āryabhaṭīyam* except for the Sun and the Moon and the later *Sūrya Siddhānta* take them as varying between two limits.

2.3 Bhujāntara correction

The true midnight of a place differs from the mean midnight by an amount of time called "equation of time". The equation of time is caused by

(i) the eccentricity of the earth's orbit; and

(ii) the obiliquity of the ecliptic with the celestial equator.

The correction to the longitude of a planet due to the part of the equation of time caused by the eccentricity of the earth's orbit is called *bhujāntara*. The other correction caused by the obliquity of the ecliptic is called *udayāntara*.

While all the *siddhāntic* texts have considered the *bhujāntara* correction the other correction-*udayāntara*-was first introduced by *Śrīpati* (about 1025 AD) and later followed by Bhāskara-II and others.

We shall discuss the *bhujāntara* correction which is mentioned in the *Sūrya Siddhānta.* The effect of eccentricity of the earth's orbit in the equation of the centre (*mandaphala*) of the sun is converted into *time* at the rate of 15° per hour or 6° per *ghatikā*. This rate of conversion is due to the fact that the earth rotates about its axis at the rate of 360° in 24 hours (or 60 *ghaṭikās.*). The resulting amount in time unit is the *equation of time* caused by the eccentricity of the earth's orbit. Thus, the equation of time (due to the eccentricity)

= [(Equation of cente of the Sun) / 15] hours

= [(Equation of centre of the Sun) / 6] ***ghaṭikās***

Now, to get the *bhujāntara* correction for the Sun or the Moon or any other planet, the equation of time obtained above must be multiplied by the motion of the planet per hour or per ***ghaṭikā*** as the case may be. That is,

Bhujāntara correction for a planet

= [(Equation of time in hours] × (Daily motion)/24]

= [(Equation of centre of the Sun)/15] × [Daily motion)/24]

= [(Equation of centre of the Sun)] × (Daily motion)/360

where the factors in the numerator are in degrees and the daily motion is that of the planet. If the time unit used is *ghaṭikā*, then

Bhujāntara correction

= (Eqn. of time in *ghaṭikā*) × (Daily motion)/60]

= [(Eqn. of centre of the Sun in degrees)/6] × (Daily motion of the planet)/60]

= [Eqn. of centre of the Sun in degrees] × [Daily motion of the planet/360] where the daily motion of the planet is in degrees and hence the *Bhujāntara* correction is also in degrees.

However, if the daily motion of the planet is in minutes of arc, then

Bhujāntara correction in degrees

= (Eqn. of centre of the Sun in degrees) $\times$ (Daily motion of the planet)/21600

Further, the *bhujāntara* correction is additive or subtractive according as the equation of centre of the Sun is so.

For example, in the case of Moon, its mean daily motion is $13^{\circ}.176352$ or $790'.58112$.

Therefore, we have (mean) *bhujāntara* correction

= (Eqn. of centre of the Sun) $\times 790'.58112/21600'$

= Eqn. of centre of the Sun/27.321674

Note : Brahamagupta takes the denominator approximately as 27 in his *Khaṇḍakhādyaka.*

It is important to note that to obtain the actual (and not the mean) *bhujāntara* correction of a planet, we have to use the true daily motion of the planet for the given day.

Example : Find the *bhujāntara* correction for the longitudes of the Sun and the Moon given that on a certain day

True daily motion of the Sun	:	$59'.65$
True daily motion of the Moon	:	$855'.23$
Equation of centre of the Sun	:	$+2^{\circ}7'32'' = 127'.53$

Therefore, we have

(i) True *bhujāntara* correction of the Sun

= (Eqn. of centre of the Sun) $\times$ (Daily motion of the Sun)/21600

$= 127'.53 \times 59'.65 / 21600' = 0'.3521835 = 0'21''$

Since the equation of centre of the Sun is additive, the *bhujāntara* correction is also additive.

(ii) True *bhujāntara* correction of the Moon

= (Eqn. of centre of the Sun) × (Daily motion of the Moon)/21600

$= 127'.53 \times 855'.23 / 21600' = 5'.0494205 = 5'3''$

Here also, the correction is additive since the Sun's equation of centre is so.

2.4 Further corrections for the Moon

We have applied so far an important correction namely, the equation of centre (*mandaphala*), to the mean position of the Moon. Besides this correction, the other two corrections applied viz, *deśāntara* and *bhujāntara* are mainly to get the true position of the moon at the true local midnight at the place of observation.

However, to get the true position of the Moon at least two more important corrections will have to be applied, of course, ignoring other minor corrections due to planetary perturbations. These are :

(i) **Evection** $= (15/4)' m e \sin(2\xi - \phi) = 76' 26'' \sin(2\xi - \phi)$

where *m* is the ratio of mean daily motions of the Sun and the Moon, *e* is the eccentricity of the Moon's orbit, $\xi = (M - S)$, the elongation of the Moon from the Sun and $\phi = M - P$, the mean anomaly of the moon (*P* being the Moon's perigee).

(ii) **Variation** = $39'30'' \sin(2\xi)$

In the above formulae, *S* and *M* are respectively the mean longitudes of the Sun and the Moon. The *Sūrya Siddhānta*, being an earlier text, does not mention these corrections. However, Mañjula (932 A.D.), text, does not mention these corrections. However, Mañjula (932 A.D.), Bhākara-II (1150 AD) and later Indian astronomers have recognized the *evection* correction in addition to the equation of centre. Besides these, the famous Orissa astronomer **Sāmanta Candraśekhara Simha** discovered independently a fourth correction called *annual equation*. According to Candraśekhara,

(iii) **Annual equation** = $(11'27''.6)$ sin(Sun's anomaly from apogee)

In fact, Candraśekhara's coefficient viz., $11'27''.6$ is very close to the known modern value. Tycho Brahe took the coefficient wrongly as $4'30''$.

Śloka 1 : In this *śloka* the *mandakendra* (anomaly) of a planet and the *bhuja* of *mandakendra* are given as follows :

Mandakendra (*MK*) = *Mandocca* of the planet – Mean planet

(i) If the *mandakendra* of the planet is less than 3 *rāśis* (i.e., $0° < MK < 90°$), then *MK* itself is the *bhuja* i.e., *Bhuja* = *MK*.

(ii) If the *mandakendra* is greater than 3 *rāśis* and less than 6 *rāśis* (i.e., $90° < MK < 180°$) then

Bhuja = 6 *rāśis* – *MK* = $180° - MK$

(iii) If *mandakendra* is greater than 6 *rāśis* and less than 9 *rāśis* (i.e., if $180° < MK < 270°$) then *Bhuja* = *MK* – 6 *rāśis* $= MK - 180°$

(iv) If *mandakendra* is greater than 9 *rāśis* and less than 12 *rāśis* (i.e., $270° < MK < 360°$) then

Bhuja = 12 *rāśis* – *MK* = $360° - MK$

i.e., *koṭi* = *3 rāśis* – *bhuja* = 90° – *bhuja*

12 *rāśis* (or 360°) have been divided into four *pādas* (quadrants) each containing 3 *rāśis* (90°). The I and III quadrants are *viṣamapādas* (odd quadrants) and II and IV are called *samapāda* (even quadrants).

Mandocca of the Sun = $78^\circ = 2^R 18^\circ$ (taken as fixed).

***Śloka* 2** : The method of finding the *mandaphala* of the Sun is explained as follows:

(i) Find the *mandakendra* (*MK*) of the Sun.

(ii) Find the *bhuja* of *MK* (hereafter denoted by *BMK*).

(iii) Subtract $\frac{bhuja}{9}$ from 20 i.e., obtain (20 – *BMK*/9)

(iv) Multiply the results of step (iii) and $\frac{BMK}{9}$

(v) Divide the result of (iv) by 9.

(vi) Subtract the result of step (v) from 57.

(vii) Express the results of step (vi) and step (iv) as *vīkalās* (seconds of arc) and divide the result of step (iv) by that of step (vi)

The result is the *mandaphala* of the Sun.

i.e., ***Mandaphala* of the Sun** = $$\frac{\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}}{57-\left\{\frac{\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}}{9}\right\}}$$

Note :

(i) If the *mandakendra* is within 6 *rāśis* from *Meṣa* (i.e., $0^\circ < MK < 180^\circ$) then the *mandaphala* is additive.

(ii) If the *mandakendra* is within 6 *rāśis* from *Tulā* ($180^\circ < MK < 360^\circ$) then the *mandaphala* is subtractive.

Rationale for the *mandaphala* of the Sun

Śrīpati Bhaṭṭa's expression for the *jyā* of the *mandakendra* is as follows :

दोः कोटिभागरहिताऽभिहताः खनागचन्द्रास्तदीयचरणोनशरार्कदिग्भिः ।
ते व्यासखण्डगुणिता विहृताः फलन्तु ज्याभिर्विनाऽपि भवतो भुजकोटिजीवे ॥

doḥ koṭibhāgarahitā bhihatāḥ khanāgacandrāstadīya-caraṇonaśarārkadigbhiḥ /
te vyāsakhaṇḍaguṇitā vihṛtāḥ phalntu jyābhirvinā'pi bhavato bhujakoṭijīve //

i.e., *Mandakendra jyā* = $\dfrac{(180 - MK)\, MK \times 120}{10125 - \dfrac{(180 - MK)}{4}\, MK}$

where *MK* stands for the *bhuja* of the *mandakendra*

i.e., *Jyā* (*MK*) = $\dfrac{(180 - MK)\, MK \times 480}{40500 - (180 - MK)\, MK}$

$$= \frac{\left(\dfrac{180 - MK}{9}\right)\dfrac{MK}{9} \times 480}{\dfrac{40500}{9 \times 9} - \left(\dfrac{180 - MK}{9}\right)\dfrac{MK}{9}} \quad \text{(dividing by } 9 \times 9\text{)}$$

$$= \frac{\left(20 - \frac{MK}{9}\right)\frac{MK}{9} \times 480}{500 - \left(20 - \frac{MK}{9}\right)\frac{MK}{9}} \qquad \ldots\ldots. \text{(i)}$$

The above derivation is based on the significant and unique formula of Bhāskara I (c.629 AD) - see Appendix-3

Now, according to the *Grahalāghavam*,

the *parama mandaphala* (i.e., maximum *mandaphala*) of the Sun

$$= \frac{125^\circ}{57} \approx 2^\circ 11' 34''.$$

$\therefore$ *Mandaphala* of the Sun = $\frac{125}{57} \times \frac{mandakendrajy\bar{a}}{120}$

$$= \frac{125}{57 \times 120} \times \left[\frac{\left(20 - \frac{MK}{9}\right)\frac{MK}{9} \times 480}{500 - \left(20 - \frac{MK}{9}\right)\frac{MK}{9}}\right] \text{ using (i)}$$

$$= \frac{\frac{125}{57}\left[\left(20 - \frac{MK}{9}\right)\frac{MK}{9} \times 4\right]}{500 - \left(20 - \frac{MK}{9}\right)\frac{MK}{9}}$$

$$= \frac{\left(\frac{500}{57}\right)\left[\left(20 - \frac{MK}{9}\right)\frac{MK}{9}\right]}{500 - \left(20 - \frac{MK}{9}\right)\frac{MK}{9}}$$

$$= \frac{\left(20 - \frac{MK}{9}\right)\frac{MK}{9}}{\frac{500}{\left(\frac{500}{57}\right)} - \frac{\left(20 - \frac{MK}{9}\right)\frac{MK}{9}}{\left(\frac{500}{57}\right)}}$$

$$= \frac{\left(20 - \frac{MK}{9}\right)\frac{MK}{9}}{57 - \frac{\left(20 - \frac{MK}{9}\right)\frac{MK}{9}}{8.7719298}}$$

i.e., ***Mandaphala* of the Sun** $\approx \frac{\left(20 - \frac{MK}{9}\right)\frac{MK}{9}}{57 - \left\{\frac{\left(20 - \frac{MK}{9}\right)\frac{MK}{9}}{9}\right\}}$

Note : The exact formula for the *mandaphala* of the Sun is $\frac{a}{R}$ sin m where $R = 360°$, a the periphery of the *manda* epicycle (degrees) and m is the Sun's anomaly (from the apogee, *mandocca*).

When $m = 90°$, sin m = 1 $\therefore$ *parama mandaphala* = $\frac{a}{R}$ *radian*

According to the *Grahalāghavam,*

$$\frac{a}{R}\times\frac{180}{\pi}=\frac{125}{57} \text{ degrees}$$

$$\Rightarrow \frac{a}{2}\times\frac{1}{\pi}=\frac{125}{57} \text{ (taking } R=360^{\circ}\text{)}$$

$$\Rightarrow a=\frac{2\pi\times 125}{57} \textit{ degrees}$$

$$\Rightarrow a=13^{\circ}.778915$$

According to the book, *Ancient Indian Astronomy* by S. Balachandra Rao, the value of a for the Sun is between $11^{\circ}.80781$ and $12^{\circ}.31284$, based on the eccentricity of the earth's orbit.

Example : *Mandocca* of the Sun = $2^{R}\,18^{\circ}$(fixed)

Mean Sun = $1^{R}\,4^{\circ}13'42''$

Mandakendra of the Sun (*MK*) = *Mandocca* – Mean Sun

$= 2^{R}\,18^{\circ} - 1^{R}\,4^{\circ}13'42''$

$= 43^{\circ}46'18''$

Since $0 < MK = (43^{\circ}46'18'') < 90^{\circ}$,

Bhuja of *MK* = $43^{\circ}46'18''$ denoted by *BMK.*

$$\therefore \; \textit{Mandaphala} \text{ of the Sun} = \frac{\left(20 - \frac{BMK}{9}\right)\frac{BMK}{9}}{57 - \left\{\frac{\left(20 - \frac{BMK}{9}\right)\frac{BMK}{9}}{9}\right\}}$$

$$= \frac{73.616558}{48.820382} = 1^\circ.5079062 = \mathbf{1^\circ 30' 28''}$$

Mandaphala is additive because $0^\circ < MK \; (= 43^\circ 46' 18'') < 180^\circ$.

Note : According to the formula, using the sine function, we have

Mandaphala of the Sun = $\frac{a}{R} \sin m$ (where $a = 14^\circ$, $R = 360^\circ$, $m = 43^\circ 46' 18''$)

$$= \frac{14}{360} \sin (43^\circ 46' 18'') = 0.0269027 \text{ radian} = \mathbf{1^\circ 32' 29''}$$

We note that the *mandaphala* of the Sun, without the sine-function, is very close to that obtained using the trigonometric function. The error is just $2'$.

Now,

Mandaphala corrected Sun = Mean Sun + *Mandaphala*

$$= 1^R \, 4^\circ 13' 42'' + 1^\circ 30' 28''$$

$$= 1^R \, 5^\circ 44' 10''$$

After the *manda* correction, the *Grahalāghavam* adopts the *cara* correction explained in *śloka* 6 of this chapter.

Śloka **3** : In this *śloka* the method of finding the *mandaphala* of the Moon is given as follows :

(i) Find the *Mandakendra* (*MK*) of the Moon.

(ii) Subtract $\frac{MK}{6}$ from 30.

(iii) Multiply the result of step (ii) by $\frac{MK}{6}$.

(iv) Divide the result of step (iii) by 20 and subtract the quotient from 56.

(v) Divide the result of step (iii) by that of step (iv).

The result gives the *mandaphala* of the Moon.

i.e., *Mandaphala* of the Moon = $$\frac{\left(30-\frac{MK}{6}\right)\frac{MK}{6}}{56-\left\{\frac{\left(30-\frac{MK}{6}\right)\frac{MK}{6}}{20}\right\}}$$

Here, *MK* is actually the *bhuja* of the *mandakendra*.

Note : In the case of the Moon, the *Grahalāghavam gives* three corrections in the order, *cara*, *bhujāntara* and *deśāntara*. After obtaining the Moon with these three corrections, finally the *manda* correction (i.e., the equation of centre) is applied.

Rationale for the *mandaphala* of the Moon

We have *mandakendra jyā* = $\dfrac{(180-MK)\,MK \times 480}{40500-(180-MK)\,MK}$

according to Śrīpati Bhaṭṭa.

Dividing the numberator and the denominator by 6×6,

$$Jy\bar{a}\ (MK) = \frac{\left(\dfrac{180-MK}{6}\right) MK \times \dfrac{480}{6}}{\dfrac{40500}{6\times 6} - \left(\dfrac{180-MK}{6}\right)\dfrac{MK}{6}} \qquad \text{..........} \ (i)$$

According to the *Grahalāghavam* the maximum *mandaphalam,*

Parama mandaphala of the Moon = 5°

∴ *Mandaphala* of the Moon = $\dfrac{5 \times mandakendrajy\bar{a}}{120}$

$$= \frac{5 \times \left(30 - \dfrac{MK}{6}\right)\dfrac{MK}{6} \times 480}{120 \times \left[1125 - \left(30 - \dfrac{MK}{6}\right)\dfrac{MK}{6}\right]} \qquad \text{using (i)}$$

$$= \frac{\dfrac{2400}{120}\left[\left(30 - \dfrac{MK}{6}\right)\dfrac{MK}{6}\right]}{1125 - \left(30 - \dfrac{MK}{6}\right)\dfrac{MK}{6}}$$

$$= \frac{20\left[\left(30-\frac{MK}{6}\right)\frac{MK}{6}\right]}{1125-\left(30-\frac{MK}{6}\right)\frac{MK}{6}}$$

$$= \frac{\left(30-\frac{MK}{6}\right)\frac{MK}{6}}{\frac{1125}{20}-\frac{\left(30-\frac{MK}{6}\right)\frac{MK}{6}}{20}}$$

$$= \frac{\left(30-\frac{MK}{6}\right)\frac{MK}{6}}{56.25-\left\{\frac{\left(30-\frac{MK}{6}\right)\frac{MK}{6}}{20}\right\}}$$

i.e., ***Mandaphala* of the Moon** $\approx \dfrac{\left(30-\frac{MK}{6}\right)\frac{MK}{6}}{56-\frac{\left(30-\frac{MK}{6}\right)\frac{MK}{6}}{20}}$

This is the expression given by the *Grahalāghavam*.

Note : The usual formula for the *mandaphala* of the Moon is $\frac{a}{R}\sin m$.

When $m = 90°$, $\sin m = 1$ so that max. *mandaphala* = $\frac{a}{R}$ *radian*

Now, according to the *Grahalāghavam*. the maximum *mandaphala*.

Parama mandaphala of the Moon = $5°$

i.e, $\frac{a}{R}\times\frac{180}{\pi}=5 \quad\Rightarrow\quad \frac{a}{360}\times\frac{180}{\pi}=5$

$\Rightarrow\quad a=2\pi\times 5 \quad\Rightarrow\quad a = \mathbf{31°.415927}$

According to the book *Ancient Indian Astronomy* by S. Balachandra Rao the value of *a* for the Moon lies between $36°.8085$ and $42°.22932$ based on the eccentricity of the Moon's orbit.

***Śloka* 4** : The method of finding true daily motions of the Sun and the Moon is given in this *śloka* as explained below.

I. True daily motion of the Sun :

We first find the '*gatiphalam*' as follows :

(i) Find the *bhuja* of the *mandakendra* (*MK*) of the Sun in degrees.

(ii) Find *koṭi* of *MK*.

(iii) Divide *koṭi* by 20.

(iv) Subtract the result of step (iii) from 11.

(v) Multiply the results of step (iii) and step (iv).

(vi) Divide the result of step (v) by 13. This gives the '*gatiphalam*' of the Sun in *kalās* (minutes of arc) i.e., *Gatiphalam* of the Sun $= \dfrac{\left[\left(11-\dfrac{koṭi}{20}\right)\left(\dfrac{koti}{20}\right)\right]}{13}$

If *the mandakendra* of the Sun is within 6 *rāśis* from *Karka* (i.e., if $90° < MK < 270°$) then add *'gatiphalam'* to the mean daily motion of the Sun to get true daily motion.

If *mandakendra of* the Sun is within 6 *rāśis* from *Makara* (i.e., if *MK* is in IV or I quadrant), subtract *'gatiphalam'* from the mean daily motion to get true daily motion of the Sun.

Example :

Mandakendra (*MK*) of the Sun = $43°46'18''$ ∴ *bhuja* = $43°\ 46'\ 18''$

Koṭi of *MK* $= 90° - bhuja$

$= 90° - 43°46'18'' = 46°13'42''$

∴ *Gatiphalam* of the Sun $= \dfrac{\left(11-\dfrac{koti}{20}\right)\dfrac{koṭi}{20}}{13}$

$$= \frac{\left(11-\dfrac{46°13'42''}{20}\right)\dfrac{46°13'42''}{20}}{13}$$

$= 1'.5448413 = 1'\ 32''\ 41'''.43$

Since *MK* of Ravi is in the first quadrant (i.e., within 6 *rāśis* from *Makara*) we have to subtract *'gatiphalam'* from the mean daily motion of the Sun

$\therefore$ True daily motion of the Sun = Mean motion - *'gatiphalam'*

$$= 59' 8'' - 1' 32'' 41'''.43 = 57' 35'' 18'''.57$$

Remark : The modern formula for correction to get the true motion of the Sun is given by

$$\Delta n = \frac{-b}{R} \cos M \left(\frac{\Delta M}{\Delta t}\right)$$

where $b = 14$, $R = 360$ and the mean daily motion of the Sun,

$\frac{\Delta M}{\Delta t} = 59'08''$ (based on the classical sine formula).

$$\therefore \ \Delta n = \frac{-14}{360}\left[\cos(43^\circ 46' 18'')\right](59'08'')$$

$$= -1'39''54'''$$

Remark : The *gatiphalam* of the Sun, obtained without the use of trigonometric function differs from that using cos M only by $7''$ of arc.

II. True daily motion of the Moon

We first find *'gatiphalam'* as follows :

(i) Find the *mandakendra MK* of the Moon and its *bhuja*.

(ii) Find *koṭi* of the *mandakendra* (*MK*); *koṭi* = 90° – *bhuja*.

(iii) Divide *koṭi* by 20.

(iv) Subtract the result of step (iii) from 11.

(v) Multiply the results of steps (iv) and (iii).

(vi) Multiply the result of step (v) by 2.

(vii) Divide the result of step (vi) by 6.

(viii) Add the results of steps (vi) and (vii).

i.e., *Gatiphalam* of the Moon $= \left[\left(11 - \frac{koṭi}{20}\right)\frac{koṭi}{20}\right]\left(2 + \frac{2}{6}\right)$ and

True daily motion of the Moon = Mean motion of the Moon $\pm$ *gatiphalam*

If *MK* is within 6 *rāśis* from *Karka* (i.e., if the *mandakendra* (*MK*) is in II or III quadrant) add *gatiphalam* to the mean daily motion.

If *MK* is within 6 *rāśis* from *Makara* (i.e., if *MK* is in I or IV quadrant), subtract *gatiphalam* from the mean daily motion.

Example :

Mandakendra of the Moon $MK = 3^R\, 25°12'17'' = 115°12'17''$

Bhuja of $MK = 180° - 115°\, 12'\, 17'' = 64°47'43'' \equiv BMK$

$\therefore$ *Koṭi* of $MK = 90° - BMK = 90° - 64°47'43'' = 25°12'17''$

$\therefore$ *Gatiphalam* of the Moon $= \left[\left(11 - \frac{Koti}{20}\right)\frac{Koti}{20}\right]\left(2 + \frac{2}{6}\right)$

$$= \left[\left(11 - \frac{25°12'17''}{20}\right)\frac{25°12'17''}{20}\right]\left(2 + \frac{2}{6}\right)$$

$$= 28'.640272 = 28'38''24''' \qquad .... (1)$$

Since $MK = 115°12'17''$ is in the II quadrant the *gatiphalam* is additive. Therefore, we have

true daily motion of the Moon = Mean motion of the Moon + *gatiphalam*

$$= 790'35'' + 28'38''24'''$$

$$\approx \mathbf{819'13''}$$

Remark : From the classical sine formula for the Moon, we have

$$\frac{\Delta n}{\Delta t} = \frac{b}{R}\cos M\left[\frac{\Delta L}{\Delta t} - \frac{\Delta A}{\Delta t}\right]$$

where A = Moon's apogee and its mean daily motion $\frac{\Delta A}{\Delta t} = 6'41''$

$b = 31°$, $R = 360°$, $\frac{\Delta L}{\Delta t} = 790'35''$, the Moon's mean daily motion

$M = BMK = 64°47'43''$, the bhuja of Mandrakendra

$$\frac{\Delta n}{\Delta t} = \frac{31}{360}\cos(64°47'43'')[790'35'' - 6'41'']$$

$$= 28'44''46''' \qquad \text{....... (2)}$$

We observe that the difference between (1) and (2) is less than $6''$.

***Śloka* 5** : In this *śloka* finding *palabhā* and *carakhaṇḍas* of a given place are explained as follows :

I. To find *palabhā* :

The day on which the true longitude of the *sāyana* (tropical) Sun becomes $0°0'0''$ (i.e. equinoctial day), determine the shadow of a 12 *aṅgula* long cone (*śaṅku*) placed on a plane surface at the noon. This shadow length is called *palabhā* or *akṣabhā*. In other words, it is the shadow of the *śaṅku* at the equinoctial noon.

II. To find *carakhaṇḍas* :

Multiply *palabhā* by 10, 8 and $\frac{10}{3}$ respectively. We get three *khaṇḍas*. They are called *carakhaṇḍas* which are in *vikalās* (seconds of arc).

Example : *Palabhā* of Almora [Long. $79^\circ E\, 40'$; Lat. $29^\circ N\, 36'$;] = $6|47$ *aṅgulas*

The three *carakhaṇḍas* are

$$(6|47)\times 10\,,\quad (6|47)\times 8 \quad \text{and} \quad (6|47)\times\frac{10}{3}$$

$$= 67.8333,\quad 54.2666 \quad \text{and} \quad 22.61111$$

$$\approx \mathbf{68''},\quad \mathbf{54''} \quad \text{and} \quad \mathbf{23''}$$

Remark : The latitude ϕ of a place can be obtained from the *palabhā* of the place as follows :

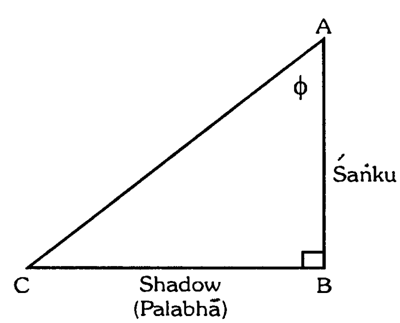

Fig. 2.2

In the right-angled Δ^{le} *ABC*, we have

$$\tan\phi = \frac{palabh\bar{a}}{sanku} = \frac{BC}{AB} = \frac{palabh\bar{a}}{12}$$

$\therefore$ *Palabhā* = 12 tan ϕ *aṅgulas*

From this we have

$$\phi = \tan^{-1}[Palabh\bar{a}/12]$$

For example, the *palabhā* of Almora ($\phi = 29°36'$) is 12 tan ϕ

$= 12 \tan(29°36') = 6^{ang}\ 49^{prat}$.

Note : 1 *aṅgula* = 60 *Pratyaṅgulas.*

***Śloka* 6** : This *śloka* gives the method of finding *cara* correction.

It is as follows :

(i) Find *sāyana* Ravi (tropical Sun).

(ii) Find *bhuja* of the *sāyana* Sun. Express it in *rāśis, amśas, kalās* and *vikalās.*

(iii) The number which represents *rāśi* gives the number of elapsed *khaṇḍas.*

(iv) Find the *bhogya khaṇḍa* (i.e., *khaṇḍa* to be covered) and multiply it by the remaining *amśas* etc.

(v) Divide the result of (iv) by 30.

(vi) Add the result of (v) to the elapsed *khaṇḍas*.

This gives the *cara in seconds* (″) of arc .

Note : During the day time

(i) if *sāyana* Ravi (*SR*) is within 6 *rāśis* from *Meṣa* (i.e., $0^\circ < SR < 180^\circ$), then *cara* is negative.

(ii) if *sāyana* Ravi (*SR*) is within 6 *rāśis* from *Tulā* (i.e., $180^\circ < SR < 360^\circ$) *cara* is positive.

During the night time

(i) *cara* is positive if $0^\circ < SR < 180^\circ$

(ii) *cara* is negative if $180^\circ < SR < 360^\circ$.

Example : *Palabhā* of Kāśī : 5 *aṅgulas* 45 *pratyaṅgulas* $= 5|45$ *aṅgulas*

The three *carakhaṇḍas* are

$$(5|45)\times 10\,, \quad (5|45)\times 8 \quad \text{and} \quad (5|45)\times\frac{10}{3}$$

$$= \mathbf{57''}, \quad \mathbf{46''} \quad \text{and} \quad \mathbf{19''}$$

Suppose *sāyana* Ravi = $1^R\,23^\circ 54' 10''$, *bhuja* = $1^R\ 23^\circ\ 54'\ 10''$

The number 1 in the *raśi* position implies that the number of elapsed *khaṇḍas* = 1

$\therefore$ Elapsed *khaṇḍa* = $57''$

Bhogya khaṅḍa = $46''$

Remaining *amśas* etc. = $23°54'10''$

Now, $\frac{23°54'10''}{30°}\times 46'' = \frac{1099}{30} \approx 36''$

∴ *Cara* = Elapsed *khaṇḍa* + $36''$

= $\mathbf{57''+36''=93''}$

Since *sāyana* Ravi is within 6 *rāśis* from *Meśa*, the *cara* is negative.

∴ *cara* corrected *nirayaṇa* Sun = $1^{R}\,5°44'10''-93''$

= $\mathbf{1^{R}\,5°42'37''}$

(the *ayanāmśa* = $18°\ 10'$)

Remark : The latitude of Kaśī (Vāraṇāsī) is $\phi = 25°N\,19'$.

Therefore, *palabhā* = $12\tan\phi = 12\tan 25°19' = 5|40$ *aṅgulas*

However, commentator Viśvanātha has taken its value as $5|45$ *aṅg.*

The *carakhaṇḍas* are

$$(5|40)\times 10, \quad (5|40)\times 8 \quad \text{and} \quad (5|40)\times\frac{10}{3}$$

i.e., $56''.766$, $45''.41$ and $18''.92$

The elapsed *khaṇḍa* = $56''.766$

Bhogyakhaṇḍa = $45''.41$

Remaining *amśas* etc. = $23°54'10''$

Now, $\frac{23°54'10''}{30°} \times 45''.41 = 36''.181$

$\therefore$ *Cara* = Elapsed *khaṇḍa* + $36''.181$

$= 56''.77 + 36''.181 = 92''.95$

Therefore , *cara* corrected *nirayaṇa* Sun

$= 1^R\ 5°44'10'' - 92''.95 \approx 1^R\ 5°\ 42'\ 37''$.

***Śloka* 7** : This *ślōka* explains the method of applying the *cara*, *bhujāntara* and *deśāntara* corrections to the Moon.

(1) *Cara* correction for the Moon :

Subtract $\left(\frac{2 \times Cara}{9}\right)'$ from the mean position of the Moon to get the *cara* corrected Moon. Here, *cara* must be taken in minutes (′) of arc.

i.e., *Cara* corrected Moon = Mean Moon − $\left(\frac{2 \times Cara}{9}\right)'$

(2) *Bhujāntara* correction for the Moon : (using *mandaphala* of the Sun).

Divide the *mandaphala* of the Sun by 27. The result will be in degrees, minutes of arc. Add this to the *cara* corrected Moon. That is

Bhujāntara corrected Moon

$= \{Cara \text{ corrected Moon}\} + \frac{mandaphala\ of\ the\ Sun}{27}$

(3) *Deśāntara* correction for the Moon :

Find the *yojanas* of the place from the Ujjayinī meridian (*rekhā*). Divide it by 6 to get the *deśāntara* correction in *kalās* (minutes of arc)(approx.).

Note : In modern astronomy, the equivalent of *deśāntara* is given by $\frac{(\lambda - \lambda_0)}{360}$ day where λ and λ_0 are respectively the longitudes of a place and of the central meridian (in degrees). Let ϕ be the terrestrial latitude of a place so that the R sine of its co-latitude CP is $3438 \sin(90^\circ - \phi)$. The diameter of the earth is taken as 1600 *yojanas* (i.e., about 8000 miles). The maximum (equitorial) circumference, $MC = 2\pi(800) = 1600\,\pi$ *yojanas.*

The corrected circumference CC at a place is given by

$$CC = \frac{MC \cdot CP}{343\,8'}.$$

From these, we get

$$Deśāntara \text{ correction} = \frac{(\lambda - \lambda_0)^\circ}{360^\circ} \cdot \frac{MC \cdot CP}{3438} \cdot \frac{DM}{CC}$$

$$= \frac{(\lambda - \lambda_0)^\circ}{360^\circ} \cdot \left(\frac{MC \cdot CP}{3438}\right) \cdot \frac{DM}{\left(\frac{MC \cdot CP}{3438}\right)}$$

$$= \frac{(\lambda - \lambda_0)^\circ}{360^\circ} \cdot DM$$

where DM is the daily motion of a planet.

Grahalāghavam gives the expression for the *deśāntara* correction in *kalās* as one-sixth of the distance in *yojanas* of a place from Ujjayinī. The commentator Viśvanātha has given the distance of Kāśī from the central meridian as 65 *yojanas*.

Let r = 4000 miles $\approx$ 800 *yojanas* (Fig. 2.3).

We have $\cos\phi = \frac{r_1}{r}$

$\therefore\ r_1 = (800\ \cos\phi)$ *yojanas*

The circumference of the small circle through a place A is given by $2\pi r_1 = (1600\ \pi)\cos\phi$ *yojanas.*

Taking the circumference of the small circle as 360°, we have

$(1600\ \pi\cos\phi)$ *yojanas* $= 360^\circ$

$\therefore\ x$ *yojanas* $= 360\ x / 1600\ \pi\cos\phi$

$$\approx \frac{(0.0716)x}{\cos\phi} \text{ degrees.}$$

$$= \frac{(0.0716)x}{15\ \cos\phi} \text{ in hours}$$

or $$\frac{(0.0716)x}{6\ \cos\phi} \text{ in } ghaṭīs.$$

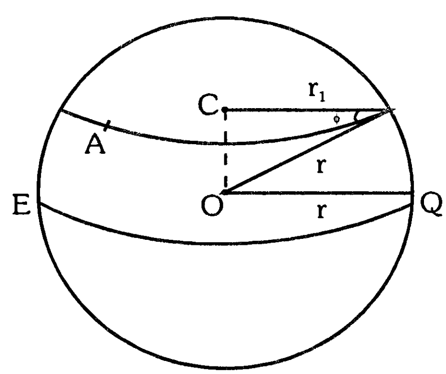

Fig. 2.3

For example, for the Moon, at Kāśī,

$$\textit{Deśāntara} \text{ correction} = \frac{(0.0716)64}{6\cos(25^\circ 36')} \times \frac{790'.35''}{60} \approx 11'9'' \quad \text{.... (1)}$$

Now, according to the *Grahalāghavam*, approximately,

$$\textit{Deśāntara} \text{ correction} = \frac{1}{6} \text{ (distance in } \textit{yojanas}\text{)}$$

$$= \frac{1}{6} \times 64 \approx 10'40'' \quad \text{.... (2)}$$

The difference between (1) and (2) is just about $29''$.

Example : Mean Moon = $6^R\ 20^\circ\ 10'\ 24''$

Candrocca = $10^R\ 14^\circ\ 54'\ 43''$

Cara = $93''$

(i) ***Cara* correction for the Moon :-**

$$\textit{Cara} \text{ corrected Moon} = \text{Mean Moon} - \frac{2 \times \textit{Cara}}{9}$$

$$= 6^R\ 20^\circ\ 10'\ 24'' - \frac{2 \times 93'}{9}$$

(taking *cara* as in minutes of arc).

$$= 6^R\ 20^\circ\ 10'24'' - 20'40''$$

$$= \mathbf{6^R\ 19^\circ\ 49'\ 44''}$$

(ii) *Bhujāntara* correction for the Moon :

Mandaphala of the Sun = $1^\circ\, 30'\, 28''$ (additive)

Bhujāntara corrected Moon = *Cara* corrected Moon +

$$\frac{\textit{mandaphala}\text{ of the Sun}}{27} = 6^R\, 19^\circ\, 49'\, 44'' + \frac{1^\circ\, 30'\, 28''}{27}$$

$$= 6^R\, 19^\circ\, 49'\, 44'' + 0^\circ\, 3'\, 21''$$

$$= \mathbf{6^R\, 19^\circ\, 53'\, 05''}$$

(iii) *Deśāntara* correction for the Moon :

Distance of Kāśī from *Rekhāpura* in *yojanas* = 64

$\therefore$ *Deśāntara* correction $\approx \left(\dfrac{64}{6}\right)' = 10'\, 40''$

$\therefore$ *Deśāntara* corrected Moon

= *Bhujāntara* corrected Moon + *Deśāntara* correction

$= 6^R\, 19^\circ\, 53'\, 05'' - 10'\, 40'' \quad = \mathbf{6^R\, 19^\circ\, 42'\, 25''}$

Now, we have

The Moon after all the three corrections = $6^R\, 19^\circ\, 42'\, 25''$

***Manda* correction for the Moon :**

Candrocca = $10^R\, 14^\circ\, 54'\, 43''$

The thrice corrected Moon = $6^R\ 19°\ 42'\ 25''$

$MK \equiv$ *Mandakendra* = $3^R\ 25°\ 12'\ 18''$

Bhuja of MK = $180° - 3^R\ 25°\ 12'\ 18''$

$= 64°\ 47'\ 42''$

Mandaphala of the Moon = $\dfrac{\left(30 - \dfrac{MK}{6}\right)\dfrac{MK}{6}}{56 - \dfrac{\left[\left(30 - \dfrac{MK}{6}\right)\dfrac{MK}{6}\right]}{20}}$ [from *Śloka* 3]

$$= \frac{\left[30 - \left(\dfrac{64°\ 47'\ 42''}{6}\right)\right]\left(\dfrac{64°\ 47'\ 42''}{6}\right)}{56 - \dfrac{\left[\left(30 - \dfrac{64°\ 47'\ 42''}{6}\right)\dfrac{64°\ 47'\ 42''}{6}\right]}{20}}$$

$= 4°\ 33'\ 38''$

Since $MK = 3^R\ 25°\ 12'\ 18'' < 180°$, the *mandaphala* is additive.

$\therefore$ True Moon = Mean Moon (after three corrections) + *mandaphala*

$= 6^R\ 19°\ 42'\ 25'' + 4°\ 33'\ 38''$

$= 6^R\ 24°\ 16'\ 03''$ = $\mathbf{204°\ 16'\ 03''}$.

***Ślokas* 8 and 9** : These two *ślokas* give the method of finding *tithi*, *nakṣatra*, *yoga* and *karaṇa*.

(1) To find *tithi* :

(i) Subtract the true longitude of the Sun from the true longitude of the Moon.

(ii) Divide the result of step (i) by 12. This (quotient) gives the number of elapsed *tithis*.

(iii) The remainder gives the elapsed part of the running *tithi*. It will be in degrees, min., etc.

(iv) Subtracting the elapsed part of the running *tithi* from 12, we get *bhogyāmśa* (i.e., the portion to be covered) of the running tithi in degrees.

(v) Convert the result of step (iii) into seconds of arc (*vikalās*).

(vi) Convert the result of step (iv) into seconds of arc (*vikalās*).

(vii) Consider the difference between true daily motions of the Moon and the Sun and convert it into seconds of arc (*vikalās*).

(viii) Multiply the result of step (v) by 60 and divide by that of step (vii). This gives the elapsed *ghaṭikās* of the running *tithi*.

(ix) Multiply the result of step (vi) by 60 and divide it by that of step (vii). This gives the *eṣyaghaṭikā* (i.e., *ghaṭikās* to be covered) of the running *tithi*.

Tithi :

In the course of a lunar month, from a new moon to the next new moon, the shape and size of the Moon changes from day to day. On an *amāvāsyā* day (newmoon day) the Moon is invisible as in *A*, (see Fig. 2.4). On the next day, a very thin "crescent" Moon *B* is visible, if the sky is clear, soon after the sunset in the western horizon.

On the succeeding days of the *śukla pakṣa* the brighter or visible part of the Moon keeps on growing until it is half (*C*) between the 7th and

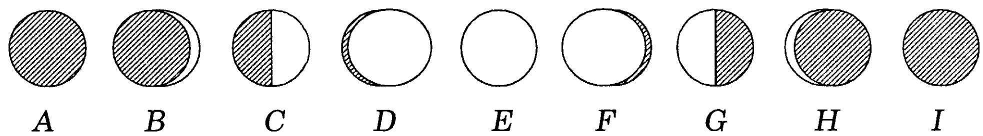

Fig. 2.4

8th day the after the new moon. Also, each day, the Moon keeps moving up in the sky at sunset since it moves farther from the Sun at the rate of about 12° per day. When the phase of the Moon is half, it will be midway in the sky between the eastern and the western horizons. Then, each succeeding day the brighter part of the Moon grows more than half when it is said to be *gibbous*, *D*. At the end of the *śukla pakṣa*, the Moon will be fully visible, *E*, when the *kṛṣṇa pakṣa* (dark half of the month) commences.

In the *kṛṣṇa pakṣa*, the phases of the Moon diminish (or wane) in the reverse order. From the full moon upto the 7th or 8th day, more than half the Moon is bright when it is said to be *gibbous*, *F*. Then, between *saptamī* and *aṣṭamī*, the Moon will be half (*G*). The bright portion of the Moon goes on decreasing till it is *crescent* (*H*) again, a day before the new moon day. At the end of the *kṛṣṇa pakṣa* the Moon is totally invisible on the new moon day, *I*.

The lunar month is divided into 30 parts called *tithis*. The bright half has 15 *tithis* and so too the dark half. The duration of a *tithi* is the time taken by the Moon to move 12° relative to the Sun. The durations of different *tithis* are not equal. In a *pakṣa* (fortnight), starting from the new moon or the full moon, there are 15 *tithis* (Table 2.2) :

Table 2.2 *Tithis*

1. *Pratipat*	6. *Ṣaṣṭī*	11. *Ekādaśī*
2. *Dvitīya*	7. *Saptamī*	12. *Dvādaśī*
3. *Tṛtīya*	8. *Aṣṭamī*	13. *Trayodaśī*
4. *Caturthī*	9. *Navamī*	14. *Caturdaśī*
5. *Pañcamī*	10. *Daśamī*	15. *Pūrṇimā or Amāvāsyā*

Therefore,

$$Tithi = [\text{Longitude of the Moon} - \text{Longitude of the Sun}] / 12^\circ$$

where both the longitudes are in degrees.

The quotient part indicates the number of *tithis* completed during the lunar month, and hence the (quotient + 1) gives the currently running *tithi*. If the running *tithi* is less than 15, then it is of the *śukla pakṣa* (bright fortnight). If the *tithi* is greater than 15, then subtract 15 from that number and the remainder gives the running *tithi* in the *kṛṣṇa pakṣa*.

If the running *tithi* is 15, then it is a *pūrṇimā* (full moon) day and if it is 30, the day is an *amāvasyā* (new moon day).

Note : If the longitude of the Moon is less than that of the Sun then add 360° to avoid the negative sign, and then divide it by 12 to get the *tithi*.

Example :

True Sun = $1^R\ 5^\circ\ 42'\ 37''$

True Moon = $6^R\ 24^\circ\ 16'\ 3''$

(i) Moon – Sun = $6^R\ 24^\circ\ 16'\ 3'' - 1^R\ 5^\circ\ 42'\ 37''$

$= 5^R\ 18^\circ\ 33'\ 26''$

$= 168^\circ\ 33'\ 26''$

(ii) Now, $\dfrac{168^\circ\ 33'\ 26''}{12} = 14 + \dfrac{0^\circ\ 33'\ 26''}{12^\circ}$

Quotient = 14; Remainder = $0^\circ\ 33'\ 26''$

Therefore, the number of elapsed *tithis* is 14 and the running *tithi* is 15th. That is the running *tithi* is *Pūrṇimā*.

(iii) Remainder = $0^\circ\ 33'\ 26''$ implies that the elapsed part of the running *tithi*, viz. *Pūrṇimā* = $0^\circ\ 33'\ 26''$.

(iv) Now, $12^\circ - 0^\circ\,33'26'' = 11^\circ\,26'34''$, the *bhogyāmśa*.

(v) The elapsed part $0^\circ\,33'26'' = 2006''$ (*vikalās*), the *gata*.

(vi) The balance, $11^\circ\,26'34'' = 41194''$ (*vikalās*), the *eṣya*.

(vii) True daily motion of the Moon – True daily motion of the Sun

$$= 819'0'' - 57'36''$$

$$= 761'24'' = 45684''$$

(viii) Elapsed *ghaṭikās* of the running *tithi* $= \dfrac{2006 \times 60}{45684}$

$$= 2^{gh}\ 38^{vgh}$$

(ix) *Eṣyagaṭikā* of the running *tithi* $= \dfrac{41194 \times 60}{45684}\,gh$

$$= 54^{gh}\ 6^{vgh}$$

Thus, we have

Elapsed *ghaṭikās* of *pūrṇimā* = $2^{gh}\ 38^{vgh}$

Eṣya ghaṭikās (i.e., balance *ghaṭikās*) of *pūrṇimā* = 54^{gh} 6^{vgh}

∴Total duration of the *tithi* = 2^{gh} 38^{vgh} + 54^{gh} 06^{vgh} = 56^{gh} 44^{vgh}

(3) To find *nakṣatra* :

(i) Find the true longitude of the Moon for the given time. Convert it into *kalās* (minutes of arc).

(ii) Divide the result ofstep (i) by 800 *kalās*.

(The extent of a *nakṣatra* = $13^\circ\ 20' = 800'$)

(iii) The quotient in step (ii) gives the number of elapsed *nakṣatras* [see Table 2.3].

(iv) The remainder in step (ii) gives the elapsed part of the running *nakṣatra*.

(v) Subtract the result of step (iv) from 800. This gives *eṣya* (i.e., *kalās* to be covered in the running *nakṣatra*). Express the results of (iv) and (v) in *vikalās*.

(vi) Consider the true daily motion of the Moon and express it in *vikalās* (seconds of arc).

(vii) Multiply the result of step (iv) by 60 and divide it by that of step (vi). This gives the (elapsed) *ghaṭikās* of the running *nakṣatra*.

(viii) Multiply the result of step (v) by 60 and divide it by the result of step (vi). This gives *eṣya ghaṭikās* of the running *nakṣatra* (i.e., *ghaṭikās* to be covered in the running *nakṣatra*).

(ix) The sum of the results of steps (vii) and (viii) gives the total duration of the running *nakṣatra*.

Table 2.3 *Nakṣatras* and their range of *nirayaṇa* longitudes

No.	*Nakṣatra*	From	To
1.	*Aśvinī*	0°0′	13°20′
2.	*Bharaṇī*	13°20′	26°40′
3.	*Kṛttikā*	26°40′	40°00′
4.	*Rohiṇī*	40°00′	53°20′
5.	*Mṛgaśira*	53°20′	66°40′
6.	*Ārdrā*	66°40′	80°00′
7.	*Punarvasu*	80°00′	93°20′
8.	*Puṣya*	93°20′	106°40′
9.	*Āśleṣā*	106°40′	120°00′
10.	*Makhā (or Maghā)*	120°00′	133°20′
11.	*Pubba (Pūrva Phālguṇī)*	133°20′	146°40′
12.	*Uttarā (Uttara Phālguṇī)*	146°40′	160°00′
13.	*Hasta*	160°00′	173°20′
14.	*Cittā* (or Citrā)	173°20′	186°40′
15.	*Svātī*	186°40′	200°00′
16.	*Viśākhā*	200°00′	213°20′
17.	*Anurādhā*	213°20′	226°40′
18.	*Jyeṣṭhā*	226°40′	240°00′
19.	*Mūlā*	240°00′	253°20′
20.	*Pūrvāṣādhā*	253°20′	266°40′
21.	*Uttarāṣādhā*	266°40′	280°00′
22.	*Śravaṇa*	280°00′	293°20′
23.	*Dhaniṣṭhā*	293°20′	306°40′
24.	*Śatabiṣaj*	306°40′	320°00′
25.	*Pūrvābhādrā*	320°00′	333°20′
26.	*Uttarābhādrā*	333°20′	346°40′
27.	*Revatī*	346°40′	360°00′

Example :

(i) True *nirayaṇa* Moon = $6^R\ 24°\ 15'\ 03''$ $= 12255'\ 03''$

(ii) $\frac{12255'\ 3''}{800'} = 15 + \frac{255'\ 03''}{800'}$

(iii) The quotient = 15 implies that the number of elapsed *nakṣatras* is 15 and the 16th *nakṣatra*, *Viṣākhā* (see Table 2.3) is running.

(iv) The remainder = $255'03''$ implies that

the elapsed part of the running *nakṣatra* (*Viśākhā*) = $255'03''$ = $15303''$

(v) Now, $800' - 255'03'' = 544'57''$ = $32697''$ (*vikalās*)

(vi) True daily motion of the Moon = $819'0''$ = $49140''$

(vii) We have $\dfrac{15303 \times 60}{49140} = 18.684982^{gh} = 18^{gh}\ 41^{vgh}$

i.e., elapsed (gata) *ghaṭikās* of *Viśākhā* $\approx 18^{gh}\ 41^{vgh}$

(viii) $\dfrac{32697 \times 60}{49140} = 39.923077^{gh} \approx 39^{gh}\ 55^{vgh}$

i.e., balance (*eṣya*) *ghaṭikās* of *Viśākhā* = $39^{gh}\ 55^{vgh}$

(ix) $\therefore$ Duration of *Viśākhā* = $18^{gh}\ 41^{vg} + 39^{gh}\ 55^{vgh}$

$= 58^{gh}\ 36^{vgh.}$

(3) The find *karaṇa* :

(i) Multiply the number of elapsed *tithi* by 2 and divide it by 7. This gives the *karaṇa* starting with *Bava*.

A half *tithi* is called a *karaṇa* i.e., it is of an angular distance of 6° between the Sun and the Moon. There are totally 11 *karaṇas*. Of these 4 are inmovable (*sthira*) and 7 are movable (*cara*). In particular, 4 half-*tithis* viz. the second half of *kṛṣṇa pakṣa caturdaśī*, two halves of *amāvāsyā*, and the first half of the *śukla pratipat* are the *sthira*, *karaṇas* named as *Śakuni*, *Catuṣpāda*, *Nāga* and *Kimstughna* in that order.

Then from the second half of the *śukla pratipat* onwards we have the movable (*cara*) *karaṇas* viz. *Bava, Bālava, Kaulava, Taitila, Gara, Vaṇij* and *Viṣṭi* (or *Bhadra*) in that order repeating the cycle 8 times.

Example 1 : Elapsed *tithi* = 14. Now, $2 \times \frac{14}{7}$ = 4 (quotient), remainder =0 i.e., the last *karaṇa* in the cycle of 7 *karaṇas* (having completed 4 cycles) namely *Viṣṭi* (or *Bhadra*).

Example 2 : March 21, 1990 at $5^h\ 30^m$ a.m. (IST). We have true Moon, $M = 262^\circ 10' 0''$ and the true Sun, $S = 336^\circ 23' 13''$. Therefore, $(M - S) = 262^\circ 10' - 336^\circ 23' 13'' = 285^\circ 46' 47''$ /(after adding 360° to avoid the negative sign).

Now, $(M - S) / 6 = 47.629953$.

Since this number is greater than 7, removing the multiples of 7, we get the remainder 5.629953. The integral part of the number is 5. Counting 5 starting with *Bava* in the list of moving (*cara*) *karaṇas*, we get *Gara* as the running *karaṇa*.

Note : Since the cycle of 7 cara *karaṇas* starts with the *second half* (and not the first half) of *śukla pratipat*, the integral part of the quotient represents the *running* (and the not the elapsed) *karaṇa*.

(4) To find yoga :

Add the *nirayaṇa* longitudes of the Sun and the Moon and convert the sum into *kalās* (minutes of arc). Divide this sum by $800'$. In the thus obtained result the integer part gives the elapsed *yoga* and the fractional part represents the elapsed portion of the running (current) *yoga* (see Table 2.4).

Table 2.4 *Yogas*

There are 27 *yogas* as listed below :

1. *Viṣkambha*	15. *Vajra*
2. *Prīti*	16. *Siddhi*
3. *Āyuṣmān*	17. *Vyatīpāta*
4. *Saubhāgya*	18. *Varīyān*
5. *Śobhana*	19. *Parigha*
6. *Atigaṇḍa*	20. *Śiva*
7. *Sukarmā*	21. *Siddha*
8. *Dhṛti*	22. *Sādhya*
9. *Śūla*	23. *Śubha*
10. *Gaṇḍa*	24. *Śukla*
11. *Vṛddhi*	25. *Brahma*
12. *Dhruva*	26. *Indra*
13. *Vyāghāta*	27. *Vaidhṛta*
14. *Harṣaṇa (or Vaidhṛti)*	

Note : If the sum of the *nirayaṇa* longitudes of the Sun and the Moon (in degrees) exceeds 360°, then subtract 360° from the sum, convert into minutes and then divide that figure by 800.

Example 1 : Sum of the longitudes of the Sun and the Moon = $7^R\,29°\,57'\,40''$ = 14397′40″. Dividing this by 800′, we get 17.9970. This means 17 *yogas* have elapsed and the 18th one viz. *Varīyān* is currently running.

Example 2 : On March 21, 1990 at $5^h\,30^m$ a.m. (IST), the *nirayaṇa* longitude of the Sun, $S = 336°\,23'$ and that of the Moon, $M = 262°\,10'$ (neglecting seconds of arc). $\therefore\ S + M = 598°\,33'$. Since the sum exceeds 360°, subtracting 360°, we get $S + M = 238°\,33'$. Converting into *kalās*, we have $S + M = 14{,}313'$. Dividing this by 800′, we get 17.89125. This means that the 17th *yoga* (*Vyatīpāta*) is over and the 18th one viz. *Varīyān* is running.