CHAPTER 3

PAÑCATĀRĀ SPAṢṬĀDHIKĀRA
(True Positions of Star-Planets)

***Ślokas* 1 to 5** : In the first five *ślokas* the *Śīghraphalas* of the five *tārāgrahas* viz. Kuja, Budha, Guru, Śukra and Śani are explained. The *Śīghrāṅkas* of the five planets at intervals of 15^0 from 0^0 to 180^0 of the *Śīghrakendras* are given as tabulated below :

Table 3.1 *Śīghrāṅkas* of *Tārāgrahas*

Planets	0	1	2	3	4	5	6	7	8	9	10	11	12
Kuja	0	58	117	174	228	279	325	365	393	400	368	249	0
Budha	0	41	81	117	150	178	199	212	212	195	155	89	0
Guru	0	25	47	68	85	98	106	108	102	89	66	36	0
Śukra	0	63	126	186	246	302	354	402	440	461	443	326	0
Śani	0	15	28	39	48	54	57	57	53	45	33	18	0

***Śloka* 6** : In the case of the superior planets viz. Kuja, Guru and Śani, the *Śīghrakendra* = *Śīghrocca* – mean planet ≡ m, say,

where *Śīghrocca* is the mean Sun for all the three planets.

To find the *Śīghrāṅka* of a planet we have the following procedure :

(1) (a) Divide the *Śighrakendra* (if $m < 180^0$) by 15 and find the quotient q (*aṅka*) and let the remainder be r.

(b) If $m > 180^0$ subtract m from 360^0 (i.e. find $360^0 - m$) and divide $(360^0 - m)$ by 15 to get q (*aṅka*) and the remainder r. We call $(360^0 - m)$ as *bhuja* for convenience.

(2) Find the *Śīghrāṅkas*, from Table 3.1, corresponding to the quotient (i.e., from the column headed by q against the planet and also from the next column [i.e., $(q+1)^{\text{th}}$ column] and take the difference between the latter and the former *Śigrāṅkas.*

(3) Multiply the above difference of *Śīghrāṅkas* by the remainder r [obtained in (1) above] and divide by 15.

(4) Add the value obtained in (3) to the *Śighrāṅka* obtained in item (2) above under the column headed by q. The result obtained by dividing by 10 is the required *Śīghraphala*. Note that the difference between two successive columns (headed by $q+1$ and q) may be negative also. In that case, subtract the result of (3) from that of (2) under q.

(5) If the *Śighrakendra m* is less than 180^0, the *Śighraphala* is posi tive. On the otherhand , if $m > 180^0$, then the *Śīghraphala* is negative.

Example : Suppose, on a certain day, mean Kuja = $11^R\, 3°\ 53'25'' \equiv MP$

and mean Sun = $11^R\, 17°\, 32'53'' \equiv MS$ which is the *Śīghrocca* of Kuja.

∴ *Śīghrakendra* (i.e., *Śīghra* anomaly) of Kuja = *Śīghrocca* – mean Kuja

$= MS - MP = 11^R\, 17°\, 32'53'' - 11^R\, 3°\, 53'25'' = 0^s\, 13°\ 39'\ 28'' \equiv SK.$

Here, $SK < 180^0$. We shall now find the *Śīghraphala* :

(1) Dividing SK by 15, quotient $q = 0$ (*aṅka*) and remainder $r =$ $13°\, 39'28''$.

(2) From Table 3.1, the entries against Kuja under the columns headed by q and $q+1$ i.e., 0 and 1 are respectively 0 and 58. Therefore,

the difference between the above *Śīghrāṅkas* = 58 – 0 (i.e., the latter – the former) = 58.

(3) Multiplying the result of (2) by r [from (1)] and dividing by 15, we get $58 \times 13°39'28''/15 = 52°48'36''$.

Adding $52°\ 48'\ 36''$ to the entry under q i.e., to 0 we get $52°\ 48'\ 36''$. Dividing the above *Śīghrānka* by 10, we get $5°16'52'' \equiv SE$, *Śīghraphala*.

5) Since $m < 180^0$, the *Śīghraphala* is additive.

According to the *Grahalāghavam*, we get the true position of a *tārāgraha*, from its mean position, by successively applying two types of corrections viz.the *Śīghra* and the *Manda samskāras* to be explained shortly.

***Ślokas* 7 and 8** : In these two *ślokas* the *Mandāṅkas* of the *tārāgrahas* are given as shown in the following table :

Table 3.2. *Mandāṅkas*

Planets	0	1	2	3	4	5	6
Kuja	0	29	57	85	109	124	130
Budha	0	12	21	28	33	35	36
Guru	0	14	27	39	48	55	57
Śukra	0	06	11	13	14	15	15
Śani	0	19	40	60	77	89	93

According to the *Grahalāghavam*, the *Mandoccas* of the five star-planets are as follows :

Table 3.3. *Mandoccas* of planets

Planets	*Mandoccas*
Kuja	$4^R = 120^0$
Budha	$7^R = 210^0$
Guru	$6^R = 180^0$
Śukra	$3^R = 90^0$
Śani	$8^R = 240^0$

Mandakendra of Planet = *Mandocca* – Mean Planet.

***Śloka* 9** : The method of finding the *Mandaphala* in the case of the five star-planets is explained as follows :

(i) Divide the *bhuja* of the *Mandakendra* by 15 and find the quotient q. Let the remainder be r.

(ii) Find the *Mandāṅka* from Table 3.2 corresponding to the quotient (i.e., from the column headed by q against the planet). Also find the next *Mandāṅka* [i.e., $(q + 1)^{th}$ column] and take the difference between the latter and the former *Mandāṅkas.*

(iii) Multiply the above difference with the remainder r (obtained in step (i) above) and divide by 15.

(iv) If the latter *Mandāṅka* is less than the former *Mandāṅka*, add the result of step (iii) to the latter *Mandāṅka.*

If latter *Mandāṅka* is greater than the former *Mandāṅka*, subtract the result of step (iii) from the latter *Mandāṅka*.

(v) Divide the result of step (iv) by 10 to obtain the *Mandaphala*.

Note : If *Mandakendra* (*MK*) is less than 180^0, the *Mandaphala* is positive. On the other hand, if $MK > 180^0$, then the *Mandaphala* is negative.

Example : Suppose on a certain day,

$$\text{Mean Kuja} = 11^R\, 3°53'25''$$

$$\textit{Mandocca} \text{ of Kuja} = 4^R$$

$$MK,\ \textit{Mandakendra} = \textit{Mandocca} - \text{Mean Kuja}$$

$$= 4^R - 11^R\, 3°53'25''$$

$$= 146°6'35''$$

$$\textit{Bhuja} \text{ of } MK = 180° - 146°6'35''$$

$$= 33°\, 53'\, 25''$$

Now, $\dfrac{33°53'25''}{15} = 2 + \dfrac{3°53'25''}{15}$

Here, quotient $q = 2$, remainder $r = 3°53'25''$

From Table 3.2, we have :

The entry against Kuja under the column headed by 2,

Gatāṅka = 57.

The entry against Kuja under the column headed by $(q + 1)$ i.e., 3,

Eṣyāṅka = 85.

Difference = 85 − 57 = 28.

Now, $\dfrac{28 \times 3^\circ 53'25''}{15} + 57 = 64^\circ 15'42''$

$\therefore$ *Mandaphala* = $\dfrac{64^\circ 15'42''}{10} = 6^\circ 25'34''$

Since $MK < 180^0$, the *Mandaphala* is positive.

***Śloka* 10** : The method of applying the *Mandaphala* and the *Śīghraphala* to attain true position of a planet is explained as follows :

(i) Find the *Śīghraphala* as explained earlier using mean planet (obtained from *Ahargaṇa*). Add (or subtract, as the case is) half of the *Śīghraphala* to (or from) the mean planet. This gives the *half-Śīghra* corrected planet.

(ii) Find the *Mandaphala* for the *half-Śīghra* corrected planet. Add (or subtract, as the case is) *Mandaphala* to (or from) the mean planet (obtained from the *Ahargaṇa*) which gives the *Manda* corrected planet.

(iii) Find the second *Śīghrakendra* using the first *Śīghrakendra* and the *Mandaphala*.

Second *Śīghrakendra* = First *Śīghrakendra* ± *Mandaphala*.

(iv) Find the second *Śīghraphala* using the second *Śīghrakendra*.

(v) Add (or subtract, as the case is) the second *Śīghraphala* to (or from) the *Manda*-corrected planet. This gives the true planet.

Example : We shall find the true positions of the planets for the example considered earlier viz., Monday, May 15, 1612 (G).

I Finding true position of Kuja :

Mean Kuja = $9^R\, 29°55'13''$

Śīghrocca of Kuja = Mean Sun = $1^R\, 4°13'42''$

(1) First *Śīghra* correction :

$SK \equiv$ *Śīghrakendra* of Kuja = *Śīghrocca* – Mean Kuja

$$= 1^R\, 4°13'42'' - 9^R\, 29°55'13''$$

$$= 94°18'29''$$

Now, $\dfrac{\textit{Śīghrakendra}}{15} = \dfrac{94°18'29''}{15} = 6 + \dfrac{4°18'29''}{15}$

so that $q = 6$ and $r = 4°18'29''$.

Therefore, from Table 3.1, we have

Gatāṅka = 325 and *eṣyāṅka* = 365

Their difference = 365 – 325 = 40

Now, $\dfrac{\text{Remainder} \times \text{difference}}{15} = \dfrac{4°18'29''}{15} \times 40 = 11|29|17$

Gatāṅka + $11|29|17 = 325 + 11|29|17$

$$= 336|29|17$$

Therefore, *Śīghraphala* = $\dfrac{336|29|17}{10} \approx 33|38|56$

The *Śīghraphala* is positive since $SK = 94^\circ 18' 29'' < 180^\circ$.

Half-*Śīghraphala* = $\dfrac{33^\circ 38' 56''}{2} = 16^\circ 49' 28''$

Therefore, the *half-Śīghra* corrected Kuja = Mean Kuja + Half-*Śīghraphala*

$$= 9^R 29^\circ 55' 13'' + 16^\circ 49' 28''$$

$$= 10^R 16^\circ 44' 41''$$

(2) *Manda* correction :

Mandocca of Kuja = $4^R 0^\circ 0'$.

Mandakendra (*MK*) of Kuja

= *Mandocca* – Half-*Śīghra* corrected planet

$$= 4^R 0^\circ 0' - 10^R 16^\circ 44' 41'' \quad = 5^R 13^\circ 15' 19''$$

Bhuja of $MK = 180^\circ - 5^R 13^\circ 15' 19'' = 16^\circ 44' 41''$

Now, $\dfrac{16^\circ 44' 41''}{15} = 1 + \dfrac{1^\circ 44' 41''}{15}$

Here, quotient $q = 1$ and remainder $r = 1^\circ 44' 41''$

From Table 3.2 we have

gatāṅka = 29, *eṣyāṅka* = 57

Difference = 57 – 29 = 28

Now, $\frac{28 \times 1|44|41}{15} + 29 = 33|15|22$

$\therefore$ *Mandaphala* = $\frac{33|15|22}{10} = 3|19|32$

Since $MK < 180^0$, the *Mandaphala* M is positive.

Manda corrected Kuja = Mean Kuja + *Mandaphala*

$= 9^R\, 29°55'13'' + 3°19'32'' = 10^R\, 03°14'45''$.

(3) Second *Śīghra* correction :

First *Śīghrakendra* $= 3^R\, 4°18'29''$

Mandaphala = $3°\, 19'\, 32''$

Second *Śīghrakendra* = First SK – *Mandaphala*

$= 3^R\, 4°18'29'' - 3°19'32''$

$= 3^R\, 0°58'57''$

$= 90°58'57''$

Now, $\frac{90°58'57''}{15} = 6 + \frac{0°58'57''}{15}$

i.e., quotient $q = 6$ and remainder $r = 0^\circ 58' 57''$

From Table 3.1, *gatāṅka* = 325, *eṣyāṅka* = 365

Difference = 40 and

$$\frac{40 \times 0^\circ 58' 57''}{15} + 325 = 327|37|12.$$

Therefore, second *Śīghraphala* = $\dfrac{327|37|12}{10} = 32^\circ 45' 43''$

∴ True longitude of Kuja = *Manda* corrected Kuja + Second *Śīghraphala*

$$= 10^R 03^\circ 14' 45'' + 32^\circ 45' 43'' = 11^R 06^\circ 00' 28'' = 336^\circ 00' 28''$$

II Finding true longitude of Budha :

(1) First ***Śighra*** **correction :**

Mean Budha = Mean Sun $= 1^R 4^\circ 13' 42''$

Śīghrakendra of Budha $SK = 1^R 17^\circ 14' 50'' = 47^\circ 14' 50''$

(*SK* of Budha is obtained in *Madhyamādhikāra* i.e., Ch. 1)

Now, $\dfrac{47^\circ 14' 50''}{15} = 3 + \dfrac{2^\circ 14' 50''}{15}$

i.e., quotient = 3 and remainder = $2^\circ 14' 50''$;

From Table 3.1, *gatāṅka* = 117, *eṣyāṅka* = 150

and the differecnce = 33.

Now, $\dfrac{2^\circ\, 14'\, 50'' \times 33}{15} + 117 = 121|56|38$

$\therefore$ *Śīghraphala* = $\dfrac{121|56|38}{10} = 12^\circ\, 11'\, 40''$

Now, Half-*Śīghraphala* = $\dfrac{12^\circ\, 11'\, 40''}{2} = 6^\circ\, 05'\, 50''$

Half-*Śīghra* corrected Budha = Mean Budha + Half *Śīghraphala*

$= 1^R\, 4^\circ\, 13'\, 42'' + 6^\circ\, 05'\, 50'' = 40^\circ\, 19'\, 32''$

(2) Manda correction :

Mandocca of Budha = $7^R = 210^\circ\, 0'\, 0''$

Mandakendra (*MK*) of Budha = *Mandocca* – Half-*Śīghra* corrected Budha

$= 210^\circ - 40^\circ\, 19'\, 32'' \quad = 169^\circ\, 40'\, 28''$

Bhuja of *MK* $= 180^\circ - 169^\circ\, 40'\, 28'' \quad = 10^\circ\, 19'\, 32''$

Now, $\dfrac{12 \times 10^\circ\, 19'\, 32''}{15} + 0 = 8|15|38$

Mandaphala $= \dfrac{8|15|38}{10} = 0^\circ\, 49'\, 34''$

Since *MK* $< 180^\circ$, *Mandaphala* is positive.

Manda corrected Budha $= 1^R\ 04^\circ\ 13'\ 42'' + 0^\circ\ 49'\ 34''$

$= 1^R\ 05^\circ\ 03'\ 16'' \quad = 35^\circ\ 03'\ 16''$

(3) Second *Śīghra* correction :

First *Śīghrakendra* $= 47^\circ\ 14'\ 50''$

Mandaphalam $= 0^\circ\ 49'\ 34''$

Second *Śīghrakendra* = First *ŚK* – *Mandaphala*

$= 47^\circ\ 14'\ 50'' - 0^\circ\ 49'\ 34'' = 46^\circ\ 25'\ 16''$

Now, $\dfrac{46^\circ\ 25'\ 16''}{15} = 3 + \dfrac{1^\circ\ 25'\ 16''}{15}$; quotient = 3 and remainder = $1^\circ\ 25'\ 16''$

From Table 3.1, *gatāṅka* = 117, *eṣyāṅka* = 150, difference = 33

Now, $\dfrac{33 \times 1^\circ\ 25'\ 16''}{15} + 117 = 120|7|35$

$\therefore$ *Śīghraphala* = $\dfrac{120|7|35}{10} = 12^\circ\ 0'\ 46''$.

Since $SK < 180^\circ$, *śīghraphala* is additive.

Therefore, we have

the true longitude of Budha = *Manda* corrected Budha + Second *Śīghraphala*

$= 1^R\ 05^\circ\ 03'16'' + 12^\circ\ 0'\ 46' \quad = 47^\circ\ 04'\ 02''$

III Finding true longitude of Guru :

(1) First *Śīghra* correction :

Śīghrocca of Guru = Mean Sun = $1^R\ 4^\circ\ 13'\ 42''$

Mean Guru = $4^R\ 8^\circ\ 15'17''$

First *Śīghrakendra* = *Śīghrocca* – Mean Guru

$= 1^R\ 4^\circ\ 13'\ 42'' - 4^R\ 8^\circ\ 15'\ 17''$

$= 265^\circ\ 58'\ 25'' \equiv SK$

Since *Śīghrakendra* > 180° subtract it from 360°.

i.e., *Śīghrakendra* argument = $360^\circ - 265^\circ\ 58'\ 25''$

$= 94^\circ\ 01'\ 35''$

Now, $\dfrac{94^\circ\ 01'\ 35''}{15} = 6 + \dfrac{4^\circ\ 01'\ 35''}{15}$; quotient = 6, remainder = $4^\circ\ 01'\ 35''$

From Table 3.1, *gatāṅka* = 106, *eṣyāṅka* = 108, difference = 2

Now, $\dfrac{2 \times 4^\circ\ 01'\ 35''}{15^\circ} + 106 = 106|32|12$

First *Śīghraphala* = $\dfrac{106\ |32\ |12}{10} = 10^\circ\ 39'\ 13''$

Since $SK = 265^\circ\ 58'\ 25'' > 180^\circ$, the *Śīghraphala* is negative.

Half-*Śīghraphala* = $\dfrac{-10°\,39'\,13''}{2} = -5°\,19'\,36''$

Half-*Śīghra* corrected Guru = Mean Guru + Half-*Śīghraphala*

$= 4^R\,8°\,15'\,17'' + (-5°\,19'\,36'') = 4^R\,8°\,15'\,17'' - 5°\,19'\,36''$

$= 4^R\,2°\,55'\,41''$

(2) *Manda* correction :

Mandocca of Guru = 6^R

Mandakendra (*MK*) = *Mandocca* – Half-*Śīghra* corrected Guru

$= 6^R - 4^R\,2°\,55'\,41'' = 1^R\,27°\,4'\,19'' = 57°\,4'\,19''$

Bhuja of *MK* = $57°\,4'\,19''$

Now, $\dfrac{57°\,4'\,19''}{15} = 3 + \dfrac{12°\,4'\,19''}{15}$; quotient = 3, remainder = $12°\,4'\,19''$

From Table 3.2, *gatāṅka* = 39, *eṣyāṅka* = 48, difference = 9

Now, $\dfrac{9 \times 12°\,04'\,19''}{15} + 39 = 46|14|35$

Mandaphala = $\dfrac{46|14|35}{10} = 4°\,37'\,27''$

The Mandaphala is positive since $MK < 180°$.

Manda corrected Guru = Mean Guru + *Mandaphala*

$$= 4^R\ 8^\circ 15' 17'' + 4^\circ 37' 27'' = 4^R\ 12^\circ 52' 44''$$

(3) Second *Śīghra* correction :

First *Śīghrakendra* = $265^\circ 58' 25''$

Second *Śīghrakendra* = First *ŚK* – *Mandaphalam*

$$= 265^\circ 58' 25'' - 4^\circ 37' 27'' = 261^\circ\ 20'\ 58'' \equiv ŚK$$

Bhuja of *SK* = $360^\circ - 261^\circ 20' 58'' = 98^\circ 39' 02''$

We have $\dfrac{98^\circ 39' 02''}{15} = 6 + \dfrac{8^\circ 39' 02''}{15}$; quotient = 6, remainder = $8^\circ 39' 02''$.

From Table 3.1, *gatāṅka* = 106, *eṣyāṅka* = 108, difference = 2

Now, $\dfrac{2 \times 8^\circ 39' 02''}{15} + 106 = 107|9|12$

$\therefore$ *Śīghraphala* = $\dfrac{107|9|12}{10} = 10^\circ 42' 55''$

Śīghraphala is negative because $SK > 180^\circ$.

$\therefore$ True longitude of Guru = *Manda* corrected Guru + *Śīghraphala*

$$= 4^R\ 12^\circ 52' 44'' + (-10^\circ 42' 55'') = 4^R\ 2^\circ 9' 49'' = \mathbf{122^\circ 9' 49''}$$

IV Finding true longitude of Śukra :

Śīghrakendra of Śukra = $3^R\ 5°\ 41'35'' = 95°\ 41'35''$ (from Ch. 1)

Mean Śukra = Mean Sun = $1^R\ 4°\ 13'\ 42''$

(1) First *Śīghra* correction :

ŚK of Śukra = $95°\ 41'35''$

We have $\dfrac{95°\ 41'35''}{15°} = 6 + \dfrac{5°\ 41'35''}{15°}$; quotient = 6, remainder = $5°\ 41'35''$

From Table 3.1, *gatāṅka* = 354, *eṣyāṅka* = 402, difference = 48

Now, $\dfrac{48 \times 5°\ 41'\ 35''}{15} + 354 = 372|12|58$

Śīghraphala = $\dfrac{372|12|58}{10} = +37°\ 13'18''$

Half-*Śīghraphala* = $\dfrac{37°\ 13'18''}{2} = 18°\ 36'39''$

Half-*Śīghra* corrected Śukra = Mean Śukra + Half-*Śīghraphala*

$= 1^R\ 4°\ 13'\ 42'' + 18°\ 36'39'' = 1^R\ 22°\ 50'21'' = 52°\ 50'21''$

(2) *Manda* correction :

Mandocca of Śukra = 3^R

Mandakendra of Śukra = *Mandocca* – Half-*Śighra* corrected Śukra

$$= 3^R - 52°\,50'21'' \;=\; 37°\,9'39''$$

Bhuja of *MK* = $37°\,9'39''$

We have $\dfrac{37°\,9'39''}{15°} = 2 + \dfrac{7°\,9'39''}{15°}$; quotient = 2, remainder = $7°\,9'39''$.

From Table 3.2, *gatāṅka* = 11, *eṣyāṅka* = 13, difference = 2

Now, $\dfrac{2 \times 7°\,9'39''}{15°} + 11 = 11|57|17$

Manda corrected Śukra = Mean Śukra + *Mandaphalam*

$$= 1^R\,4°\,13'42'' + 1°\,11'44'' = 1^R\,5°\,25'\,26'' \;=\; 35°\,25'\,26''$$

(3) Second *śīghra* correction

First *SK* = $95°\,41'35''$

Second *Śīghrakendra* = First *SK* – *Mandaphala*

$$= 95°\,41'35'' - 1°\,11'44'' \;=\; 94°\,29'51''$$

We have $\dfrac{94°\,29'51''}{15°} = 6 + \dfrac{4°\,29'51''}{15°}$;quotient = 6, remainder = $4°\,29'51''$.

From Table 3.1, *gatāṅka* = 354, *eṣyāṅka* = 402 and their difference = 48

Now, $\dfrac{48 \times 4°\,29'51''}{15°} + 354 = 368|23|31$

$\therefore$ *Śīghraphala* = $\dfrac{368|23|31}{10} = +36°\,50'21''$.

$\therefore$ True position of Śukra = *Manda* corrected Śukra + *Śīghraphala*

$$=1^R\ 5^\circ\ 25'\ 25'' + 36^\circ\ 50'\ 21'' = \mathbf{72^\circ\ 15'\ 46''}$$

V Finding true longitude of Śani :

We have, from Chapter 1,

Mean Śani = $11^R\ 0^\circ\ 36'\ 45''$

Śīghrccca of Śani = Mean Sun = $1^R\ 4^\circ\ 13'\ 42''$

Śīghrakendra SK = *Śīghrocca* – Mean Śani

$$= 1^R\ 4^\circ\ 13'\ 42'' - 11^R\ 0^\circ\ 36'\ 45'' \ = 63^\circ\ 36'\ 57''$$

We have $\dfrac{63^\circ\ 36'\ 57''}{15^\circ} = 4 + \dfrac{3^\circ\ 36'\ 57''}{15^\circ}$; quotient = 4, remainder = $3^\circ\ 36'\ 57''$

From Table 3.1, *gatāṅka* = 48, *eṣyāṅka* = 54, difference = 6

Now, $\dfrac{6 \times 3^\circ\ 36'\ 57''}{15^\circ} + 48 = 49|26|47$

$$\textit{Śīghraphala} = \frac{49|26|47}{10} = +\,4^\circ\ 56'\ 41''$$

$$\text{Half-}\textit{Śīghraphala} = \frac{4^\circ\ 56'\ 41''}{2} = 2^\circ\ 28'\ 21''$$

Half-*Śīghra* corrected Śani = Mean Śani + Half-*Śighraphala*

$$=11^R\ 0^\circ\ 36'\ 45'' + 2^\circ\ 28'\ 21'' \ = 11^R\ 03^\circ\ 05'\ 06''$$

2) *Manda* correction :

Mandocca = 8^R

Mandakendra MK = *Mandocca* – Half-*Śīghra* corrected Śani

$$= 8^R - 11^R\ 03^\circ\ 05'\ 05'' = 8^R\ 26^\circ\ 54'\ 55'' = 266^\circ\ 54'\ 55''$$

Bhuja of *MK* = $266^\circ\ 54'\ 55'' - 180^\circ = 86^\circ\ 54'\ 55''$

We have $\dfrac{86^\circ\ 54'\ 55''}{15^\circ} = 5 + \dfrac{11^\circ\ 54'\ 55''}{15^\circ}$; quotient = 5, remainder = $11^\circ\ 54'\ 55''$.

From Table 3.2, *gatāṅka* = 89, *esyāṅka* = 93, difference = 4

Now, $\dfrac{4 \times 11^\circ\ 54'\ 55''}{15^\circ} + 89 = 92|10|39$

$\therefore$ *Mandaphala* = $\dfrac{92|10|39}{10} = 9^\circ\ 13'\ 04''$

The Mandaphala is negative since *MK* > 180°

Manda corrected Śani = Mean Śani + *Mandaphala*

$$= 11^R\ 0^\circ\ 36'\ 45'' + (-9^\circ\ 13'\ 04'')$$

$$= 10^R\ 21^\circ\ 23'\ 41''$$

(3) Second *Śīghra* correction :

Second SK = First SK – *Mandaphala*

$$= 63^\circ\ 36'\ 57'' - (-9^\circ\ 13'\ 04'') = 63^\circ\ 36'\ 57'' + 9^\circ\ 13'\ 04'' = 72^\circ\ 50'\ 01''$$

We have $\dfrac{72^\circ\ 50'\ 01''}{15^\circ} = 4 + \dfrac{12^\circ\ 50'\ 01''}{15^\circ}$; quotient = 4, remainder = $12^\circ\ 50'\ 01''$

From Table 3.1, *gatāṅka* = 48, *eṣyāṅka* = 54, difference = 6

Now, $\dfrac{6 \times 12^\circ\ 50'\ 01''}{15^\circ} + 48 = 53|8|0$

$\therefore$ *Śīghraphala* = $\dfrac{53|8|0}{10} = 5^\circ\ \ 18'\ \ 48''$

Hence, True Śani = $10^R\ 21^\circ\ 23'\ 41'' + 5^\circ\ 18'\ 48'' = 326^\circ\ 42'\ 29''$.

We shall now consider a modern example :

True positions of planets for August 11, 1998.

I. True position of Kuja :

For the given date 11th August 1998, we have

Ahargaṇa A = 2033 and *Cakra* C = 43

For Kuja, *Kṣepaka* $K = 10^R\ 7^\circ\ 8'$ and *Dhruvaka* $D = 1^R\ 25^\circ\ 32'$

$$\text{Mean } Kuja = \left\{\left(\frac{10A}{19}\right)^\circ - \left(\frac{10A}{73}\right)'\right\} - C \times D + K$$

$$= \left\{ \left(\frac{10 \times 2033}{19} \right)^{\circ} - \left(\frac{10 \times 2033}{73} \right)' \right\} - 43 \times 55^{\circ}\ 32' + 10^{R}\ 7^{\circ}\ 8'$$

$$= \mathbf{64^{\circ}\ 33'\ 30''}$$

(1) First *Śīghra* correction :

Śīghrocca of Kuja = Mean Sun = $115^{\circ}\ 9'\ 59''$

Deśāntara corrected *Śīghrocca* of Kuja = $115^{\circ}\ 9'\ 42''$

Śīghrakendra = *Śīghrocca* – Mean Kuja

$$= 115^{\circ}\ 9'\ 42'' - 64^{\circ}\ 33'\ 30''$$

$$= 50^{\circ}\ 36'\ 12''$$

Bhuja of *Śīghrakendra* = $50^{\circ}\ 36'\ 12''$

Now, $\dfrac{Bhuja}{15^{\circ}} = \dfrac{50^{\circ}\ 36'\ 12''}{15^{\circ}} = 3 + \dfrac{5^{\circ}\ 36'\ 12''}{15^{\circ}}$

$\therefore$ quotient = 3, remainder = $5^{\circ}\ 36'\ 12''$

From Table 3.1, *gatāṅka* = 174, *eṣyāṅka* = 228 and difference = 54.

$$\therefore \frac{\text{Difference} \times \text{Remainder}}{15} + Gatāṅka$$

$$= \frac{54 \times 5^\circ\,36'\,12''}{15^\circ} + 174$$

$$= 194|10|19$$

$$\therefore\ \textit{Śīghraphala} = \frac{194|10|19}{10} = 19^\circ\,25'\,2''$$

$$\text{Half-}\textit{Śīghraphala} = \frac{19^\circ\,25'\,2''}{2} = 9^\circ\,42'\,31''$$

Half-*Śīghra* corrected Kuja = Mean Kuja + Half *Śīghraphala*

$$= 64^\circ\,33'30'' + 9^\circ\,42'\,31'' = 74^\circ\,16'\,01'.$$

(2) *Manda* correction :

Mandocca of Kuja = $4^R = 120^\circ$

Mandakendra = *Mandocca* – Half-*Śīghra* corrected Kuja

$$= 120^\circ - 74^\circ\,16'\,01'' = 45^\circ\,43'\,59''$$

Bhuja of *Mandakendra* = $45^\circ\,43'\,59''$

Now, $\dfrac{45^\circ\,43'\,59''}{15^\circ} = 3 + \dfrac{0^\circ\,43'\,59''}{15^\circ}$

$\therefore$ quotient = 3, remainder = $0^\circ\,43'\,59''$

From Table 3.2, *gatāṅka* = 85, *eṣyāṅka* = 109, difference = 24

$$\therefore \textit{Mandaphala} = \frac{\textit{Gatāṅka} + (\text{difference} \times \text{remainder})/\ 15}{10}$$

$$= \frac{85 + (24 \times 0^\circ\ 43'\ 59'')\ /\ 15^\circ}{10}$$

$$= 8^\circ\ 37'\ 2''$$

$\therefore$ *Manda* corrected Kuja = Mean Kuja + *Mandaphala*

$$= 64^\circ\ 33'30'' + 8^\circ\ 37'\ 02''$$

$$= 73^\circ\ 10'32''.$$

(3) Second *Śīghra* correction :

First *Śīghrakendra* = $50^\circ\ 36'\ 12''$

Mandaphala = $8^\circ\ 37'\ 2''$

$\therefore$ Second *Śīghrakendra* = $50^\circ\ 36'\ 12'' - 8^\circ\ 37'\ 2''$ = $41^\circ\ 59'\ 10''$

Now, $\dfrac{41^\circ\ 59'\ 10''}{15^\circ} = 2 + \dfrac{11^\circ\ 59'\ 10''}{15^\circ}$

so that quotient = 2 and remainder = $11^\circ\ 59'\ 10''$

From Table 3.2, *gatāṅka* = 117, *eṣyāṅka* = 174, difference = 57

$$\textit{Śīghraphala} = \frac{\left[\dfrac{(\text{difference} \times \text{remainder})}{15} + \textit{gatāṅka}\right]}{10}$$

$$= \frac{\left[\frac{(57 \times 11^\circ 59' 10'')}{15^\circ} + 117\right]}{10}$$

$$= 16^\circ 15' 17''$$

∴ True Kuja = *Manda* corrected Kuja + *Śīghraphala*

$$= 73^\circ 10' 32'' + 16^\circ 15' 17''$$

$$= \mathbf{89^\circ 25' 49''}.$$

II. To find true position of Budha :

For the given date 11th Augutst 1998, we have

A = 2033 and C = 43

Mean Budha *Kendra* = $197^\circ 7' 47''$ (obtained in Chapter 1).

(1) First *Śīghra* correction :

Śīghrakendra of Budha SK = $197^\circ 7' 47''$

Mean Budha = Mean Sun = $115^\circ 9' 42''$

Since *SK > 180 °, we have*

Bhuja of *Śīghrakendra* = $197^\circ 7' 47'' - 180^\circ \;\; = 17^\circ 7' 47''$

Now, $\dfrac{17^\circ 7' 47''}{15^\circ} = 1 + \dfrac{2^\circ 7' 47''}{15^\circ}$

so that quotient = 1 and remainder = $2^\circ\, 7'\, 47''$

From Table 3.1, *gatāṅka* = 41, *eṣyāṅka* = 81, difference = 40

$$\therefore \textit{Śīghraphala} = \frac{\left[\dfrac{(\text{difference} \times \text{remainder})}{15} + \textit{gatāṅka}\right]}{10}$$

$$= \frac{\left[\dfrac{\left(40 \times 2^\circ\, 7'\, 47''\right)}{15^\circ} + 41\right]}{10}$$

$$= -4^\circ\, 40'\, 4''$$

Since *Śīghrakendra* = $197^\circ\, 7'\, 47'' > 180^\circ$, *Śīghraphala* is negative.

$\therefore$ Half-*Śīghraphala* = $-2^\circ\, 20'\, 2''$

Half-*Śīghra* corrected Budha = Mean Budha + Half-*Śīghraphala*

$$= 115^\circ\, 9'\, 42'' - 2^\circ\, 20'\, 2'' = 112^\circ\, 49'\, 40''.$$

***Manda* correction :**

Mandocca of Budha = $7^R = 210^\circ$

Mean Budha = Mean Sun = $115^\circ\, 9'\, 42''$

Half-*Śīghra* corrected Budha = $112^\circ\, 49'\, 40''$

Mandakendra = *Mandocca* – Half-*Śīghra* corrected Budha

$$= 210^\circ - 112^\circ 49' 40'' = 97^\circ 10' 20''$$

Bhuja of *Mandakendra* = $180^\circ - 97^\circ 10' 20''$

$$= 82^\circ 49' 40''$$

Now, $\dfrac{82^\circ 49' 40''}{15^\circ} = 5 + \dfrac{7^\circ 49' 40''}{15^\circ}$

so that quotient = 5 and remainder = $7^\circ 49' 40''$

From Table 3.2 *gatāṅka* = 35, *eṣyāṅka* = 36, difference = 1

$$\therefore \textit{Mandaphala} = \frac{\left[\dfrac{(\text{difference} \times \text{remainder})}{15} + \textit{gatāṅka}\right]}{10}$$

$$= \frac{\left[\dfrac{(1 \times 7^\circ 49' 40'')}{15^\circ} + 35\right]}{10} = 3^\circ 33' 8''$$

Manda corrected Budha = Mean Budha + *Mandaphala*

$$= 115^\circ 9' 42'' + 3^\circ 33' 8''$$

$$= 118^\circ 42' 50''.$$

(3) Second *Śīghra* correcion :

First *Śīghrakendra* = $197^\circ 7' 47''$

Mandaphala = $3°\,33'\,8''$

$\therefore$ Second *Śīghrakendra* = $197°\,7'\,47'' - 3°\,33'\,8'' = 193°\,34'\,39''$

Bhuja of *Śīghrakendra* = $193°\,34'\,39'' - 180° = 13°\,34'\,39''$

Now, $\dfrac{13°\,34'\,39''}{15°} = 0 + \dfrac{13°\,34'\,39''}{15°}$

so that quotient = 0, remainder = $13°\,34'\,39''$

From Table 3.1, *gatāṅka* = 0, *eṣyāṅka* = 41, difference = 41

$$\therefore \textit{Śīghraphala} = -\frac{\left[\dfrac{41 \times 13°\,34'\,39''}{15°} + 0\right]}{10} = -3°\,42'\,40''$$

True Budha = *Manda* corrected Budha + *Śīghraphala*

$= 118°\,42'\,50'' - 3°\,42'\,40''$

$= 115°\,0'\,10''$.

III. To find true position of Guru :

For the given date 11th August 1998,

$A = 2033$ and $C = 43$

Mean Guru = $330°\,17'\,57''$

After *Dēśantara* correction Mean Guru = $330°\,21'\,40''$

Mean Sun = $115°\,9'\,42''$

(1) First *Śīghra* correction :

Śīghrakendra of Guru = *Śīghrocca* – Mean Guru

= Mean Sun – Mean Guru

$= 115^\circ\, 9'\, 42'' - 330^\circ\, 21'\, 40'' = 144^\circ\, 48'\, 2''$ (by adding 360°)

Bhuja = $144^\circ\, 48'\, 2''$

Now, $\dfrac{144^\circ\, 48'\, 2''}{15^\circ} = 9 + \dfrac{9^\circ\, 48'\, 2''}{15^\circ}$

so that quotient = 9 and remainder = $9^\circ\, 48'\, 2''$

From Table 3.1 *gatāṅka* = 89, *eṣyāṅka* = 66, difference = – 23

$$\therefore \textit{Śīghraphala} = \frac{\left[\dfrac{\left(-23 \times 9^\circ\, 48'\, 2''\right)}{15^\circ} + 89\right]}{10} = 7^\circ\, 23'\, 50''$$

Half-*Śīghraphala* = $3^\circ\, 41'\, 55''$

$\therefore$ Half-*Śīghra* corrected Guru = $330^\circ\, 21'\, 40'' + 3^\circ\, 41'\, 55'' = 334^\circ\, 3'\, 35''$

(2) *Manda* correction :

Mandocca of Guru = $6^R = 180^\circ$

Mandakendra = *Mandocca* – Half-*Śīghra* corrected Guru

$= 180^\circ - 334^\circ\, 3'\, 35''$

$= 205^\circ\ 56'\ 25''$ (by adding 360°)

'Bhuja' = $25^\circ\ 56'\ 25''$

Now, $\dfrac{25^\circ\ 56'\ 25''}{15^\circ} = 1 + 10^\circ\ 56'\ 25''/15^\circ$

so that quotient = 1 and remainder = $10^\circ\ 56'\ 25''$

From Table 3.2, *gatāṅka* = 14, *eṣyāṅka* = 27, difference = 13

$$\therefore Mandaphalam = \frac{\left[\left(\dfrac{13 \times 10^\circ\ 56'\ 25''}{15^\circ}\right) + 14\right]}{10} = -2^\circ\ 20'\ 53''$$

Mandaphala is negative since *Mandakendra* > 180°.

Manda corrected Guru = Mean Guru + *Mandaphala*

$= 330^\circ\ 21'\ 40'' - 2^\circ\ 20'\ 53'' = 328^\circ\ 0'\ 47''$

(3) Second *Śighra* correction :

Mandaphala = $-2^\circ\ 20'\ 53''$

First *Śīghrakendra* = $144^\circ\ 48'\ 2''$

Second *Śīghrakendra* = $144^\circ\ 48'\ 2'' + 2^\circ\ 20'\ 53''$ = $147^\circ\ 8'\ 55''$

Bhuja argument = $147^\circ\ 8'\ 55''$

Now, $\dfrac{147^\circ\ 8'\ 55''}{15^\circ} = 9 + \dfrac{12^\circ\ 8'\ 55''}{15^\circ}$

From Table 3.1, *gatāṅka* = 89, *eṣyāṅka* = 66 and difference $d = -23$

We have $\left(\dfrac{-23 \times 12^\circ 8' 55''}{15^\circ}\right) = -18|37|40$

$\therefore$ *Śīghrakendra* = $\dfrac{89 - 18|37|40}{10} = 7^\circ 2' 13''$

True Guru = Manda corrected Guru + *Śīghrakendra*

$= 328^\circ 0' 47'' + 7^\circ 2' 13'' = 335^\circ 3' 0''$

IV. To find true position of Śukra

For the given date 11th August 1998 we have

$A = 2033$ and $C = 43$

Mean *Śīghrakendra* of Śukra = $310^\circ 9' 46''$

Mean Śukra = Mean Sun = $115^\circ 9' 46''$

(1) First *Śīghra* correction :

Śīghrakendra of Śukra = $310^\circ 9' 46''$

Bhuja = $49^\circ 50' 14''$

Now, $\dfrac{49^\circ 50' 14''}{15^\circ} = 3 + \dfrac{4^\circ 50' 14''}{15^\circ}$

From Table 3.1, *gatāṅka* = 186, *eṣyāṅka* = 246 and difference = 60

Śīghraphala = $\dfrac{\left[\left(\dfrac{60 \times 4^\circ 50' 14''}{15^\circ}\right) + 186\right]}{10} = 20^\circ 32' 5''$.

Since $SK > 180^\circ$, the *Śīghraphala* is negative.

Half-*Śīghraphala* = $-10^\circ 16' 3''$

Half-*Śīghra* corrected Śukra = Mean Śukra + Half-*Śīghraphala*

$$= 115^\circ 9' 42'' - 10^\circ 16' 3'' = 104^\circ 53' 39''$$

***Manda* correction :**

Mandocca of Śukra = $3^R = 90^\circ$

Mandakendra MK = *Mandocca* – Half-*Śīghra* corrected Śukra

= $90^\circ - 104^\circ 53' 39'' = 345^\circ 6' 21''$ (adding 360°) > 180°

Bhuja = $14^\circ 53' 39''$

Now, $\dfrac{14^\circ 53' 39''}{15^\circ} = 0 + \dfrac{14^\circ 53' 39''}{15^\circ}$

From Table 3.2, *gatāṅka* = 0, *eṣyāṅka* = 06 and difference = 06

$$\therefore \textit{Mandaphala} = -\frac{\left[\dfrac{14^\circ 53' 39'' \times 06}{15^\circ} + 0\right]}{10} = -0^\circ 35' 44''$$

The *Mandaphala* is negative since $MK > 180^\circ$.

Manda corrected Śukra = Mean Śukra – *Mandaphala* = $114^\circ 33' 57''$

(3) Second *Śīghra* correction :

First *Śīghrakendra* = $310^\circ 9' 46''$

Mandaphala = $-0^\circ 35' 44''$

Second *Śīghrakendra* = $310^\circ 9' 46'' + 0^\circ 35' 44'' = 310^\circ 45' 30''$

'Bhuja' = $49^\circ\ 14'\ 30''$

Now, $\dfrac{49^\circ\ 14'\ 30''}{15^\circ} = 3 + \dfrac{4^\circ\ 14'\ 30''}{15^\circ}$

From Table 3.1, *gatāṅka* = 186, *eṣyāṅka* = 246 and difference = 60

$$\therefore\ \textit{Śīghraphala} = -\frac{\left[\dfrac{60\times 4^\circ\ 14'\ 30''}{15^\circ} + 186\right]}{10} = -20^\circ\ 17'\ 48^\circ$$

being negative since $SK > 180^\circ$.

True Śukra = *Manda* corrected Śukra + *Śīghraphala*

= $114^\circ\ 33'\ 57'' - 20^\circ\ 17'\ 48'' =$ $\mathbf{94^\circ\ 16'\ 09''}$.

V. To find true position of Śani :

For the given date 11th August 1998 : $A = 2033$, $C = 43$

Mean Śani = $8^\circ\ 14'\ 2''$

Mean Sun = *Śīghrocca* of Śani = $115^\circ\ 9'\ 42''$

(1) First *Śīghra* correction :

Śīghrakendra of Śani = *Śīghrocca* – Mean Śani

= $115^\circ\ 9'\ 42'' - 8^\circ\ 14'\ 2'' = 106^\circ\ 55'\ 40''$

Bhuja argument = $106^\circ\ 55'\ 40''$

Now, $\dfrac{106^\circ\ 55'\ 40''}{15^\circ} = 7 + \dfrac{1^\circ\ 55'\ 40''}{15^\circ}$

From Table 3.1, *gatānka* = 57, *eṣyāṅka* = 53, difference = – 4

Now , *Śīghraphala* = $\dfrac{\left[\dfrac{-4 \times 1^\circ\ 55'\ 40''}{15} + 57\right]}{10} = 5^\circ\ 38'\ 55''$

Half-*Śīghraphala* = $2^\circ\ 49'\ 27''$

Half-*Śīghraphala* corrected Śani = $8^\circ\ 14'\ 2'' + 2^\circ\ 49'\ 27'' = 11^\circ\ 03'\ 29''$

(2) *Manda* correction :

Mandocca of Śani = $8^R = 240^\circ$

Mandakendra = *Mandocca* – Half-*Śighra* corrected Śani

$= 240^\circ - 11^\circ\ 03'\ 29'' = 228^\circ\ 56'\ 31''$

Bhuja argument = $48^\circ\ 56'\ 31''$

Now, $\dfrac{48^\circ\ 56'\ 31''}{15^\circ} = 3 + \dfrac{3^\circ\ 56'\ 31''}{15^\circ}$

From Table 3.2, *gatāṅka* = 60, *eṣyāṅka* = 77, difference = 17

$\therefore$ *Mandaphala* = $-\dfrac{\left[\dfrac{17 \times 3^\circ\ 56'\ 31''}{15^\circ} + 60\right]}{10} = -6^\circ\ 26'\ 48''$

Mandaphala = $-6^\circ\ 26'\ 48''$

being negative since $MK > 180^\circ$.

Manda corrected Śani = Mean Śani + *Mandaphalam*

$= 8^\circ\ 14'\ 2'' - 6^\circ\ 26'\ 48'' = 1^\circ\ 47'\ 14''$.

(3) Second *Śīghra* correction :

First *Śīghrakendra* = $106°\,55'\,40''$

Second *Śīghrakendra* = $106°\,55'\,40'' + 6°\,26'\,48'' = 113°\,22'\,28''$

Now, $\dfrac{113°\,22'\,28''}{15°} = 7 + \dfrac{8°\,22'\,28''}{15°}$

From Table 3.1, *gatāṅka* = 57, *eṣyāṅka* = 53, difference = – 4

$$\therefore\ \textit{Śīghraphala} = \frac{\left[\left(\dfrac{-4 \times 8°\,22'\,28''}{15°}\right) + 57\right]}{10} = 5°\,28'\,36''$$

True Śani : = *Manda* corrected Śani + *Śighraphala*

$= 1°\,47'\,13'' + 5°\,28'\,36'' = \mathbf{7°\,15'\,49''}$.

Comparison of true positions of planets with the ephemeris values.

Date : August 11, 1998

Planets	**According to *Grahalāghavam***	**According to *Indian Ephemeris***
Ravi	113°51′ 33″	114°18′ 19″
Candra	331°55′ 27″	330°02′
Kuja	89°25′ 49″	89°48′ 40″
Budha	115°0′ 10″	119°37′ 01″
Guru	335°03′ 0″	333°18′ 07″
Śukra	94°16′ 09″	93°15′
Śani	7°15′ 49″	9°46′ 21″
Rāhu	44°21′ 03″	–

Note : The *Ayanāṃśa*, according to the *Grahalāghvam* for the given year *Śa.Śa*. 1920 is $(1920' - 444)' = 1476' = 24^\circ\, 36'$. According to the Indian Ephemeris, the *Ayanāmśa* = $23^\circ\, 50'\, 08''$. We find fairly a good agreement between the two sets of values for the positions of planets except in the case Śani and Budha.

True daily motions of the five star-planets

(*Gatispaṣṭīkaraṇam*)

Ślōka 11 : This *śloka* explains the method of obtaining *Manda* corrected motions of Śani, Kuja Guru, Budha and Śukra.

(i) Divide the difference of *Mandāṅkas* of Śani, Kuja and Guru by 75, 5 and 30 respectively.

(ii) Multiply the difference of *Mandāṅkās* of Budha and Śukra by 2 and divide by 5 respectively.

(iii) Step (i) and step (ii) give *Manda gatiphala* of respective planets which will be in minutes of arc *(kalās).*

(iv) Add (or subtract) the *Mandagatiphala* to mean motion of planets to get *Manda* corrected motion.

Note : If the *Mandakendra* of a planet is within 6 *Rāśis* from *Karka* (i.e., if the planet is in II or III quadrant) the *gatiphala* is additive.

If the *Mandakendra* of the planet is within 6 *Rāśis* from *Makara*, (i.e., if the planet is in IV or I quadrant) then the *gatiphala* is subtractive.

Ślōka **12** : The method of obtaining true daily motions of planets is explained as follows.

1. True daily motion of Kuja :

(i) Obtain the *Manda*-corrected motion as explained in the previous *śloka.*

(ii) Take the difference between *Śīghrāṅkas* which are obtained in finding the second *Śīghraphala.*

(iii) Divide the difference obtained in step (ii) by 5. The result will be in minutes of arc *(kalās)*

(iv) Add (or subtract) the above *kalās* to *Manda* corrected motion. This gives the true daily motion of the planet.

Note : If the first *Śīghrāṅka* (i.e., elapsed *Śīghrāṅka*) is less than the second *Śīghrāṅka* (i.e., *Śīghrāṅka* to be covered) then add the result of step (iii) to the *Manda* corrected planet. On the otherhand, if the first *Śīghrāṅka* is greater than the second then subtract the result of step (iii) from the *Manda* corrected planet.

The same rule holds in the case of other planets also.

Example : In the case of Kuja, we have

(i) The difference between the *Mandāṅkas* = 28 (see the first example under *Śloka* 9)

(ii) *gatiphalam* = $\frac{28}{5} = 5'\,36''$

(iii) Since *Mandakendra* is within 6 *Rāśis* from *Karka,* the *gatiphalam* is additive.

$\therefore$ *Manda* corrected motion of *Kuja* = Mean motion of Kuja + *gatiphalam*

$= 31'\,36'' + 5'\,36'' = 37'\,12''$

(iv) The difference between the *Śīghrāṅkas* = 40 since the elapsed *Śīghrāṅka* = 325 and the *Śīghrāṅka* to be covered = 365.

(v) Since the elapsed *Śīghrāṅka* is less than the *Śīghraṅka* to be covered,

add $\frac{40}{5} = 8'\,0''$ to the *Manda* corrected planet.

$\therefore$ True motion of Kuja = *Manda* corrected motion of Kuja $+ 8'\,0''$

$= 37'\,12'' + 8'\,0'' = 45'\,12''$.

2. True daily motion of Budha

(i) Obtain the *Manda* corrected Budha's motion as explained earlier.

(ii) Consider the difference between the elapsed *Śīghrāṅka* and the *Śīghrāṅka* to be covered, which are obtained in finding the second *Śīghraphala.*

(iii) Divide the difference obtained in step (ii) by 5; the result will be in *kalās*, (minutes of arc).

(iv) Add the result of step (iii) to the difference obtained in step (ii).

(v) Add (or subtract as the case may be) the result of step (iv) to the *Manda* corrected motion of Budha.

Example : The difference of *mandāṅkas* = 12

$$\therefore gatiphalam = \frac{12 \times 2}{5} = 4' 48''$$

The *Manda* corrected motion = Mean motion + *gatiphalam*

$= 59' 8'' + 4' 48'' \ = 63' 56''$

Difference between *Śīghrāṅkas* = 33

$$\therefore\ ghatiphalam = \frac{33}{5} = 6' 36''$$

Now, the difference between *Śighrānkas* + $6' 36'' = 33' + 6' 36'' \ = 39' 36''$

$\therefore$ True motion of Budha = *manda* corrected motion $+39' 36''$

$= 63' 56'' + 39' 36'' \ = 103' 32''$

3. True Motion of Guru :

(i) Obtain the *Manda* corrected motion of Guru as explained earlier.

(ii) Obtain the difference of *Śīghrāṅkas* which are obtained in finding the second *Śīghraphala.*

(iii) Divide the above difference by 3. The result will be in *kalās.*

(iv) Add (or subtract) the above result to the *manda* corrected motion of Guru.

Example :

The difference of the *Mandāṅkas* = 9

$$gatiphalam = \frac{9}{30} = 0'18''$$

The *Manda* corrected motion = Mean motion – *gatiphalam*

$$= 5' - 0'18'' = 4'42''$$

Now, the difference between *Śīghrāṅkas* = 2 and

$$gatiphalam = \frac{2}{3} = 0'40''$$

True motion of Guru = The *Manda* corrected motion + $0'40''$

$$= 4'42'' + 0'40'' = 5'22''$$

4. True motion of Śukra :

(i) Obtain the *Manda* corrected motion of Śukra as explained in the previous *śloka.*

(ii) Obtain the difference between the *Śīghrāṅkas* of the second *Śīghraphala.*

(iii) Divide the above difference by 4. The result will be in *kalās.*

(iv) Add (or subtract) the result of step (iii) to the *Manda* corrected motion of Śukra.

Example : The difference between the *Mandāṅkas* = 2

$$\therefore gatiphalam = 2 \times \frac{2}{5} = 0'48''$$

The *Manda* corrected motion of Śukra = Mean motion of Śukra – *gatiphalam*

$$= 59'08'' - 0'48'' = 58'20''.$$

Difference of the *Śīghrāṅkas* = 48

$$\therefore \textit{gatiphalam} = \frac{48}{4} = 12'0''$$

True motion of Śukra = *Manda* corrected motion of Śukra + $12'0''$

$$= 58'20'' + 12'0'' = 70'20''$$

5. True daily motion of Śani :

(i) Obtain the *Manda*-corrected motion of Śani as explained earlier.

(ii) Obtain the difference of the *Śīghrāṅkas* of the second *Śīghraphala*

(iii) Multiply the above difference by 2 and divide it by 5. The result will be in *kalās*.

(iv) Add (or subtract) the above *kalās* to the *Manda*-corrected motion of Śani. This gives the true motion of Śani.

Example : The difference between the *Mandāṅkās* = 4

$$\therefore \textit{gatiphalam} = \frac{4}{75} = 0'03''$$

Now, the *Manda* corrected motion = Mean motion + *gatiphalam*

$$= 2' + 0'03'' = 2'03''$$

The difference between the *Śīghrāṅkas* = 6

$$\text{Now, } \textit{ghatiphalam} = 6 \times \frac{2}{5} = 2'24''$$

$\therefore$ True daily motion of Śani = *Manda* corrected motion + $2'24''$

$$= 2'03'' + 2'24'' = 4'27''$$

***Śloka* 13** : In the case Kuja and Śukra, due to large differences in their *Śīghrāṅkas* an additional correction, in each case, to the *Śīghraphala* is given.

In the process of determining the second *Śīghra* correction, when the *Śīghrakendra* is divided by 15°, let the quotient be Q and the remainder R. Now, between R and 15–R, take the lesser one and call it R'. Divide R' by 5 in the case of Kuja and by 3 in the case of Śukra. The result in degrees etc. should be added to or subtracted from the *Śīghra* corrected planet according as the *Śīghraphala* is additive or subtractive. In the case of the other three *tarāgrahas* viz. Budha, Guru and Śani, this second difference is negligible and hence omitted.

Example:

Considering the example worked out earlier for Kuja, we have

Śīghrakendra = $91^\circ\ 4'\ 57''$.

Between $\left(1^\circ\ 4'\ 57''\right)$ and $15^\circ - \left(1^\circ\ 4'\ 57''\right)$ i.e., $13^\circ\ 55'\ 03''$, the former is less.

Dividing the lesser value by 5, we get $1^\circ\ 4'\ 57''/5 = 12'\ 59''$. Since the *Śīghraphala* is additive, we add $12'\ 59''$ to the *Śīghra* corrected Kuja to get:

the true Kuja = $336^\circ\ 00'\ 28'' + 12'\ 59'' = 336^\circ\ 13'\ 27''$.

In the case of Śukra, the *Śīghrakendra* = $94^\circ\ 29'\ 52''$. Dividing by 15° the quotient is 6 and the remainder is $4^\circ\ 29'\ 52''$. Between this value and its difference from 15° viz., $10^\circ\ 30'\ 08''$, the former is less. Dividing the lesser value by 3, we get $4^\circ\,29'\,52''/3 = 1^\circ\,29'\,57''$. Since the *Śīghraphala* is additive, by adding $1^\circ\,29'\,57''$ to the *Śīghra* corrected Śukra, we get corrected true Śukra $72^\circ\,15'\,46'' + 1^\circ\,29'\,57'' = 73^\circ\,45'\,43''$.

***Śloka* 14** : While finding the final *Śīghrāṅka* of Kuja, Budha and Śukra, the remainders obtained must be multiplied by 10 and then divided respectively by 7, 7 and 3; the result be added respectively to 35, 97 and 53. This gives the corrected *gatiphala* (of Kuja, Budha and Śukra). These must be accepted and not the one obtained earlier.

Example : Suppose the second *Śīghrāṅka of* Kuja is 11 and the remainder, $11^\circ 50' 0''$. Now, multiplying the remainder by 10 and dividing by 7, we get $11|50 \times \frac{10}{7} = 16'54''17'''$ as the *Śīghra gatiphala*. This is subtractive since the *Śīghra gatāṅka* and the *eṣyāṅka* are respectively 249 and 0 corresponding to the quotients 11 and 12 and hence decreasing. Therefore, subtracting $51'54''17'''$ from the already obtained *Manda spaṣṭagati* of $25'43''$, we get the final true daily motion of Kuja as $25'43'' - 51'54''17''' = -26'11''17'''$ (*retrograde* since negative).

***Śloka* 15** : Retrograde motion of planets is explained. The *Śīghrakendras* of the planets for (the commencement of) retrograde motion (*vakragati*) are $163^\circ, 145^\circ, 125^\circ, 167^\circ, 113^\circ$. The retrogression continues till the *Śīghrakendras* become 360° *minus* the above values (i.e, 197°, 215°, 235°, 193°, 247°) respectively for Kuja, Budha, Guru, Śukra and Śani.

Retrograde Motion of Star-Planets

The star-planets move from west to east, relative to the fixed stars, as seen from the earth, due to their natural motion. However, during certain periods each of these planets appears to move backwards, i.e., from east to west. Their celestial longitudes keep on decreasing instead of increasing, day by day, for some time. This apparent backward motion is called *vakragati* (retrograde motion).

The phenomenon of retrograde motion is caused by the difference in the angular velocities of the earth and the planet, i.e, the relative velocity. This phenomenon is demonstrated in Fig. 3.1.

In Fig. 3.1 the motion of Mars (Kuja) relative to the earth is shown in the heliocentric model. The earth's linear speed is 18.5 miles per second while that of Mars is 3.5 miles less, i.e, 15 miles per second. As the earth overtakes Mars, the latter appears to move backwards, when seen from the earth. The direct motion of Mars eastward is shown at positions 1, 2 and 3, the retrograde at 4 and 5 westward and again direct motion eastward at 6 and 7.

The rule for determining the retrograde motion of a planet is given in the *Sūryasiddhānta* as follows :

The retrograde motion (*vakragati*) of the different star-planets commences when the *Śīghrakendra* (i.e., *Śīghra* anomaly), in the fourth process of determining true positions, is as follows :

Kuja 164°, Budha 144°, Guru 130°, Śukra 163°, Śani 115°

That is, the retrograde motion of Kuja, for example, commences when its

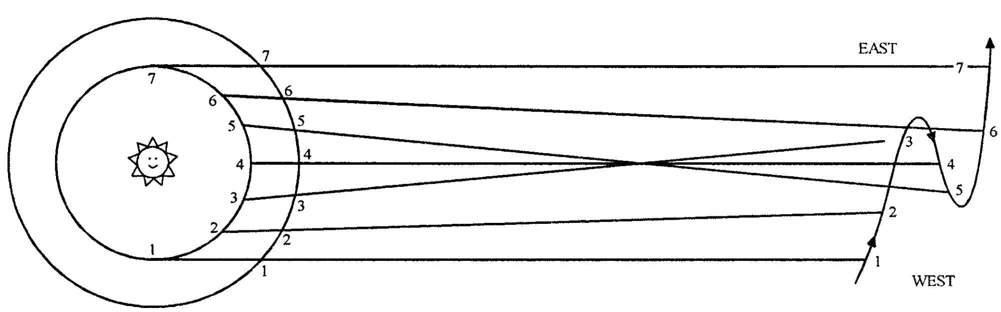

Fig. 3.1 Retrograde motion of Kuja

The point at which the motion of a planet changes from direct to retrograde is called a "*stationary point*". The planet remains retrograde for some days and then, again, its motion changes from retrograde to direct. This point of change is the second stationary point. At both the stationary points the planet has no apparent motion (i.e., the relative velocity is zero).

Bhāskara II (b. 1114 A.D.) in his *Siddhānta Śiromaṇi* (*Spaṣṭādhyāya*) mentions that at the maximal point the (angular) velocity of the planet vanishes :

यत्र ग्रहस्य परमं फलं तत्रैवगतिफलाभावेन भवितव्यम् ।

yatra grahasya paramaṃ phalaṃ tatraivagati phalābhāvena bhavitavyam /

Rationale for the Stationary Point

In Fig. 3.2 let M be the mean planet, P be the true planet on the epicycle of radius p, E be the earth and S the Sun, If n is the mean daily motion of the Sun, let t be the number of days since the Sun S was at the first point of *Meṣa*, $P\hat{M}K = \theta$ and $P\hat{E}M = E$, then the celestial longitude L of the planet is given by $L = nt - \theta + E$ where nt is the longitude of the Sun. Therefore,

$$\frac{dL}{dt} = n - \frac{d\theta}{dt} + \frac{dE}{dt} \quad \ldots (1)$$

Let $PM = p$ and $EM = r$, where the radii p and r are constants. In Fig. 3.2, we have $MA = p\cos\theta$ and $PA = p\sin\theta$. Therefore,

$EA = EM + MA = r + p\cos\theta$ and hence

$$\tan E = \frac{PA}{EA} = \frac{p\sin\theta}{(r + p\cos\theta)} \quad \text{so that} \quad E = \tan^{-1}\left[\frac{p\sin\theta}{(r + p\cos\theta)}\right]$$

Differentiating this expression with respect to t we get

$$\frac{dE}{dt} = \frac{\left(\frac{d\theta}{dt}\right)\left[r^2 + rp\cos\theta\right]}{\left[r^2 + p^2 + 2rp\cos\theta\right]} \qquad \ldots (2)$$

Substituting (2) in (1), we get

$$\frac{dL}{dt} = n - \left(\frac{d\theta}{dt}\right)\left[\frac{\left(r^2 + rp\cos\theta\right)}{\left(r^2 + p^2 + 2rp\cos\theta\right)}\right] \qquad \ldots (3)$$

If n is the mean daily motion of the Sun and n' is that of the planet and α is a suitable constant, then

$$\theta = (n - n')(t + \alpha)$$

so that $\frac{d\theta}{dt} = (n - n')$ .... (4)

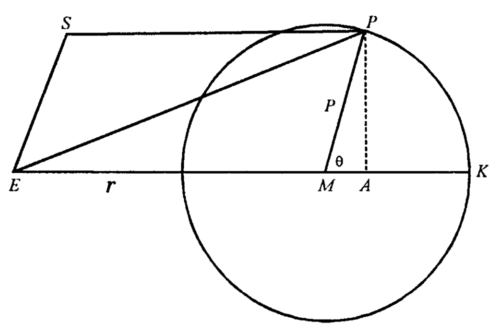

Fig.3.2. Stationary points

Substituting (4) in (3), we get

$$\frac{dL}{dt} = \frac{\left[np^2 + n'r^2 + rp(n+n')\cos\theta\right]}{\left[r^2 + p^2 + 2rp\cos\theta\right]}$$

At the stationary point where the retrograde motion begins, $\frac{dL}{dt} = 0$.

Therefore, $np^2 + n'r^2 + rp(n+n')\cos\theta = 0$

so that $\cos\theta = -\frac{(np^2 + n'r^2)}{pr(n+n')}$

For example, in the case of Kuja, considering the mean values, we have $n = 0^\circ.98560265$, $n' = 0^\circ.5240193$, $p = 233^\circ.5$, $r = 360^\circ$. Here, p and r are taken as peripheries of the planet's *Śīghra* epicycle and of the mean orbit, which are proportional to their radii. Substituting these values, we arrive at

$$\cos\theta = -\frac{[0.98560265\ (233.5)^2 + 0.5240193\ (360)^2]}{[(233.5)(360)(1.5096219)]}$$

$$= \frac{-[53737.271 + 67912.901]}{[126898.82]} = \frac{-12650.17}{126898.82} = -0.9586391$$

Therefore,

$$\theta = 180^\circ - \cos^{-1}(0.9586391) = 163^\circ.4636$$

The *Sūryasiddhānta* has taken this value as 164° while *GL* takes it as 163°. The other stationary point is given by $360^\circ - \theta$, noting that

$$\cos\theta = \cos(360^\circ - \theta)$$

In the above example, since $\theta = 164^\circ$ according to the *Sūryasiddhānta*, the second stationary point is $360^\circ - 164^\circ = 196^\circ$. This means that Kuja will be retrograde during the peirod when its *Śīghrakendra* (or *Śīghra* anomaly) lies between 164° and 196°. Similarly, the corresponding limits for other planets can be calculated.

Remark : According to modern astronomy, the stationary value of the angle θ is given by

$$\cos\theta = \frac{-\left[a^{1/2}\, b^{1/2}\right]}{\left[a - a^{1/2}\, b^{1/2} + b\right]}$$

where a is the mean distance of the planet from the Sun and b is the mean distance of the earth from the Sun. Now, taking b as one astronomical unit (a.u.) we have

$$\cos\theta = \frac{-\left[a^{1/2}\right]}{\left[a - a^{1/2} + 1\right]}$$

where a is the mean distance of the planet from the Sun, in astronomical unit (a.u.).

Note : 1 astronomical unit (a.u.) = Earth's mean distance from the Sun.

The stationary points θ, given in Table 3.5, are those at which the respective planets change their motion from direct to retrograde, i.e., the beginning of the retrograde motion (*vakrārambha*). The other stationary points, where the retrograde motion ends (*vakratyāga*), are given by $(360^\circ - \theta)$.

It is noteworthy that Bhāskara II gives the correct value $\theta = 167^\circ$ for Śukra in his *Karaṇa kutūhalam.*

For comparison, we tabulate the critical values of the *Śīghrakendras* for the retrograde motion of the planets according to the different texts :

Table 3.5 *Śīghrakendras* for retrograde motion

Planet	*Sūrya-Siddhānta*	Brahma-gupta	Bhāskara II and Lalla	*Graha-lāghavam*	Modern
Kuja	164°	164°	163°	163°	163°.215
Budha	144°	146°	145°	145°	144°.427
Guru	130°	125°	125°	125°	125°.565
Śukra	163°	165°	165°	167°	167.005
Śani	115°	116°	113°	113°	114°.466

Note : Brahmagupta has given the stationary value θ for Guru as 125° in his *Brahmasphuṭa siddhānta* and as 130° in his *Khaṇḍakhādyaka.*

***Śloka* 16** : This *śloka* tells the *Śīghrakendra* of *udayāsta* (rising and setting) of Kuja, Guru and Śani.

When the *Śīghrakendra* of Kuja, Guru and Śani becomes 28°, 14° and 17° respectively, these planets rise in the east. Subtracting the above *Śīghrakendra* values from 360° we get the *Śīghrakendra* of setting of these planets in the west respectively. That is Kuja, Guru and Śani, set in the west when their *Śīghrakendras* become 332°, 346° and 343° respectively.

***Śloka* 17** : This *śloka* gives the *Śīghrakendra* of *udayāsta* of Budha and Śukra.
When the *Śīghrakendra* of Budha and Śukra become $50°$ and $24°$ respectively they rise in the west and when their *Śīghrakendras* become $155°$ and $177°$ they set in the west.

Similarly, when *Śīghrakendras* for Budha and Śukra become respectively $205°$ and $283°$ they *rise in the east* and for $310°$ and $336°$ they *set in the east.*

***Śloka* 18** : The *śloka* explains the number of days for the commencement of or the elapsed or balance of the phenomena of retrogression, rising and setting.

(1) Find the *Śīghrakendra (SK)* of the planet for the given day.

(2) Find the difference between the above *Śīghrakendra* and the presribed *Śīghrakendra* of retrograde motion or of rising or setting of the concerned as desired.

(3) This difference is multiplied by 2 for Kūja, divided by 3 for Budha, divided by 9 and added to itself (the difference) for Guru, multiplied by 10 and divided by 6 for Śukra and the difference divided by 1 (i.e., the difference itself) for Śani gives the *gata* or *eṣya* (elapsed or balance) days of retrogression or rising or setting as the case many be.

(ii) If the current *SK* is greater than the prescribed *SK* for the particular phenomenon, then the planet will be undergoing the said phenonemon and the number of days elapsed since the commencement of the same is as explained above.

***Śloka* 19** : Now, for Budha and Śukra the phenomena of retrogression, rising and setting and the number of days of the same are explained.

Thirty two days after the setting in the east Budha rises in the west, 32 days later Budha becomes retrograde, *vakrī*, 3 days after that sets in the west, 16 days later Budha rises in the east and becomes direct (*mārgī*) and 32 days after this the planet sets in the east.

Similarly, Śukra, 2 months after setting in the east rises in the west, 8 months later becomes retrograde and $\frac{3}{4}$ month (i.e., about 22 ½ days) after this sets in the west; after this $\frac{1}{4}$ month (7½ days) later Śukra rises in the east. Further, $\frac{3}{4}$ month (22 ½ days) later becomes direct and 9 months after this, Śukra sets in the east. Thus, the cycles for Budha and Śukra repeat.

***Śloka* 20** : Now, rising, retrogression and setting days for Kuja, Guru and Śani are explained.

Kuja rises (in the east) four months after setting (in the west). The planet becomes retrograde ten months after rising. Kuja resumes direct motion two months later and sets ten months after that.

Guru rises one month after his setting. He becomes retrograde four-and-a-quarter months later. After four months of becoming retrograde, Guru becomes direct. Finally, he sets four-and-a-quarter months later.

Śani rises, becomes retrograde, resumes direct motion and sets respectively after $\frac{5}{4}$, $\frac{7}{2}$, $\frac{9}{2}$ and $\frac{7}{2}$ months successively (i.e. rises $\frac{5}{4}$ months after setting etc.).

CHAPTER 4

TRIPRAŚNĀDHIKĀRA

(Three Problems of Direction, Place and Time)

This chapter deals with issues connected with direction, place and time (*dik*, *deśa* and *kāla*). The shadow of the gnomon (*śaṅku*) occupies an important role in determining the altitude of the Sun, the direction, the time, Sun's declination and the latitude of a place. In this chapter, besides these, the ascending point of the ecliptic at the eastern horizon (*Lagna*, Ascendant), the rising and setting of the Sun etc. are discussed.

***Śloka* 1** : The durations of the rising of *Meṣa*, *Vṛṣabha* and *Mithuna* at *Laṅkā* are respectively 278, 299 and 323 in *vighaṭīs*. These figures written in the reverse order are the durations of risings of the next three *rāśis*. That is, the durations of risings of *Karkaṭaka*, *Simha* and *Kanyā* are respectively 323, 299 and 278.

The durations of the rising (*udayamāna*) of *Meṣa*, *Vṛṣabha* and *Mithuna*, diminished by the *cara khaṇḍas* are the durations of the risings of these three *rāśis* at one's place [see Ch. 2, *Śl.* 5].

The durations of the risings of *Karkaṭaka*, *Simha* and *Kanyā* increased by the *carakhaṇḍas* are the durations of the risings of these three *rāśis* at one's place.

The durations of risings of these six *rāśis*, that is from *Meṣa* to *Kanyā* written in the reverse order give the durations of the risings of the six *rāśis* from *Tulā* to *Mīna* [see Table 4.1].

Table 4.1 *Udayamānas (vig.)* of *Rāśis* at *Laṅkā*

Rāśis	Durations of risings
Meṣa	278
Vṛṣabha	299
Mithuna	323
Karkaṭaka	323
Simha	299
Kanyā	278
Tulā	278
Vṛścika	299
Dhanu	323
Makara	323
Kumbha	299
Mīna	278

The sum of the durations of the risings (*udayamānas*) of the six *rāśis* at a place is 1800 *palas* (*vig.*)

The sum of the durations of the risings of all 12 *rāśis* is 3600 *palas* [60 *palas* = 1 *ghaṭī*].

The durations of the risings of rāśis for Kāśī

The *palabhā* of *Kāśī* = 5|45 *aṅgulas.*

Carakhaṇḍas of *Kāśi* are 57, 46, 19 *palas.*

The durations of the risings of *rāśis* are given in Table 4.2 :

Table 4.2 ***Udayamānas*** **of** ***Rāśis*** **at Kāśī**

Meṣa	278 - 57 = 221	*Mīna*
Vṛṣabha	299 - 46 = 253	*Kumbha*
Mithuna	323 - 19 = 304	*Makara*
Karkaṭaka	323 + 19 = 342	*Dhanu*
Simha	299 + 46 = 345	*Vṛścika*
Kanyā	278 + 57 = 335	*Tulā*

Note : According to the *Sūryasiddhānta* the durations of the risings of *rāśis* (starting from *Meṣa*) are respectively 1670, 1795 and 1935 *asus* where 6 *asus* = 1 *pala* (*vighaṭī* or *vināḍī*)

Therefore, 1670 *asus* = 278.33 *palas*

1795 *asus* = 299.17 *palas*

1935 *asus* = 322.5 *palas*

Now, 1670 + 1795 + 1935 = 5400 *asus* = 900 *palas*

According to the *Grahalāghavam*, we have 278 + 299 + 323 = 900 *palas*.

Ślokas **2 and 3** : These *ślokas* explain the method of finding *lagna* (ascendant) at a given time. It is as follows :

(i) Determine the *sāyana* Ravi at the given time.

(ii) Find the *amśas* to be covered (*bhogyāmśa*) by dividing the true position (in degrees) of the Sun by 30 and subtracting the remainder from 30^0.

(iii) Multiply the above remainder (degrees) by the *udayamāna* (duration of rising) of that particular *rāśi* and divide the product by 30. The result gives the *bhogya kāla* (the duration to be covered) in *palas* (*vighaṭīs*) of that *lagna*.

(iv) Express the given time (after sunrise) in *palas* and subtract the *bhogya kāla* (in *palas*) from it.

(v) Subtract the total duration (in *palas*) of the *śuddha rāśis* (i.e. already completed *lagna rāśis*) from the result of item (iv).

(vi) *Bhuktāṃśa* (ellapsed portion) of *lagna*

$$= \frac{\text{Remainder from (v)} \times 30^\circ}{\textit{Udayamāna} \text{ of the current } \textit{lagna rāśi}}$$

(vii) Adding the above *bhuktāṃśas* to the *śuddha rāśis* (i.e., completed *rāśis*)

we get *sāyana spaṣṭa lagna*. Subtract the *ayanāṃśa* to get the *nirayaṇa lagna*.

Example : Given time (*Iṣṭakāla*) = $10^{gh}\ 30^{vig}$ from sunrise at *Kāśī*

Ayanāṃśa = $18^\circ\ 0'$

(i) *Sāyana* Sun at the given time = $54^\circ\ 2'\ 40''$

(ii) Dividing by 30, $\dfrac{54^\circ\ 2'\ 40''}{30^\circ} = 1 + \dfrac{24^\circ\ 2'\ 40''}{30^\circ}$

i.e., quotient = 1 and remainder = $24^\circ\ 2'\ 40''$

The *amśas* to be covered $= 30^\circ - 24^\circ\ 2'\ 40'' = 5^\circ\ 57'\ 20''$

in the 2nd *rāśi* i.e., *Vṛṣabha*.

This implies that the Sun is in the *Vṛṣabha rāśi*, with remainder

$5^\circ\, 57'\, 20''$.

At *Kāśī*, the duration of rising of *Vṛṣabha rāśi* = 253 *palas*.

$$\therefore \quad Bhogyakāla = \frac{5^\circ\, 57'\, 20'' \times 253}{30^\circ} \approx 50.225\ palas \approx 50\ palas$$

Now, the given time = $10^{gh}\ 30^{vig}$ = 630 *palas* (after sunrise).

(iii) The given time – *bhogyakāla* = 630 – 50 = 580 *palas*.

(iv) The duration of the rising *udayamāna* of *Mithuna* = 304 *palas*.

Now, 580 – 304 = 276 *palas*

The duration of the rising of *Karkaṭaka* = 342 *palas*.

which cannot be subtracted from 276 (and hence the *aśuddha rāśi*).

We have $\dfrac{276 \times 30^\circ}{342} = \dfrac{8280^\circ}{342} = 24^\circ\, 12'\, 37''$

The number of elapsed *rāśis* from *Meśa* = 3

$$\therefore \quad Sāyana\ lagna = 3^R + 24^\circ\, 12'\, 37'' = \mathbf{3^R\ 24^\circ\ 12'\ 37''}$$

and hence *nirayaṇa lagna* = *Sāyana lagna* – *Ayanāṃśa*

$$= 3^R\ 24^\circ\, 12'\, 37'' - 18^\circ\, 10' = \mathbf{3^R\ 6^\circ\ 2'\ 37''}$$

i.e., *Karkaṭaka* $6^\circ\, 2'\, 37''$.

***Śloka* 4** : This *śloka* explains the method of finding the *lagna* when the time to be covered is greater than the given time (i.e., if *bhogyakāla* > *iṣṭakāla*) and also gives the method of finding the time from *lagna*. The procedure is as follows.

(i) Find the true longitude of the (*sāyana*) Sun at the given time.

(ii) Find the time to be covered (*bhogyakāla*) (as explained in *Ślokas* 2 & 3).

(iii) If the time to be covered is greater than the given time, multiply the given time by 30 and divide by the duration of the rising of the *rāśi* in which the *sāyana* Sun lies.

(iv) Add the quotient obtained in step (iii) to the true (*sāyana*) longitude of the Sun. This gives the *sāyana lagna* at the given time.

Example : The given time = $0^{gh}\ 40^{vg}$. At this instant, we have

The *sāyana* Sun = $1^R\ 24^\circ\ 02'\ 40''$

Now, $$\frac{54^\circ\ 02'\ 40''}{30} = 1 + \frac{24^\circ\ 02'\ 40''}{30^\circ}$$

Here, the quotient = 1 and the remainder = $24^\circ\ 02'\ 40''$

The *amśas* to be covered = $30^\circ - 24^\circ\ 02'\ 40'' = 5^\circ\ 57'\ 20''$

This implies that the Sun is in the *Vṛṣabha rāśi* with remainder $5^\circ\ 57'\ 40''$.

The duration of rising of the *Vṛṣabha rāśi* is 253 *palas.*

Now, $$\frac{5^\circ\ 57'\ 40'' \times 253}{30^\circ} = 50.225185\ palas \approx 50\ palas$$

i.e., *Bhogyakāla* = 50 *palas.*

The given time (*Iṣṭakāla*) = 40 *palas.*

Since *Bhogyakāla* > *Iṣṭakāla,* multiplying the given time by 30 and dividing by 253 we get

$$\frac{40 \times 30^\circ}{253} = 4^\circ\ 44'\ 35''$$

Sāyana Ravi is in *Vṛṣabha rāśi* $5^\circ\ 43'\ 15''$

$\therefore$ *Sāyana lagna* = *Sāyana* Ravi + $4^\circ\ 44'\ 35''$

$$= 1^R\ 5^\circ\ 43'\ 15'' + 4^\circ\ 44'\ 35'' = 1^R\ 10^\circ\ 27'\ 50''$$

$\therefore$ *Nirayaṇa lagna* = $40^\circ\ 27'\ 50'' - 18^\circ\ 10' = 22^\circ\ 17'\ 50''$ (i.e., in *Meṣa*)

Finding the time from *lagna* :

(i) Find the *bhogyāṃśa* of the *sāyana* Ravi.

(ii) Find the covered portion of the *lagna* (i.e., *bhuktakāla* of the *lagna*).

(iii) Take the sum of the results of step (i) and step (ii).

(iv) Consider the duration of rising of the *rāśi* that lies between the *sāyana* Ravi and the *sāyana lagna*. Add this to the sum obtained in step (iii). This gives the required time in *palas*. Divide it by 60 to convert the time into *ghaṭīs* etc.

Example : *Nirayaṇa lagna* = $3^R\ 6^\circ\ 2'\ 37''$

Sāyana lagna = $3^R\ 24^\circ\ 12'\ 37''$

i.e., *Sāyana lagna* is in *Karka rāśi* with elapsed portion = $24^\circ\ 12'\ 37''$.

The duration of rising (*udayamāna*) of *Karka rāśi* = 342 *vig.* at *Kāśī.*

Now, the elapsed time = $\dfrac{24^\circ\ 12'\ 37'' \times 342}{30^\circ}$ *palas* = 276 *palas.*

Similarly the *bhogyakāla* of Ravi = 50 *palas.*

Now, $276 + 50 = 326$ *palas.*

We note that the *sāyana lagna* is in *Karka rāśi* and the *sāyana* Ravi is in *Vṛṣabha rāśi.* We have *Mithuna rāśi* in between these two.

The durations of the rising of *Mithuna* = 304 *palas.*

Now, 326 + 304 = 630 *palas* i.e., $10^{gh}\ 30^{vg}$

i.e., the required time at the given *lagna* = $10^{gh}\ 30^{vg}$ from the sunrise.

***Śloka* 5** : The method of finding the time from *lagna* when the *śāyana* Ravi and *sāyana lagna* are in the same *rāśi,* is explained as follows :

1. When the Sun and *lagna* are in the same *rāśi*, consider the difference between the Sun and the *lagna.*

2. Multiply the above difference by the duration of rising of the *rāśi* in which both the Sun and the *lagna* are present.

3. Divide the product obtained in step (2) by 30. This gives the time in *palas* at the given *lagna.* Divide it by 60 to convert it into *ghaṭīs.*

Note : If *lagna* < the Sun, then subtract from 60 the time obtained in step (3).

Example 1 : *Sāyana lagna* = $1^R\ 28^\circ\ 37'\ 50''$

Sāyana Ravi = $1^R\ 23^\circ\ 53'\ 15''$

(i) *Lagna* – Ravi = $4^\circ\ 44'\ 35''$

We note that Ravi and *lagna* both are in *Vṛṣabha rāśi.*

The duration of rising of *Vṛṣabha* = 253 *palas.*

(ii) Now, $\dfrac{4^\circ\ 44'\ 35'' \times 253}{30^\circ}$ *palas* ≈ 40 *palas.*

$\therefore$ The time = 40 *palas* = $0^{gh}\ 40^{vig}$

Example 2 : When *Lagna* < Sun

The given time after sunrise = 59 *ghaṭīs.*

True longitude of Ravi at this instant = $1^R\, 6^\circ\, 39'\, 07''$.

Adding 6 *rāśis* and the *ayanāṃśa* we get

$$1^R\, 6^\circ\, 39'\, 07'' + 6^R + 18^\circ\, 10' = 7^R\, 24^\circ\, 49'\, 07''$$

Bhogyakāla = 59 *palas* (obtained as explained earlier)

The given time = 59 *ghaṭīs;* the duration of half day = $33^{gh}\ 10^{vg}$

Therefore, the given time from the sunset

$$= 59^{gh} - 33^{gh}\ 10^{vg}$$

$$= 25^{gh}\ 50^{vg}$$

= 1550 *palas*

Now, we have the given time from the sunset – *Bhogyakāla* = (1550 – 59) *palas* = 1491 *palas*

Nirayaṇa lagna = $0^R\ 29^\circ\ 37'\ 11''$ (obtained earlier)

Sāyana lagna = $1^R\ 17°\ 47'\ 11''$

Sāyana Ravi = $1^R\ 24°\ 49'\ 7''$

Here, *Lagna* < Ravi

Ravi – *Lagna* = $7°\ 1'\ 56''$

Given time = 59 *ghaṭīs*

Balance = 1 *ghaṭī*

Duration of a day = 60 *ghaṭīs*

∴ 60 – Balance = 59 *ghaṭīs* (of the previous day)

i.e., the time from sunrise = 59 *ghaṭīs*.

Note :

(i) When the Sun and *Lagna* (both *sāyana*) are in the same *rāśi*,

$$\text{the given time} = \frac{Udayam\bar{a}na \times Difference}{30}$$

(ii) When *Lagna* < Sun, the Sun will be below the horizon. Then the given *iṣṭakāla*, since it is before the next sunrise and after the sunset of the previous day, it should be subtracted from 60 *ghaṭīs* of the previous day.

***Śloka* 6** : This *śloka* tells about the *Āyanagola* (hemisphere), *dina rātri māna* (duration of day and night) and *akṣāṁśa* (latitude).

(i) If the *sāyana* planet is within 6 *rāśis* from *Meṣa* to *Kanyā*, (i.e., from 0° to 180°) then it is said to be in the *uttaragola* (northern hemisphere).

(ii) If the *sāyana* planet is within 6 *rāśis* from *Tulā* to *Mīna* (i.e., from 180° to 360°) then it is said to be in the *dakṣiṇagola* (southern hemisphere).

(iii) If the *sāyana* Ravi is from *Karka* to *Dhanu*, then that period is called

Dakṣiṇāyana (southern course). If the *sāyana* Ravi is from *Makara* to *Mithuna*, then that period is called *uttarāyaṇa* (northern course). In fact, the *dakṣiṇāyana* of the Sun is from about June 22 to December 22 and the *uttarāyaṇa* is from December 22 to June 22 (approxly.).

(iv) When Ravi is in the *uttaragola*, adding *carapalas* to 15 *ghaṭīs*, we get *dinārdham* (half of the duration of the day time i.e., from the sunrise to noon). When Ravi is in the *dakṣiṇagola*, subtracting the *carapala* from 15 *gh.* we get the *dinārdham*.

(v) Subtracting *dinārdham* from 30^{gh}, we get *rātryārdha* (one half of the duration of the night). Multiplying *dinārdham* or *rātryārdham* by 2, we get *dinamāna* (i.e., duration of the day-time) or *rātrimāna* (i.e., duration of the night-time) respectively.

(vi) Multiply the *palabhā* of one's place by 5, and subtract $\frac{1}{10}^{th}$ of the square of the *palabhā* from the above product to get *akṣāmśa* (latitude) of one's place.

i.e., $Akṣāmśa = 5 \times palabhā - \dfrac{(palabhā)^2}{10}$

Remark : Let p be the *palabhā* and ϕ be the latitude (*akṣāmśa*) of a place. Then from the right-angled triangle *ABC* (Fig. 4.1), we have

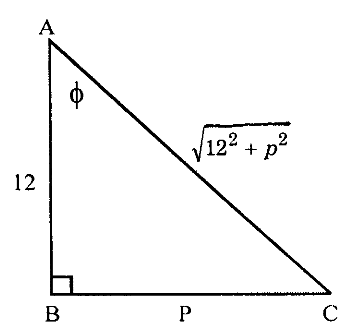

Fig. 4.1 *Akṣāmśa*

$\tan\phi = \frac{p}{12}$ where $AB = 12$ *aṅgulas*, the height of the *śaṅku* (gnomon).

$$\therefore \quad \phi = \tan^{-1}\left(\frac{p}{12}\right)$$

By infinite series expansion, we have

$$\theta = \frac{p}{12} - \frac{(p/12)^3}{3} + \ldots\ldots\ldots\ldots (\textit{in radian})$$

$$= \frac{180}{\pi}\left[\frac{p}{12} - \frac{p^3}{3 \times 12^3} + - \ldots\ldots\ldots\ldots\right] (\textit{in degrees})$$

$$\approx 5p - \frac{180}{\pi \times 3 \times 12^3} p^3$$

where the second term is of order p^3. In *GL* the second term viz. $\frac{p^2}{10}$ appears to be a small error. Thus, to a first approximation, the formula $5p$ for the latitude, is correct.

Example : Consider *palabhā*, $p = 2^{aṅg.}\ 46^{pra.} = 2.7666$ *aṅg.*

By the *GL* formula, *akṣa* $\phi = 5p - \frac{p^2}{10}$

$$= \frac{50(2.7666) - (2.7666)^2}{10} = 13^\circ.0679 = 13^\circ\ 04'$$

In fact, the *palabhā* and *akṣa* refer to Bangalore.

Example :

When *sāyana* Ravi is in the *uttaragola, carapala* = $93^{palas} = 1^{gh}\ 33^{p}$

(i) *Dinārdham* $= 15^{gh} + carapala$

$= 15^{gh} + 1^{gh}\ 33^{p} = 16^{gh}\ 33^{p}$

(ii) *Dinamānam* $= Din\bar{a}rdham \times 2$

$= 16^{gh}\ 33^{p} \times 2 = 33^{gh}\ 06^{p}$

(iii) *Rātryārdha* $= 30^{gh} - Din\bar{a}rdham$

$= 30^{gh} - 16^{gh}\ 33^{p} = 13^{gh}\ 27^{p}$

(iv) *Ratrimānam* $= 2 \times R\bar{a}try\bar{a}rdham$

$= 2 \times 13^{gh}\ 27^{p} = 26^{gh}\ 54^{p}$

(v) *Palabhā* $= 5^{ang.}\ 54^{pr.}$

Akṣāṃ śa $= 5 \times palabh\bar{a} - \dfrac{(palabh\bar{a})^2}{10}$

$$= 5 \times (5|45) - \frac{(5|45)^2}{10} = \mathbf{25^{\circ}\ 26'\ 42''}$$

***Śloka* 7** : This *śloka* explains the *nata* and the *unnata*. It also gives the method of finding *akṣakarṇa*.

When the given time is before the noon it is *pūrva kapāla* and when it is after the noon, *paścimakapāla*.

(i) ***Unnatakāla*** : The time after sunrise in *ghaṭīs* before noon. After the noon, the time remaining for the sunset is *unnata* (see Fig. 4.2).

(ii) ***Natakāla*** : The balance after subtracting the *unnatakāla* from the *dinārdham*.

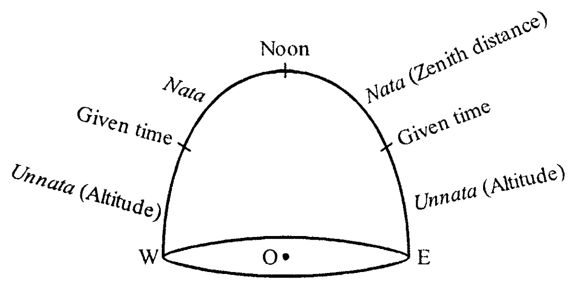

Fig. 4.2 *Natakāla* and *Unnatakāla*

Note : Zenith distance = 90° – Altitude

$$Nata = 90^\circ - Unnata$$

Example : Suppose the given time = $10^{gh}\ 30^{vg}$ from sunrise. Since the given time is before noon, it is *pūrvakapāla*.

Therefore, *Unnatakāla* = $10^{gh}\ 30^{vg}$, *Dinārdham* = $16^{gh}\ 33^{vg}$ (given)

$$Natakāla = Dinārdham - Unnatakāla$$

$$= 16^{gh}\ 33^{vg} - 10^{gh}\ 30^{vg} \quad = 6^{gh}\ 03^{vg}$$

(iii) **To find *Akṣakarṇa* :**

By adding 12 to $\dfrac{(palabh\bar{a})^2}{25}$, we get *Akṣakarṇa* in *aṅgulas*.

i.e., $Akṣakarṇa = 12 + \frac{(palabh\bar{a})^2}{25}$ *ang.*

Example : *Palabhā* = 5|45 *aṅg.*

$\therefore Akṣakarṇa = 12 + \frac{(5|45)^2}{25} = 13^{ang.}\ 19^{pr}$

Remark : We have from Fig. 4.3,

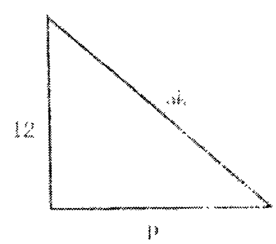

Fig. 4.3 Akṣakarṇa

$$(ak)^2 = 12^2 + p^2 = 12^2 \left[1 + (p/12)^2\right]$$

where *ak* ≡ *akṣakarṇa* and *p* ≅ *palabhā*

$$\therefore\ ak = 12\left[1 + (p/12)^2\right]^{1/2} = 12\left[1 + \frac{1}{2}\,\frac{p^2}{12^2} + \ldots\ldots\right]$$

$$\approx 12 + p^2 / 24$$

GL has taken $p^2 / 25$ instead of $p^2 / 24$ with a small error.

Śloka 8 : This *śloka* explains the method of finding *hāra* and *sama* in order to get the *hara* (denominator). Then dividing the *bhājya* (numerator), explained in the next *śloka*, we obtain the *palakarṇa* (or *akṣakarṇa*). From this the *iṣṭacchāyā* is got from the corresponding right angled triangle.

(1) To find *hāra* :

(i) Divide *carapala* by 5.

(ii) If *sāyana* Ravi is in the *uttaragola*, then add 114 to the quotient obtained in step (i). The sum is called *hāra*.

(iii) If *sāyana* Ravi is in the *dakṣiṇagola*, then subtract the quotient obtained in step (i) from 114. This result gives *hāra*.

(2) To find *sama* :

(i) Add *ghaṭikārdham* (i.e., 30^{vg}) to *nata* ; take the square of the sum.

(ii) Divide the result of (i) by 2. The resulting quotient is called *sama*.

i.e., $$Sama = \frac{(30 + nata)^2}{2}$$

(3) To find *hara* :

Consider the difference between *hāra* and *sama*. Divide this difference by *Akṣakarṇa*. The result is called *hara*.

i.e., $$hāra = \frac{hāra - sama}{akṣakarṇa}$$

Note : If *nata* (in *gh.*) is greater than $13^{gh}\ 30^{p}$ then take *nata* $- 13|30$. Multiply this difference by 4. Subtract the product from *sama* obtained earlier in (2) to get true *sama*.

Example : *Cara* = 93 *palas.*

Since *sāyana* Ravi is in the *uttaragola*,

(1) *Hāra* $= 114 + \dfrac{cara}{5} = 114 + \dfrac{93}{5} = 132|36$

(2) *Nata* = $6|3$ *ghaṭīs*

$$Sama = \frac{(30^{vig} + \text{nata})^2}{2} = \frac{(30^{vig} + 6|3^{gh})^2}{2} = \frac{(6|33)^2}{2} = 21|27$$

(3) *Hara* is given by :

$$hara = \frac{h\bar{a}ra - sama}{ak\d{s}akar\d{n}a}$$

$$= \frac{132|36 - 21|27}{13|19} = 8|20$$

***Śloka* 9** : This *śloka* explains the method of finding *the bhājya* (dividend) and the *iṣṭacchāya* (from the numerator and the divisor), the shadow of the gnomon at the given instant.

(1) To find *bhājya* :

(i) Divide *cara* by 10 × *palabhā*. Consider the square of the resulting quotient and multiply the square by 2.

(ii) Add 1/5 of the result obtained in step (i) to itself.

(iii) Add 114 to the sum obtained in step (ii). This result is called *bhājya* (dividend or numerator).

i.e., $$Bh\bar{a}jya = \left\{ 2\left(\frac{cara}{10 \times palabh\bar{a}}\right)^2 + \frac{2\left(\frac{cara}{10 \times palabh\bar{a}}\right)^2}{5} \right\} + 114$$

$$= \frac{6}{5} \times 2\left(\frac{cara}{10 \times palabh\bar{a}}\right)^2 + 114$$

(2) To find *iṣṭacchāyā* :

(i) Divide *bhājya* obtained as above by *hara*. The resulting quotient is called *iṣṭakarṇa* or *palakarṇa*.

(ii) Subtract 12^2 from the square of *palakarṇa*.

(iii) Take the square root of the above difference. The result obtained is called *iṣṭacchāyā*.

i.e., $$I\d{s}\d{t}acch\bar{a}y\bar{a} = \sqrt{\left(\frac{Bh\bar{a}jya}{hara}\right)^2 - 144} \text{ aṅg.}$$

Example : *Palabhā* = 5|45 *aṅgulas*

Cara = 93 *palas*

(i) $$bh\bar{a}jya = \frac{6}{5} \times 2\left(\frac{cara}{10 \times palabh\bar{a}}\right)^2 + 114$$

$$= \frac{6}{5} \times 2\left(\frac{93}{10 \times 5|45}\right)^2 + 114 = 120|16$$

(ii) *hara* = 8|20 (obtained in the previous *śloka*)

$$\therefore\ iṣṭacchāyā = \sqrt{\left(\frac{bhājya}{hāra}\right)^2 - 144}$$

$$= \sqrt{\left[\frac{120|16}{8|20}\right]^2 - 144} = 7|59|22 \ aṅgulas$$

***Śloka* 10** : Finding *akṣakarṇa* from *iṣṭacchāya* and hence *nata* is explained in this *śloka* as follows.

(i) Add 12^2 to $(chāyā)^2$. Take the square root of this sum. The result obtained gives *akṣakarṇa.*

i.e., $akṣakarṇa = \sqrt{12^2 + (chāya)^2}$

(ii) Divide *bhājya* (obtained earlier) by the above *akṣakarṇa*. The quotient gives *iṣṭa hara.*

i.e., $iṣṭa\ hara = \frac{bhājya}{akṣakarṇa} = \frac{bhājya}{\sqrt{(chāya)^2 + 12^2}}$

(iii) Multiply *iṣṭahara* by *akṣakarṇa*. Subtract the product from *madhyahara* (i.e., *hāra*, obtained earlier). The result gives *sama.*

i.e., $sama = madhyahara - (iṣṭahara \times karṇa)$

(iv) Multiply the *sama* by 2, and take the square root. This gives *nata.*

Subtract $\frac{1}{2}$ *ghaṭī* (i.e., 30 *palas*) to get true *nata*.

i.e., $nata = \sqrt{2 \times sama}$

True $nata = \left(\sqrt{2 \times sama}\right) - 30$ *palas.*

Note : If $2 \times sama$ is greater than 194 (i.e., if *nata* is greater than $13|30$)

then, $nata = \sqrt{\frac{2 \times sama - 194}{3} + (2 \times sama)}$

True $nata = nata - \frac{1}{2}$ *ghaṭī*

$= nata - 30$ *palas*

Example :

(i) *Iṣṭacchāyā* $= 7|59|22$ *aṅgulas*

$aks\d{}akar\d{n}a = \sqrt{(chaya)^2 + 12^2}$

$= \sqrt{(7|59|22)^2 + 12^2} = 14|25$ *aṅgulas* $\equiv$ ***Karṇa***

(ii) *Bhājya* $= 120|16$

$$iṣṭahara = \frac{Bhājya}{Karṇa} = \frac{120|16}{14|45} = 8|20$$

(iii) *Madhyahāra* = *hāra* = $132|36$ and *akṣakarṇa* = $13|19$

sama = *madhyahara* – (*iṣṭahara* × *akṣakarṇa*)

$$= 132|36 - (8|20 \times 13|19) \qquad = 21|33$$

(iv) *nata* = $\sqrt{2 \times sama} \qquad = \sqrt{2 \times (21|33)} = 6|33$ *gh.*

Subtracting $\frac{1}{2}$ *ghaṭī* (30 *palas*) we get

True *nata* = $(6|33 - 30)$ *palas* = $6|03$ *ghaṭīs*.

Example : When given *nata* > $13|30$ *ghaṭīs*, finding *iṣṭacchāyā* and conversely finding the *nata*.

Suppose *nata* = $15|10$ *ghaṭīs*

Adding $\frac{1}{2}$ *ghaṭī* i.e., 30 *palas*, we get

$$15|10^{gh.} + 30^{pa.} = 15|40 \ ghaṭīs$$

$$sama = \frac{(15|40)^2}{2} = 122|43$$

$$nata - 13|30 = 15|10 - 13|30 = 1|40$$

Spaṣṭa sama = *sama* – $(4 \times 1|40) = 122|43 - 6|40 = 116|3$

hāra = $132|36$ and *akṣakarṇa* = $13|19$ *aṅgulas* (obtained earlier)

$$hara = \frac{\textit{hāra - spaṣṭa sama}}{\textit{akṣakarṇa}}$$

$$= \frac{132|36 - 116|3}{13|19} = 1|14$$

We have, *bhājya* = 120|16

$$\textit{iṣṭakarṇa} = \frac{\textit{bhājya}}{\textit{hara}} = \frac{120|16}{1|14} = 97|29 \ \textit{aṅgulas}$$

$$\textit{iṣṭacchāyā} = \sqrt{(97|29)^2 - 12^2} = 96|44|30 \ \textit{aṅgulas}$$

Conversely, finding *nata* :

Suppose *chāyā* = 96|44|30 *aṅg.*

$$\textit{Karṇa} = \sqrt{(\textit{chāyā})^2 + 12^2}$$

$$= \sqrt{(96|44|30)^2 + 12^2} = 97|29 \ \textit{aṅg.}$$

Bhājya = 120|16 (given)

$$hara = \frac{\textit{bhājya}}{\textit{karṇa}} = \frac{120|16}{97|29} = 1|14$$

palakarṇa (*akṣakarṇa*) = 13|19 *aṅg.*

madhyahara (hāra) = 132|36

Sama = *madhyahara* − [(*palakarṇa* × *hara*)]

$= 132|36 - [(13|19) \times (1|14)] = 116|11$

Now, $2 \times sama = 2 \times (116|11) = 232|22 > 194$

Since $2 \times sama > 194$ we proceed as folows :

$$nata = \left[\frac{(2 \times sama) - 194}{3} + (2 \times sama)\right]^{1/2} = \left[\frac{232|22 - 194}{3} + 232|22\right]^{1/2}$$

$= 15|40$ *gh.*

True *nata* = $15|40$ *gh.* – 30 *palas* = $15|10$ *ghaṭīs.*

***Śloka* 11** : This *śloka* explains the method of finding *krānti* (declination) by using *krānti khāṇḍas*. It is as follows :

The nine *Krāntikhaṇḍas* are 40, 40, 37, 34, 30, 25, 18, 12, 4.

(i) Divide the *bhujāmśa* of *sāyana* Ravi by10.The quotient gives the number of elapsed *khaṇḍas* called *gatakhaṇḍas.*

(ii) Multiply the remainder obtained in step (i) by the *khaṇḍa* to be covered (i.e., *eṣyakhaṇḍa*). Divide the product by 10.

(iii) Add the quotient obtained from step (ii) to the sum of all elapsed *khaṇḍas.*

(iv) Divide the above sum by 10. The result gives the *krānti* (declination).

i.e., $$Krānti = \frac{\left[\begin{array}{l} Sum\,of \\ gatakhaṇḍas + \dfrac{Remainder \times Eṣyakhaṇḍa}{10}\end{array}\right]}{10}$$

(see Table 4.3)

Example : Ravi = $1^R\ 5°\ 52'\ 41''$

Ayanāṃśa = $18^\circ\, 10'$

Sāyana Ravi = $1^R\, 24^\circ\, 2'\, 41'' = 54^\circ\, 2'\, 41''$

Bhujāṃśa of *sāyana* Ravi = $54^\circ\, 2'\, 41''$

Now, $\dfrac{\textit{Bhujāṃsa}}{10^\circ} = \dfrac{54^\circ\, 2'\, 41''}{10^\circ} = 5 + \dfrac{4^\circ\, 2'\, 41''}{10^\circ}$

remainder = $4^\circ\, 2'\, 41''$, the quotient = 5

This implies that the number of *gatakhaṇḍas* is 5.

i.e., the *khaṇḍas* which are over are 40, 40, 37, 34 and 30.

Eṣyakhaṇḍa, the *khaṇḍa* to be covered = 25

$$\frac{\textit{Eṣyakhaṇḍa} \times \text{Remainder}}{10^\circ} = \frac{25^\circ \times 4^\circ\ 2'\ 41''}{10^\circ} = 10^\circ\, 6'\, 42''$$

Sum of the *gatakhaṇḍas* = 40 + 40 + 37 + 34 + 30 = 181

$$\textit{Krānti} = \frac{\left[\textit{Sum of gatakhaṇḍas} + \dfrac{\textit{Eṣyakhaṇḍa}}{10}\right]}{10}$$

$$= \frac{(181^\circ + 10^\circ\, 6'\, 42'')}{10} = 19^\circ\, 06'\, 40''$$

Note : For finding the declination *krānti* of a heavenly body, the argument (λ) is the *sāyana* longitude of the body. For intervals of 10° of λ, the *Grahalāghavam* gives the declinations (δ) as in Table 4.3.

Table 4.3 Declination *(krānti)*

λ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
δ	0	40	80	117	151	181	206	224	236	240
Difference		40	40	37	34	30	25	18	12	4

The modern formula is $\sin\delta = \sin\varepsilon \sin\lambda$. for a body on the ecliptic (latitude $\beta = 0$). Now, if δ, ε and λ are in degrees, converting them into radians, we have for $\lambda = 90°$,

$$\sin\left(\frac{\delta_{max}}{180} \times \pi\right) = \sin\left(\frac{\delta \times \pi}{180}\right)$$

Since the argument is small,

$$\frac{\delta_{max}}{180} \times \pi \approx \frac{\delta \times \pi}{180}$$

i.e., $\delta_{max} \approx \varepsilon \approx 24°$

$\therefore$ $\sin\delta = 24° \sin\lambda$ for small δ.

Here, λ is the *sāyana* longitude, δ the declination and ε the obliquity of the ecliptic.

Ganeśa Daivajña has multiplied the RHS by 10 (to avoid fractions) and then the final value is divided by 10. Thus his expression is $\delta = 240 \sin\lambda$ (to be divided by 10 later). Of course, for $\sin\lambda$ he has used an approximation formula.

***Śloka* 12** : This *śloka* gives a method for finding approximate declination (*sthūla krānti*) for easy computation by using *laghu khaṇḍas*. It is as follows.

At intervals of $15°$, the six *krānti khaṇḍas* known as *laghu khaṇḍas* are given as 6, 6, 5, 4, 2, 1.

(i) Divide the *bhujāṃśa* of *sāyana* Ravi by 15. The quotient gives the number of *gatakhaṇḍas.*

(ii) Consider the product of the remainder obtained in step (i) and the *eṣyakhaṇḍa.* Divide this product by 15.

(iii) Add the sum of all the *gatakhaṇḍas* to the above quotient. This sum is the *sthūla krānti.*

$$\text{i.e.,}\quad \textit{Krānti} = \left[\begin{matrix}\text{Sum of} \\ \textit{gatakhaṇḍa}\end{matrix} + \frac{\textit{Esyakhaṇḍa} \times \text{Remainder}}{15}\right]$$

Example : *Sāyana* Ravi = $1^R\ 24°\ 2'\ 41''$, *bhujāṃśa* = $54°\ 2'\ 41''$

$$\text{Now,}\quad \frac{\textit{bhujāṃśa}}{15°} = \frac{54°\ 2'\ 41''}{15°} = 3 + \frac{9°\ 2'\ 41''}{15°}$$

Here, quotient = 3 and remainder = $9°\ 2'\ 41''$

This implies, that the number of *gatakhaṇḍas* is 3. They are 6, 6 and 5.

$\therefore$ *eṣyakhaṇḍa* = 4.

$$\textit{Krānti} = \left[\begin{matrix}\text{Sum of} \\ \textit{gatakhaṇḍas}\end{matrix} + \frac{\textit{Remainder} \times \textit{Eṣyakhaṇḍa}}{15}\right]$$

$$= (6° + 6° + 5°) + \frac{9°\ 2'\ 41'' \times 4°}{15°}$$

$$= 17° + 2°\ 24'\ 43'' = \mathbf{19°\ 24'\ 43''}$$

Remark :

The formula for finding declination is given by $\delta = \sin^{-1}(\sin\varepsilon \sin\lambda)$ for the Sun. If $\lambda = 54^\circ\ 2'\ 41''$ and $\varepsilon = 24^\circ$ we get

$$\delta = \sin^{-1}(\sin 24^\circ \sin 54^\circ\ 2'\ 41'') = \mathbf{19^\circ\ 13'\ 22''}$$

The declination obtained using *krāntikhaṇḍas* is $19^\circ\ 06'\ 40''$.

The approximate declination obtained using *laghukhaṇḍas* is $19^\circ\ 24'\ 43''$.

***Śloka* 13** : This *śloka* explains the method of finding the *bhujāṃśa* of Ravi when *krānti* is known by reverse process as follows.

(i) Subtract the sum of elapsed *khaṇḍas* from *sthūlakrānti* (*krānti* obtained using *laghukhaṇḍas*).

(ii) Multiply the above difference by 15.

(iii) Divide the above product by *eṣyakhaṇḍa*.

(iv) Take the product of the number of elapsed *khaṇḍas* and 15.

(v) Add the result of step (iii) to that of step (iv). This sum gives the *bhujāṃśa* of Ravi. That is

bhujāṃśa

$$= \frac{\left(\textit{Krānti} - \text{Sum of elapsed } \textit{khaṇḍas}\right) \times 15}{\textit{Eṣyakhaṇḍa}} + (\text{no. of elapsed } \textit{khaṇḍas} \times 15)$$

Example : *Krānti* obtained using *laghukhaṇḍas, sthūlakrānti* = $19^\circ\ 24'\ 43''$

and the number of the elapsed *khaṇḍas* = 3

Sum of the elapsed *khaṇḍas* = 6 + 6 + 5 = 17

Eṣyakhaṇḍa = 4

$$\textit{Bhujāṃśa} = \frac{(19^\circ\ 24'\ 43'' - 17) \times 15}{4} + (3 \times 15) = \mathbf{54^\circ\ 2'\ 41''}$$

Śloka **14** : The method of finding *krānti* by knowing only the *dina māna* (duration of day time) without the knowledge of the position of Ravi is explained as follows.

(i) Subtract 15 *ghaṭīs* from *dinārdham* (in *gh.*). Multiply this difference by 60 to get *cara* (in *palas*).

(ii) Add $^1/_8$ of *carapala* to itself, and divide the sum by *palabhā*.

(iii) Add 25 *kalās* to the resulting quotient. This gives the *krānti* of the sun.

i.e., $$Kr\bar{a}nti = \frac{\left(carapala + \dfrac{carapala}{8}\right)}{palabh\bar{a}} + 25\ kal\bar{a}s$$

$$= \frac{9}{8} \times \frac{carapala}{palabh\bar{a}} + 25\ kal\bar{a}s$$

Note :

(i) If *dinārdham* $> 15^{gh}$, then the *krānti* is north.

(ii) If *dinārdham* $< 15^{gh}$, then the *krānti* is south.

Example : $Din\bar{a}rdham = 16^{gh}\ 33^{vig}$

$$carapala = (din\bar{a}rdham - 15^{gh}) \times 60$$

$$= (16^{gh}\ 33^{vig} - 15^{gh}) \times 60$$

$$= 93\ palas$$

$Palabh\bar{a} = 5|45$ *aṅgulas* (given)

$$Kr\bar{a}nti = \frac{9}{8} \times \frac{carapala}{palabh\bar{a}} + 25'$$

$$= \left(\frac{9}{8}\times\frac{93}{5|45}\right)^{\circ} + 25' = \mathbf{18^{\circ}\ 36'\ 44''}$$

Since *dinārdham* = $16^{gh}\ 33^{vig} > 15^{gh}$, the *krānti* is north.

***Śloka* 15** : The methods of finding *natāmśa* and *unnatāmśa* using *dinārdham* and *krānti*, without using *krānti khaṇḍas* are explained in this *śloka* as follows.

(1) Determine *krānti* and *akṣāṃśa*. Then *natāṃśa* = *krānti* $\pm$ *akṣāṃśa* = $\delta \pm \phi$, the zenith distance. Here, for northern latitude ϕ, the negative sign and for the southern ϕ, positive sign are taken. In other words, *natāṃśa* = $\delta - \phi$ where ϕ is $+^{ve}$ or $-^{ve}$ according as the place is in the northern or southern hemisphere. Similarly, the declination may also be positive or negative.

(2) *Unnatāṃśa* = [90° – *natāṃśa*], the altitude.

(3) To find *krānti* without using *krāntikhaṇḍas* :

(i) Find the *bhujāṃśa* of *sāyana* Ravi. Divide it by 10.

(ii) Subtract the result of step (i) from 18°. Multiply the difference by the result of step (i).

(iii) Divide the result of step (ii) by 72.

(iv) Subtract the result of step (iii) from $4^{\circ}\ 30'$.

(v) Divide the result of step (ii) by the result of step (iv). The quotient obtained gives the *krānti* (in *degrees*).

i.e.,
$$Krānti = \frac{\left(18^\circ - \dfrac{bhujāṃśa}{10}\right)\dfrac{bhujāṃśa}{10}}{4^\circ\ 30' - \dfrac{\left\{\left(18^\circ - \dfrac{bhujāṃśa}{10}\right)\times\left(\dfrac{bhujāṃśa}{10}\right)\right\}}{72}}$$

We shall express the above formula in a simpler form :

Let $x = \dfrac{Bhujāṃśa}{10}$. Then

$$Krānti = \frac{(18^\circ - x)\,x}{4^\circ.5 - \dfrac{[(18^\circ - x)\,x]}{72}} \text{ degrees.}$$

Example : (1) *Bhujāṃśa* of *sāyana* Ravi = $54^\circ\ 2'\ 41''$

Using the above formula,

$$Krānti \approx \frac{\left(18^\circ - \dfrac{54^\circ\ 2'\ 41''}{10}\right)\dfrac{54^\circ\ 2'\ 41''}{10}}{4^\circ\ 30' - \dfrac{\left\{\left(18^\circ - \dfrac{54^\circ\ 2'\ 41''}{10}\right)\dfrac{54^\circ\ 2'\ 41''}{10}\right\}}{72}} = \mathbf{19^\circ\ 9'}$$

(2) *Krānti* $\delta = 19^\circ\ 06'\ 40''$ north

Akṣāmśa $\phi = 25^\circ\ 26'\ 42''$ in the northern hemisphere

$\therefore$ *Natāṃśa* = *Krānti* – *akṣāṃśa* $\equiv \delta - \phi$

$$= 19^\circ\ 6'\ 40'' - 25^\circ\ 26'\ 42'' = \mathbf{6^\circ\ 20'\ 2''} \textbf{ south.}$$

(3) *Unnatāṃśa* = 90° – *natāṃśa*

$$= 90^\circ - 6^\circ\, 20'\, 2'' = \mathbf{83^\circ\, 39'\, 58''}$$

To find *parākhya* of *natāṃśa* :

(i) Divide *natāmśa* by 6. Add the quotient to the square of itself. Divide the resulting sum by 2.

(ii) Subtract the result of step (i) from 114. The resulting difference is called *parākhya*.

i.e., $$parākhya = 114 - \frac{\left\{\left(\frac{natāṃśa}{6}\right) + \left(\frac{natāṃśa}{6}\right)^2\right\}}{2}$$

Example : *Natāṃśa* = $6^\circ\, 20'$

$$\therefore\ parākhya\ = 114 - \frac{\left\{\left(\frac{6^\circ\, 20'}{6}\right) + \left(\frac{6^\circ\, 20'}{6}\right)^2\right\}}{2} = 112^\circ\, 54'\, 55''$$

To find *parākhya* of *unnatāṃśa* :

The declination (*krānti*) obtained by using *laghukhaṇḍas* of *unnatāmśa* is known as *parā* (or *parākhya*) of *unnatāṃśa*.

Example : We have *unnatāṃśa* = $83^\circ\, 39'\, 58''$; its *bhujāṃśa* = $83^\circ\, 39'\, 58''$

Now, $$\frac{83^\circ\, 39'\, 58''}{15^\circ} = 5 + \frac{8^\circ\, 39'\, 58''}{15^\circ}$$

Here, quotient = 5 and remainder = $83^\circ\, 39'\, 58''$.

Therefore, from *śloka* 12 we have

the number of *gatakhaṇḍas* = 5. They are 6, 6, 5, 4 and 2. *Eṣyakhaṇḍa* = 1.

The sum of the *gatakhaṇḍas* = $6 + 6 + 5 + 4 + 2 = 23°$.

Therefore, *parā* i.e., the *krānti* of the *unnatāmśa* is given by

$$Krānti = para = \text{Sum of } gatakhaṇḍas + \frac{Eṣyakhaṇḍa \times \text{remainder}}{15}$$

$$= 23° + \frac{1° \times 8°\ 39'\ 58''}{15°}$$

i.e., *parā* of *unnatāṃśa* = **23° 34′ 39″**

To find *yantrabhāga* from *nata* :

(i) Add $^1/_2$ *ghaṭī* to the *nata*. Multiply the sum by 15 and divide the product by *dinārdham*.

(ii) Multiply the square of the result obtained in step (i) by *parākhya* of *natāṃśa*.

(iii) Divide the result of step (ii) by 114 and add 228 to the resulting quotient.

(iv) Subtract from the above result, the *parākhya* multiplied by 2 and consider the square root of the difference obtained.

(v) Multiply the result of step (iv) by 6 and subtract 3 from this product.

The result gives *yantrabhāga*.

If $p = parākhya$, $n = nata$, $d = dinārdham$

and since $^1/_2$ *ghaṭī* = 30 *palas*, then

$$Yantrabhāga = \left[\left\{\frac{\left(\frac{(n+30)\times 15}{d}\right)^2 \times p}{114} + 228\right\} - 2p\right]^{1/2} \times 6 - 3 \text{ in degrees.}$$

Example : *Nata* $n = 6^{gh}\ 03^{vg}$ (obtained in *Śloka* 10)

dinārdham $d = 16^{gh}\ 33^{vg}$

parākhya of *natāṃśa* $p = 112^\circ\ 54'\ 55''$

$\frac{1}{2}$ *ghaṭī* = 30 *palas*

$$Yantrabh\bar{a}ga = \left[\left\{\frac{\left(\left(\frac{n+30}{d}\right)\times 15\right)^2 \times p}{114} + 228\right\} - 2p\right]^{\frac{1}{2}} \times 6 - 3$$

$$= \left[\left\{\frac{\left(\left(\frac{6|03+30}{16|33}\right)\times 15\right)^2 \times 112^\circ\ 54'\ 55''}{114} + 228\right\} - 2\times 112^\circ\ 54'\ 55''\right]^{\frac{1}{2}} \times 6 - 3$$

$$= \mathbf{33^\circ\ 32'}$$

To find *nata* using *yantrabhāga* :

(i) Add 3 to the *yantrabhāga*, and divide the sum by 6. Take the square of the result.

(ii) Add *parākhya* multiplied by 2 to the above square.

(iii) Subtract 228 from the result of step (ii).

(iv) Multiply the difference obtained in step (iii) by 114 and divide the product by *parākhya* (of *natāṁśa*).

(v) Consider the square root of the above result. Multiply it by *dinārdham.*

(vi) Divide the result of step (v) by 15 and subtract 30 *palas* from it. This gives *nata* (in *ghaṭīs*).

i.e., if $p = parākhya,\ d = dināradham$ and $y = yantrabhāga,$ then

$$nata = \left[\frac{\left\{\left(\left(\frac{y+30}{6}\right)^2 + 2p\right) - 228\right\} \times 114}{p}\right]^{\frac{1}{2}} \times \frac{d}{15} - 30$$

Example : Suppose *Yantrabhāga* = $33|32|0$, *parākhya* $p = 112^\circ\ 54'\ 55''$

and *dinārdham* $d = 16|33$ *gh.*

Using the above formula, we get

$$nata = \left[\frac{\left(\left(\frac{33|32+30}{6}\right)^2 + 2 \times 112^\circ\ 54'\ 55'' - 228\right) \times 114}{112^\circ\ 54'\ 55''}\right]^{\frac{1}{2}} \times \frac{16|33}{15} - 30$$

$$= \mathbf{6^{gh}\ 03^{vig}}$$

Śloka 16 : This *śloka* gives a different method for finding *iṣṭakarṇa* (*akṣakarṇa* or *palakarṇa*) from *unnatam* (in *gh.*) at a given time. It is as follows :

(i) Consider the product of *unnatam* at a given time and $90°$. Divide this product by *dinārdham.*

(ii) Considering the *bhujāmśa* of the quotient obtained in step (i), find *krānti* using *laghukhaṇḍas.*

(iii) Multiply the *krānti* by *parākhya* (or *para* which refers to the diclination of *unnatāmśa* obtained using *laghukhaṇḍas*)

(iv) Divide the number 6912 by the result of step (iii). This gives the *akṣakarṇa* or *iṣṭakarṇa* in *aṅgulas.*

Example : *unnatam* at a given time = $10^{gh.}$ $30^{vig.}$; *dinārdham* = 16^{gh} 33^{vig}

(i) Multiplying *unnatam* by 90 and dividing by *dinārdham*

we get

$$\frac{(10|30)\times 90}{dinardham} = \frac{945}{16|33} = 57°\ 5'\ 59''$$

(ii) *bhujāṃśa* of the above = $57°\ 5'\ 59''$;

(iii)*krānti* of this, using *laghukhaṇḍas*,is given by

Krānti = $20°\ \ 13'\ \ 35''$ (obtained as explained in *Śloka* 13)

(iv) ***parākhya*** = $23°\ 34'\ 39''$ (obtained earlier)

Now, ***parākhya*** $\times$ ***krānti*** = $23°\ 24'\ 39'' \times 20°\ 13'\ 35''$ = $476|53|12$

(v) *palakarṇa* = $\dfrac{6912}{476|53|12}$ = **14|29 *aṅgulas***

Śloka **17** : The inverse process of finding *unnatam* from *palakarṇa* is explained as follows.

(i) Divide the number 6912 by *palakarṇa*

(ii) Divide the result of step (i) by *parākhya* (declination of *unnatāmśa*) The result gives *krānti*.

(iii) Find *bhujāṃśa* of the *krānti* obtained above as given by *Śloka* 13.

(iv) Multiply the *bhujāṃśa* by *dinārdham* and divide by 90. The result gives *unnatām*.

i.e., $$unnatam = \frac{dinardh\bar{a}m \times bhuj\bar{a}\dot{m}\acute{s}a \text{ of } kranti}{90}$$

$$\text{where } kr\bar{a}nti = \frac{\left(6912 \big/ palakar\underset{\cdot}{n}a\right)}{par\bar{a}khya}$$

Example : *Dinārdham* = 16|33 *gh.*

and *Palakarṇa* = 14|29 *aṇg.*

Parākhya = 23° 34′ 39″

$$Kr\bar{a}nti = \frac{\left(6912 \big/ 14|29\right)}{23^\circ\ 34'\ 39''} = 20^\circ\ 14'\ 37''$$

The *bhujāṃśa* for the above *krānti* = 57° 9′ 15″ (obtained by inverse process)

$$Unnatam = \frac{16|33 \times 57^\circ\ 9'\ 15''}{90^\circ} = \mathbf{10^{gh}\ 30^{vig}}$$

Śloka 18 : The method of finding *unnatakāla* when the *unnatāmśa* obtained from *yantrabhāga* is known is explained as given below.

(i) Find the declination (*krānti*) of *unnatāmśa* obtained from *yantrabhāga*. Multiply this *krānti* by 24 and divide by *parākhya* (declination of *unnatāmśa*).

(ii) Find the *bhujāṃśa* of the above result and multiply it by *dinārdham* and divide the product by 90. The result gives *unnatakāla*.

For the *pūrva kapāla* (eastern hemisphere) the *unnatakāla* obtained will be the elapsed *ghaṭīs* (*gataghaṭī*) and for the *paścima kapāla* (western hemisphere) the *unnatakāla* is the remaining *ghaṭīs* to be covered.

Example : *yantrabhāga* = $55|45|48$

declination of the above = $19^\circ\ 52'\ 13''$

parākhya = $23^\circ\ 34'\ 39''$

$$\text{Now, } \frac{\text{declination} \times 24}{\textit{parākhya}} = \frac{(19|52|13) \times 24}{23|34|39} = 20^\circ\ 13'\ 25''$$

bhujāṃśa from the above *krānti* = $57^\circ\ 05'\ 52''$

$$\text{Now, } \frac{\textit{bhujāṃśa} \times \textit{dinārdham}}{90} = \frac{(57|05|52) \times (16|33)}{90}$$

unnatakāla = $\mathbf{10^{gh}\ 30^{vig}}$

Since we have obtained *unnatakāla* in *pūrva kapāla*, it gives the elapsed *ghaṭīs*.

***Śloka* 19** : This *śloka* explains the inverse process of finding *yantrabhāga* using *unnataghaṭī*. It is explained below.

(i) Multiply *unnataghaṭī* (i.e., *unnata* in *ghaṭīs*) by 90°, and divide it by *dinārdham*.

(ii) Determine the *krānti* of the above quotient using *laghukhaṇḍas*.

(iii) Multiply the *krānti* and *parākhya* and divide by 24.

(iv) The *bhujāmśa* for the above result is called *yantrabhāga*.

Example : *Unnatam* = $10^{gh}\ 30^{vig}$, *dinārdham* = $16^{gh}\ 33^{vig}$

(i) $\dfrac{unnatam \times 90}{din\bar{a}rdham} = \dfrac{10^{gh.}\ 30^{vig} \times 90^\circ}{16^{gh.}\ 33^{vig}} = 57^\circ\ 5'\ 58''$

(ii) *bhuja* of the above = $57^\circ\ 5'\ 58''$

krānti (of this using *laghukhaṇḍas*) = $20^\circ\ 13'\ 35''$

(iii) *parākhya* = $23^\circ\ 34'\ 39''$

$$\frac{par\bar{a}khya \times kranti}{24} = \frac{23^\circ\ 34'\ 39'' \times 20^\circ\ 13'\ 35''}{24} = 19^\circ\ 52'\ 13''$$

(iv) *bhujāṃśa* for the result of (iii) [See *Śloka* 13]

yantrabhāga = **$55^\circ\ 45'\ 48''$**

***Śloka* 20** : The method of finding *iṣṭakarṇa* from *yantrabhāga* and the inverse process of finding *yantrabhāga* from *iṣṭakarṇa* is explained in this *śloka* as follows.

(i) Find the declination of the given *yantrabhāga*. Divide 288 by the declination of *yantrabhāga* obtained above. The result gives the *iṣṭakarṇa*.

i.e., $\text{\textit{Iṣṭakarṇa}} = \dfrac{288}{kr\bar{a}nti\ of\ yantrabh\bar{a}ga}$

(ii) Divide 288 by the given *iṣṭakarṇa*. Find *bhujāmśa* of the resulting quotient. This gives *yantrabhāga*.

i.e., *yantrabhāga* = *bhujāṃśa* of $\left(\dfrac{288}{i\d{s}\d{t}akar\d{n}a}\right)$

Example :

(i) given *yantrabhāga* = 55|45|48

Krānti (declination) of the *yantrabhāga* = 19° 52′ 12″

iṣṭakarṇa = $\frac{288}{19° \ 52' \ 12''}$ = 13| 56| 38 *aṅgulas*

(ii) given *iṣṭakarṇa* = 13|56|38 *aṅgulas*

Now, $\frac{288}{13|56|38}$ = 19|52|12

bhujāṃśa for (19|52|12) = 55|45|48

i.e., *yantrabhāga* = **55|45|48**

Remark :

Here the maximum declination (*parama krānti*) is 24°. Therefore,

Yantrabhāga = *Bhujāṃśa* of $\frac{24 \times 12}{Iṣṭakarṇa}$

and *Iṣṭakarṇa* = $\frac{288}{Yantrabhāga}$

***Śloka* 21** : Now, determination of directions is explained.

On an even level ground a circle is drawn. The *śaṅku* (gnomon) is fixed at the centre. The line joining the entry and exit points of the shadow (on the circle) determines the *west* and the *east* directions respectively. The perpendicular line, through the centre, to the east-west line provides the north and south directions.

***Śloka* 22** : A different method of finding *bhuja* in order to know the *direction* (*dik*) is explained as follows.

(i) Find the declination of the Sun at the given time and multiply it by *palakarṇa*.

(ii) Multiply the above product by *chāyākarṇa*.

(iii) Divide the product obtained in step (ii) by 350, to get *bhuja* in the direction of the Sun.

(iv) The above *bhuja* is added to or subtracted from the product $2 \times$ *palabhā*. This gives the corrected *bhuja* (*spaṣṭa bhuja*).

Note : In step (iv) take the sum if *palabhā* and *bhuja* are in the same direction and the difference if they are in different directions.

Example : Given time from sunrise = $10^{gh}\ 30^{vgh}$

Sāyana Sun at the given time = $1^{R}\ 5°\ 52'\ 41''$

Declination (*krānti*) of the Sun = $19°\ 6'\ 40''$

Palakarṇa = $13|19$ *aṅgulas*

Chāyākarṇa = $14|25$ *aṅgulas*

We have

(*Krānti* $\times$ *Palakarṇa*) $\times$ *Chāyākarṇa*

$$= (19|6|40) \times (13|19) \times (14|25) = 3668|59|8$$

i.e., $$bhuja = \frac{3668|59|8}{350} = 10|28$$

palabhā = $5|45$ *aṅgulas* (given)

Since *sāyana* Ravi and *palabhā* are in different directions, we have

$$\textit{spaṣṭa bhuja} = (2 \times \textit{palabhā}) - \textit{bhuja}$$

$$= (2 \times 5|45) - 10|28$$

$$= \mathbf{1|02\ (south)}$$

***Śloka* 23** : A different method of finding *digaṃśa* in order to find *dik* is explained as follows.

(i) Multiply the difference between *dinamāna* and 30 by 11. This result is positive or negative according as *dinamāna* is greater than or less than 30 *ghaṭīs* respectively.

(ii) Find the *krānti* (declination) of *yantrabhāga* using *laghukhaṇḍas*. This declination is always negative (south).

(iii) If the results of step (i) and step (ii) have the same sign, consider their sum ; if they are in different signs take the difference between them.

(iv) Multiply the result obtained in step (iii) by 8, and divide this product by the declination of the difference between 90° and *yantrabhāga*.

(v) The *bhujāṃśa* of the above result is called *digaṃśa*.

i.e.,*digaṃśa* = *Bhujāṃśa* of

$$\frac{[(\textit{dinamāna} - 30) \times 11 \pm \textit{declination of yantrabhāga}] \times 8}{\textit{declination of}(90^\circ - \textit{yantrabhāga})}$$

Example : *Dinamāna* = $33|6$ *ghaṭīs*

(i) $[(\textit{dinamāna}) - 30] \times 11 = \left(33^{gh}|6^{vgh} - 30^{gh}\right) \times 11$

$$= 3|6 \times 11$$

$$= +34|6$$

Since *dinamāna* > 30, the result is positve.

(ii) *yantrabhāga* = $55° \ 45' \ 48''$ (given)

declination of the above = $-19° \ 52' \ 13''$ which is always taken south.

(iii) Now, $34|6 - 19|52|13 = 14|13|47$

since the results of (i) and (iii) are of opposite signs,

(iv) declination of $(90° - 55|45|48) = 13|24|44$

and $\dfrac{(14|13|47) \times 8}{13|24|44} = 8|29|15$.

(v) *Digaṃśa bhujāṃśa* of $8|29|15 = \mathbf{21° \ 13''}$ (as explained in *Śloka* 13).

***Śloka* 24** : This *śloka* explains the method of finding *dik* (directions) from *digamśa*.

Tūriya yantra is an instrument with a circular base, calibrated in degrees, and two mutually perpendicular strips (indicating the east-west and north-the south lines). For fixing the (four) directions, circular base of this instrument is placed horizontally on an even ground surface. Then, with the foot of the *śaṅku* (gnomon) coinciding with the centre of the *tūriya yantra*, measure an angle equivalent to the *digamśa* (obtained from earlier *śloka*) from the shadow line to the *śaṅku*. Thus, the line at that angle will be the actual east-west line. The direction perpendicular to it will be the north-south direction.

***Śloka* 25** : For the fixing of *nalikā* (tube) instrument finding the *bhuja* and *koṭi* is explained.

For the planet to be observed, its true declination (*spaṣṭa krānti*) has to be determined. The true declination is multiplied by the *iṣṭa karṇa* and *pala karṇa* (i.e., *akṣa karṇa*). The product is divided by 700. The result is the required *bhuja* in the direction of the *krānti* which should be corrected with the southern *palabhā*. This gives the *spaṣṭa bhuja*. Subtract the square of the *spaṣṭa bhuja* from the square of the *iṣṭa chāyā* and take the of square-root of this difference. This gives the *spaṣṭa koṭi*.

Example 1 : *Samvat* 1669, *Śaka* 1534, *Vaiśākha Śukla Paurṇimā* (15th *tithi*), Monday at 57 *ghaṭīs* from the sunrise, the fixing of the *nalikā* for observing *Kuja* is explained. The place of observation is Kāśī.

After the usual corrections, we have

true Sun is $1^R\ 6^\circ\ 37'\,12''$ and true Kuja is $11^R\ 6^\circ\ 37'\,29''$.

Karṇa = $11|48|40$, *Krānti* = $23^\circ\ 44'\ 59''$ (south), *Śara* (south) = $46|14|34$ *aṅg.*

Subtracting 3^R (i.e., 90°) from true Kuja, we get $8^R\ 6^\circ\ 37'\ 29''$.

For this, the *Krānti* $\delta = 23^\circ\ 47'\ 29''$ (south). *Akṣāmśa* (latitude) of kāśī ϕ = $25^\circ\ 26'\ 42''$ (north).

$\therefore$ *Natāṃśa* = $\delta - \phi = 49^\circ\ 14'\,11''$ (south)

Dṛkkarma = $118'\,44''$. The *dṛkkarma* corrected *Kuja* = $11^R\ 8^\circ\ 36'\,13''$.

For this position of *Kuja*, the *Krānti* = $1|17|30$

Śara corrected *Krānti* i.e., *spaṣṭa krānti* = $3|1|33$

Iṣṭaghaṭī = 57 *gh.*, *dinamānam* = $33|10$ *gh.*

Ravibhogyakāla = 59 *gh.*, *Lagna* = $0|15|23|21$

Bhogyakāla of *dṛkkarma* corrected *Kuja* = 18 *gh.*

dinagatakāla of *Kuja* = $4|29$ *gh.*

Cara of the corrected Kuja = 6

Corrected *cara* = 14.

Dinamānam = $29|32$ *gh.*

Cor. *Natāṃśa* = $28^\circ\ 28'\ 15''$

Unnatāṃśa = $61^\circ\ 31'\ 45''$ (complement of *Natāṃśa*)

Parākhya = $21|12|14$

Unnatam = $4|29$

Multiplying this by 90 and dividing by *dinārdham* (i.e., $14|46$ *gh.*) we get *phala* = $27|19|37$ degrees.

Krānti of the above *phala* = $10^\circ\ 42'\ 36''$.

Multiplying this *krānti* by *parākhya*, we get $227|5|37$. Dividing this by 6912, we have

Iṣṭakarṇa = $30|26$

Akṣakarṇa = $13|19$

$$Bhuja = \frac{(Akṣa\ karṇa \times (Iṣṭakarṇa) \times krānti}{700}$$

$$= \frac{(13|19)\,(30|26)\,(3|1|33)}{700} = 1|45 \text{ aṅgulas}$$

Since the *krānti* is south, the *bhuja* is also south. *Palabhā* = $5|45$ *aṇgulas.*

Palabhā corrected *spaṣṭa bhuja* = $1|45 + 5|45 = 7|30$ *aṅgulas.*

Squaring the *spaṣṭa bhuja*, we get $56|15$.

Iṣṭa chāyā $= \sqrt{(Iṣṭakarṇa)^2 - (12)^2}$

$= \sqrt{926|11 - 144}$

$= 27|58$ *aṅgulas*

$(Chāyā)^2 = 782|11$

Koṭi $= \sqrt{(Chāyā)^2 - (Spaṣṭabhuja)^2}$

$= 26|56$ *aṅgulas*

***Śloka* 26** : Now, fixing of the *nalikā* is explained.

Knowing all the directions on an even level ground, according as the planet is in western or eastern hemisphere, the angle equal to the *koṭi* is measured westward or eastward from the east-west line. From this, equal to the *bhuja*, towards the north or south, the length is measured. The line joining the end points of the *karṇa* and *bhuja* gives the *chāyā*. (This is the hypotenuse). The line joining end of the *chāyā* and the top of the *śaṅku is* the direction for the *nalikā yantra* to observe the planet in the sky. The line joining the top of the *śaṅku*, fixed at the central point, and the end of the *chāyā* (shadow) gives the direction for fixing the *nalikā* to observe the planet's reflection in water.