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VEDĀṄGA JYOTIṢA – WHERE AND WHEN?

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The observations of the length of the days in a year and the heliacal rising of the star α Delphini (*Śraviṣṭhā / Dhaniṣṭhā*) are given in *Vedāṅga Jyotiṣa*. These have been analyzed to identify the possible location and the time of composition of this text. It is shown that this (post-) Vedic text was composed at a location between 25° north and 30° north latitude. It is very likely that this was in South Asia. The visibility of α Delphini at dawn indicates that this text must have been composed before 600 BC, and most probably was composed between 1150 BC and 600 BC.

1. INTRODUCTION

The oldest (known) text of mathematical astronomy from South Asia is *Vedāṅga Jyotiṣa*. This is one of the six 'limbs' (*aṅga*) of Veda that had to be mastered by the Vedic priests in order to be able to compute times for the performance of prescribed sacrifices. The text has survived in two recensions – a *Ṛgveda* recension called *Ārca- Jyotiṣa* and a *Yajurveda* recension called *Yājuṣa-Jyotiṣa*. There are minor differences between the two recensions, the *Ṛgvedic* recension is smaller, it has thirty six verses. Thirty of these are repeated in the *Yajurvedic* recension and this recension has thirteen additional verses. The *Ṛgvedic* recension is attributed to Lagadha but unfortunately nothing else is known about him. The author of the *Yajurveda* recension is not known, this recension, however, does have a (rather unsatisfactory) *bhāṣya* or commentary by Somā kara. The *Ṛgvedic* recension is considered to be much earlier than the *Yajurvedic* recension and in this paper *Vedāṅga Jyotiṣa* should be taken to mean the *Ṛgvedic* recension. *Vedāṅga Jyotiṣa* is culmination of a process of calendar making that must have started at a much earlier date in South Asia. It is not a thesis on calendar but a hand-book for ritual officiates who were educated orally to make the necessary observations and calculations to determine the correct times for

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ceremonies. The contents of the text are not systematically arranged and topics on the same subject are distributed in different places suggesting that the original text has not come down to us. It is possible that the text was used mostly to jog the memory of a priest who was familiar with the detail. This coupled with the terse *sūtra* style makes the interpretation of the text very difficult. The *Jyotiṣa* was noticed by early (western) Indologists like Sir William Jones and Colebrooke as it appeared to offer the prospects of determining the Vedic chronology. The *Ṛgvedic* recension was first published in 1834 by (Captain) Jarvis[1], who was investigating the measurement of time in India. In 1862 the German Indologist Weber[2] published the *Yajur*-recension from the manuscripts available to him. Apart from a few simple verses, he was unable to interpret the text. A second stab at the *Yajur*-recension was taken by Thibaut[3] who was able to interpret some of the difficult verses but left out several. Translation (in Marathi) and interpretation of thirty-six verses of *Yajur*-recension were presented by Dikshit[4] but he also left-out a number of difficult verses. Further attempts to translate and interpret the *Jyotiṣa* were made in the late nineteenth and early twentieth century culminating in the Sanskrit commentary and English translation by Shamasastry[5]. In 1979 a translation and interpretation of the *Jyotiṣa* was produced by Kuppanna Sastry; this, along with critical editions of both the *Ṛgvedic* and *Yajurvedic* recensions (from twenty manuscripts) was produced by Sarma and published in 1984 by the Indian National Science Academy[6]. References to *Vedāṅga Jyotiṣa* in the present paper are from this translation and interpretation.

The location and the time of composition of either recensions of *Jyotiṣa* are not known. Pingree[7] has argued that much of mathematical astronomy of *Vedāṅga Jyotiṣa* has been borrowed/copied from Mesopotamian sources during the Achaemenid occupation of parts of northern South Asia between 513 BC and 326 BC. Pingree also contends that the borrowed elements were inserted into *Jyotiṣa* without proper understanding or correction for the location and the time of the composition of *Jyotiṣa*. A great deal of Pingree's argument is based on comparison of *Jyotiṣa* with the post-Vedic (but pre-medieval) South Asian texts on mathematical astronomy. This line of reasoning does not amount to much and will not be considered here. There are however two pieces of data in the *Jyotiṣa* that are amenable to mathematical analysis and can provide independent evidence for the location and epoch of composition of *Vedāṅga Jyotiṣa*. These data are considered in this paper.

2. Location and Epoch

2.1. Location

The verses *RJ* 7 (i.e. the 7th verse of *Ārca-Jyotiṣa*) and *YJ* 8 (i.e. 8th verse of *Yājuṣa-Jyotiṣa*) state that the difference in the length of the longest and the shortest day in a year is "six *muhūrta* in an *ayana*" or 4.8 hours (a *muhūrta* is 48 mins.) in six months (an *ayana* is from solstice to solstice, about 183 days). This gives the ratio of the length of the longest to the shortest day to be 3:2. This ratio is also found in a number of Babylonian astronomical texts[8]. In Fig. 1 the ratio of the length of the longest to the shortest day in a year is plotted as a function of the latitude. The curve in this figure is for the length of the day defined as the interval from sunrise to sunset. Sunrise and sunset times are defined as the time when the solar zenith distance is 90.8°. The effects of atmospheric refraction have been included in these computations. Pingree has pointed out that a ratio of 3:2 is only possible around latitude of 35° north thus only in the most northern parts of South Asia and thus excludes most of South Asia (Fig. 1.) as the home of composition of *Jyotiṣa*. He has argued that this ratio was copied into

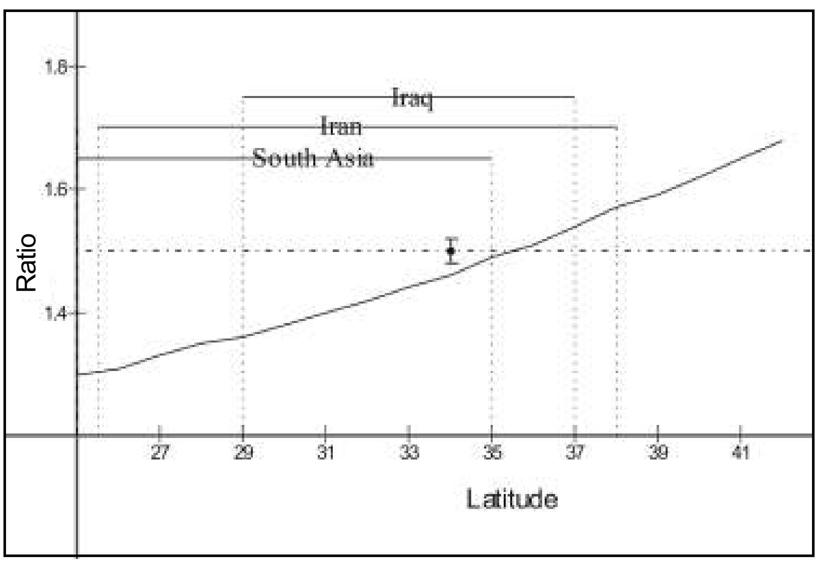

Fig. 1. The ratio of the longest day to the shortest day is a function of the latitude. The error bar shown is for 2min. error in determining the time of sunrise and sunset. The dot-dash line indicates the ratio 3:2. The latitudinal boundaries of modern South Asia, Iraq and Iran are shown by dotted lines.

Vedāṅga Jyotiṣa and no effort was made to correct for the change of location. But if this line of reasoning is followed then this ratio also excludes all locations in two-thirds of Iraq (the home of Sumerian, Babylonian and Chaldean cultures) and Iran (a possible conduit for transfer of Babylonian astronomy to South Asia) as can be seen from Fig. 1. It certainly excludes major Chaldean cultural centers like Uruk, Ur, Eridu, Lagash (latitude between 30° and 32°) and Babylon (latitude about 34°) but would include Nineveh and Nimrud (latitude about 35°). It would also exclude Persepolis and Pasargadae (latitude less than 30°), the cultural centers of the Achaemenid empire. Even an error of 2 mins. (the transit time from the first contact to the last contact of the solar disc with the horizon) in determining the time of sunrise and sunset will not bring these Middle Eastern sites into the region where this ratio of 3:2 could have been measured.

The narrowing down to just couple of cultural sites in the Middle East where a ratio of 3:2 for the length of the longest to the shortest day could have been measured warrants a more detailed examination of the 'length of the day' in *Vedāṅga Jyotiṣa* (and possibly also in mul.Apin). This has been briefly considered by Ôhashi[9]. The change in the length of the day from winter solstice to summer solstice is given in *RJ* 22 and *YJ* 40 and can be expressed in (modern notation) as

$$\text{The length of daytime} = \left(12 + \frac{2}{61}n\right) \textit{muhūrta},$$

where n is the number of days after (or before) winter solstice. This is known as the 'linear zig-zag' function and shows that the *āryā* considered the length of the day to change linearly from winter solstice to summer solstice (and *vice versa*). The length (in hours) of the days between the solstices, as given by this function, is plotted in Fig. 2. Plotted in this figure is also the computed length of daytime for three latitudes that cover the northern half of South Asia and most of Iraq and Iran. The length of the day is defined as the interval between the points in a day when the zenith distance of the sun is 90.8° and in these computations also the effects of atmospheric refraction have been included. These data show that from (about thirty days after) winter solstice to spring equinox the zig-zag function matched the curve for 30° north and from spring equinox to summer solstice the zig-zag function does not match any of the three curves but comes close (for the first sixty days) to the curve for 25° north. An error of 2 min., in the observations of sunrise and sunset times, marginally improves the agreement between the zig-zag function and the curves for 25° north. Compared to these two curves the

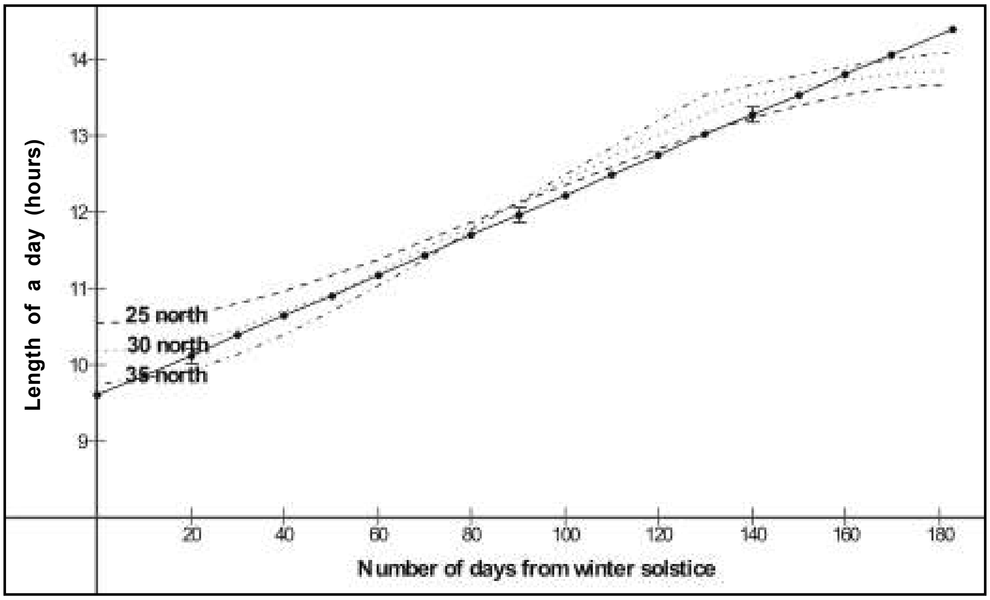

Fig. 2. The length of a day from winter solstice to summer solstice. The linear zig-zag function is the dot-line curve. The error bars are for a 2 min. error in determining the time of sunrise and sunset (see text for detail). Curves for the length of a day at latitudes of 25°, 30° and 35° north are also shown, effects of atmospheric refraction have been included in these computations.

match of the zig-zag function to the curve for 35° north is poor at all times. This suggests that the linear zig-zag function was not obtained by measuring the length of the day at the solstices and then interpolating it to the days between these two points. The function was most probably derived from observations made around spring equinox, at a location between latitudes 25° and 30° north. The curve was then extrapolated to the solstices and it is entirely fortuitous that the ratio of the length of the longest to the shortest day matches the value that could have been measured at 35° north. This is in agreement with Ôhashi's conclusion. This discussion, of course, does not prove that this ratio was measured in South Asia but it considerably weakens the case for a Mesopotamian origin for this ratio in *Vedāṅga Jyotiṣa*.

2.2. Epoch

The ratio of 3:2 for the length of the longest to the shortest day is only found in the Babylonian astronomical texts after 700 BC, and Pingree asserts that since this ratio was copied into *Vedāṅga Jyotiṣa* this text must have been composed after 700 BC. His most likely date for the *Ṛgvedic* recension being fifth or fourth century BC, and third or fifth century AD, for the *Yajurvedic* recension.

The verses *RJ* 5-6 and *YJ* 6-7 state that "When the sun and the moon occupy the same region of the zodiac (sky) together with the asterism of *Śraviṣṭhā,* at that time begins the *yuga*, the (synodic) month of *māgha*, the (solar seasonal) month of *tapas* etc., ... the sun and the moon begin to move north....." that is, the five year intercalation period (*yuga*) starts when the star (or star group) *Śraviṣṭhā* rises helically at winter solstice. It is worth emphasising that this verse describes an observation not some computed association between a star and the sun. The principle star (*yogatārā*) in the constellations (*nakṣatra*) of *Śraviṣṭhā* (also called *Dhaniṣṭhā*) has been identified as α Delphini (Pingree and Morrissey [10]). The star α Delphini is 3.77 magnitudes bright and the brightest star in this constellation is β Delphini at 3.63 magnitudes, but, as will be shown later, the slight difference in the position of these two stars will not affect this discussion. The *Jyotiṣa* verses above suggest that α Delphini rose helically at winter solstice or the ecliptic longitude of α Delphini at the time of composition of this verse (if not *Jyotiṣa*) was about 270°. The ecliptic longitude of this star in 1950 was 316° 41′ and precession of 1° in 72 years gives the time of composition of *Vedāṅga Jyotiṣa* to be 1413 BC (Saha and Lahiri[11]). From the *nakṣatra* at winter solstice given in *Pañcasiddhāntikā* and *Sūryasiddhānta*, Kuppanna Sastry[12] has suggested the period from 1370 BC to 1150 BC for composition of *Vedāṅga Jyotiṣa*. Pingree[13] makes a valid point that errors in observation are to be expected and these would affect the calculations of the epoch of *Vedāṅga Jyotiṣa*. These uncertainties are difficult to quantify. Ôhashi[14] has assumed an error of quarter of a *nakṣatra* segment (a *nakṣatra* segment is about 13° wide) in the position of the solstice and an error of 4 days in the date of the solstice to conclude that there could be an error of 500 to 600 years in the date of the *Vedāṅga Jyotiṣa* determined by just allowing for precession (but see discussion below). Pingree[15] assumes an error of about 10° in the placement of the beginning of *nakṣatra* or about 10 days in the (computation of the) date of solstice. He does not justify this error and he is being rather disingenuous as an error of 10° (and precession of 1° in 72 years) would bring the date of composition of the *Jyotiṣa* from twelfth century BC to fifth century BC, in agreement with his preconception (or prejudice).

But in his rush to establish his bias Pingree has either overlooked or ignored the realities of (astronomical) observations.

In the later astronomical texts from South Asia the *nakṣatra* meant a sector (about 13° wide) on the sky and a star or stars in this sector identified this sector. The start of the *nakṣatra* sector is from the star(s) identifying the sector. Ôhashi (above) refers to the uncertainty in the part of the sector rising helically at winter solstice. There is no reason to believe that *RJ* 5-6 and *YJ* 6-7 refer to observations of a portion of a sector in the sky. Observations of a point source, like a star, would have been easier then an imaginary line in the sky in the days when (perhaps) the only astronomical instruments available were the gnomon and the (water) clock[16]. The error of 4 days in the day of the solstice, assumed by Ôhashi, comes from the discrepancy in the number of days in a *yuga* (1830) and the number of days in five tropical years (1826.2). However it is unlikely that the Vedic Ārya would have identified the start of a new *yuga* by naively counting the number of days in the previous *yuga*. In *Kauṣītakī Brā hmaṇa* (xix.3)[17] there is a detailed description of the apparent motion of the sun at solstice. The Ārya could have (and probably did) identify the solstice days from observations of the sun. Also the helical rising of α Delphini at winter solstice would have occurred over a number of years and any error in the day of the solstice could have been corrected. The day of winter (and summer) solstice can be determined by observing the position of the sun on the horizon. In this type of observations an error of one day in the day of the solstice is possible.

The visibility of a star as it rises above the horizon is determined by the brightness of the star relative to the sky glow in the vicinity of the horizon above which the star rises. Before sunrise (and after sunset) there is an interval of time when natural light is provided by sunlight scattered in the atmosphere. This is called the twilight. The beginning of the (civil) twilight is defined as the time when the sun is geometrically 6° below the horizon. Bright astronomical objects like the Moon, Venus and Sirius are visible during this time. The visibility of fainter objects is determined by the transparency of the sky which depends on the weather conditions, pollution etc. Observations (made in the twentieth century, at a North American site) suggest that in the absence of morning haze a dense cluster of 3rd magnitude stars like the Pleiades is visible when the sun is about 16° (or more) below the horizon[18]. In Fig. 3., the times, at winter solstice, of sunrise (zenith distance 90.8°), of start of morning (civil) twilight (zenith distance 96.0°) and when the zenith distance of the sun is 105.0° (i.e. the sun is 15° below the horizon) are shown from 2000 BC, to 2000 AD. The data shown are for longitude 77.2° east and latitude 28.6° north (New Delhi) and the times shown are local times (the effects of atmospheric refraction have been included). Ideally the rise time of a star is considered to be the time when it becomes (just) visible above the astronomical horizon, that is, its zenith distance is 90.0° or its altitude 0.0°. This is only possible when looking out towards the horizon at sea; it is generally not possible on land. The rise times (altitude 0.0° or zenith distance 90.0°) at winter solstice of α Delphini are shown in Fig. 3, also from 2000 BC to 2000 AD (the effects of atmospheric refraction have been included). Around 500 BC, this star rises in the morning twilight. A 4th magnitude star would not be visible under these conditions and an observer around and after 500 BC, could not have seen *Śraviṣṭhā* / *Dhaniṣṭhā* rise helically at winter solstice. This star would have been visible at the horizon, just before the onset of morning twilight, only before 600±75 BC. The error in this epoch is due to a possible error of one day in determining the day of the solstice. If instead of α Delphini the Vedic calendar makers had observed the brighter β Delphini then the curve shown by the dotted line in Fig. 3 would have been obtained. As can be seen this makes no difference to the epoch when the calendar makers would have observed the helical rising of these stars. It would, however, be unrealistic to ignore topological features when considering the visibility, from a land-based site, of a star near the horizon. It is also impossible to access the impact of topological features on the visibility of *Śraviṣṭhā* /*Dhaniṣṭhā* at the time of *Jyotiṣa* as the exact location where this text was composed is not known. A line-of-sight at an altitude of 5.0° will probably clear all (reasonable) topological features. The times when α Delphini reaches an altitude of 5.0° (zenith distance of 85°) at winter solstice between 2000 BC and 2000 AD, are shown in Fig. 3. (effects of refraction have been included). On (and before) the winter solstice of about 1150±75 BC, α Delphini would have been at an altitude of 5.0° just before the onset of morning twilight and would have been clearly visible. The altitude of α Delphini at winter solstice of 1150 BC, and for the following twelve months is shown in Fig. 4. This altitude is calculated for the time just before the onset of morning twilight (i.e. the time when the sun gets to a zenith distance of 96°). From winter solstice the star appears further up in the sky and reaches its maximum altitude just after the vernal equinox. The altitude then begins to decrease and from about the last quarter of July the star does not rise above the horizon. It reaches its minimum altitude just after the autumnal

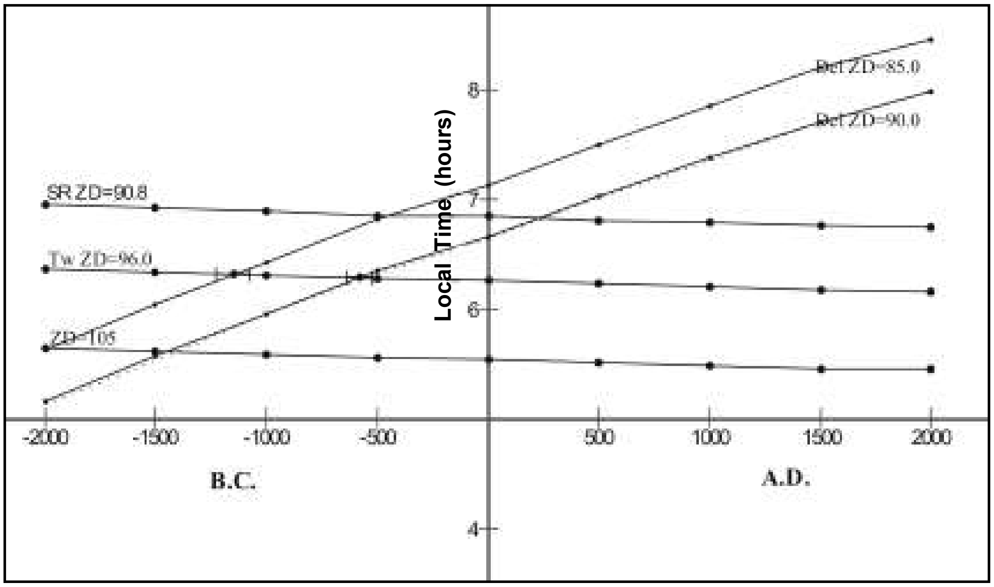

Fig. 3. The visibility of α Delphini. The local times, from 2000 BC to 2000 AD, of sunrise (ZD=90.8), civil twilight (ZD=96.0) and when the sun is 15° below the horizon (ZD=105), are shown by the dot-line curves. The local times when α Delphini is at a zenith distance of 90.0° (at the horizon) and 85° (5° above the horizon) are shown by the full curves. The error bars are for an uncertainty of 1 day in determining the winter solstice. The dotted curve shows the rise time of ß Delphini. The data are computed for longitude 77.2º east and latitude 28.6º north (New Delhi).

equinox and reappears in the morning sky from about mid-December. In the above description of the visibility of α Delphini, a very clear and transparent sky is assumed. If the sky transparency, similar to that observed in the twentieth century (described above) is assumed then an earlier date for the visibility of α Delphini would have to be considered (Fig. 3). It is reasonable to assume that on average the sky transparency in early second millennium BC, was considerably better than that in the twentieth century. Also the separation in azimuth (distance along the horizon) at sunrise, between the sun and α Delphini is about 30º at winter solstice (of 1150 BC) and it is possible that the visibility of the star would not have been significantly impaired. From the analysis of the data on the helical rising, at winter solstice, of *Śraviṣṭhā* /*Dhaniṣṭhā* given in *Vedāṅga Jyotiṣa* it can be concluded that this text was composed sometime between 1150 BC and 600 BC.

Narahari Achar[19] has proposed a revision of the composition of the *Jyotiṣa* to about 1800 BC. This is based on his contention that the identification of *Śraviṣṭhā* /*Dhaniṣṭhā* with α or β Delphini is wrong as these stars are too far from the ecliptic. He proposes δ Capricorni (2.81 magnitudes) as a more likely candidate and interprets verses *RJ* 5-6 and *YJ* 6-7 accordingly. However, he also ignores the realities of astronomical observations. At and around the winter solstice of 1752 BC (his fiducial epoch) δ Capricorni would have risen about 17 min. after sunrise and this star would not have been visible. In principle, it is possible

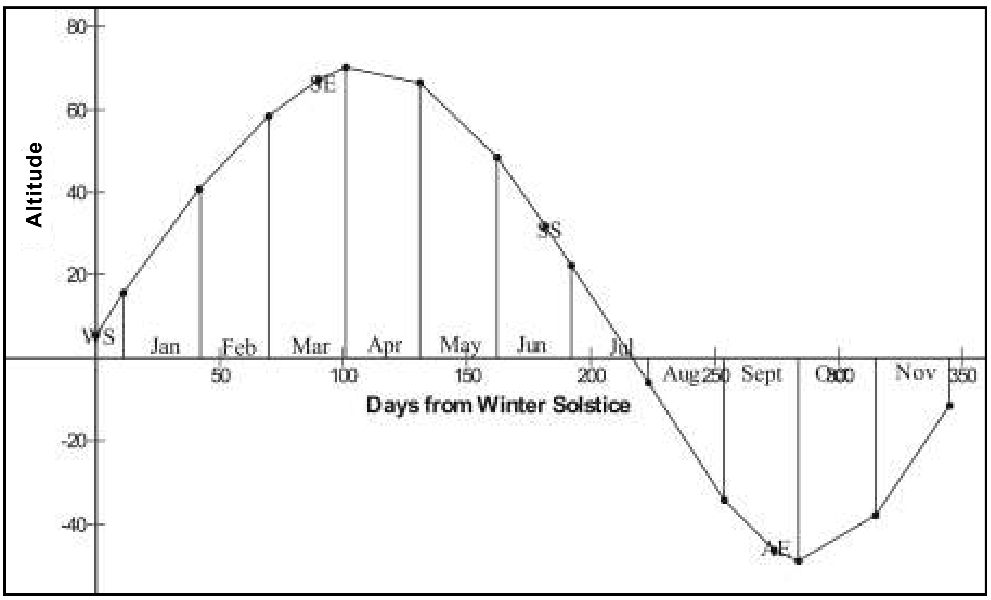

Fig. 4. The altitude of α Delphini at the start of twilight. The data are computed from winter solstice to the end of following November. The four cardinal points on the ecliptic are shown by WS (winter solstice), SE (spring equinox), SS (summer solstice) and AE (autumn equinox).

for the composer (Lagadha) of *Vedāṅga Jyotiṣa* to have known that the sun and a star would be close in the sky but when he states that the sun and *śraviṣṭhā* are together in the sky he is very likely referring to an observation and it would have been impossible for him to have observed δ Capricorni.

Observations of helical rising of a star to determine the appropriate time for performance of a sacrifice or a rite are mentioned in texts earlier then *Vedāṅga Jyotiṣa* like the *Brāhmaṇas* (e.g. *Taittirīya Brāhmaṇa* I.5.2.1). Thus observations of helical rising of a star or a group of stars to mark the start of a calendrical cycle would have been entirely appropriate for the observer. This *nakṣatra*, that marked the beginning of a *yuga,* is called *Śraviṣṭhā* - most famous, or *Dhaniṣṭhā* - very swift. It is worth asking why a rather inconspicuous group of stars (most stars in this group are around or fainter than 4^{th} magnitude) was given these rather conspicuous names. The answer may be in the sky at the time when *Vedāṅga Jyotiṣa* was composed. A montage of images of the sky at sunrise on winter solstice at four epochs is shown in Fig. 5. Around 2 BC, the Delphini star-group would have risen with the sun and would not have been seen. Around 500 BC, although the star group would have risen before sunrise it would have been in the full glare of the morning twilight and again would not have been visible. Only around 1000 BC, this group of stars could have been considered to rise helically as it would have been visible just before morning twilight. Between 2 BC and 500 BC, a marginally brighter star, η Aquilae (3.5 magnitudes) and a considerably brighter star, α Aquilae (0.77 magnitudes) could have been used to mark the sunrise as these stars would have risen at about 25 mins., and 36 mins., respectively before sunrise. Around 1000 BC, these rise times would have been 44 mins., and 55 mins., respectively and a more precise time of sunrise could only have been provided by the Delphini star-group (rise time 15 mins., before the onset of twilight). The only other brighter star that would have risen before sunrise at about this date would have been β_1 Capricorni (3.08 magnitudes), but at the on-set of twilight this star would have been only 1° above the horizon and would not have been visible in the glow of the morning twilight.

It would appear that around 1000 BC, the Ārya calendar makers had arrived at a critical stage in either formulating or re-formulating their calendar and had decided to identify or mark the start of their intercalation period (*yuga*). They required a celestial marker to signal the start of the *yuga* and they chose the Delphini group of stars. This was because a single bright star that would rise helically at winter solstice was not visible in the sky at that time and it is easier to identify a group of faint stars (for example the Pleiades) then a single faint star and the Delphini group entered the Ārya *nakṣatra*. The importance of this unique moment may account for the conspicuous name(s) given to this group of stars.

3. Conclusions

The Vedic astronomical text *Vedāṅga Jyotiṣa* states that the star *Śraviṣṭhā* /*Dhaniṣṭhā* α Delphini 'rises with the sun at winter solstice and this

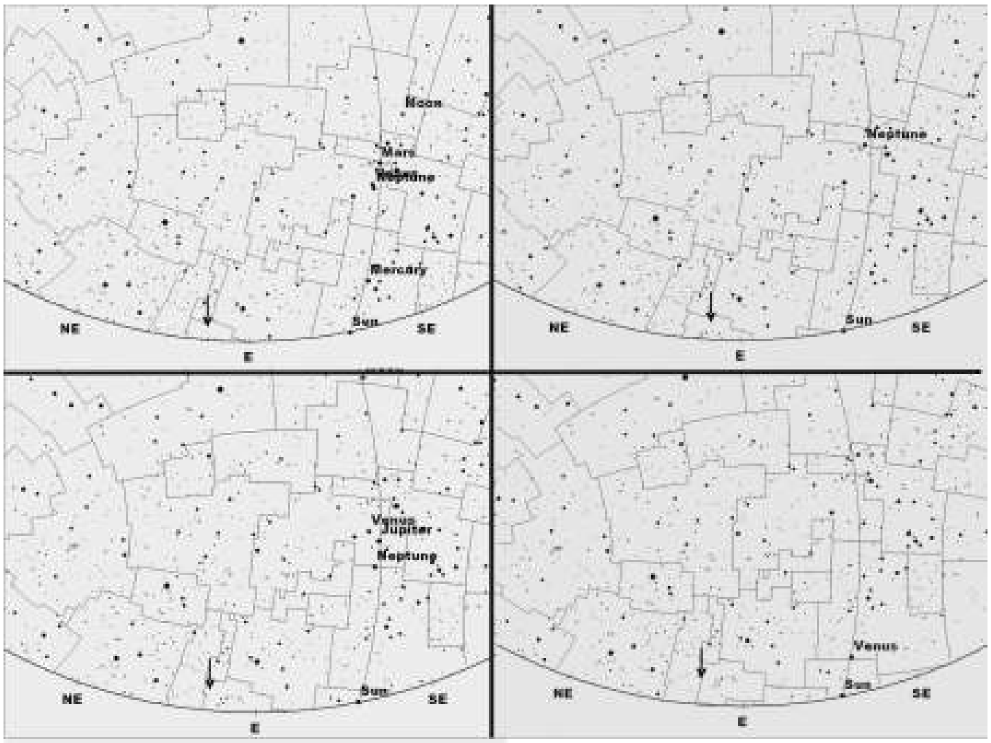

Fig. 5. Montage of position on the sky of the constellation of Delphini (arrow) at sunrise on winter solstice. The maps shown are for 2 BC (top left), 500 BC (top right), 1150 BC (bottom left) and 2000 BC (bottom right).

is the start of a *yuga* or the star α Delphini rose helically at winter solstice and this marked the start of a *yuga*. This information/data have been used to identify the epoch of composition of this text. Earlier efforts have suffered from unknown differences in the coordinate systems, unquantifiable errors in observations and personal prejudices. But the *Jyotiṣa* describes an observation and the visibility of α Delphini at sunrise or just before sunrise can be replicated to identify the epoch of this observation. This observation must have been made before the on-set of morning twilight as α Delphini and the stars in this constellation are too faint to be visible in the glow of the twilight. In other words, the Ārya must have observed this star 24 min. before dawn. It is shown that this observation is possible only before 600 BC, assuming there are no obstructing features in the line-of-sight. After this date, at winter solstice, this star would have been masked by the glow of twilight and direct sun-light. Topological features cannot be ignored in an analysis of this nature and in this analysis an elevation of 5° is assumed to clear all obstacles in the line-of-sight. Along this line-of-sight the helical rising of α Delphini would have been visible at and before 1150 BC. It is concluded that the *Ṛgvedic* recension of *Vedāṅga Jyotiṣa* was composed before 600 BC and possibly between 1150 BC and 600 BC.

The 'linear zig-zag' function in *Vedāṅga Jyotiṣa* that describes the 'length of the day' from winter solstice to summer solstice (and *vice versa*) is analysed to show that this function must have been determined from observations made around vernal equinox. Also these observations must have been made at a location(s) between 25° north and 30° north latitude. The function must have been extrapolated to the solstices and the ratio of 3:2 for the length of the longest to the shortest days in the *Jyotiṣa* is a consequence of this unsatisfactory extrapolation. There is no reason to believe that *Vedāṅga Jyotiṣa* was not composed in South Asia.

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