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ENLARGEMENT OF *VEDIS* IN THE *ŚULBASŪTRAS*

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The paper highlights the great works of Baudhāyana Āpastamba and Kātyāyana in the field of construction and enlargement of *Vedis*.

Key words: *Prakarma, Puruṣas, Samāsavidhi, Yajña*

Introduction

In the early vedic period, people performed ritual activities by the execution of *yajña*. For the accomplishment of these *yajñas*, they need to construct certain sacrificial altars and for the precise construction of these complex altars they had to attain some specific geometrical methods. These methods are written in the form of aphorisms or *sūtras*, and are found in the *śulbasūtras*. In this way from the *śulbasūtras* we get a glimpse of the knowledge of geometry, which the Vedic people had.

The *śulbasūtras*, which therefore represents the old and traditional material with further elaboration of vedic mathematics, generally classified as the part of *kalpa* or *śrauta-sūtras* or are separate. Though the names of several *śulbasūtras* are known, the *śulbasūtras* of Baudhāyana, Āpastamba and Kātyāyana are the most representative texts available to us.

Further it has been observed that for the performance of different rituals they need to construct distinct *Vedis*, which differ in shapes and sizes from one another by a specified amount. For the success of these sacrificial rituals the altars should be of very accurate measurements, thus mathematical accuracy was seen to be of the utmost importance at that time. Hence they derived and used some

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significant methods for the attainment of enlarged or contracted *vedis*. They affix the size of the altar according to the size (length) of the performer of the *yajña*, thus they called the word *puruṣa*, as the unit of measurement. Similarly *aṅgula*, *aratni*, *padas*, *prādeśa*, *akṣa* and other units of measurements, were used in making altars.

They were very accurate in their measurement that with the help of only a *śulba* (or cord) they not only made the altar as per size but also carried out the proportion to enlarge or contract the size of their vedis, according to the requirement. This paper presents the various methods and equations involved in the enlargement of *vedis* and fire-altars.

1. *Vedis* and *Agnis*

A *vedi* is constructed on a definite raised area on which the sacrifice is to be performed and on which persons performing the ceremony namely the sacrificer, the *hotā*, the *adhvaryu*, the *ṛtvik* and others are to be seated. Some of the main *vedis* include the *mahāvedi*, *aśvamedha*, *paitṛkī*, the *uttara* and the *śautrāmaṇi*.

Vedi is a raised altar, which is made by bricks for keeping the fire. The fire-altars were of two types, the *nitya* (or perpetual) and the *kāmya* (or optional). The *kāmya agnis* intended for wish fulfillment, included the *śyenacit*, *kaṅkacit*, *droṇacit*, *praugacit*, *alajacit* and so on.

Areas of Different *Vedis*

The specific methods for the construction of these *vedis* and their areas have been given in Baudhāyana, Āpastamba and Kātyāyana *śulbasūtras*. From these, we find the methods of construction of these *vedis* by using one cord (*ekarajjuvidhi*) and by using two cords (*dvirajjuvidhi*). Areas of different *vedis* are as follows:

Mahāvedi

From *Āpastamba Śulbasūtra* we find that Mahāvedi is an isosceles trapezium having face 24, base 30 and altitude 36 *padas* (Fig. 1). Hence its area will be:

$$A_M = 36 \times \frac{(24+30)}{2}$$

$$= 36 \text{ x } 27$$

$$= 972 \text{ sq. } \textit{padas}$$

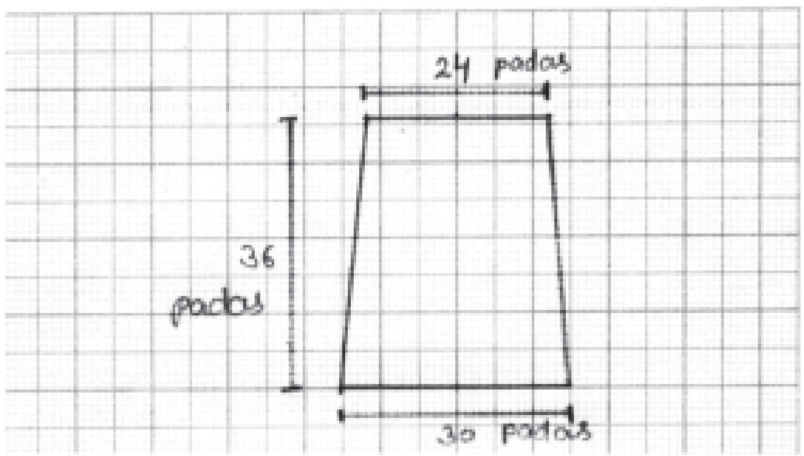


Fig. 1. Area of *mahāvedi*

Thus area of *mahāvedi* (A_M) is 972 square *padas*.

From *Āpastamba Śulbasūtra*, we get the following *śloka* which specify the area of *Mahāvedi*:

v’Vkfo'kR;u inlgl egkosfn%A vki 'k- i- 83

‘The area of *Mahāvedi* is 1000-28 i.e. 972 square *padas*.’

Aśvamedha Vedi

The *Aśvamedha vedi* covers an area of 1944 sq. *padas*. This is double of the size of *Mahāvedi*. From *Āpastamba Śulbasūtra* we find hints for the construction of similar isosceles trapezium of area 1944 sq. *padas* for the *Aśvamedha vedi*. Its area will be as follows:

$$A_A = 36\sqrt{2} \text{ x } \tfrac{1}{2} (24\sqrt{2} + 30\sqrt{2}) \text{ sq. } \textit{puruṣas}$$

$$= 1944 \text{ sq. } \textit{puruṣas}$$

where A_A = Area of *aśvamedha vedi*

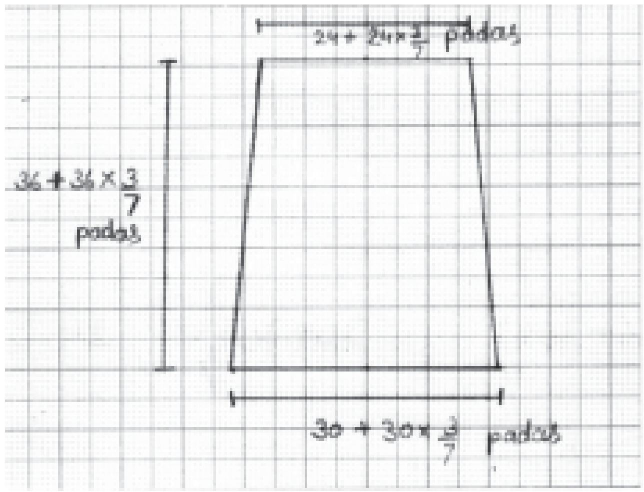


Fig. 2. *Aśvamedha vedi* (Area of enlarged vedi)

About the area of *aśvamedha vedi* we get the following *śloka* from *Āpastamba Śulbasūtra*

f}Lrkok osfnHkorhR;'oes/ks foKk;rsA vki 'k- i- 86

'Area of the altar for *Aśvamedha* is double the area of *Saumikī vedi* (*Mahāvedi*) i.e. 1944 sq. *padas*.'

Śautrāmaṇikī Vedi

Now the area of the *Śautrāmaṇikī vedi* is one-third of the area of the *Mahāvedi*. It is also in the form of an isosceles trapezium having face 24/√3 or 8√3, base 30/√3 or 10√3, and altitude 36/√3 or 12√3,and has an area 324 sq. *padas*. The area of *Śautrāmaṇikī vedi* (A_S) can be calculated as follows:

$$A_S = \tfrac{1}{2}\,(24/\sqrt{3} + 30/\sqrt{3}) \times 36/\sqrt{3}$$
$$= 27/\sqrt{3} \times 36/\sqrt{3}$$
$$= 972/3$$
$$= 324 \text{ sq. } padas$$

From *Āśl* and *Bśl*.1.88 we found the following verse

=hf.k prfo'kfu in'krkfu lk=kef.kdh osfn%A vki- 'k- 8-86

=hf.k prfo'kf/kdkuh in'krkfu osfnrh;eAA ck- 'k- 1-86

'The area of *Śautrāmaṇikī vedi* is (1/3 of *mahāvedi* = 1/3 x 972=324) 324 square *padas*.'

Paitṛkī Vedi

According to Baudhāyana, the area of *Paitṛkī vedi* (A_P) is one-ninth of the area of *Mahāvedi*.

Area of *Paitṛkī vedi* = 1/9 the area of *Mahāvedi*.

$A_P = 1/9\ A_m$

$A_P = 1/9 \times (972)$

$= 108$ sq. *padas*

Hence area of *paitṛkī vedi* is 108 sq. *padas*.

About the area of *paitṛkī vedi*, we find the following *śloka* from *Bśl*

egkosssr rh;s leprjlÑrk;kLrrh;dj.kh Hkorhfr uoeLr HkeHkkxks HkofrA

ck- 'k 1-82

'The altar for the *Pitṛyajña* is to be formed with the third part, of the side of the *Mahāvedi*, so that its area will be equal to the ninth part of the *Mahāvedi*".

(*Bśl* 1.82)

Uttara Vedi

The *uttara vedi*, according to Baudhāyana is a square pit of side 10 *padas*. Hence the area of *Uttara vedi* is 100 sq. *padas*.

$A_U = (10)$

$= 100$ sq. *padas*

Where A_U = area of *uttara vedi*

We get nothing much about *uttara vedi* from Āpastamba and Kātyāyana *śulbasūtras*.

After studying thoroughly about these main *vedis*, we found the following relation between their areas:

$A_M = 3A_S = A_A/2 = 9A_P = 9.72A_U$

Hence it is clear from the above relation that the areas of all the *vedis* are in proportion to each other. Thus if we know the area of only the *mahāvedi* we can derive the area of any of the above vedi by using this relation. For instance to make *aśvamedha vedi* they used the method of enlargement of *mahāvedi*, because *aśvamedha vedi* is double in area of *mahāvedi*.

2. Enlargement

It has been observed that in the sacrificial rituals of the early Hindus it is often necessary to construct a *vedi* differing in area from another by a specified amount. For instance, to make *Aśvamedha vedi* they enlarged the size of *Mahā vedi*. The specific methods adopted by them for the enlargement of *vedis* are described here.

2.1 Methods of Enlargement of *Vedis* in Early Period

Earliest evidences of the enlargement of *vedis* are found from *Śatapatha Brāhmaṇa*. From this we found that they enlarge the size of their *vedis* by increasing the length of the unit of measurement while keeping the shape (of the enlarged *vedi*) similar to the original one.

To construct a *vedi* 14 or $14^3/_7$ times as large as the *mahāvedi*, and which will be similar to it, *Śatapatha Brāhmaṇa* says :

If one wants to double the size of *mahāvedi* without changing its magnitude then operations are implied in following way:[1]

He measures (by means of) a cord 36 *prakramas* long, folds it into 7 equal parts ;of these three parts he adds to the east-west line and leave the rest. Similarly he measures 30 *prakramas* and 24 *prakramas* and folds them both (30 and 24) into 7 equal parts of these three parts he adds to the hind (transverse line) and throws out 4. This then is the alternative measurement of the (enlarged) *vedi*.

The measurement for enlarged *vedi* is as follows:

Face = 3/7 of 24 + 24

= (3/7 x 24 + 24) = 24(3/7+1)

= 24(10/7) *padas*

base = 3/7 of 30 +30
= 30(3/7+1)
= 30(10/7) *padas*

altitude = 3/7 of 36+36
= 36(3/7+1)
= 36(10/7) *padas*

Thus the area of enlarged *vedi* using the above measurement will be

$$= 36(10/7) \times \frac{\{24(10/7)+30(10/7)\}}{2}$$

= 36(10/7) x 54/2(10/7)

$= 36 \times 27(10/7)^2$

= 972(100/49)

= 972 x 2.04

Thus it is clear that the area of new enlarged *vedi* is approximately double of *mahāvedi*, where all the constituent sides received increment in equal proportions. This double of *mahāvedi* is *Aśvamedha vedi*.

2.2 Methods of enlargement of *Vedis* in *Śulbasūtras*

Vedic Hindus adopted some traditional methods for the enlargement of *vedis*, as they have employed for the construction of original *vedi*, only they replace the unit of measurement of the *vedi* by √N times. And hence they obtained new enlarged *vedi*, which is n-times, the original one. This will be clear from the succeeding instances:

For enlargement of *Mahāvedi* or to double its size they adopt the following method:

According to Ācarya Āpastamba the methods of construction of the new enlarged isosceles trapezium for the *aśvamedha vedi* will be the same as that of the given isosceles trapezium but here "√2 of a *prakramas* should be taken in the place of one *prakrama* therein ". Baudhāyana also has given the same method (vide p. 178).[1]

The *Śautrāmaniki vedi*, according to Baudhāyana, may be a square of 18 *padas* or an isosceles trapezium of area one-third of the area of *mahāvedi*, but he had not given any method for its construction. Āpastamba also made use of this method in the contraction of *vedis*. He constructed it by using $1/\sqrt{3}$ of the units used in the *mahāvedi* (vide p. 178).

2.3 Methods of Enlargement of *Vedis* Leading to the Quadratic Equation

The first plan of enlargement of a figure in which all the constituent parts are affected in equal proportions leads to the quadratic equation of the type[1]

$$ax^2 = c$$

And the second plan leads to the complete quadratic equation

$$ax^2 + bx = c$$

let x denote the length of enlarged unit of *puruṣa* and m denote the total increment in area. Then in the case of the enlargement of the isosceles trapezium i.e. *mahāvedi* on the 1st plan, we shall have

$$36\,x \times (24\,x+30\,x)/2 = 36 \times (24+30)/2 + m$$

$$36\,x \times (54\,x)/2 = 36 \times 54/2 + m$$

$$36\,x \times 27\,x = 36 \times 27 + m$$

or $$972\,x^2 = 972+m$$

$$x^2 = 1+m/972$$

or $$x = \sqrt{(1+ m/972)}$$

If m = 972(n-1) so that the area of the enlarged trapezium is n-times its original area we get $x = \sqrt{n}$ as given in the *śulba*.

Similarly for the enlargement of *aśvamedha vedi* we have

$$36\sqrt{2}\,x \times (24\sqrt{2}\,x + 30\sqrt{2}\,x)/2 = 36\sqrt{2} \times (24\sqrt{2} + 30\sqrt{2})/2$$

$$36\sqrt{2}\,x \times (54\sqrt{2}x)/2 = 36\sqrt{2} \times (54\sqrt{2})/2 + m$$

$$(36 \times 54)(\sqrt{2} \times \sqrt{2})\,x^2 = (36 \times 54)(\sqrt{2} \times \sqrt{2}) + m$$

$$1944\,x^2 = 1944 + m$$

$$x^2 = 1 + m/1944$$

Hence $x = \sqrt{(1+m/1944)}$

$x = \sqrt{n}$ if $m = 1944(n-1)$

In this way we see that the area of enlarged trapezium is n-times its original area.

Now it is clear to us that why vedic people used $\sqrt{2}$ to double the size of any *vedi* and $\sqrt{3}$ to triple the size of any *vedi*, because the area of enlarged figure is equal to n-times its original area, and the sides of enlarged *vedi* is equal to $\sqrt{n}$ times the sides of original one.

2.4 Equations Involved in the Enlargement of *Vedis*

Thus from above we get the following equations involved in the enlargement of *vedis*.

The rules of enlargement of *mahāvedi* leads to the equation

$972x^2 = 972 + m$

And the rules of enlargement of *Aśvamedha vedi* lead to the equation

$1944x^2 = 1944 + m$

Where x = side of small squares which will be used to draw the *vedi*

m = no. of squares which will increase with increase in area

Thus we find that the geometrical construction described here are of considerable algebraic significance. They indeed form the seed of the Hindu geometrical algebra whose developed form and effect we found from the *Bījagaṇita* of Bhāskara II (born 1114 AD). The quadratic equations involved in the enlargement of *mahāvedi* and *Aśvamedha vedi* shows that the seeds of quadratic equations are embedded in the rules and constructions of *Śulbasūtras*.

From equation of enlargement of *mahāvedi* we have

$972x^2 = 972 + m$

where

m	x
0	$\sqrt{1}$
972	$\sqrt{2}$
1944	$\sqrt{3}$
2916	$\sqrt{4}$

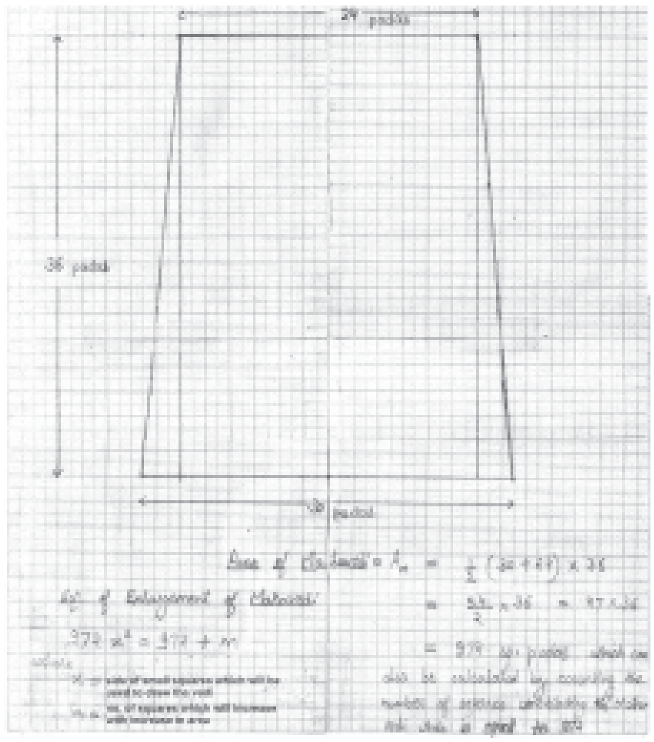


Fig. 3. Diagram of *mahāvedi*

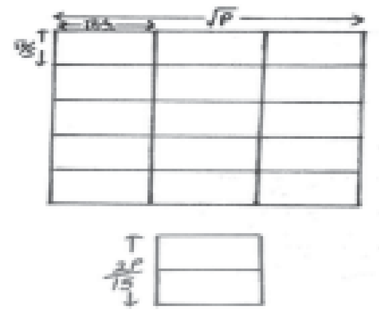
In this way we found that it is a mathematical fact that, only when m will be a multiple of 972 we get exact values of x.

2.5(a) Enlargement of Fire-Altar

Above we have read about the enlargement of *vedis*. Enlargement has been observed in the case of fire-altars also. For the fulfillment of any desire they start their *yajña* from the fire-altar of area 7½ sq *puruṣa*. Once their wish is fulfilled, they enlarged the size of fire-altar by 1 square *puruṣa* for holding another *yajña*. Similarly they enlarge the size of fire-altar by one square at each construction and this continues upto 101½ sq. *puruṣa*. Thus at the time of first construction the fire-altar should have an area of 7½ sq. *puruṣas*. At the second construction its area shall have to be 8½ sq. at the third construction 9(½) sq., this process continues upto 101½ sq. *puruṣas*.[6]

Baudhāyana gave the following methods for the geometrical operations applied in this method of construction:

At first is drawn a square of an area equal to p sq. *puruṣas*, whose side is equal to √p *puruṣa*. Divide horizontal side into 3 equal parts and vertical side into 5 equal parts. After drawing crosswise lines as shown in the diagram we get 15 rectangles, which are equal in area.Two of the rectangular portions are then combined together with the help of *samāsavidhi*. This portion is again added to a unit sq. *puruṣa* so as to form a third square. So the resulting square is [1+(2/15)] p long. On constructing an altar taking this length as unit we get the required figure of area 7½[1+(2/15) p].



Hence the area of enlarged fire-altar = 7½[1+(2/15) p]

or [7½])

And its unit will be = 1 + (2/15)p

Baudhāyana gives the side of a square (2/15)[7½+p] sq. *puruṣas* (where p=1 sq. *puruṣa*)[6] Āpastamba and Kātyāyana (*kśl* 5.5) also have given the same values as that of Baudhāyana.

Baudhāyana suggested an another method of dividing the original square into 15 equal parts, which is as follows:

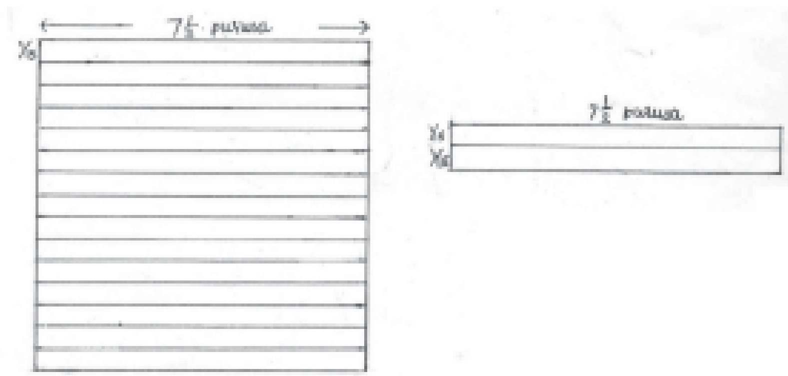
First of all a square of area 7½ sq. *puruṣa* is drawn. Now dividing it into 15 equal rectangles whose one side is equal to that of the side of the square of 7½ sq. *puruṣa* area and other side is one-fifteenth the length of the side of the square.[5]

Thus the area of each rectangle = 1/15 **X** 7½ = ½ sq. *puruṣa*

On combining the above area of two rectangles = ½ + ½ = 1 sq. *puruśa*

Adding this area to the given square = 7½ + 1 = 8½ sq. *puruṣa*

We get new fire-altar of area 8½ sq. *puruṣa*.



Similarly to obtain the square of 9½ sq. *puruṣa* area convert four numbers of these rectangles into one square of 2 sq. *puruṣa* area and by adding this square first square one can obtain a square of 9½ sq. *puruṣa* area. This can be shown as:

On combining the area of four rectangles = ½ + ½ + ½ + ½ = 2 sq. *puruṣa*

Adding this area to the given square = 7½ + 2 = 9½ sq. *puruṣa*

In this way they obtain a new fire-altar of area 9½ sq. *puruṣa*.

This procedure continues up to a fire-altar of area 101½ sq. *puruṣa*.

2.5(b) Enlargement of Fire-altar also Lead to the Quadratic Equation

Here we will discuss the enlargement in the case of falcon shaped fire-altar.

First Plan: Considering x is the enlarged unit in *puruṣa* and m the total increment in area for the enlargement of the falcon shaped fire-altar on the first plan.

It can be written in the form of a quadratic equation as follows:

$$\text{Body} + 2\text{ wings} + \text{tail} = 7\tfrac{1}{2} + m$$

$$(2x \times 2x) + 2x(x+x/5) + x(x+x/10) = 7\tfrac{1}{2} + m$$

$$4x^2 + 12/5x^2 + (11/10)\,x^2 = 7\tfrac{1}{2} + m$$

$$15/2\,x^2 = 7\tfrac{1}{2} + m$$

$$x^2 = 1 + 2m/15$$

or $x = \sqrt{(1 + 2m/15)}$

if we consider $n = 1 + 2m/15$

Then $x = \sqrt{n}$

It is similar, as we have obtained in the case of enlargement of *vedis*.

From *Śatapatha Brāhmaṇa* we found that m = 94 for the maximum enlargement of the fire-altar, that is

$x^2 = 13 + 8/15 = 14$ (approximately)

Second Plan: The rules of enlargement of the *śyenaciti* leads to the equation[3]

$$7x^2 + (x/2) = 7½ + m$$

or

$$7x^2 + x = 7½ + m$$

Multiplying both sides by 7 and completing the square on the left hand side

$$(7x + ¼)^2 = 841/16 + 7m$$

Taking square-root on both side

$$(7x + ¼) = \sqrt{(841/16 + 7m)}$$

$$x = 1/28 \{\sqrt{(841+112m)} - 1\}$$

$$x = 1/28 \{29(1+56m/841) - 1\}$$

$$x = 1+2m/29$$

On squaring and neglecting higher powers of m we get

$$x^2 = 1 + 4m/29 \text{ (approximately)}$$

Śyenaciti, the most ancient and primitive form of the "fire-altar for the sacrifices to attain special requirement", was in the form of a falcon.

The remains of the most striking altar, the *śyenaciti* still survive at Kausambi (near Allahabad).

From the above illustration it is quite clear that the construction of enlarged altars according to the second plan does undoubtedly depend preliminary on the solution of the complete quadratic equation

$ax^2 + bx = c$

Conclusion

It is very clear now that in the early vedic period mathematics was brought into the service of both secular and ritual activities. Indeed, *Śulbasūtras* laid the foundation of modern geometry, arithmetic and algebra, which has been further flourished, by our Indian mathematicians and scholars.

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