# HISTORICAL NOTES 

Asymmetrical Vedis in Śulbasūtras

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The Śulbasūtras describe complicated geometrical constructions showing mathematical knowledge of the ancient Indians. The objectives of some of these constructions are not very clear. Here an attempt is made to understand the Dārśapaurnamāsikī Vedi, which, as per the literal meaning should be connected to the new and full moon events. The construction of this seems to be through observational technique used for prediction of eclipses prior to establishing the mathematical rules.

## Introduction

Vedis are the altars constructed for the sacrificial fires. A variety of these altars with complicated geometrical constructions are described in the Śulbsūtras. Most of them are symmetrical and appear quite intricate. They are testimonies to the depth of mathematical knowledge apart from the technological advances involved in the making of bricks.

The Śulbsūtras available today are Baudhāyana, Apastamba, Kātyāyana and Mānava, which are dated between 800 BC to 400 BC (Sen and Bag, 1983). Among the various patterns that are described in these texts, two Vedis appear to deviate from the general symmetric patterns. One is the Dārśapaurṇamāsikī Vedi and the other is the positioning of the dakṣināgni with reference to āhavanīya and gārhapatya. The latter idea seems to be representing a method of identifying the onset of uttarāyaṇa (Shylaja, 2010). This was important since it marked the beginning of the year during the vedic period. Here we consider the astronomical significance of the Dārśapaurṇamāsikī Vedi.

[^0]The literal meaning of the name implies that it has something to do with the full moon and new moon. This aspect has not been discussed in detail in any of the previous studies. Dārśa is a word found often in stone inscriptions to refer to solar eclipses*. For example, the inscription no 398 of Epigraphia Carnatica Vol III states "kālayuktākhye mārgaśira māsi ca sūryoparāga samaye punye darśe samanvite...."; this corresponds to the solar eclipse of December 13, 1498AD. Another record dated $16^{\text {th }}$ January 1665, (EC, Vol V no 100, Mysore) states "śálivāhana varśeṣu śaṇashtā śarabhūmiśu gateśu krodhivarśe asmin pauṣe darśe ravi grahe ravendu kūja jevajñya ketu yoge. $\qquad$ .".

## Construction of Dārśapaurnāmsikī Vedi

As described in the Baudhāyana śulbasūtra the Vedi is in the form of isosceles trapezium. The precise dimensions are also provided (in Sen and Bag, 1983). This is reproduced in Fig. 1.


Fig. 1. The Dārśapaurnamāsikī Vedi as defined by the Baudhāyana śulbasūtra reproduced from Sen and Bag, 1983, Fig. 28.

[^1]In what follows the procedure to draw is described in detail.
The Vedi is placed west of āhavanīya having altitude of 96 añgulas, bases are 48 and 64 añgulas. In the figure $\mathrm{AD}=48$ arigulas, $\mathrm{BC}=64$ añgulas and EW = 96 añgulas (vide Fig. 1).
"A cord of length 2 BC is taken and a mark is made at its midpoint. The ends of the cord are fixed at the southern poles A and B and is stretched to the south by the middle mark and a pole is fixed at it. Fixing two ends of the cord at this pole an arc is drawn through AB by the middle mark of the cord. Similar arcs are drawn on the other sides. This is the Vedi."

The important question that needs to be addressed is the purpose of the Vedi. The clue comes from the name itself. Going back to the era of Vedas we confront a very big question on the observational tools they had for preparing the calendar and predicting eclipses. Ohashi (1994) has studied this aspect very carefully and shows that various types of sundials were used for marking the time and the angle. The standard which appears to have been in wide use was a horizontal dial with a 12 " gnomon. It must have been indeed a challenging task to make accurate measurements and apply it efficiently. Now let us see how the shadow paths are of relevance to determine the declination or the north-south coordinates of the sun and the moon.

## Shadow paths

The horizontal sundial has a "dial" with the shadow paths of the 12" gnomon marked from East to West. This simple device reveals the rather slow North-South motion of the sun after careful monitoring of the shadow day after day. Such diagrams can now be generated with trigonometric formulae since the variation of the declination of the sun (North - South coordinate) are precisely known. Fig. 2 gives an example of the shadow paths generated for Varanasi.

The longer duration of the days during the summer and shorter paths in winter is immediately noticeable. The path of the sun on equinox day will be more or less parallel to the E-W line.

The same dial can also be used for the moon for a few days around the full moon. Typically, it clearly shows similar tracks. However a careful examination shows that it differs as is indicated in Fig. 3. It deviates from the sun-shadow at the edges.


Fig. 2. The shadow paths for Varanasi as generated from sunearthtools.com


Fig. 3.The path of the tip of shadow for moon; notice that it can go the northern extreme beyond that of sun; At the southern edge a typical trace can be either of the two dashed curves. The template was created with the help of sunearthtools.com

Since the movement of the moon is much faster, the "day", in principle, will be shorter. However the light will not suffice to trace the shadow from rise to set. Thus the exercise will be limited to in and around the meridian transit. Extrapolation also is difficult as is clear from the Fig. 3. However, in the year 2001, experiments were conducted to trace the shadow and the result is depicted in Fig. 4. Here the markings 3, 2 and 1 indicate the trajectories obtained.

- 3 days before the full moon follows the sun shadow of $18^{\text {th }}$ March.
- 2 days before the full moon follows the sun shadow of $14^{\text {th }}$ April.
- 1 day before the full moon follows the sun shadow of $10^{\text {th }}$ May.


Fig. 4. The shadow of the moon traced for 3 days prior to the full moon in January 2001. The observed paths are superposed on the template created as in Fig. 3.

This shows that the declination of the moon varies rapidly and if we were to trace the shadow for the entire "moon day" the deviation would become immediately apparent. This is because the moon traverses the entire zodiacal belt within 27.32 days reaching the northern and southern maximum limits. In the example shown in Fig. 4, the moon's declination increases from -1 degree to +9.5 degrees to +18 degrees. The declination of the full moon was +22 degrees. The declination of the sun was -22 degrees and therefore that of the shadow was +22 degrees. This is depicted in the Fig. 5. Although, the declination of the full moon fell short by a few minutes from the centre of shadow of earth, there was a lunar eclipse.


Fig. 5. The geometry of the moon and the shadow of earth on January 9, 2001. Note that the node (point of intersection of the equator and the ecliptic) is well outside the shadow

The limits of declination values of the moon within a month are not the same as for the sun. The annual variation of declination for the sun range from +23.5 degrees to -23.5 degrees. For the moon it can have a larger range from +28.5 to -28.5 and it changes every year. In the year 2000 it was 21 degrees; in 2001 it was 22.6 degrees. In fact this is the most difficult thing to ascertain from observations and a tool like this Vedi could serve this purpose.

The declination of the sun also changes within a month, but slowly. In fact the corresponding correction for this movement from sunrise to sunset has been shown (Ohashi 1994) to be indicated in a verse in Mānasāra. It gives the range of correction for all the 12 months of the year. A similar correction is also described in Tantrasanigraha of Nīlakanṭha Somayājī (Ramasubramanyam and Sriram, 2010).

In the Fig. 4 the path of the full moon is shown in dashed lines; it was an eclipse and so no shadow could be marked.

We notice that on the full moon days (when there is no eclipse) the shadow path clearly gives an indication of how far the full moon was from the sun in the North-South direction. The daily motion of the moon can also be inferred. This can be extrapolated to the forthcoming full moon. For example, if the full moon was away from the shadow this month by a very
small angle, there is every possibility that it will coincide with the shadow next month. On the other hand, if the full moon is almost 90 degrees away there is no chance of an eclipse in the next couple of months. The possibility of the eclipse is decided by the location of the node, the point of intersection of the moon's orbit and the earth's orbit. This point itself has a retrograde motion and therefore fixing it is a very difficult job.

The Vedi can tell us the path of the moon around full moon nights, which in turn can be used to fix the position of the node. In fact this information itself will tell us about the maximum value of the declination of the moon attainable during that month. Let us extrapolate the above example for the previous month namely December 2000. The sun has a declination of -23.5 and the maximum possible value the moon can attain is only 21 , there is no possibility of eclipse. This information comes from the full moon shadow. At the same time it hints at the possibility of an eclipse in the next month when the sun's declination will change.

One may notice that the shadow dial reads out the declination of the moon at meridian transit. It may cross the value of the sun's declination during the day. In that case although the declination of the full moon was equal to that of the shadow the eclipse could not be seen. It happened during our day time. The instant at which the declination of the moon and that of the shadow were equal was called vyatipāta and recorded perhaps by interpolation. Extrapolating such records of vyatipāta and no visibility of eclipse, we find that those eclipses were visible from the other side of the globe. (Shylaja and Geetha, 2012).

Thus the north-south as well as the east-west difference of the node from the full moon is readable. The situation for the next 15 days is predictable and therefore the eclipses can be predicted within a month or two.

## The practical use of the Vedi

Now we consider the practical aspects of these observations. To match the dimensions of the Vedi a gnomon of 48 anigulas is needed. This is uncommon - all available texts give a value of 12 arigulas. It appears to be in use in Thailand. (See www.sundial.thai-isan-lao.com.htm)

The construction procedure, described above for the Vedi as delineated in Śulbasūtras, is similar for the shadows is described as follows by the first approximation to a circle.

The motion of the (tip of) the shadow on a desired day is to be determined as follows:
"The bāhu, koṭi and chāyā are determined at the desired instant as described earlier. With three sticks whose lengths are equal to bāhu, koṭi and chāyā at some instant a triangle is formed such that koti is along the eastwest direction with one tip at the centre of the circle. The bāhu also gets aligned in the appropriate direction (north-south). A point is marked at the intersection of bāhu and chāyā. A similar point is marked in the afternoon also. The tip of the midday shadow is taken as the third point. With these points three circles are drawn such that two fish figures are formed. The lines passing through the fish figures are extended and their point of intersection is found. With this point as the centre, draw a circle passing through the above three points. The (tip of the) shadow is along the circle drawn. $\qquad$ .."
(Ramasubramanian and Sriram, 2011, p. 179)
It is now very well known that the path is not a circle. It is mentioned by Nīlakanṭha as well.
"The statement made here that it is a circle is only approximate since it has not been proved $\qquad$ It is stated here simply to maintain concordance with what has been stated by the earlier teachers"
(Ramasubramanian and Sriram, 2011, p. 185)
This reference is of interest because it describes the method of marking the shadow path in exactly the same manner as has been described in the Śulbsūtras above for the Dārśapaurṇamāsikī Vedi.

Fig. 1 now needs a second look. The Vedi will serve the above mentioned purpose only if it is turned by 90 degrees. The 96 anigulas will now refer to the north-south direction. The northern side is longer than the southern side. The revised figure is shown in Fig. 4. It matches with a latitude of about 24 degrees and hence the comparison was done with Varanasi. (25 degrees; Ujjain with 23.5 degrees could have been a better choice, but considering the antiquity of 800 BC Vārānasi may be a better choice)

The original Sanskrit verse states as (words in parenthesis do not exist in the original):
3.6 To the west of ahavaniya, as per tradition, is the altar for the (new and full moon sacrifices) dārśapaurṇamāsa measuring (96 añgulas) yajamānamātrī (in the east west direction) 3.7 This measure less its third
(64 anigulas) forms the western side (of the altar) and half the measure (48 angulas) the eastern side; after making in this way a rectangle shorter on one side, poles are fixed at the four corners.
It is implied that the dimension of the Vedi is yajamānamātrī interpreted as 96 añgulas. The direction that it should be along east-west is not specified. The rest of the construction can now continue in the same way. There are some errors in the procedure - the two South pegs need to be used for marking the path by stretching the cord along South - where as the shape of the figure requires that the cord be stretched to the north and vice versa. Whether a similar construction is necessary along the east- west direction is not clear from the verse. Since no diagrams are available a direct verification is not possible.

It is interesting to note that the Vedi serves well for a latitude of about 24 degrees. Therefore the diagrams presented here are constructed for Varanasi. Application of this a place like Ujjain (latitude +23.5) will not lead to a great error.

That the Dārśapaurṇamāsikī Vedi was indeed oriented this way and the observations were done mounting a śañku at the point indicated in the Fig. 6, requires independent verification.


Fig. 6. Dārśapaurnamāsikī Vedi corrected for carrying out observations; the curves are for the tips of shadows for the moon when its declination is 28.5 N and 28.5 S

An indirect reference is available in the hundreds of stone inscriptions. The instant when the declination of the moon and the sun was same in magnitude was called vyatipāta. Its observation was very important task and on such days donations were made by the king or the ruling authority. A study of inscriptions from the $8^{\text {th }}$ century to about $15^{\text {th }}$ century AD shows that these events indeed were observationally recorded. (Shylaja and Geetha Kaidala, 2011, p.335).

There are two questions that remain unanswered:
i. Purpose of the curvature of the East - West boundary
ii. 48 añgulā gnomons were ever used.

## Conclusions

Dārśapaurṇamāsikī Vedi was a tool for marking the path of the full moon and the sun and comparing their relative positions in the sky. This helped in fixing the position of the node, which in turn decided the maximum possible declination of the moon and therefore the possibility of eclipses.

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www.sundial.thai-isan-lao.com.htm
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[^0]:    *Jawaharlal Nehru Planetarium, High Grounds, Bangalore 560001; email: shylaja.jnp@gmail.com

[^1]:    * The word darśe refers to the new moon day and not solar eclipse and not a solar eclipse. In other quotation from Vol III to EC too, it is pretty evident that the word darśa refers to new moon day. Rather than trying to understand what is intended the author has somehow tried to give an interpretation that is quite far-fetched. (Referee's opinion)

