# MATHEMATICS AND MATHEMATICAL RESEARCHES IN INDIA DURING FIFTH TO TWENTIETH CENTURIES - PROFILES AND PROSPECTS* 

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The Birth Centenary Celebration of Professor M. C. Chaki (19122007), former First Asutosh Birth Centenary Professor of Higher Mathematics and a noted figure in the community of modern geometers, took place recently on 21 July 2012 in Kolkata. The year 2012 is also the $125^{\text {th }}$ Birth Anniversary Year of great mathematical prodigy, Srinivas Ramanujan (1887-1920), and the Government of India has declared 2012 as the Year of Mathematics. To mark the occasion, Dr. A. K. Bag, FASc., one of the students of Professor Chaki was invited to deliver the Key Note Address. The present document made the basis of his address.

Key words: Algebra, Analysis, Binomial expansion, Calculus, Differential equation, Fluid and solid mechanics, Function, ISI, Kerala Mathematics, Kutṭaka, Mathemtical Modeling, Mathematical Societies Calcutta, Madras and Allahabad, Numbers, Probability and Statistics, TIFR, Universities of Calcutta, Madras and Bombay, Vargaprakrti.

India has been having a long tradition of mathematics. The contributions of Vedic and Jain mathematics are equally interesting. However, our discussion starts from $5^{\text {th }}$ century onwards, so the important features of Indian mathematics are presented here in phases to make it simple.

500-1200
The period: 500-1200 is extremely interesting in the sense that this is known as the Golden (Siddhäntic) period of Indian mathematics. It begins

[^0]with Āryabhaṭa I,who was born in 496 AD in Kusumpura (near Patna), a pioneer mathematician known for his systematic collection and systematization of knowledge, and ends with Bhāskara II born in 1114 AD in Bijjalabiḍa (Maharastra, close to Ujjain) who put the knowledge of mathematics on a solid foundation. The mathematicians in between themVarāhamihira (c. 505), Bhāskara I (c. 600), Brahmagupta (c. 628), Mahāvīra (c. 850), Śrīdhara (c. 850), Śrīpati (c. 1039), were equally famous, but I will take only a few examples from three mathematicians- Āryabhaṭa I, Brahmagupta and Bhāskara II to explain the importance of the period. In this context it will not be out of place to mention that the work, Aryabhat $\bar{y} y a$ of Āryabhaṭa I has two sections—Daśagītikā ( some essential parameters on decimal scale \& discovery of zero, elements of trigonometry) \& Gaṇita (eight fundamental operations, plane geometry, algebraical equations and their solutions), Brahmagupta has two sections in his Brähmasphuṭasiddhānta —Ganita (mathematics) and Kutṭaka (pulverizer), where as Bhā skara II wrote two separate works, Līlāvatī (mathematics) and Bījagaṇita (algebra), which shows how the knowledge of mathematics has expanded in volumes in this period. Of course I have avoided the discussion on the texts and restricted rather to their contributions in mathematics only.

## Zero and Operations with zero

The symbol for zero was discovered by Āryabhata I in connection to the decimal expression of numbers. Āryabhata I just says, 'the vacant places should be filled up with a circle’ which looks like 'śünya’. This has been illustrated by his commentator Bhāskara I. This indeed brought a revolution in mathematical computation and simplified whole technique of expressing number with nine numerical symbols and zero. The operation is carried out
 was taken to Central Asia with the business people which has been exemptified by al-Khwārizmi (c. 850). This subsequently passed on to Latin Europe and it was accepted as a part of the universal system. Āryabhata I gave details of eight fundamental operations but did not explain any operations with zero. Brahmagupta defines;

$$
\begin{aligned}
& x+(-x)=0 ; x \pm 0=x ; 0 \pm 0=0 \text { (addition with zero); } \\
& x \times 0=0=0 \times 0 \text { (multiplication with zero) }
\end{aligned}
$$

French mathematician ALG Demonville gave $0 \times 0=1$ even in 1831 . The division by zero was a problem with Indian mathematicians. So is other mathmaticians in Europe at the time. However, Indians gave : x / 0 = taccheda (Brahmagupta) or ; khahara (Bhāskara II); Bhāskara II had no idea whether khahara means infinity and got confused as far as operation is concerned.

## Geometry

Āryabhaṭa I gave the correct formulae for area or perimeter of all the common geometrical figures except surface area and volume of a sphere. Brahmagupta gave correct formulae for area of the triangle; also of the cyclic quadrilateral when sides are known, diagonals of the cyclic quadrilateral, but could not improve Āryabhaṭa's values for the surface area and volume of a sphere. It is Bhāskara II who gave the correct value.

## Trigonometry

Āryabhaṭa I who for the first time defined a right triangle within a quarter-circle and expressed perpendicular and base as function of angle, and expressed perpendicular, jyā or $j \bar{\imath} v a$ as $\mathrm{R} \operatorname{Sin} \theta$, and base, ko-jyā as $R \operatorname{Cos} \theta$, satisfying the relation
$(R \operatorname{Sin} \theta)^{2}+(R \operatorname{Cos} \theta)^{2}=R^{2}$,
where R is the radius of the circle. He gave all the values,
$R \operatorname{Sin} 0^{\circ}=0, R \operatorname{Sin} 30^{\circ}=R / \sqrt{ } 3, R \operatorname{Sin} 45^{\circ}=R / \sqrt{ } 2, R \operatorname{Sin} 60^{\circ}=R \sqrt{ } 3 / 2$, $R \operatorname{Sin} 90^{\circ}=R$.

He further suggested that all the Sin values increases in the interval $0 \leq \theta \leq 1$, and $\operatorname{Cos}$ values decreases in the same way, i.e. $\mathrm{R} \operatorname{Cos} 0^{0}=\mathrm{R}$ and $\operatorname{Cos}$ $90^{\circ}=0$.He also knew that first-order sine difference of Sine table gradually decreases, but he had no idea of the importance and use of second-order sine difference, which Bhāskara II gives correctly, and that it gradually increases. Further Bhāskara II deduced all the the important trigonometrical formulae correctly. The Sine table of Āryabhaṭa I was gradually improved by Bhā skara II by using of second- order difference function with interpolation which is equivalent to Newton-Stirling formula discovered in Europe in the $17^{\text {th }}$ century. Indian Sine concept is almost the same as the modern concept,
mathematically, if we consider Modern arc $\alpha$, the Indian arc as $\theta$, then the Modern and Indian function may be written as, $\operatorname{Sin} \alpha=\mathrm{R} \operatorname{Sin} \theta$

Both the relations become equal when $\mathrm{R}=1$ radian. Āryabhata I gave the value of $\pi=3.1416$ (correct to 3 places of decimal), considered also a great achievement in his time.

## Algebra

Kuttaka (the process of pulverization method or repeated division as in HCF process) was another technique used by Āryabhaṭa I for the solution of indeterminate equation of first degree of the type: by $=a x \pm c$.This has wide application in the solution of astronomical problems where 'a' represents revolution number of planets in a yuga, and 'b' the civil number of days in a yuga. Both these numbers were astronomical numbers and are as big as seven digits and are difficult to handle. Since the decimal system of fixing the value of a/b was not known before, the Kuttaka system was a process of mutual division as in a HCF process to find the approximate values which are closer to $\mathrm{a} / \mathrm{b}$, known as convergents in modern connotation, for $\mathrm{a} / \mathrm{b}$. It was also noted by Āryabhaṭa that all the odd convergents lie towards left and even convergents towards right, and higher the convergents, the more close it is to the actual value. The method is nothing but the process of continued fraction, which is a general method, discovered later by European scholars Bombelli \& Cataldi in 1548. With this method Bhāskara I solved as many as 117 mathematical and astronomical problems by pulverizer process. The method becomes extremely popular with later mathematicians including Bhāskara II who further simplified it. It became also popular in China when Itsing came to India in the seventh century and carried the knowledge to China.

Varga-prakrti or Solution of second degree equation of the type : $\mathrm{N} \mathrm{x}^{2} \pm 1=\mathrm{y}^{2}$, or $\mathrm{N} \mathrm{x}^{2} \pm \mathrm{c}=\mathrm{y}^{2}$, when N is a non-square integer, was given first by Brahmagupa, now popularly known as Pell's equation. The modern method of solution is to find the approximate values for $\sqrt{ } N$, since $y / x=\sqrt{ } N$ (approx.). What Brahmagupta did in the early $7^{\text {th }}$ century, from two adopted values, he obtained the solution by repeating a simple method, known as principle of crosswise multiplication \& addition (samāsa-bhāvanā) and principle of crosswise multiplication \& subtraction (viśleṣa-bhāvanā), which says,
if $(x, y)=\left(a_{1}, b_{1}\right)$ be an arbitrary sol of $N x^{2}+c_{1}=y^{2}$ for a suitable value of $c_{1}$,
and $\left(a_{2}, b_{2}\right)$ be another set of solution of $N x^{2}+c_{2}=y^{2}$ for suitable value of $\mathrm{C}_{2}$,
then $(x, y)=\left(a_{1} b_{2} \pm a_{2} b_{1}, b_{1} b_{2} \pm N a_{1} a_{2}\right)$ is the sol of $N x^{2}+c_{1} c_{2}=y^{2}$. Brahmagupta suggested that the integral sol of $\mathrm{N}^{2} \pm \mathrm{c}=\mathrm{y}^{2}$ is always possible when $\mathrm{c}= \pm 1, \pm 2$ or $\pm 4$. The general solution was however obtained by Jayadeva, Udayadivākara, but concretized by Bhāskara II on the basis of the extension of Brahmagupta's process, known as Cakravāla. From two set of values, one assumed value ( $\mathrm{a}, \mathrm{b}, \mathrm{k}$ ) and one identity $\left(1, \mathrm{~m}, \mathrm{~m}^{2}-\mathrm{N}\right)$ for smaller-root x, higher-root y and ksepa c respectively, the method forms a new set of root $\left(a_{1}, b_{1}, k_{1}\right)$ such that $a_{1}=(a m+b) / k, b_{1}=(N a+b m) / k$, $k_{1}=\left(m^{2}-N\right) / k$, $m$ should be so selected that $k_{1}$ (new kṣepa) should be an integer as small as possible. The method has a deep minimization property and can achieve the value of $x$ and $y$ much faster than Euler and Lagrange's regular expansion method. The Cakravāla method is found to have been based completely on a new algorithm which is much faster and better than that of Euler and Lagrange in the $18^{\text {th }}$ century.
Binomial expansion as used by Āryabhaṭa I is another important contribution. It had the following expressions and were applied for finding operations on indices, square, square-root, cube, cube-root etc,

$$
\begin{aligned}
& (a+b)^{0}=1 \\
& (a+b)^{1}=1 \cdot a+1 \cdot b \\
& (a+b)^{2}=1 \cdot a^{2}+2 \cdot a b+1 \cdot b^{2} ; \\
& (a+b)^{3}=1 \cdot a^{3}+3 \cdot a^{2} b+3 \cdot a b^{2}+1 \cdot b^{3} ; \text { and so on. }
\end{aligned}
$$

The coefficients in the right hand side, $(1,1),(1,2,1),(1,3,3,1)$, ( $1,4,6,4,1$ ).. and so on, form a triangular array when placed down below starting from the top, and were derived from the combination of musical sounds ( $\mathrm{a}, \mathrm{b}$ ), where $\mathrm{a}=$ laghu sound, and $\mathrm{b}=$ guru sound. That means there are (a,b), (aa, ab, ba, bb), (aaa, aab, aba, baa, abb, bab, bba, bbb) ...., or $(a+b),\left(a^{2}+2 a b+b^{2}\right),\left(a^{3}+3 a^{2} b+3 a b^{2}+b^{3}\right) \ldots$ varieties depending on eka-, dvi-, tri-mātrik sounds and so on. This triangular arrary was known as merū-prastāra from the time of Pingala (2 ${ }^{\text {nd }}$ century AD). He says, 'Make
a square and put 1 in it; make two squares under the first square exposing half-way on both sides and put 1 in each; make three squares under them exposing half-way on both sides and put 1 on the two ends of the three squares, and the middle square should have the number equal to the sum of the numbers of the upper two squares and so on.' This helped the expansion of $(a+b)^{n}$ when $n$ is a positive integer. The expansion was also known when is a negative integer in the Kerala school in the $15-16^{\text {th }}$ century. This triangular array is known as Pascal triangle several centuries later, and he says it was obtained by him in connection to discovery of combination of musical sounds.

## 1201-1800

The period: 1201-1800 is known as the Islamic period in India. It is actually the period of commentaries of the earlier texts or traditions. The period witnessed unique contributions in two areas- Kerala school of mathematics in the development of infinite series, and the establishment of five huge stone observatories-in Delhi, Jaipur, Benares, Mathura and Ujjain by Sawai Jai Singh(1688-1743). After the death of Aurangzeb (1658-1707), Jai Singh who had knowledge in Hindi, Sanskrit, Persian and Arabic got these done under Portugese \& Missionary influences, which was an attempt to revive interest in observational astronomy to improve time-measuring and astronomical co-ordinate measuring devices in India.

## Kerala Mathematics

The Kerala school developed during $15-16^{\text {th }}$ centuries with Mādhava (c.1400), Parameśvara (c.1430), Nīlakaṇṭha (c.1500), Citrabhānu (c.1510), Sañkara Vāriyar (c.1530), Jyeṣṭhadeva (c.1550), Accuta Piṣāroṭi (c.1570) and so on. The second and third scholars were major commentators of the earlier sanskrit texts and both call Mādhava as parama-guru, made great improvements of the earlier knowledge by introducing infinite series for $R \operatorname{Sin} \theta, R \operatorname{Cos} \theta$ when arc $s$ is taken as $s=R \theta$, and also for values of $\pi$. The Kerala School discovered these results about two hundred years before Gregory and others as follows:

$$
\operatorname{Sin} \theta=\theta-\frac{\theta^{3}}{3!}+\frac{\theta^{5}}{5!}-\frac{\theta^{7}}{7!}+\frac{\theta^{9}}{9!}-\ldots \ldots, 0 \leq \theta \leq \frac{\pi}{2}
$$

$$
\begin{aligned}
& \operatorname{Cos} \theta=1-\frac{\theta^{2}}{2!}+\frac{\theta^{4}}{4!}-\frac{\theta^{6}}{6!}+\frac{\theta^{8}}{8!}-\ldots, \frac{\pi}{2} \leq \theta \leq 0 \\
& \text { And, } \frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots \ldots, 0 \leq \frac{\pi}{4} \leq 1
\end{aligned}
$$

The value of $\pi$ has extremely slow converging properties, and were used to compute the Sine table. Consistent approach was also made to improve better and better values of $\pi$. From the series of $\pi$ to get the value correct to 8 decimal places, one has to compute 500 to 600 steps in the series. Mādhava made corrections at its n-th term to improve Āryabhata’s value of 3.1416 to 3.1415926536 (correct to 9 places of decimals). This helped Nīlakaṇṭha to prepare a better Sine table. The Sin, Cos series were established algebraically about 200 years later by Gregory (1638-1675), Newton (16421727), De-Moivre (1707-38), and Euler (1748).

## Stone Observatories

The five stone observatories in Delhi, Jaipur, Benares, Mathura and Ujjain were built by Sawai Jai Singh under the patronization of Mughal Emperor Mohammad Shah. This is also a great event in the observational history of India. The instruments like, Samrāt yantra (Equinoctial Sundial for measuring apparent solar time or local time of the place), Șaṣthamśayantra (60 degree meridian chamber for measuring the declination, zenith distance and diameter of the Sun); Dakșiṇottara vitti (Meridian dial or graduated semicircle for measuring meridian altitude or zenith distance of celestial object) are precision instruments for correction varying $\pm 10$ arc second. Some other instrument, Kapāla-yantra (Concave hemispherical bowl for measuring coordinates of the Sun in the horizon and equatorial systems, Lagna or sign rising in the eastern horizon) and a few others, were also known. But the most versatile instrument was Astrolabe, which became extremely popular for finding the longitude and latitude of stars, the time of day and night from the altitude of Sun, stars and for their use as perpetual calendar.

## Period of $19^{\text {th }}$ and $\mathbf{2 0}^{\text {th }}$ Centuries

The period of the $19^{\text {th }}$ century is a continuation of colonial set-up, where there were a few salaried scientists and technicians in government
scientific services, but as such there was little or no research effort in general ${ }^{1}$. The $20^{\text {th }}$ century saw the real exploration of knowledge in both mathematics teaching and mathematical researches, the major effort being made by the Indian themselves. They are discussed separately.

## 1801-1900

Idea of scientific research through a national institution came through the effort of Dr Mahendralal Sircar (1833-1904) which resulted in the establishment of Indian Association for the Cultivation of Science (IACS) in Calcutta in 1876. Organized research came under the initiative of J.C. Bose (1858-1937), P.C. Ray (1861-1944), Sir Asutosh Mookerjee, (1864-1934), C.V. Raman (1888-1970) and others. In mathematics Sir Asutosh Mookerjee was a luminary. He stood first in MA (mathematics) in 1885, passed MA in Physical sciences and Mixed mathematics in 1886, gave as many as 30 lectures based on his original research in IACS (Indian Association for the Cultivation of Science) as Hon. professor (1887-89), and published 12 research articles between1881 to 1990 in the Messenger of Mathematics, Quarterly Journal of Pure and Applied Mathematics (published from Cambridge) and the Journal \& Proceedings of the Asiatic Society of Bengal, Calcutta. All are well appreciated. His research investigations covered areas of both pure and applied mathematics - Theory of Elliptic functions, Differential equations relating to all types of Conics; and problems of hydrokinemetic equations relating to rotational and irrotational motions of fluids. He was elected Fellow of the American Mathematical Society in 1900.

## 1901-1960

The period: 1900-1960 brought a new change in the university circle when the three old universities of Calcutta, Madras and Bombay were allowed by the University Act of 1904 to conduct teaching and research. Calcutta university started first in 1907, Madras started also in 1907 but systematic research in Madras university began in 1927, while Bombay university began its research program in mathematics as late as 1941. However, ISI (Calcutta) began its activities in 1931. By 1950, the researches in mathematics expanded quickly, with the support by Research Centers, IITs, Advanced Centers, DAE, Atomic Energy, and others. Mathematical research also germinated in IIT Kharagpur (established in 1951), TIFR (established in 1951), and a few
other research centers as well as universities. To have an idea of status of mathematical knowledge and researches in the period, a zone-wise survey and investigation is carried out which may be found interesting:

## Eastern Zone

The Eastern Zone covers the activities of the Calcutta University, ISI Calcutta, IIT Kharagpur, and Jadavpur university.

## Calcutta University (CU)

Sir Asutosh served two terms of Vice-Chancellorship during 1906-14 (eight years) and again in 1921-23 (two years), which dominated the university affairs. He was instrumental in establishing Indian Mathematical (Club) Society in 1907 and its two serials, Journal and Bulletin (1908) to augment the research activities in the region. He was President of Calcutta Mathematical Society for about 16 years (1908-1923) from its inception to the time of his death (B.R. Seth became president next in 1924 and Syamadas Mukhopadhyay in 1937). The first set of PhD theses also came from this University. Two faculties, the Hardinge Professorship of Higher mathematics (alias Pure Mathematics) was established in the Calcutta University in 1912, and the Ghosh Professorship in Applied Mathematics in 1913. For Hardinge Professorship, W.H. Young, FRS, an expert in Measure theory, Furier series, Differential calculus, Functions of complex variables, was appointed as its first professor in 1913. For Ghosh Professorship, Asutosh invited Ganesh Prasad who had his master degree both from Allahabad and Calcutta, DSc. from Allahabad, and education and research in Cambridge under great professors. Ganesh Prasad had established himself as a reputed expert both for Pure and Applied mathematics.

The second Hardinge Professor C.E. Cullis who was experts on Matrices and Determinoids, continued up to 1923. The rigorous teaching and research were carried out. In Geometry-systematic contributions were made by Syamadas Mukhopadhyay (Properties of plane curves, parametric coefficients in the differential geometry of curves, setting minimum number of cyclic and sextactic points on a convex oval conic), Surendra Mohan Ganguly (Hyper-spaces), Haridas Bagchi (Curves of the third order, cubics and quartics, cycloid and hyper cycloids), D.N. Mallik (Fermat's law), R.C. Bose (Euclidean geometry of four dimensions, Non-Euclidean geometry,

Hyperbolic geometry, Hyperbolic space), R.N. Sen (Differential geometry, analysis of an arc in n-space, Spherical simplexes in n-dimensions, Rotation of hyper-surfaces, Connection between Levi-Civita parallelism and Einstein's teleparallelism), M.C. Chaki (Differential Geometry in Harmonic spaces, Symmetric spaces, Pseudo symmetric manifold, Psedo Ricci symmetric manifold, Quasi-Einstein manifold) ; in Analysis- F.W. Levi (Combinatorial analysis of Incidence graphs or Levi graphs of Various types-MobiusKantor graph of 8 points and 8 lines, Pappus graph of 9 points and 9 lines, Desargues graph of 10 points and 10 lines), H.M. Sengupta (Elasticities and analysis of functions), N.C.Bose, P.L.Ganguly ; and in Algebra- Nripendra Nath Ghosh (Roots of Numerical equations), B.C.Bhattacharjee (Abstract algebra), A.C. Chowdhury and others.

Ganesh Prasad was the first Ras Behari Ghosh Professor in Applied mathematics who served the Calcutta University during 1914-18, later he was invited again to hold Hardinge Professorship of Higher mathematics in 1923 which he accepted and continued till the time of his death in 1937. Ganesh Prasad made significant contributions on Function of real variables, Fourier series, Curvature of surfaces, Potential theory, Spherical and Ellipsoidal Harmonics. The next Ghosh professor, Sudhansu Kumar Banerjee (1917-1921), contributed on Sound waves on Spherical surfaces. N.R. Sen was the third Ghosh Professor who served it almost for 35 years from1924 to 1959, contributed immensely on Propagation of Waves in Canals \& Elastic mediums, Heisenberg's Spectrum of turbulence, Isotropic turbulence, Equilibrium Configuration of a Rotating Fluid. S.N. Bose (FRS, Padmabhusan on Bose-Einstein Condensate, Bose- Einstein Statistics, Quantum Mechanics, Boson) and Megnad Saha (Thermal Ionization equation, Stellar atmosphere) were in the faculty at the time. Other contributors are : C.V. Raman(Nobel Lareaute, became Palit professor in 1917, contributed on New methods of kinematic theory, on new features of Fermat's law, quantum nature of light, scattering of light and its changing of wavelength when passing through a transparent medium leading to Raman effect, etc), B.M. Sen, (Principal Presidency College during 1931-42 on Tidal oscillation on a spheroid), Suddodhan Ghosh (on Solid and Fluid Mechanics), Subodh Mitra (on Differential equations), Nripendra Nath Sen (Vortex Rings on Compressible \& Incompressible Fluids, Problems of Tidal oscillations), B.B. Datta ( Vortices on both Compressible and Incompressible Fluids \& Hist of Indian math),

Avadesh Narayan Singh (Non-differentiable functions, \& Hist of Indian Math),, and others. The Applied mathematics mainly covered Classical and Continuum Mechanics including Elasticity, Fluids Dynamics (both compressible and incompressible fluids), Rigid Dynamics, Transforms and Topology.

## Indian Statistical Institute (ISI Kolkata) and Faculty of Statistics (CU)

The foundation of Indian Statistical Institute (ISI) by P.C. Mahalanobis, the Professor of Physics, Presidency College Calcutta, a Cambridge Tripose, DSc, FRS in 1931, may be considered a glorious event. The publication of its journal, Sankhya, in 1933 was planned after Karl Pearson’s Biometrika, which brought international fame. Mahalanobis, a well-known expert on anthropometric measurements of Anglo-Indians in Calcutta; metrological problems; university exam results; large-scale sample surveys; crop and administrative surveys, consistently worked with his his collaborators who helped him to build up ISI. He took intiative to establish Calcutta University Department of Statistics as its honorary Head in 1941, and continued up to 1945.Mahalanobis was appointed Honorary Statistical Advisor to Indian Cabinet in 1949 and with his guidance the National Sample Survey (NSS in 1950) and the Central Statistical Organisation (CSO, in 1951) were set up as full-fledged department of statistics under Government of India and later on transferred to Ministry of Planning and Programme Implementation. After Mahalanobis, the Calcutta University Professorship was taken over by R.C.Bose (head, ISI) as full-fledged professor. Bose was systematic researcher and contributed on Design Theory, Coding method-BCH code (with D.K. Ray-chaudhri) which is named after him, Greco-Latin square (along with Shrikhade and E.T. Parkar). S.N. Roy (Joined ISI) and became professor of Statistics at Calcutta University during 1949-50 and contributed in the multivariate statistical analysis mainly on the Jacobian transformation \& Barlette decomposition; produced 15 doctorates; The Journal of Planning and Inference has produced a special issue on the centenary celebration of S.N. Roy in 1906..Well-known students of the Department were: H.K. Nandi \& C.R. Rao First batch) P.K. Bose, who made considerable popularity.. C.R.Rao (Director of ISI, FRS, student of R.C. Bose at University, went to USA in 1947) contributed on Statistical Inference, Linear Models, Multivariate Analysis, Biometry, Functional equations. Gopinath Kallianpur was a
distinguished probabilist and a professor of ISI in 1950's, went out to USA and came back and became its Director during 1976-79. J.K.Ghosh (another Director of ISI, worked on Modeling, Data analysis, Bayesian inference, Mathematical statistics and applications; R. Bahadur of ISI worked on Mathematical Statistics, Concept of Efficiency, Algorithm relating to Covariance matrices the latter two are known after his name, Deva Brata Lahiri of ISI (Number Theory, Survey Sampling, Partition Functions). J.M. Sengupta, S. Raja Rao, M.N. Murthy worked for NSS; B. Ramamurti, S. Subramanian, P.C. Mathew, K.R. Nair worked for CSO made names.

## Indian Institute of Technology, Kharagpur (IIT-Kh)

B.R. Seth became the first head in the Mathematics faculty of IIT Kharagpur. He had his Graduation from Delhi University \& D.Sc from London University. He joined IIT in 1951 as its Head, contributed on Solution to the Flexure of a bar problem, known as Saint-Vanaut's problem,Transition theory to unify the elastic and the plastic behavior of materials, and Continuum Mechanics. Gaganbehari Bandyopadhyay, a brilliant student of the Applied Mathematics Department of Calcutta University joined the faculty position of IIT Kharagpur and contributed on Flow Formation in an Elasto-Viscous Fluids, Transformation in Compressible Fluid with Heat transfer, Adiabasy and Periodicity in heat conducting gas, and Non-Newtonian Fluids. A.K. Gayen, another bright student of Calcutta University Statistics Department joined the IIT faculty and made names for his contribution on Statistics \& Quality Control. He was instrumental to publish the Journal, Journal of Science and Engineering Research. D.N. Mitra made great names and became the faculty member and contributed on Elasticity and Plasticity.

Four other IITs at Mumbai (1958), Chennai (1959), Kanpur (1959) and Delhi (1961), and Roorkee (2001) were established as institutes of national importance for higher technical education, basic and applied research. The contribution of these centers is quite marginal since the period gives coverage upto 1960.

## Jadavpur University (JU)

The Jadavpur university faculty of Applied Mathematics was started in 1956 with B. Sen as head. He was a brilliant student of applied mathematics
of Calcutta university and made enormous contribution in solid mechanic. 50 PhD theses were completed under his guidance from the Jadavpur university. Second head was R.N. Bhattacharya who was also product of Calcutta university applied mathematics. He studied problems related to Wave produced by a pressure system on the surface of deep and shallow water, Ship waves, Wave propagation and resistance and many other areas of fluid mechanics. Three students got their PhD under his guidance from Jadavpur university. Jadavpur established with in few years time a center of both solid and fluid mechanics.

## South Zone

The South Zone covers the activities of Srinivas Ramanujan, Indian Mathematical Society, Madras University, IMS and CMI.

## Srinivas Ramanujan (1887-1920)

He was a mathematical prodigy of Madras, and a young contemporary of Sir Asutosh of Calcutta University, needs a special mention. He was genius, contributed significantly in number theory and often compared with those of Euler, Gauss, Jacobi and others. He had no formal education in mathematics, joined the Cambridge group of mathematicians at the invitation of Prof Hardy, and left behind about 4000 original theorems. He was honored with B.A. Degree for his research by Cambridge University ( 1916), Fellow of the Royal Society (FRS of London in 1918). His ‘Collected Papers’, containing 21 of his earlier published papers including those with Hardy (edited by Profs. G.H. Hardy, P.V. Seshu Aiyar and B.M. Wilson) is published by Cambridge University Press, 1927. Three Note Books (Edited by: Prof. Bruce C. Berndt of the University of Illinois and published by SpringerVerlag in five parts) are now available. His contributions in number theory, elliptic functions, continued \& 'Mock' theta functions, infinite series are considered as gems and accepted as valuable contributions in world mathematics.

## Indian Mathematical Society, Madras

Indian Mathematical Society, Madras, was established in 1907 with its headquarters in Madras. The first President of the Society is B. Hanumantho

Rao (1907-1912), Professor of Mathematics, Punjab University, Lahore. Others Presidents are: B. Ramachandra Rao (1915-17), V. Ramaswamy Iyer (1926-30), P.V. Seshu Iyer (1932-34), R.P. Paranjape (1936-40, Senior Wrangler at the University of Cambridge, Librarian in IMS, College Principal, Vice-Chancellor in Bombay and Calcutta University). In its early stages the Indian Mathematical Society was nurtured by a band of enthusiastic persons who were however not researchers. The Journal consisted mainly problems posed and solutions given to these problems. This created an excellent climate for research but it delayed the PhD programme considerably.

## Madras University (MU)

It started serious research with R. Vaidyanathswamy (1894-1960; FRS, Edinburg) who had his education in Benaras and joined the reorganized mathematics research department of Madras University in 1927. He contributed significantly in the Point-set topology, Rational norm curve, and Algebra of Cubic residues. Ananda Rau (1893-1966),, a contemporary of Ramanujan and a Professor of Mathematics, Presidency College, Madras, contributed on number theory. T. Raghavan (1902-1955) worked with G.H. Hardy and contributed on Pure Mathematics. S.S. Pillai (1901-50) from Madras was also well known for his work on number theory, waring problem in particular in the determination of exact values for $g(n)$. S. Minakshisundaram (1913-68), educated in Madras University and \& professor of Andhra University in 1950 contributed on Tauberian theorems, Summability results of Classical Fourier analysis, P.K. Menon(1917-79) was from Alathur, Kerala, educated in Madras University contributed on Number theory, Combinatorics, Classical inequality theory. The contribution of M.R. Siddiqui (1949-50), in the University of Madras, are equally well known.

## Institute of Mathematical Sciences (IMS)/ Chennai Mathematical Institute (CMI), Chennai

IMS is another national institute which started in 60s by Allady Krishnaswamy on Theoretical Physics, Pure Mathematics and Stochastic processes. It followed a Visiting Professors program under DAE. R. Ramachandran, a distinguished theoretical physicist has been its head. R. Balasubramanian, the present Director of IMS made important contributions on theory of numbers. C.S. Seshadri (FRS) the present Director of the CMI
contributed significantly on Projective modules over polynomials, Group and Ring theory with various applications.

## North Zone

The North Zone has taken into account of ALU, BHU, AMU, ICAR, DU, PU, and Mehta Research Institute of Mathematics and Education.

## Allahabad University (ALU)

It is quite an old university which started teaching mathematics at the Muir College. Amiya Charan Banerjee who was a toper in mathematics from Presidency College, Calcutta, became a wrangler from Cambridge and joined as professor of mathematics in ALU when Meghnad Saha (Physics) and Nilratan Dhar (Chemistry) were at the University. Allahabad University Mathematical Association was established in 1927. A.C. Banerjee made contribution to astro-physics, galactic dynamics, and made lot of noise in favor of Indian observatories. Harish-Chandra, FRS,the renowned Indian mathematician was born in Kanpur, educated at Allahabad university worked with, Paul Dirac \& Wolfgang Pauli (both Nobel Laureates), obtained PhD in Mathematics from Cambridge University, contributed largely on Discrete series, Representation of semi-simple Lie group (analogues of Peter-Weyl group (with Anand Borel), finite group analogues, Automorphic forms, and Harmonic analysis of p-adic groups.

## Benaras University (BHU)

Mathematics got a new boost when Ganesh Prasad worked here from 1917-23. He gave new dynamics to research and founded the Benaras Mathematical Society which later changed its name to Bharat Ganit Parishad. The Society played an important role in the development of mathematics in India. V.V. Narlikar served the university for a long time (Pioneer researcher in General theory of relativity), and R.S. Misra (Gorakhpur, Aligarh University \& BHU) contributed to differentiable and metric manifolds of $n$ dimensional space during their associations in the University.

## Aligarh Muslim University (AMU)

AMU got the status of the residential university in 1920 with Sir Ziauddin Ahmad as the first head, followed by Andre Weil (1930-33),A.M.

Kureishy (1935-47), S.M. Shah(1953-58), Jamil A. Siddiqui (1958-66), also joined as head. Avadesh Narayan Singh and R.S. Misra also worked for sometime. Nothing much is known about its research activities.

Andre Weil wrote a book on Mathematics in India giving an interesting picture, as to why Indians are interested in mathematics, says, (1936, pp.12931): It says,
'Ramanujan's examples have served to give scope to the ambition of many young men and direct them towards the field of scientific research. This is the new generation working in Indian universities. They gradually try to change the spirit there and lift the level to that of universities in the west......For centuries they (Indians) have devoted themselves tirelessly to the most abstract subtleties of grammar and theology. It is hardly surprising the younger generations, when their time came, turned towards the sciences, and preferably the most abstract of them (mathematics)'.

## Indian Council of Agricultural Research (ICAR)

P.V. Sukhatme (1911- ) is originally from Pune, studied at the university of college of London (1933-36), got PhD (1936), and subsequently DSc (1939), joined Statistics Division of ICAR in 1940. He studied his research in method of yield of crops by developing survey technique of yield estimation by random sampling. V.G. Panse, Director of the Institute of Plant Industry, Indore, joined ICAR after him, and adopted the randomized block and split-plot design to plant bleeding material. He also recommended sampling method for estimating the yield per acre of cotton produce for Indian Central Cotton Committee. He was one of the founders of Indian Society of Agricultural Statistics.

## Delhi University (DU)

Post Graduate teaching started in Delhi university in 1947, Ram Behari was appointed Head of the Department for his expertise on Theory of ruled surface, Rectilinear congruences, Riemannian spaces, Einstein's Unified field theory and Differential geometry; followed by, P.L. Bhatnagar (joined later as Director to Mehta Research Institute and contributed on Fourier \& Allied series, Astrophysics—Origin of Solar system, Dwarf stars); J.N. Kapur (originally of IIT Kanpur) contributed on Fluid dynamics, Operation research, Mathematical modeling, followed by R.N. Verma, and others.

## Punjab University (PU)

Punjab university Chandigarh started its program of mathematics under R.P. Bambah who made significant contribution on number theory and discrete geometry. It became an advanced center in mathematics in 1963 to deal realistic problems of physical and experimental importance and solutions of non-linear equations, ordinary, partial and functional differential equations.

Mehta Research Institute of Mathematics and Education or Harish-Chandra
Institute of Mathematical Sciences (at Allahabad):
It started by Mehta Trust but mainly funded by DAE.A few distinguished scholars like P.L. Bhatnagar, S.S. Shrikhande, acted as directors and contributed. The name was later on changed to Harish-Chandra Institute of Mathematical Sciences. Well known names attached to this center are: Ashoka Sen, and others.

## Western Zone

The West Zone covers MU, and TIFR. IIT Mumbai, was started in 1958 and its contribution is not included in the period. So is IISc Bangalore which started late in mathematical researches.

## Bombay/Mumbai University (MU)

Bombay university in the Western India started at the same time as in Calcutta and Madras universities. But first PhD in mathematics was produced as late as 1942. The reason for such delay is that most of mathematics student from western region went to Cambridge and got fascinated by the Cambridge - Tripose. Mahajani and Pavate became senior wranglers, came back, wrote good text books, and showed no interest in research. Moreover, R.P. Paranjpe (1876-1966), Senior Wrangler, Head of Mathematics, Fergusson College, Vice-Chancellor (Bombay \& Lucknow Univ) was also not much interested in research. The University department started as late as 1941. Serious mathematical work started under the Headship of S.S. Srikhande when the department was approved as advanced center of mathematics with work on Algebra, Analysis, Combinatorics, Graph theory and Number theory.

## Tata Institute of Fundamental Research, Bombay (TIFR)

It is the most prestigious school of mathematics established in 1950.Originally hundreds of noted mathematicians have been invited to give lectures and interact with scholars. The school recruited bright students and gave them an opportunity to interact with mathematicians of the school and with visiting mathematicians. The school also organized occasionally summer schools and gave facilities to research workers to stay for short period at the school. Department of Atomic Energy (DAE) took the responsibility of TIFR when late H.J. Bhabha was the Secretary of DAE. The responsibility of looking after the interest of mathematics was given to DAE, which in its policy statement emphasized that that the main effort of the Department would be on the nurturing and development of mathematical talent in Indian Universities. After his death, not much was done on the improvement of university teaching. Further DAE depended much on TIFR which was not a teaching university and did not have much contact with students and teachers of Indian universities. However, TIFR got the maximum benefit from DAE.Total man-months spent by the foreign mathematicians here stand no comparison with those of the Indian Universities. The mathematical alumni here were: K.G. Ramanathan (Analytic number theory), K. Chandrasekharan (Analysis and theory of numbers), K. Ramachandra (Classical analytic theory of numbers), Roddam Narasimha (FRS on Atmospheric fluid mechanics and aerospace), S.Raghavan (Transcendental and Analytic number theory), M.S. Raghunathan (FRS, contributed on Algebraic and Discrete groups),R. Narasimhan (Fracture Mechanics \& Computational Solid Mechanics), Jayant Narlikar (astro-physicist). TIFR also started a program on Applications of Mathematics at Indian Institute of Science Bangalore. Here also, a large number of distinguished applied mathematicians were invited to give lectures, but in the absence of a core mathematical faculty at the centre, the Bangalore centre did not develop as expected. However, M.S. Narasimhan of Indian Institute of Science contributed substantially on Analysis, Differential Geometry \& Algebraic geometry.

## 1961-2000

Lineage of mathematicians in the period: 1961-2000 are listed below along with their professors, discipline-wise and zone-wise. The list is tentative and incomplete and requires further input. Discussion on the period is avoided,
since the last 50 years history is considered obligatory under IUHPS. The reference is only made when it is absolutely essential in the interest of the mathematics:

## Lineage of Mathematicians ( $\mathbf{2 0}^{\text {th }}$ century)

## East Zone

This has taken into account CU, ISI, IIT-Kh, and Jadabpur Univ. (JU).
(a) Fluid Dynamics, Solid Mechanics, Biomathematics and Relativity

Ganesh Prasad ${ }^{2}$ (1866-1937; CU, BHU \& ALU)—-Gorakh Prasad (ALU), B.N. Prasad (ALU), A.N. Singh (LU.), Braj Mohan (BHU), Hariprasanna Banerjee (CU), R.S. Verma (D U).
B.M. Sen (1890-60; Presidency College, Kolkata)— S.K. Banerjee (18931966; CU)—Shashadhar Dasgupta, Bijon Datta
N.R. Sen (1894-1963; CU)— M.V. Laue, N.K. Chatterjee, U.R. Burman, T.C. Roy, K.K. De, M. Ray, H.K. Ganguly, N.L. Ghosh; Inspired B.B. Datta (CU, Dsc in fluid mechanics and a great historian of Hindu Math.) \& G. Bandyopadhyay (IIT-Kh)
B.R. Seth (1907-79; IIT- Kh)— M.K. Jain (IIT-Del), Late Y.D. Wadhwa, P.D.S. Verma, A.C. Srivastava
G. Bandyopadhaya (IIT- Kh)—A.S. Gupta (IIT Kharagpur), Late J.R. Rao, C.N. Kaul;
D.N. Mitra (IIT-Kh)—A. Sanyal, S.D. Nigam;
A.S. Gupta (1932-2012, IIT- Kh)— L. Rai, B.S. Dandapath, G.C. Layek, M. Reza, S. Sengupa;
B. Sen (JU)—S.K. Datta, S.C. Dasgupta, A.K. Mal, M. Mitra, A.R. Sen, A.K. Mitra, D.K.Sinha, and so on (produced 50 PhD );

Suddodhan Ghosh (CU)—S.R. Khamrui;
R.N. Bhattacharya (JU)—R. Chakraborty, Iva Basu, Bandana Goswami;

Ambarish Ghosh (CU \& ISI-Kol)— P. Bhattacharya, K.D. Debnath, A.K. Majumder;
H.P. Majumder (CU \& ISI-Cal)—Swarup Poria (CU), Kripanath Dey (Maharaja Manindra Chandra College, CU), Nurul Islam (Narendrapur RK Mission, CU),A.K. Chakraborty, S.C. Ghosh, U.N. Ganguly;
S.K. Chakrabarty (Bengal Eng. College, Cal)-
(b) Algebraic/ Differential Geometry

Syamadas Mukhopadhyay (1866-1937; CU)—R.C. Bose, R.N. Sen, Gurudas Bhar;
S.M. Ganguli (1881-1931)-

Haridas Bagchi (1888-1968)-
R.N. Sen (1896-1974; CU)—M.C. Chaki, Hrishikesh Sen;
M.C. Chaki (1912-2007)—A.K. Bag (Editor, IJHS), Bandana Barua (CU), Manjusha Majumdar (CU), AN. Roychowdhury, D. Ghosh, A.K. Ray, S.K. Kar, G. Kumar, B. Chaki, S.K. Saha;

Bandana Barua (CU)—Ashok Kumar Ray (Dibrugarh U.), Soumitra Mukhopadhyay (Maharaja Manindra Chandra College, CU);

Manjusha Majumdar (CU) - Musa Jawarneh, Ashtapada Mayrah, Amalendu Ghosh;

## (c) Calculus and Analysis of Functions

W.H. Young (CU)—
F.W. Levi (CU)—
H.M. Sengupta(1902-1960, CU)—P.L. Ganguly, S. Mukhoti, S. Chatterjee, N.C. Basumajumdar, B.K. Lahiri
(d) Statistics, Probability, Combinatorics, Quality Control
P.C. Mohananobis (1893-1972; ISI \&CU)—
S.N. Bose (1894-1974; Dacca \& CU)-
R.C. Bose (1901-87; ISI \&CU)—C.R. Rao, H.K. Nandi, D.P. Raychaudhuri, P.K. Bose, A. Bhattacharya, S.S. Srikhande;
S.N. Roy (1906-1964)—A.K. Gayen;
C.R. Rao (1920- ; ISI-Cal; Penn State U. USa) - TES Raghavan (ISI-Cal, Illinois Univ. USA); W. Yu Linnik, Lau Ka-Sing, Z.D. Bai;
Raghu Raj. Bahadur (1924- ; ISI-Cal)—Jayaram Sethuraman (ISI-Cal), Johannes Venter (Chicago Univ., USA);
G. Kallianpur (1925- ; ISI-Cal) - ;
S.K. Mitra (1932-2003; ISI-Cal — M.L. Puri, C.G. Khatri;
A.K. Gayen (IIT-Kh)—A.K. Bhattacharya, G.P. Bhattacharya;
J.K. Ghosh (1937-; ISI-Cal)— B. Basu, K. Subramanyam, B.K. Sinha, R. Mukherjee;
K.B. Sinha (ISI-Cal) - D. Goswami (ISI-Cal), A. Mohari, W.O. Amrein, A. Mahapatra, R. Bhatia;
S. Dasgupta (1951- ; ISI-Cal) - G.S. Mdholkar, M.D. Parlman, R. Mukherjee, A. Bhandari, A. Goswami, B.V. Rao;
R. Mukherjee (1956- ; ISI-Cal) - ;

Arup Bose (1959- ; ISI-Cal) - ;
P. Chaudhuri (1963- ; ISI-Cal) - ;
(e) Differential Equations

Jyoti Das (CU)—V. Krishna Kumar (NISER, Bhuvaneswar), J. Sett (CU), G. Laha (Gurudas College, CU);

## South Zone

This covers besides MU, IMS, CMI, the other organisations like Hyderabad University (HYDU), Andhra Univ (ANU), Annamalai Univ (ANNU) and other research centres.
(a) Number \& Algebraic Fields/Mathematical Analysis

Srinivas Ramanujan (1887-1920)— Ananda Rao, S.S. Pillai;
K. Ananda Rao (1893-1966)—.K.S. Chandrasekharan (TIFR);

RSV Vaidyanathaswamy (1894-1960; MU)—P.K. Menon
S.S. Pillai (1901-1950)—
S. Minakshisundaram (1913-1968, ANU)—
P.K. Menon (1917-1979, ANNU)-
C.S. Seshadri (1932- CMI)— V.B. Mehta, T. Oda, V. Lakshmibai, C. Musili, K.R. Nagara;
R.S. Sridharan (1935- ; CMI)—
V. Kannan (1939- ; HYDU)— M. Rajagoplan, T. Soundararajan, S. Ravichandran;
R. Balasubramanian (1957- ; IMS)-
V.S. Sunder (1952- IMS)—
(b) Statistics\& Probability
B. Ramamurti ( ) -
T. Parthasarathy (1941; HYDU)-

BLS Prakash Rao (1942- ; HYDU)— M. Siva Prasad, Arusharka Sen;
R.L. Karandikar (1956- CMI)-
(c) Differential Geometry
S. Ramanan (1937- ; CMI)-
R. Narasimhan (1960- ; IISc)-
(d) Relativity/Astrophysics/Cosmology
S.Chandrasekhar (1910-1995;)—
J.V. Narlikar (1938- ; TIFR \& IUCCA-Pune)—
A.K. Das ( ; Osmania Nizamia Observatory)—

## North Zone

Covers University of Allahabad, Gorakhpur, Patna, Aligargh, Lucknow, Delhi, Punjab, ISI-Del, IIT-Del, Mehta Inst and others.
(a) Astrophysics/ Relativity
A.C. Banerjee (1891-1968; ALU)—P.L. Bhatnagar, Harish-Chandra;
V.V. Narlikar (1908-1991; BHU)—P.C. Vaidya;
(b) Number/Algebraic Theory/Geometry
R.P. Bambah (b.1925-, PU)—) S.K.Aggarwal, Gurnam Kaur, V.C.Damir;

Hansaraj Gupta ( ; PU)-
R.S. Misra ( ; Gorakpur U., BHU)—

Nagendranath ( ; Patna U)-
IBS Passi (1939- ; PU)— S.K. Sehgal, A.W. Hales, A.K. Bhandari;
R.J. Hans-Gill (1943-; PU)—
(c) Analysis / Functions

Harish-Chandra (1923-1983; ALU);
Ajit. Iqbal. Singh (1943- ;DU, ISI-Del)— Ajay Kumar (DU)
Rajendra Bhatia (1952- ; IIT-Del)— Tirthankar Bhattacharyya (IISc);
(d) Differential Equations

Ram Behari (1897-1981;DU)—
J.N. Kapur (1923-2002; DU\& IIT-Kan)— H.K. Kesavan, P.K. Sahoo, G.B. Gaur;
R.N. Verma (DU)-
P.C. Jain (1930- , IIT-Mum, DU)— B. G. Lohar, D.N. Holla, D. Greenspan, T.V. Singh;
M.K. Jain (1932- ; IIT-Del)— V.K. Srivastava, Tariq Aziz, Anantha Krishnaiah U.;
O.P. Bhutani (1935- ; IIT-Del)— Rama Shankar, P. Chandran, Santosh Sharma, Neelam Arora, G. Chasekaran, K. Vijaykumar;
S.K. Malik (1943- ; PU)— M. Singh, H.K. Khosla, R. Kant;
(d) Statistics \& Probability and Mathematical Modeling
P.V. Sukhatme (1911-1997; ICMR)—
B.G. Panse ( ; ICMR)-

PVK Iyer (1909-2009; PU)— M.N. Kapur, B.N. Singh, N.S. Shakuntala, M.N. Bhattacharyya;
S.S. Shrikhande (1917- ; Mehta I; MU)— D. Raghavarao, S.K. Tarthare, V.N. Bhatt, N.N. Singhi;

Ram Ballabh (LU) )—
B. Ramachandran (1932- ; ISI-Del)— K.S. Lau, H.M. Gu;
K.R. Parthsarathy (1936- ;ISI-Del)— Inder Kumar Rana (IIT-Mu) Rajendra Bhatia (ISI-Del), S. Ramasubramanian (ISI-Bang), B.V. Rajaram Bhat (ISIBang), Arup Kumar Paul (ISI-Del);

Alok Dey (1945- ; ISI-Del)— Priyanka Grover (ISI- Del),
(d) Solid Mechanics, Fluid Dynamics
P.L. Bhatnagar (1912-76; ALU)— P. Prasad (IISc);

Mahendra Pal Singh (1931- ; AIT, Gurgaon) - K. Khetarpal, Maithili Sharan, Indu Sud, Aminataci, S. Selvakumar, Manju Kumari, S. Ghosh;
S.K. Trehan (1931-2003 PU)— N.K. Nayyar, B. Buti, R.K. Chhabra;

Sarva Jit Singh (1939- ; DU)—H.R. Wason (IIT-Roorki), N.R. Garg (Dayanand U., Rohtak), Kuldip Singh (Guru Jambheshwar U., Hissar), J.S.Sikka (Dayanand U., Rohtak), Sunita Rani (Guru Jambheshwar U., Hissar)

Maithili Sharan (1939- ; DU)-
N.K. Gupta (1942- ; IIT-Del)—

## West Zone

This includes TIFR, IISc, IIT-Mum, universities of Mumbai, Gujarat, Karnataka, Bangalore and others.
(a) Number Theory, Algebraic Geometry, Analysis
K.G. Ramanathan (1920-92; TIFR) — K. Ramachandra, S.A. Raghavan, V.C. Nanda,;
K.S. Chandrasekharan (1920- 2009; TIFR)—M.Narasimhan,Raghavan Narasimhan (TIFR), C. Seshadri (MU), Max-Albert Knus (Zurich, Manuel Ojanguren (Zurich), John Steinig (Zurich);
M.S. Narasimhan (1932- ; IISc)— S. Ramanan, T. Kotake, T.R. Ramadas, Shrawan Kumar;
K. Ramachandra (1933- ; TIFR)—R. Balasubramanian (IMS, Chennai), T.N. Shorey, Srinivasan, Shankaranarayan, K. Srinivas, P. Erdos, G.J. Babu; Srinivas Raghavan (1934- ; TIFR)— J.S. Dani (MU), T.C. Vasudevan (RKM Vivekananda College, Chennai), Juliet Britto (KU) , U. Balakrishnan (Zurich, Singapore), S.S. Rangachari;
V.C. Nanda (TIFR) -Goutumi Adhikari, Asha Narang;
R. Sridharan (1935- ; TIFR, CMI)—Shrikant M. Bhatwadekar, R. Parimala, M.A. Kaus;
M.G. Nadkarni (1939- ; MUMU)— V. Mandrekar, J.M. Bagchi, J. Mathew, J. Aaronson, P. Chaube;

Balwant Singh (1940; MUMU Campus) - L.G. Roberts, L. Reid;
M.S. Raghunathan (1941- ; TIFR)— G. Prasad; A. Ramanathan;
R. Parthasarathy (1945- ; TIFR)— R. Hotta, T. Enright;
G. Prasad (1945- ; Miahi U, USA)— A. Borel, A. Moy;
T.N. Shorey (1945- ; IIT-Mum)— N. Saradha, R. Tijdeman;

Shrikant M. Bhatwadekar (1946- ; TIFR)—Amarta Kumar Datta (ISICal),Manoj Kumar Kehari (IIT-Mum), Sarang S. Sane (Kansas U., USA);
V.B. Mehta (1946- ; TIFR)-
S.G. Dani (1947- ; TIFR)— M.M. Crudden, A.G. Marqulis;
R. Parimal (1948- ;)— M. Kneswer, J.L. Colliot-Thelene, M. Ojanguren;
M.V. Nori (1949- ; Chicago U.,USA)—

NKM Singhi (1949- ; TIFR)— M. Deza, C.K. Praneshachar, G.R. Vijaykumar, S.B. Rao, D.K. Raychaudhuri;

Rajendra V. Gurjar (1950- ; TIFR)—S.K.Rouson (TIFR), Vinay Wagh (IIT-Guwahati), Alok Maharana (IISER Mohali), Shameek Paul (Post Doctoral position, France), Sagar Kolte (Post Doctoral position in KIAS, South Korea);
G. Misra (1956- ; IISc)-
S. Thangavelu (1957- ; IISc)-
T.N. Venkataraman (1958- ; TIFR)—

Vasudevan Srinivas (1958- ; TIFR)—A.J.Parameswaran (TIFR), Jishnu G. Biswas, Amalendu Krishna(TIFR), Vivek M. Mallick (Centre de Recerca Mathematica, Barcelona, Spain),Ronnie Mani Sebastain (Humbolt U, Berlin, Germany);
D. Prasad (1960- ; TIFR)-
(b) Differential equations
S. Ramanan (1937- ; TIFR)— A. Ramanathan, N. Mestrano;
P. Prasad (1944- ;IISc)— Renuka Ravindran, K.W. Morton, A. Sau;
V.D. Sharma (1949-;IIT-Mum)—
A. Adimurthi (1952- ; TIFR)-
(c) Fluid Dynamics/Solid Mechanics/Seismology/Aero-dynamics
V.R. Thiruvenkatachar (1908-88; Defence Ministry, Bang)— K. Viswanathan, P.K. Khosla, I.J. Kumar, S. Bhansali;
N. Rudraiah (1932- ; Gularga U.) - M. Venkatachalappa, B.G. Chandrasekhara, P. Kandeswami, P.K. Srimani, E.S. Shivaraya, P.L. Sachdev;
B. Siddappa (1932- ; Gulbarga Univ.) - N.M. Bujurke

Roddam Narasimhan (1933- ; NAL \& NIAS)—
P.L. Sachdev (1937-; IISc)— M. Lobo, Neelam Gupta, Philip Varughise, K.R.C. Nair;
N. Mukunda (1939- ; IISc)—
N.M. Bujurke (1944- ; KARU)— S.N. Biradar (Science \& Commerce College, Bidar), N.B. Nadurinmani (Gulbarga Univ), N.P. Pai (MIT, Manipal, R.P. Kudanalli (Bang U.), Shival Shetty (Karnatak College, Dharwad);
M. Venkatachalappa (1947- ; BANU)-
V.D. Sharma (1949-;IIT-Mum)-
R. Narasimhan (1960-; IISc)-
(d) Relativity/Astrophysics
P.C. Vaidya (1918- ;GUJU)— I.K. Patel;
J.V. Narlikar (1938-; TIFR \& INU-Pune)—

## Discussion

The modern period of $20^{\text {th }}$ century in Indian mathematics went through a lot of conceptual changes, specially in the first half, because of British and European interactions in mathematics. It is not an accident but a chance situation because of the Colonial rule that the Indian mathematicians came closer to British and European mathematics and felt it as a national urge to prove that they are equally competent and contribute to higher field of mathematics, and in the process created a good deal of qualitative changes in mathematics and mathematical researches in India.

## Field of Numbers

The field of numbers has passed through the concept of real number, square-root of non-square integers, imaginary numbers (square-root of negative numbers), complex numbers, hyper complex numbers, analytic or algebraic numbers (root of an algebraic equation whose coefficients are rational integers, e.g. $0, \pm 1, \sqrt{ }-2$, or 4.5 ) and transcendental numbers (not algebraic numbers, e.g. $\pi$ or e, $\log _{\mathrm{e}} \mathrm{x}$, and so on). The complex numbers, transcendental numbers throw ample light on the theory of algebraic equations. On the theory of prime numbers, French mathematician Fermat's algebraic equation led to, $x^{n}+y^{n}=a^{n}$ (for $n>2$, and not zero) created lot of interest among the mathematicians and is still a mystery. Gauss developed a whole thesis in order to obtain prime numbers less than a fixed number. Some of the Ramanujan's conjectures in number theory are still puzzles to many scholars and some American mathematicians are presently busy to find methodologies for thirty of his unsolved problems. His conjectures of cusp form (tau function) has got some connections with A Weil's algebraic geometry in the study of modular group which has opened new areas for investigation. So his well-known asymptotic formula for the partition of $n$ along with Hardy-Littlewood (partition theory) is being used to attack 'Circle Method Problems'. This method has been further discussed and refined by S.S. Pillai of Madras university. The theory of sets (two sets contain the same number of things if, and only if, the things in them can be paired off one-to-one), nondenumerable sets (set of all points in a line), subsets, point set topology (logic of Vaidyanathswamy) etc. came in the process to make the concepts of real numbers, continuity, limit and infinity precise and
consistently usable. The work on analytic and transcendental number theories, however, has been further extended by TIFR group and the geometry of these numbers by the Punjab university school of mathematic under R.P. Bambah.

## Fields of Algebra

The field of algebra has basically moved on the fundamental concept that every equation has a root and an equation of degree $n$ has $n$-number of roots. The search for roots has led to the concept of 'Determinants', an algebraic method to solve simultaneous equations; Vector algebra as a row or column of quantities and 'Matrix', a type of arrangement into rows and columns following algebraic rules to solve linear and other type of equations; detailed investigation of Complex numbers of the form $\mathrm{a}+\mathrm{i} \mathrm{b}$ ( $\mathrm{a}, \mathrm{b}$ are real numbers) and so on. The concepts of Groups, Rings and Fields consisting of a set of elements having one or more specific operations were also considered as a part of the mathematical system for investigations. C [a, b] the class of continuous functions (Banach algebra), $\mathrm{P}[\mathrm{a}, \mathrm{b}]$ the set of all polynomials, C* restricted class of Banach algebra are all Group algebra and are being studied for their various applications. The concept of 'Groups' was also attempted to solve the general equation of fifth degree. (The same group turns up slightly to anticipate the theory of elliptic functions). The matrices was recognized as an important tool by Heisenberg in 1925 to explain his revolutionary work in quantum mechanics which was further extended on the basis of commutative relations by him jointly with BornJordan. So is the concept of vector (a line segment having both length and direction introduced in algebra with the help of complex numbers) with its various laws used in two dimensions which were generalized and brought meaningful changes for rotation in space for three dimensions. The Boolean algebra based on certain postulates was an attempt to translate the whole of classical logic symbolically (symbolic logic) did not make much impact here in India in spite of support by Whitehead and Russell. However it is being used to electrical circuits which form basic structure of business machines. The theory of Groups is being used in the 'System of Rays in Optics' consisting of a straight, bent or curved lines to investigate problems (without figures or diagrams) by functions and formulas in order to construct an algebraic theory for such systems. This type of application of algebra to

Optics has many applications to problems of dynamics in mechanics. TIFR is busy for the construction of 'ray class fields' over 'imaginary fields of elliptic units' and appears possibly one of the major achievements in the field.The field of algebraic geometry with reference to vector bundles has been an important area for study for the TIFR group. Vector bundles have led to moduli of parabolic bundles, principal bundles, algebraic differential equations having various other applications in physics. Fields are special types of rings. Seshadri has made considerable studies on Ring algebra and their applications.

## Fields of Geometry

Geometry today is largely a matter of analysis. The old terminology of 'points', 'lines', 'distances' and so on have become meaningful in suggesting interesting things with sets of co-ordinates. The Concept of nonEuclidean geometry developed by Bolayai, Lobachevsky and Gauss got a new meaning in the hands of Riemann who gave us the concept of a manifold, defining it as a set of ordered ' n -tuples' of numbers ( $\mathrm{x}_{1}, \mathrm{x}_{2}, \ldots \mathrm{x}_{\mathrm{n}}$ ). It is said to be of $n$ dimensions, as there are $n$ numbers occurring in this manifold of ordered n-tuples. Distance and Curvature have also been defined and generalized from their definition of two or three dimensions (i.e. when $n=2$ or 3).Taking $n=4$, the distance between two neighboring points, say, ( $x_{1}, x_{2}$, $\left.\mathrm{x}_{3}, \mathrm{x}_{4}\right)$ and $\left(\mathrm{x}_{1}+\mathrm{x}^{\prime}{ }_{1}, \mathrm{x}_{2}+\mathrm{x}^{\prime}{ }_{2}, \mathrm{x}_{3}+\mathrm{x}_{3}^{\prime}, \mathrm{x}_{4}+\mathrm{x}_{4}^{\prime}\right)$ in a space of four dimensions of the manifold is found as the square root of :

$$
\begin{aligned}
& g_{11} x_{1}^{\prime}{ }_{1}^{2}+g_{22} x_{2}^{\prime}{ }^{2}+g_{33} x_{3}^{\prime}{ }^{2}+g_{44}^{\prime}{ }^{2} \\
& +g_{12} x_{1}^{\prime} x_{2}^{\prime}+g_{13} x_{1}^{\prime} x_{3}^{\prime}+g_{14} x_{1}^{\prime} x_{4}^{\prime} \\
& +g_{23} x_{2}^{\prime} x_{3}^{\prime}+g_{24} x_{2}^{\prime} x_{4}^{\prime} \\
& +g_{34} x_{3}^{\prime} x_{4}^{\prime}
\end{aligned}
$$

in which the ten coefficients, $g_{11}, g_{22}, \ldots \ldots, g_{34}$ are functions of $x_{1}, x_{2}, x_{3}, x_{4}$.
For particular choice of g's, one space is defined. We can even consider a space in which all the g's except $g_{44}$ and $g_{34}$ were zero and so on. In relativity, a space is of this general kind in which all the g's except $g_{11}, g_{22}$, $g_{33}, g_{44}$, are zero. For two dimensions, a straight line has zero curvature, and the amount of curvature for a curved line is assessed by its departure from straightness. For the curvature of a surface, it is measured similarly by the
amount of departure from a plane, which has zero curvature. Mathematically, the curvature at any point of the surface could be calculated in terms of given functions $g_{11}, g_{12}, g_{22}$. The measure of curvature in space has likewise been generalized from the expression involving g's, as it was considered as part of the general case of n-dimensional space. With the use of concept of tensor, the definition of n-dimensional Riemannian space has been further extended to conformally symmetric, and conformally non-symmetric spaces depending on the curvature defined by metric tensor gij for n-dimensional space. Space has however been defined as plane of recurrent carvature. This is possibly a revolutionary step in geometrical thought which has opened up the creation of 'spaces' and 'geometries' for unlimited number of specific purposes for its use in dynamics, in pure geometry, in physical science and so on. It also clarifies our conception of 'space', and allows us not to disbelieve in any geometry or any space, as a necessary mode of human perception. It would be wise if we look at 'the higher space' or 'hyperspace' of R.N. Sen, or 'Ricci Symmetric \& Pseudo-symmetric manifold, Harmonic \& Symmetric Space of Prof M.C. Chaki from this perspectives. This, however, brings significant improvement in the geometry of manifolds, net result being modern geometry enters into a representation of figureless and formless (not characterless) configurations of n-tuples.. The Lie theory (after the name of Scandanavian mathematician, M.S. Lie, professor of mathematics in Oslo) by which modern geometry of point coordinates could be transformed to differential geometry of plane coordinates, a right-line to a sphere, correspondences of one space to another and vice versa attracted the attention of Harish-Chandra who contributed a work on 'the infinite dimensional representation theory' which has created a lot of interest in India. This type of algebraic geometry remain topologically invariant and are being used to study Galois Module structure. After an international conference in TIFR in 1960, the subject as well as the rigidity question of lattices having deformations in group theory other than well-known classical cases became the areas of great interest.

## Fields of Calculus, Analysis and Functions

The method of finding areas (in integral calculus)), and drawing of tangents \& measuring curvature at a point in any continuous curve (differential calculus) are essentially the key problems of the both calculus and function.

The expression, dy/dx was defined analytically as the limiting value of $\Delta y / \Delta x$ (provided such a value exits) or as rate of change (derivative) of $y$ with respect to $x$, or when $d y / d x=f(x), y=\int f(x) d x$, were treated as the first mathematical step to the discovery of both differential and integral calculus. Monge made a systematic use of calculus for the investigation of surfaces. With this it also came out that it is impossible to represent equations with a graph by finite, closed, mathematical expressions. For investigation of such graphs mathematically, the classical Fourier series was also taken into account and it was checked whether the given function exists within a certain interval, or has only a finite of discontinuities in the interval, or has only a finite number of turning- points in the interval, or could be expressed as infinite sum of Sines or Cosines functions or both.

As a result, the importance of investigation to the properties of simple (single) and multiple periodic functions (having two or more) and their boundary limits (boundary-value problems) was given in studies and researches. For example, the trigonometric functions, Sin $x$ or $\operatorname{Cos} x$ has simple period $2 \pi$, since $\operatorname{Sin}(x+2 \pi)=\operatorname{Sin} x$, or $\operatorname{Cos}(x+2 \pi)=\operatorname{Cos} x$, which means the cycle in the graph comes to the same point on the axis. The functions, Sin $2 x$, and $\operatorname{Sin} 3 x$ have periods, $2 \pi / 2,2 \pi / 3$ respectively as they go through its complete period twice and thrice as fast as $\operatorname{Sin} \mathrm{x}$, and so on. The elliptic function was also taken into account, since it has two distinct periods, say $\mathrm{p}_{1}$ and $\mathrm{p}_{2}$, such that $\mathrm{E}\left(\mathrm{z}+\mathrm{p}_{1}\right)=\mathrm{E}\left(\mathrm{z}+\mathrm{p}_{2}\right)=\mathrm{E}(\mathrm{z})$. the automorphic functions were taken up, since the function, $\mathrm{F}(\mathrm{z})$ remains invariant under an infinite group of linear transformations. It was found that trigonometric functions are special cases of elliptic functions, which in turn are special cases of automorphic function, there by indicating that periodicity is merely a special case of a more general property. All these properties have applications in problems of heat, sound and fluid motion. Similarly the properties of theory of functions of complex variables was also considered important from the stand point of convergence of infinite series. Similarly, the study of Zeta function, and Epstein Zeta function when $\mathrm{z}=\mathrm{u}+\mathrm{i} \mathrm{v}$ were taken up with reference to their boundary limit and convergence. Banach space, C [a, b] was defined as a norm of general vector space and used in all stages of rotation of functional analysis. Harmonic series is the modern version of classical Fourier analysis, the complex form of representation of Fourier series was also introduced. However, the convergence of Fourier series is far
from being automatic. Harmonic analysis is a possible extension of Lie's theory on 'continuous transformation groups' which is implemented for the consideration of more abstract situations. Set Theory as an abstract system of element of set, relations, operations, postulates etc is also identified with other mathematical systems like field, vector space, group, topology, ring, etc. Topology, which deals with the properties of geometric objects remaining unaltered under deformation have similar such analogus properties. Mathematical tools like limit, convergence, continuity are used to study closeness of points in a set. The necessity of studying exceptional subsets was the main purpose for the introduction of abstract set theory. Poincare's recurrence theorem that all points in a subset of the phase space eventually revisit the set concept creating lot of interest in India. This created interest in Erodic theory which emphasizes that under certain conditions, the time average of a function along the trajectories exists almost everywhere and is related to space averages. The metric classification of this type of dynamical system is another important part of the abstract Erodic theory. Attempts are also being made to replace $\mathrm{C}[\mathrm{a}, \mathrm{b}]$ by $\mathrm{C}(\mathrm{X})$ where X is a topological space. The Wavelet analysis which is Fourier series like sums is being used as an important tool for scrutinizing localized phenomena of various natures. Wavelets are able to analyze signals discontinuities of fractal structures. This modern Fourier series is an important tool, like integration by parts, and is being used again and again for its fundamental role. Studies of various types of functions of these types have been started to find their limiting conditions in IIT's, TIFR and Madras University and other places. The main objective, however, is to couple geometry \& function with calculus.

## Differential Equations

Differential equations arise in many areas of science and technology, whenever any problem involving some continuously varying quantities expressed by functions and their rate of change in space or in time is related. Newtonian variables like positions, velocity, accelerations and their forms acting on the body are often expressed dynamically as a differential equations for unknown position of body as a function of time. The knowledge of partial derivatives likewise came, when the concept $\delta \mathrm{f}(\mathrm{t}, \mathrm{p}) / \delta \mathrm{t}$ represent partial with respect to $t$ when $p$ is constant, and $\delta f(t, p) / \delta p$ when $t$ is constant, or, $\delta(\mathrm{u}, \mathrm{v}) / \delta \mathrm{x}=\mathrm{u} \delta \mathrm{v} / \delta \mathrm{x}+\mathrm{v} \delta \mathrm{u} / \delta \mathrm{x}$ as rate of change of (uv) with respect to x were in use. The question of solution of differential equations first came into
force possibly to settle the initial boundary condition of the functional representation of these figures.

The partial differential equations of second degree like, $\delta^{2} u / \delta x^{2}+$ $\delta^{2} u / \delta y^{2}+\delta^{2} u / \delta z^{2}=0$ for 'perfect fluid in the theory of fluid motion' indicating the fact that the fluid has no vortices, had been widely used in Newtonian gravitation, electricity and magnetism and in a way was starting point of Indian researches in fluid dynamics. In the process, large number of linear and non-linear transformation or operators based on abstract system like vector space, groups or under any applied force lead to differential equations of various types and order. Their solution sometimes require special function or orthogonal type polynomial functions. Likewise, the problems of fluid motion under stressed conditions, and that of motions of three bodies involved a large number of simultaneous differential equations. The solution, of course, was not to be expected in finite terms, if a solution exists at all, it will be given by infinite series. The solution will exit if these series satisfy the equations, and moreover convergence for certain values of the variables need to be satisfied. Solutions are at times approximately found by using computers. Many numerical methods have been developed for their solutions. An Indian Society of Nonlinear Analysis (ISNA) was established in June 2000, with the aim of providing a forum for collaborative study and research in nonlinear analysis for all branches of science.

The problem of L-function (after the name of Lie) arising out of the eigen-values of discrete spectrum of differential equations got an important attention in India. Differential equations leading to Epstein Zeta function for the connected compact Riemanian manifold, according to Minakshisundaram (with A.Pleijel), appears to have great potential. IIT Kharagpur has tackled heat equations on regularity of weak solutions, which has been further improved by Indian scholars, specially by TIFR and IISc. in their joint undertaken with the help of non-linear partial differential equations. Quite a few interesting results have been established. Various attempts have been made to solve non-linear differential equations relating to physical and engineering system with special reference to Navier-Stabes equations and cross-viscous forces in non-Newtonian fluids. The Selected Papers on Numerical solution of Numerical Solutions of Equations in Fluid Dynamics, published by the Physical Society of Japan, has quoted these attempts as important contributions, which has been widely referred in many prestigious
volumes on Classical Mechanics and Field Theory. Various soft-wares on the solutions of differential equations are now available. One should be careful in their uses, otherwise, it would lead to wrong conclusions.

## Mathematical Modeling

In mathematical modeling, the concept of function with simple(single) periodicity has great importance, for it helps in the prediction of natural phenomena, the tides, the phases of the Moon, the seasons and for other things which are periodic in character. The recurrence of Sunspots are closely approximated by superposition of certain number of graphs having simple periodicity taking the original as the resultant. For analysis of a musical sound into its fundamental and successive harmonics, the process is mathematically same. As a very crude approximation to 'quality of sound', only the fundamentals are considered, the superposition of only a few harmonics usually suffice. Long periods (the fundamentals) have also been attempted to detect the recurrence of earthquakes and annual rainfall. The notion of simple periodicity is as important in pure mathematics as in applied and other IITs mathematics. J.N. Kapur and IIT Delhi have applied both functional method of simple and multiple periodicity \& differential equations in mathematical modeling, and achieved considerable success. The BGK model of differential equations (by P.L. Bhatnagar, Gross \& Krook) in collision process in gas, and Ramdas Paradox by Ramdas of NPL Delhi in applied mathematics (that the temperature minimum happens about 30 cm above the surface) are undoubtedly interesting hypothesis in mathematical modeling.

## Field of Probability and Statistics

The mathematical side of probability had developed contemporaneously with economic and official statistics, though the two movements were entirely unrelated. The $\mathrm{D}^{2}$-Statistic of Mahalanobis used for distinguishing populations remains a powerful and fundamental tool in multivariate analysis, classification problems and cluster analysis. The theory of errors based on normal distribution had also been used to minimize the observational errors in astronomy and physical sciences, and to the analysis of anthropological and other problems. On the request of Director General of Observatories, Mahalanobis looked at the correlation between the upper
air variables. His idea of correlation was accepted also as an important tool and began to be used in problems of biological sciences with interesting results. The idea of least squares plays an important role in the scientific study of social, economic, biological and psychological problems. So is the inequality problem based on mean values efficiently used in the theory of dispersion. The Chi-square test on the distribution of $\chi^{2}$, was accepted as a tool for assessing agreement between the theoretical expectations and actual observations. Mahalanobis always emphasized on the standardization in measurement, and rectified about 133 discrepancies in Risley's famous data set (1891) just by cross-examination of data and internal consistency check. The 'circular cut' recommended by ISI, and 'rectangular cut' by ICAR created some controversy, but it has been proved that the yield rates for both the cuts are more or less the same. However, Mahalanobis's crop-cutting experiments and yield estimates paved way for R.C. Bose in the construction of design experiments. India remained a leading contributor in this area. The role of ISI in the analysis of variance, types of experiments, sample surveys, statistical methods are efficiently designed for tackling problems of demography, psychology and education, industrial quality control and economics which have created lot of confidence in India and Abroad and in the history of modern statistics in India. The Boson of Bose-Einstein statistics leading to discovery of Higgs Boson at CERN (Geneva), C.R. Rao's contribution on quality control, Statistical inference, Linear Model \& mathematical biometry), S.N. Roy on multivariate statistical analysis, mainly of Jacobian transformations and Bartlett decompositions, Ghosh-Bahadur-Kiefer representation and Ghosh-Pratt identity, are some of the landmarks in Indian statistics.

The dynamic leadership provided by P.C. Mahalanobis led to exponential growth of Mathematical Statistics in India which ultimately placed India firmly on the world map of Mathematical Statistics. ISI is also largely responsible for the development of Operations Research in India, since it is here this technique was applied to national planning at the instance of Pandit Nehru.

## Fluid Dynamics/ Solid Mechanics/Applied Mathematics

The study of fluids, whether compressible or incompressible, viscous or invisid, steady or unsteady, laminar or turbulent and under different
thermodynamic conditions of various types has been the major interest of the applied mathematics group of the Calcutta University and IIT Kharagpur. So are the experiments in solid mechanics were taken up very seriously by the Jadabpur university under B. Sen who produced more or less forty-eight PhDs from the university. It covers various areas on the subject including study of motions and position of solid bodies in a situation of four dimensions-three Cartesian coordinates and one time coordinate, that is to locate a moving particle in both space and time. The problem of motion of 2-bodies was originally solved by Newton, problem of 3-bodies brought many agonies to scholars. The problem of n-bodies still remains a puzzle.This is a way of looking at mechanics that has become popular when Einstein exploited it for general relativity in 1915. The objective was to solve nonlinear partial differential equations governing varieties of physical and engineering problems. The major research interests of both Calcutta and Kharagpur were on the areas of non-linear mechanics, rheological fluid mechanics, elasticity-plasticity, hydro-elasticity, thermo-elasticity, magnetohydrodynamics and high-temperature gas dynamics, numerical analysis, theory of relativity, cosmology etc. These have been extended further by other IITs, Research laboratories, Technical Institutes and other centers. Indian scholars have also contributed to areas like biomechanics, chaotic dynamics, theory of turbulence, porus media, magnetic fluids and so on. However, the researches of B.R.Seth on Saint-Venant's problem on elastic deformations and transition theory unifying elastic-plastic behavior of materials which earned him prestigious Euler Medal of the Soviet Academy of Science,1957. Mach number of the flow of fluid and Reynolds number for assessing friction in fluid motion played also important role. Turbulent flow was also decomposed into various components by Reynolds or Navier-Stokes equations or by direct numerical simulation. N.R. Sen and his student H.P. Majumdar gave solution of equations for early stage of turbulence, and Chandrasekhar a numerical solution of Heisenberg's homogeneous isotropic turbulence. Chandrasekhar number (Indian Nobel Laureate) an important dimensionless number of magneto-hydrodynamics ; Ashoke Sen’s (Harish-Chandra Institute) S-Duality or Weak Coupling Duality in Particle physics (String theory) to reconcile quantum mechanics to general relativity are some of the landmarks in the studies of applied mathematics.

To summarize these activities, it may be said that the twentieth century first-half in India has been the phase of explosive expansion, and it is impossible for any one to familiarize himself with the entire mathematical mass that has been dumped. Of course, some tendency is found at times towards the contraction of this vast knowledge, e.g. in algebra the wholesale postulational methods are being introduced to make the subject more abstract, more general, and less disconnected on the basis of unexpected similarities amounting to disguised identity. So is the growing use of tensor calculus in preference to numerous special brands of vector analysis, which may be cited as another example of this contraction process. Many of these particular and difficult things might have been subsumed under simpler general principles. Such generalizations and condensations are part of the procedures and often hard to grasp. But in the end it is usually realized that that general methods are essentially simpler and easier to understand than miscellaneous collections of ingenious tricks devised for special problems.

## Concluding Remarks

As to Prospects, let me give you a few quotes from great mathematicians. Indian mathematician Lagadha said, 'Mathematics is like the diamond on the head of nāga snake' ('gaṇitam nāgānām manoyor yath $\bar{a}$ '). Gauss, the German mathematician, said, 'Mathematics is the Queen of the Sciences'. Einstein, one of the greatest scientists of the $20^{\text {th }}$ century, justified it by saying, 'There is another reason for high repute of mathematics. It is mathematics that offers exact natural sciences a certain measure of security which, without mathematics, they could not attain'.

But, the spontaneous generation of mathematicians is not automatic. It requires a good soil to seed, a good environment and a liberal society to nurture the young talents. It is becoming increasingly difficult to germinate even first rate mathematicians, not to talk about genius. Genius, of course, according to Thomas Alva Edison, is 'ninety-nine percent perspiration and only one percent inspiration'.

In fine, it may be said that the historian of mathematics has its own limitation. Only a picture of mathematics and mathematical researches in India is attempted the way it is understood.

## Appendix

## Copy of the Draft letter sent to mathematicians for Lineage program

Dear Sir,
We are making a survey of lineage of the well-known Indian Mathematicians of the $20^{\text {th }}$ century, specially in the second half. In this context we are looking for the name of your teacher (with whom you have worked for your doctorate program, may be Indian or Foreigner or from whom you have got the maximum help or guidance). We also like to have the name of your students (preferably completed Ph.D. program with short title if possible under your guidance), of course the number of students may be restricted to maximum of five. This is extremely important to have a picture from national perspective and to have an idea as to how each area of mathematics has grown in India. I wish to have your kind cooperation.

## Lineage of Indian Mathematicians

1. Name of the Mathematician -
(a) Present status, affiliation if any -
(b) Area of specialization -
2. Name of your Ph.D. Guide/ Professor (Indian or Foreigner) (time period if possible) -
(a) Status -
(b) Specializations -
3. Name of your Ph. D. students -
(a) Name -

Title of thesis in short -
(Status of the student ) -
(b) Name -

Title of thesis-
(Status) -
(c) Name-

Title of thesis-
(Status) -

The reply may kindly be sent to the address given below by e-mail or post.

With kind regards,
(Dr. A K Bag, FASc., Editor, Indian Journal of History of Science, Indian National Science Academy, Bahadur Shah Zafar Marg, New Delhi110002. E.mail : akbag99@gmail.com, Mobile: 9654023365)

## Acknowledgements

I am personally thankful to all mathematicians who have responded to our request to Lineage Program. There is a plan for updating the list. The proforma as given in the Appendix may be utilized for upgradation.

## Notes

1. The general Committee of Public Instruction, appointed by Colonial Government of India in July 1823 suggested measures for introduction of useful knowledge, including Sciences and Arts of Europe in the Curriculum. Gradually, mathematics classes were started in Kolkata Madrassa, Sanskrit College and other institutions. The Despatch of 1854 recommended the establishment of Universities defining their functions as examining bodies in various branches of learning including science. Sen (1972) has made a survey of Sanskrit works translated into foreign languages and vice versa in the $18^{\text {th }}$ and $19^{\text {th }}$ centuries. Many more such translation work was taken up also in regional languages in $19^{\text {ih }}$ century. However no steps were taken in the field of scientific education and research.
2. Professor Ganesh Prasad has been described as father of modern mathematics in India by J.N. Kapur.

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[^0]:    * The Key Note Address was delivered at the Indian Association for Cultivation of Science (Central Hall of IACS, Kolkata) organized on behalf of the M.C. Chaki Centre for Mathematics and Mathematical Sciences on 21 st July 2012.
    **Editor, Indian Journal of History of Science, INSA, New Delhi

