PIONEER MATHEMATICIANS AND THEIR ROLE IN CALCUTTA UNIVERSITY DURING THE NINETEENTH AND TWENTIETH CENTURY*

Mala Bhattacharjee, Purabi Mukherji and Nandita Mallik**

Calcutta University has the unique distinction of being the first University in India, where Post Graduate teaching and research in Mathematics was initiated. The objective of our study was to investigate the contributions of the pioneer mathematicians who worked there in the late nineteenth and till the middle of the twentieth century. In fact Indian contributions to Mathematics during that period have remained to a large extent unacknowledged and even unknown to our own countrymen because of limited circulation of journals where they were originally published. So the main aim of this project was to assess and document the research contributions of some pioneer mathematicians who were associated with the University of Calcutta either as an important functionary or as a faculty member or as an illustrious student. For making the assessment a scientific methodology have been adopted.

The contributions of fifteen mathematicians of Bengal have thus been analyzed, whose names are given below in a chronological order.

1. Sir Asutosh Mookerjee (1864-1924), 2. Prof. Ganesh Prasad (1876-1935), 3. Dr. Shyamadas Mukhopadhyay (1866-1937), 4. Dr. Bibhuti Bhusan Datta (1888- 1958), 5. Prof. Haridas Bagchi (1888-1968), 6. Prof. Meghnad Saha (1893-1956), 7. Prof. Satyendra Nath Bose (1894-1974), 8. Prof. Nikhil Ranjan Sen (1894- 1963), 9. Prof. Rabindra Nath Sen (1896-1974), 10. Prof. Bibhuti Bhusan Sen (1898-1976), 11. Prof. Raj Chandra Bose (1901-1987), 12. Prof. Subodh Kumar Chakrabarty (1909-1987), 13. Prof. Manindra Chandra Chaki (1913-2007), 14. Prof. Anadi Shankar Gupta (1932-2012) and the great lady mathematician, 15. Prof. (Mrs) Jyoti Das.

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^{**}Department of Mathematics, Gokhale Memorial Girls' College, 1/1 Harish Mukherjee Road, Kolkata-700020; Email: rubym1945@gmail.com

The project was planned in two sections, which had further sub sections in it.

In the first Section, apart from the Introduction, it has a chapter, which is a "prelude" to the main part. Then there are fifteen chapters dedicated to the fifteen mathematicians mentioned above. Sir Asutosh Mookerjee has been discussed over three sub-sections which ultimately combines to one whole chapter.

In Section II extended bibliographies in the following branches of Mathematics have been compiled: (1) Real and Complex Analysis, (2) Fluid Mechanics, (3) Solid Mechanics, (4) Bio-Mathematics, (5) Mathematical Statistics, (6) Geometry, (7) General Relativity and Cosmology (8) Quantum Physics and Plasma Physics (including Quantum Mechanics) and (9) Differential Equations and Eigenfunction Expansions.

Sir Asutosh Mookerjee was a versatile man, who almost singlehandedly started the school of mathematical research in the country. He did some original research mainly in the areas of Geometry, Differential Equations and Hydrokinetics. The number of his personal research papers are seventeen which were published in different national and international journals of repute. One of his important research paper is entitled "On the Differential Equation of a Trajectory" [JASB, 56, 1887, 117-120]. In this paper, Sir Mookerjee considered the solution of the differential equation of oblique trajectory of confocal ellipses as given by the Italian mathematician Mainardi. This solution being extremely complicated and cumbersome, it was impossible to trace the curve from it. Asutosh showed by an ingenious process that Mainardi's inelegant solution could be replaced by a pair of remarkably simple equations. From this, interesting geometrical interpretations could also be made. These elegant equations as established by Sir Asutosh have been incorporated by Prof. A. R. Forsyth (1858-1942) in his classical book on Differential Equations in latter editions. In another of his paper entitled "Remarks on Monge's Differential Equations to all conics" [PASB., Feb, 1888], Asutosh successfully gave geometrical interpretation on Monge's Differential Equations to all conics. His interpretation was that "the radius of curvature of the aberrancy curve vanishes at every point of every conic". Edwards in his book "Differential Calculus" has quoted this interpretation. Asutosh Mookerjee's research papers aroused a lot of interest amongst the

British school of mathematicians of those times. Sir Mookerjee was a trendsetter in the fields of research in Geometry as well as Fluid Mechanics. We have compiled his complete bibliography of mathematical research papers.

Professor Ganesh Prasad, who is referred to as the "father of mathematical research" served Calcutta University in two phases and was involved both with the Departments of Pure as well as Applied Mathematics as the Head. In the Final Report, we have tried to show how he inspired young researchers and also highlighted his personal contributions. Prof. Prasad has published about fifty research papers in national and international journals and written a number of books. We shall selectively discuss a few of his more important publications. His paper entitled "Constitution of Matter and Analytical Theories of Heat" [Abhandulgen d.k., GWZG, 1903] is considered as an authoritative solution of a difficult problem in mathematical physics. Great German mathematician Felix Klein considered it as a very satisfactory solution. Another of his paper entitled "Expansion of arbitrary functions in a series of Spherical Harmonics" [MA, 1912] is considered to be a very important result and has been quoted in Hobson's book named "Theory of Spherical and Ellipsoidal Harmonics". Prof. Prasad did a lot of original work on the summability of the Fourier Series. Another research paper by Prof. Prasad entitled "On the differentiability of the integral function" [CJ, Vol.160, 1929] roused a lot of interest at that time. According to the mathematicians of the time it was an epoch making publication. Amongst the books written by Prof. Prasad, the following are still considered as classic contributions.

- A treatise on Spherical Harmonics and the Functions of Bessel and Lame (1930-1932)
- Six Lectures on recent researches in the theories of Fourier Series [Calcutta University Publications, 1928]
- Some Great Mathematicians of the Nineteenth Century (2 volumes)

In the Final Report, a list of his important research papers and books have been made.

Dr. Shyamadas Mukhopadhyay was the first Indian to obtain a doctorate degree in Mathematics in India. Inspired from his student days by his teacher Prof. W. Booth of Presidency College and later by Sir Asutosh

Mookerjee, after joining the Calcutta University as a faculty in the Pure Mathematics Department, he seriously took up research in various branches of Geometry and made very valuable contributions. As remarked by another great geometrician of Calcutta University, Professor M. C. Chaki, "Dr. S. Mukhopadhyay's contributions in Geometry stand out prominently on account of their novelty and originality". Dr. Mukhopadhyay, while dealing synthetically with properties of a plane curve, specially in infinitesimal regions, developed new methods. These methods led to a number of interesting theorems on the existence of cyclic and sextactic points on a convex oval. The theorem which states that the minimum number of cyclic points on a convex oval is four, is now well known and is referred to in Geometry as Mukhopadhyay's "Four Vertex Theorem", the world over. The proof of the celebrated theorem was published in the Bulletin of the Calcutta Mathematical Society in 1909 ["New Methods in the Geometry of a Plane Arc, I, Cyclic and Sextactic Points", BCMS., Vol.1, 31-37, 1909]. In the paper entitled "Geometrical Investigations on the Correspondence between a right-angled triangle, a three right angled quadrilateral and a rectangular pentagon in Hyperbolic Geometry" [BCMS., Vol.13, No.4.211-216, 1922- 1923], Dr. Mukhopadhyay established an important result and he concluded "We have thus the closed series of 5 associated right-angled triangles and the Engel-Napier Rules are shown to posses a real geometrical basis in the rectangular pentagon." In 1926, with the publication of the paper entitled "On General Theorems of Co-Intimacy of Symmetries of a Hyperbolic Triad" [BCMS.,18,No.1,39-55, 1926] started a very fruitful collaboration between Dr. S. Mukhopadhyay and his legendary student R. C. Bose. We have compiled a complete list of Dr. Mukhopadhyay's 30 research publications.

In the backdrop of the early twentieth century mathematical world of India, Dr. **Bibhuti Bhusan Datta's** research contributions in Fluid Mechanics is rich, but his contributions in the field of History of Mathematics is stupendous. Inspired by his teacher and colleague, Prof. Ganesh Prasad, Dr. Datta pursued serious research in History of Indian Mathematics and wrote nearly 60 articles on the contributions of ancient Hindu and Jaina Mathematicians. In 1931, Dr. B. B. Datta delivered six lectures about ancient Hindu Geometry. In 1932, Calcutta University published these lectures in the form of a book entitled "*The Science of Sulba – A Study of Hindu Geometry*". These lectures were the fruit of intensive research by B. B.

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Datta. He revealed that the Vedic priests were concerned with practical problems involved in the construction of alters for carrying out religious rituals. In this way they developed a type of "esoteric geometry as their secret property". Dr. Datta's classical book "*The History of Hindu Mathematics*" Part I and II was published in 1935. Dr. A. N. Singh of Allahabad University was the co-author. In our Final Report, we have analyzed Dr. Datta's contributions and have also made a complete bibliography of his research publications in Fluid Mechanics as well as in the History of Mathematics.

Professor M. N. Saha, an illustrious student of the Department of Applied Mathematics, Calcutta University was elected an F.R.S. in London for his outstanding research in the fields of Astrophysics and Electromagnetic theory. He is world famous for his "Thermal ionization Theory". This discovery known the world over as "Saha's Thermal ionization Theory" transformed spectroscopy into a precise, quantitative method of investigating the physical and chemical conditions inside the Sun and the stars. Saha's work was instrumental in inspiring hectic scientific activity across the world. Following Saha's inventions, scientists of Cambridge in U.K. and Harvard in U.S.A. made important contributions.

Professor S. N. Bose, the first M.Sc. from Calcutta University and a star product of the Department of Applied Mathematics was also elected an F.R.S. from London. S.N. Bose worked on a number of topics in various branches of science. But his most outstanding work was "Planck's Law and the Light-Quantum Hypothesis" which was published in Z. Physik in 1924 with a note from Albert Einstein. As acknowledged by both Dirac and Einstein, the new statistics derived by Bose in this research paper laid the foundation of Quantum Statistics. With the discovery of this statistics known as "Bose-Einstein Statistics", Bose is considered a giant among mathematical physicists of the world. Einstein adopted this idea and extended it to atoms. This led to the prediction of the existence of the phenomena which became known as "Bose-Einstein Condensate". This condensate is a dense collection of Bosons (which are particles with integer spin, named after Bose by Dirac). With recent observations of Bose-Einstein Condensation in the dilute atomic gases at temperatures in mili-kelvin and micro-kelvin ranges, S. N. Bose's contributions has brought his name to the frontiers of contemporary

Theoretical Physics. Particle Statistics as derived by S. N. Bose is considered by the scientific community as one of the top ten achievements of the 20th Century Indian Science.

Professor Nikhil Ranjan Sen did research over a wide range of subjects and topics. He did pioneering work in the fields of turbulence and cosmology. In 1924, Prof. Sen took over as the "Rashbehary Ghosh" Professor of Applied Mathematics. Apart from his own brilliant research contributions, he introduced new subjects in the Post-Graduate curriculum and inspired his young colleagues and research associates to take up original and challenging problems and solve them in modern areas like Relativity, Astrophysics, Quantum Mechanics, Geophysics, Statistical Mechanics, Fluid Dynamics, Magneto-Hydrodynamics, Elasticity and Ballistics. He was the fountainhead of inspiration to research workers in Calcutta University and under his dynamic leadership the Department of Applied Mathematics became a vibrant centre of teaching and research and earned a reputation throughout the country, which would be hard to match. He was very rightly called the "Father of Applied Mathematics" in India. We have compiled the complete list of his personal research contributions.

Professor Rabindra Nath Sen was a student and later a faculty member of the Department of Pure Mathematics. From 1954, for seven years he served the Department as the "Hardinge Professor" of Pure Mathematics. In his personal research, he has made enormous contributions in the fields of Differential Geometry of Riemannian and Finsler spaces. M. C. Chaki, the most famous student of Prof. R. N. Sen has stated, "Sen's investigation on the behaviour of an arbitrary parallel displacement in a metric space resulted in the discovery in 1949-50, of an algebraic system of affine connections in which Levi-Civita parallelism could be identified." This work is considered as highly significant and has been referred to by I. M. H. Etherington of Edinburgh University as "Senian Geometry". We have compiled his complete bibliography.

Professor Raj Chandra Bose was a student of the Pure Mathematics Department and became a legendary research worker under the guidance and inspiration of Dr. Shyamadas Mukhopadhyay. He is famous for his contributions in Geometry and Mathematical Statistics. The father of Statistics in India, Prof. P.C. Mahalanobis persuaded R. C. Bose to join the newly set

up Statistical Institute in the late thirties of the twentieth century. R. C. Bose was a pioneer in using finite geometries, finite fields and the combinatorial methods in the construction of designs. In 1938, his paper entitled "On the application of properties of Galois fields to the construction of hyper Graeco-Latin Squares" [S, Vol.3, 1938, 323-339] is very significant. In the introduction of that paper, Raj Chandra Bose had written "It is hoped that the properties of Galois fields and the finite geometries connected with them, will prove useful in many problems of experimental design and the author hopes to pursue this matter in subsequent papers". Indeed until the middle of 1950, Professor Bose was mainly preoccupied in the development of a mathematical theory of design. But perhaps his most important work in the field of Mathematical Statistics was written in collaboration with another famous mathematician Prof. S. N. Roy and the paper is titled "The Distribution of the Studentised D2-staistic" [S,4,1938, 19-38]. It was critically acclaimed by R. A. Fisher. According to him, the above mentioned paper has carried the work a distinct step forward. Prof. P. C. Mahalanobis too wrote that the paper supplied the necessary mathematical tool to use the D2-statistic when only the sample values of dispersion were known. We have very carefully compiled Prof. R. C. Bose's complete bibliography.

Professor Bibhuti Bhusan Sen, a notable student of the Department of Applied Mathematics had the good fortune of being taught by such stalwarts as Prof. S. N. Bose and Prof. N. R. Sen. Inspired particularly by Prof. S. N. Bose, he was the man who practically initiated modern type of research in the field of Solid Mechanics. His research papers entitled "Two dimensional boundary value problems of elasticity" [*PRSL*, A 187(1946), 87-101] and "Note on the direct determination of steady state thermal stresses in circular discs and spheres" [*BCMS*, 56 (1964) 77-81] deserve special mention. Apart from his own rich contributions in various branches of Solid Mechanics, he built a very strong school of research workers as well. He has the unique distinction of guiding more than fifty students for their Ph.D. degrees. His complete list of research publications has been compiled.

Professor Subodh Kumar Chakrabarty was a student and later a faculty in the Department of Applied Mathematics of the University of Calcutta. In 1963 he was made the "Rashbehary Ghosh" Professor of Applied Mathematics and served there till his retirement in 1974. In his personal

research, he collaborated with H. J. Bhabha and made remarkable contributions in field of Cosmology. Prof. Chakrabarty is well known in the scientific community for his researches on "Cascade Theory" and Theoretical Seismology. During his tenure as the Head of the Department, he inspired many young students to take up research in the field of Theoretical Seismology and Geophysics. His full list of research publications has been compiled.

Professor Manindra Chandra Chaki, a student and later a faculty of the Department of Pure Mathematics has made notable contributions in the fields of Riemannian Geometry, Classical and Modern Differential Geometry, Theoretical Physics, General Relativity, Cosmology and History of Mathematics. Perhaps a very noteworthy contribution of Prof. Chaki was in 1987, when he introduced the notion of Pseudo-symmetric Manifolds, which is known in mathematical literature as Chaki Manifold. He guided more than twenty research students for their Ph.D. degrees in various branches of Mathematics.

Professor Anadi Shankar Gupta, a student of the Department of Applied Mathematics and a much admired faculty of I.I.T., Kharagpur is very renowned for his contributions in the field of Magneto-hydrodynamics and the stability of flows of Newtonian and Non-Newtonian fluids. He was responsible for building up a strong school of research in Fluid Mechanics. We have compiled Prof. A. S. Gupta's complete bibliography.

Professor (Mrs.) Jyoti Das has made important contributions in the fields of Special Functions, Eigen Function Expansions and Ordinary Differential Equations.

Conclusions

The contributions of the above mathematicians are indeed remarkable. Apart from their personal brilliance and commitment, a thing, which should definitely come into consideration, is the fact that many of them, particularly, in the early part of the twentieth century worked under very difficult conditions. They did not have access to any modern gadgets, not even to calculators. But in spite of such impediments, their research works made a mark in the world of science and it was critically acclaimed by the mathematical community both in India as well as abroad. It is needless to

mention, that these pioneer mathematicians played a very important role in context of Calcutta University.

It is also evident from Section II of research report containing the extended bibliographies that the trend of research initiated more than hundred years ago is still going strong in both the Departments of Pure as well as Applied Mathematics of the University of Calcutta. The documentation of the research work unfolded by our investigations will be a rich source of information to both the present day researchers as well as the future generation of scholars of Mathematics.

Abbreviations

JASB	Journal of the Asiatic Society of Bengal
PASB	Proceedings of the Asiatic Society of Bengal
MA	Mathematical Annals
CJ	Crelle's Journal
BCMS	Bulletin of the Calcutta Mathematical Society
S	Sānkhya (Indian Journal of Statistics)
PRSL	Proceedings of the Royal Society of London

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