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HISTORICAL NOTES

Dakṣiṇāgni in *Śulbasūtras* – An Astronomical Interpretation

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The placement of altars in *vedic* rituals have been described in the *Śulbasūtras*, the ancient texts of mathematics and sciences in India. The *sūtras* offer different geometrical constructions whose purpose has not been discussed widely. Here an attempt has been made to attribute the constructions of the altar for astronomical purposes.

Key words: *Dakṣiṇāgni*, Vedic altars, *Śulbasūtras*, Declination measures of the sun

1. Introduction

Śulbasūtras are ancient texts for altar constructions and associated mathematics. The knowledge has been debated to be of the period extending from about BC 3000 to about BC 300. Extensive studies have been made on these *Śulba* texts which offer excellent definitions for mathematical concepts like surds; the visualisation of the Pythagoras theorem and so on. Many aspects of the *Śulbasūtras* have been studied in great detail (Saraswathi Amma, 1979, Kulkarni, 1983, Sen and Bag, 1983, Delire 1993 and Plofker, 2009) and by many authors. Four of the eight texts have been edited and translated in to English (Sen and Bag, 1983).One of the interesting constructions corresponds to the location of *Dakṣiṇāgni*. The main platform where the vedic altar is constructed is called the *Mahāvedi*, which was constructed in an elevated place. This had an annexure called *prācinavaṃśa* which has three altars in three specified positions. They are called *Gārhapatya*, *Āhavanīya* and *Dakṣiṇāgni*. The *Dakṣiṇāgni* is sometimes referred to as *Anvāharyapācana* (a place for cooking, Kak, 1993).

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Specific formulae for fixing the location of *Dakṣiṇāgni* are given in *Baudhāyana Śulbasūtra* (*B Śl*), *Katyayana Śulbasūtra* (*K Śl*) and *Mānava Śulbasūtra* (*M Śl*). The sections had the same measure of the area although *Āhavanīya* was a square, *Gārhapatya*, a circle and square while *Dakṣiṇāgni* was a semi circle (Bag, 1990). They appear to be aimed at achieving a specific accuracy and the purpose is not specified anywhere. One of the statements reads

> *"the* intention *of the Sūtrakāras not to locate Dakṣiṇāgni; but to fix the value √2 and / or √5 .... However the approximate values obtained by these constructions are so much in error …the same Sūtrakāras who gave the value of √2 so accurately (elsewhere) would not tolerate here.......... All the same, none of the two statements possibly give the* intention *of Sūtrakāras as the error is still large"*
>
> (Kulkarni, 1983)

This prompted to us to study another possible scenario for the need of such apparently complicated constructions. Here we try to interpret these rules as may be specified for astronomical observations.

2. The formulae

The earliest work by Apte (1926, cited from Kulkarni, 1983) who interpreted the word *Vrithiya* as "one third removed" has been used by all later authors.There are altogether six different methods of construction for fixing the location of *Dakṣiṇāgni*, of which two appear different but yield identical results. Let us consider each of them one by one:

The fundamental rule can be stated as follows - Let *Gārhapatya* be denoted by G and *Āhavanīya* by A. (See Fig. 1). These lie along the East - West line. (A to the East and G to the West). Let the separation between them be x units. The location of *Dakṣiṇāgni* is at point D such that

Rule 1: the ratio AE : ED :: 2 : 1 and
Rule 2: D is to the South East of G.

Now let us see how these rules are satisfied by the constructions.

Method 1

This is from *B śl*. Divide the distance x into 3 parts so that AE = 2x/3. (See Fig. 1) From E draw ED so that it is perpendicular to AG and ED = x/3. Considering the right angled triangle EGD we can see that D is exactly to the south east of G.

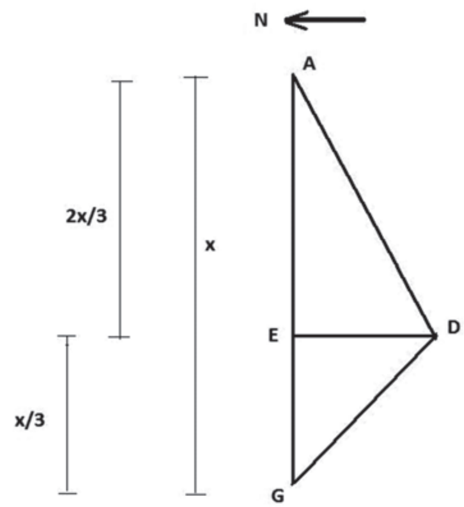

Fig. 1. Graphical representation of the Method 1

The rule 1 is not applicable here since the ratio AD : GD will now be √ 5 / √ 2.

Method 2

This is also from *B śl*, which offers two possibilities - If the length AG is x take a rope of length either x/6 or x/7 to be divided into two parts of ratio 2:1. Let us consider here the first option. Increasing the length by

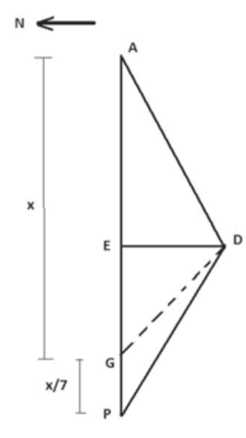

Fig. 2. The methods 2, 3 and 4 can be represented by extension of the chord to an extra length denoted by GP, which may be 1/5 or 1/6 or 1/7

x/6 and marking two parts with specified ratio yields the point D such that its perpendicular on AG is at E and E is given by EG = 0.27682 x. Thus rule 1 is followed; but not rule 2 namely the point D is not exactly to the south east of G.

Method 3

Here we consider the second option that length be increased by x/7 before measuring out the two parts with ratio 2:1. This procedure yields the point EG such that EG = 0.28231x. Again it is not to the south east of G - rule 2 is defied.

Method 4

This is from *B śl*. Here the length is increased by x/5 and the point E is fixed so that EG = 0.26 x. Again rule 1 is satisfied but not rule 2.

Method 5

This is from *M śl*. The construction is similar to Method 1. This satisfies rule 2 namely the point D is exactly to the south east of G - but defies rule 2. The ratio AD:GD is $\sqrt{5} / \sqrt{2}$.

Method 6

This is from *K śl*. Here AG is divided in to three parts to fix E. ED is constructed so that ED = 2x/3 . Here we notice that D is not to the southeast of G but exactly southwest of A. The ratio of AD:GD is now $\sqrt{5} / \sqrt{2}$.

Thus one wonders why there was a dire need for these constructions. If the purpose was to achieve the value $\sqrt{5} / \sqrt{2}$ the other methods in the same text yield more accurate results.

There is a unique set of points that can satisfy the rule (1). It can easily be shown that they lie along a semicircle. However, rule (2) namely the point D should exactly to the south east of G needs to be interpreted correctly. This has not been attempted so far. Since the ratios in constructions according to methods 2, 3 and 4 give an angle other than 45 degrees for angle DGE the rule is considered to be defied. Here we try to argue on this point.

3. Discussion

We may now try to understand the procedural differences which appear to be adjusting the location of point D to suit some specific need. Here is an attempt to interpret this in the context of observational astronomy.

The high standard of the astronomical knowledge of the ancient Indians is very well known. This naturally was based on accurate observations. One of the most important tasks was to fix the time of the day, month and the year. For this purpose it was essential to monitor the equinoxes and solstices. The ritual of marking winter solstice (*Uttarāyaṇa*) has been discussed extensively in *Kauśitakī Brāhmaṇa* and the *Yajurveda*. The corresponding text translates as

> "They perform the Ekaviṃśa day, the Viṣuvan, in the middle of the year; by this day Gods raise the sun.......therefore he going between these 10 days does not waver.''

The year-long Vedic sacrifices were begun on the days following winter solstice, according to Sengupta (1947). He further discusses the need for observation of 21 days the midpoint of which considered the solstice to an accuracy of about 0.05″ of the noon shadow. In the footnote he refers to a second method as "it is also possible to observe the sun's amplitude during summer solstice, which will remain constant for about 10 days''. Here the word amplitude is the azimuth of the rising sun.

This provides a clue on the possible method of observation that might have been adopted for fixing solstice days. A study of the architecture of many temples has shown that marking solstices was one of the important purposes of a temple (Shylaja, 2007, 2008). The basic requirement is drawing a line for the sun rise / sun set points from a reference point. Now we try to find if this procedure of marking *Dakṣiṇāgni* aimed at this. In the absence of the definition of the reference point (it may be G or A or even D) we proceed to test all three possibilities.

The azimuth, θ of rising sun is decided by the declination of the sun, δ as well as the location of the observer or the latitude, ϕ as

$$\cos \theta = \{\sin \delta\} / \{\cos \phi\}$$

Using either A as the reference we get a set of values for the azimuth and an equal number of values for reference as G instead of A. However, it should be noted that the azimuth values vary only as θ or 360-θ depending on whether the sun rise or sun set is being considered.

Let us begin with Method 1. Here the angle EGD is 45 degrees and the angle GAD is 26.56 degrees. We consider the observer to be at A and looking south; the point D then may correspond to the sunset along AD on winter solstice. Using this idea we get a value of the latitude as 26.9 N. This is true also for an observer sitting at D and observing sun rise along DA on summer solstice.

We may consider yet another possibility when the observer is at G and is looking at sun rise on winter solstice along GD. This gives us the latitude of 55.67 N, which is unrealistic.

In this calculation we have assumed that the maximum value of the declination of the sun is 23.5 degrees.

Let us consider method 2. If the observer is at A and looking at the sun set along AD or the observer at D looking at sun rise along DA does not give a valid result. Same is the case with method 3 and method 4. In these three cases we have yet another point for the observer namely the extension of AG to a point P as decided by the fraction of choice. This results in values of latitude as 41.53 N, 39.3 N and 53.5 N respectively. Method 5 is identical to method 4; Method 6 gives a latitude of 55.74 N

It may be noticed that most of these values, shown in Table 1, put the location well beyond the Indian latitudes. Moreover, point P shown in Fig. 2 may not be marked but is used only for fixing the length of the rope.

Some temples have been identified with a special orientation so as to mark the meridian noon passage of the sun. This is considered an auspicious day and also fixes the latitude of the place. Thus all through the year there

Table 1: The latitudes (N) of observer assuming a maximum declination of 23.5

Observer	**at A**	**at G**	**at P**
Method 1	26.9	55.7	
Method 2			41.5
Method 3			39.3
Method 4			33.5
Method 5	26.9	55.7	

are two specific dates or one as the case may be, for the noon passage of the sun (Jagadish, 2009, personal communication).

This gives us yet another possibility on fixing the latitudes as 19.58 N, 18.55 N, 28.8 N and 33.8 N, all of which lie within the boundary of India.

Since there is an option in our calculation to change the declination, let us fix the latitude at 26.9 N and try to find the declinations of the sun which satisfies the angles of construction. This latitude is well within the zone of the region known in the past as "*Brihmavartha*", a seat of knowledge and religion (Ganesh, 2010, personal communication).

For a latitude of 26.9 N the declinations range from 23.5 to 17.42; it is likely that the sun ever had these values at solstices? Since we have a choice on latitudes let us attempt fitting other values as well; the results are shown in Table 2 for 24.54 N, 22.17 N, 15.44 N and 10 N. It may be seen now that methods 2, 3 and 4 give a range of declinations for different latitudes as included in Table 2; we get a range of declinations ranging from 17.42 to 26.56.

Table 2. The southern maximum values of declination for different latitudes

Method / ?	**26.9**	**24.5**	**22.2**	**15.4**	**10**
1	23.5	24	24.4	25.5	26.6
2	18.5	18.9	19.2	20.1	20.5
3	17.4	17.8	18.0	18.9	19.3
4	19.8	20.2	20.6	21.5	22.0

This interpretation leads us to an interesting question on whether the maximum southern declination of the sun varied during the period of *Śulbasūtras*. The value of the obliquity of the earth's orbit has been steadily decreasing from 24.40 in BC 3800 to today's value. An extrapolation to BC 9000 shows the opposite trend (Abhyankar, 2008); the value increases to 25.53.

Attention has been drawn towards the secular variations of earth's orbital inclination and eccentricity from the days of Lagrange in 18th century. A precise solution taking into account the effect of all planets except Neptune was provided by Le Verrier in 1856. The obliquity was particularly addressed by Pilgrim (1904). Subsequently the interest in the topic has been retained especially in the context of long term variation of the climatic parameters. There have been several numerical solutions of planetary orbits in general and earth in particular, in the context of understanding the long term climatic variations, where in small corrections to the orbital parameters lead to noticeable changes over millions of years. Particularly more attention is paid to the earth - moon system (Varadi *et al*., 2009; Berger, 1978 and Laskar *et al.*, 2004). All these simulations point to the gradual changes in the eccentricity, orbital inclination. Simulations have been extended to other planets as well.

The increase in obliquity and the consequent change in the value of declination is thus a very realistic situation. In light of this, the different constructions offered in the *Śulbasūtras* may be considered as possible options to fit an observed parameter by geometrical constructions. Interestingly, the solution by Laskar et al (2004) shows a continuous increase in the obliquity for the past 250 million years with a singularity at the present epoch. Thus there is a small decrease by about 0.4 degrees and it is expected to increase again.

Bāudhāyana Śulbasūtra traditions are believed to be the earliest and dated to BC 3000. *Kātyayana Śulbasūtra* is the most recent among the four (Plofker, 2009). A direct application of the simulations to the positioning of *Dakṣiṇāgni* is yet to be explored since the above cited references consider time scales of the order of millions of years.

Conclusion

Different locations for the placement of *Dakṣiṇāgni* are described in the different texts of *Śulbasūtras* with no explicit mention of the purpose. Here we have discussed the possibility of its use

1. As tracer of the azimuth of sunrise on winter / summer solstice. It appears that marking this was an important ritual and the changes in its location may reflect on the actual variation in the obliquity of the earth's orbit.
2. As an indicator of the change in the location of the observer. This may imply the gradual change in the maximum value of the declination also.

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Appendix

The verses describing fixing *Dakṣiṇāgni* are (the numbers are from Sen and Bag, 1983)

a. *Āpastamba Śulbasūtra* (*Āśl*)

4.3 According to tradition, the (place of the) *Dakṣiṇāgni* is near the south-east corner of the third part of the distance of the *Gārhapatya* (from the *Āhavanīya*).

4.4 The distance between the *Gārhapatya* and the *Āhavanīya* is divided into five or six (equal) parts, a sixth or a seventh part is added, the whole (of the cord measuring the original distance plus the added part) is divided into three (equal) parts, and a mark is given at the end of the third part from the western end. (With two ties) fastened to (poles at) the two ends of (the distance between the *Gārhapatya* and the *Āhavanīya*, the cod is stretched to the south by the mark and a pole fixed (at the point reached by the mark). This is the place of the *Dakṣiṇāgni*. This is according to Sruti.

b. *Baudhāyana Śulbasūtra* (*Bśl*)

3.1 Now, the placement of the *Āhavanīya* from the *Gārhapatya* in the arrangement for the laying of sacrificial fires (will be discussed). According to tradition, the Brahmana has to place this fire (*Āhavanīya*) (at a distance of) 8 *prakramas*, the prince 11 *prakramas* and the merchant 12 *prakramas* (from the *Gārhapatya* towards east).

3.2 Three squares of side one-third the distance (between the *Āhavanīya* and the *Gārhapatya*) are made so as to be in contact with each other (along the east-west line); the *Gārhapatya* (fire) lies at the north-west and the *Dakṣiṇāgni* (*anvāhāryapacana*) at the south-east corner of the western square; the north-east corner of the eastern square marks the place of the *Āhavanīya*.

3.3 Alternatively, the distance between the *Gārhapatya* and the *Āhavanīya* is divided into five or six (equal) parts, a sixth or a seventh part is added, the whole (of the cord measuring the original distance plus the added part) is divided into three (equal) parts, and a mark is given at the end of the second part from the eastern extremity. (With two ties) fastened to (poles at) the two ends of the (the distance between) the *Gārhapatya* and the *Āhavanīya*, the cord is stretched to the south by the mark and a pole fixed at (the spot reached by) the mark. This is the place of the *Dakṣiṇāgni*.

c. *Kātyayana Śulbasūtra* (*Kśl*)

1.10 We shall explain in what follows how to find the southern agni by (the method of) the third. (A cord of a length equal to) the distance between the *Gārhapatya* and the *Āhavanīya* is increased by one-sixth or one-seventh (of its length) and the length so increased is divided into three equal parts; the cord is stretched towards the south by the mark given at one-third from the other (western) end; at the point (thus obtained) the fire (is to be placed). The opposite point in the north is the place for the utkara (pit).

1.11 Alternately, with a cord of length equal to the distance (between the *Āhavanīya* and the *Gārhapatya* fire) reduced by one-third, a square is drawn in the eastern half; the fire (*Dakṣiṇāgni*) (is placed) at the *śroṇi* (that is, at the south-western corner of the square). By reversing, the rubbish heap (*utkra*) (is placed) at the *aṃsa* (that is, the north-east corner).

d. *Mānava Śulbasūtra* (*Mśl*)

1.4-1.6 (the cord) for the altar of the new and full moon sacrifice (*darsikya*) is 6 *aratni*s long, each (*aratni*) having the mesure of 24 *aṅgulas*. The east-west line, east and west corners (*praci*, *aṃsas* and *śroṇis*) of the altar are fixed by means of drawing arcs with the help of a cord (marked) at 24 (=7+17) *aṅgulas* and 8 (=1+2+5) *aṅgulas*. The cord is (then) placed from south-east corner (*aṃsas*) to the south-western corner (*śroṇi*) and with this distance and with south-western corner as center, is drawn an arc in the east. Having taken the center of the circle at south-east corner, an arc is similarly drawn in the west. One end of the cord is now placed at this point of intersection and the portion from the south-east corner to south-west corner is cut off by means of drawing arcs. The same is repeated in the north, east and west.

1.7 The mound (*khara*) for the eastern fire (*Āhavanīya*) is a square of one *aratni*, that for the western fire (*Gārhapatya*) is in the shape of a chariot wheel (*rathacakrākṛti*) and that for the southern fire (*Dakṣiṇāgni*) in the form of the half moon (*candrardha*).

1.8a,1.8b. A circle is drawn from the middle (point of a square drawn for *Āhavanīya*) with *koṭi* measure. With third part of the length which liles outside (the square) together with the original (inside) length (i.e., half of the side of the square) is described another circle (for *Gārhapatya*). Draw another circle with the half of the square circumscribing (second circle). The half of this circle is (*Dakṣiṇāgni*).

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