## SANKRAMAVĀKYAS OF THE VĀKYAKARAŅA

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(Received 15 December 2012; revised 15 November 2013)

#### Abstract

Computation of the time of transit of the Sun from one zodiacal sign to the next is one of the requisites for performing Hindu rituals. *Sankramavākyas* have been instrumental in this task since the latter half of 13th century AD. Our effort in this article is to throw light on *sankramavākyas* of the *Vākyakaraņa* (c. 1300 AD) as they have a very specific structure. We will explore whether the *sankramavākyas* can be called numerical tables and how they are obtained. The translation of the *sankramavākyas* of *Vākyakaraṇa* have also been provided in this content and that of the commentary of Sundararāja (c. 1500 AD).

Key words : Ahargaņa, Dhruva, Kaţapayādi, Rāśi, Sankrama, Vākya

#### **1.** INTRODUCTION

In Indian tradition the transmission of knowledge from generation to generation was primarily oral. Writing was not the preferred system and it would not have been, given the fact that the writing tools (palm leaf and a metal stylus) weren't quite convenient. Even when something had to be written, the principle of parsimony was upheld unanimously by almost all the authors irrespective of the discipline or field in which they wrote their texts. The following adage may be quoted in support of what has been stated:

ardhamātrālāghavena putrotsavam manyante vaiyyākaranāh

Grammarians consider [the joy produced] by reducing half a syllable [to be equal to] the celebration of the birth of a son.

In the field of astronomy and mathematics also, several tools were invented to make the contents precise and memorable. We find many numerical data which were presented in the form of verses or sentences. The *Grahacāranibandhana* (c. 683 AD) [Sarma KV 1954], the *Vākyakarana*  (c. 1300 AD) [Sastri and Sarma 1962] etc. present various numerical tables which can be memorised easily. These texts provide numerical data in the form of  $v\bar{a}kyas$ . The case of *sankrama* is an example in this regard, which we are going to discuss in detail.

Here in this paper, we provide the translation of Sanskrit text of *sankramavākyas* of the *Vākyakaraņa* and also that of Sundararāja's explanation. The structure of the *vākya* is highlighted in the context of numerical tables. Under the light of the commentary of Sundararāja we explore the method by which the values of the *sankramavākyas* would have been obtained.

## 2. THE VĀKYA TRADITION

The word  $v\bar{a}kya$  literally means a sentence. It may simply consists of either a single word that can run to several lines or a group of words. When used in the context of astronomy, the word  $v\bar{a}kya$ does not refer to any ordinary sentence. It has a very special connotation. They are short

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meaningful sentences which when decoded present certain numerical data that either correspond to the longitudes of planets or something related to that. Usually they are constructed using the *kaṭapayādi* system [Plofker 2009, pp.76-77], one of the three major systems employed for denoting numerals in Indian astronomical texts. The words and phrases for composing were chosen with great care so that they become meaningful and convey ethical values and worldly wisdom. In other words, they had dual purpose of encoded numerical coding as well served a poetic and asthetic appeal.

## 2.1 Types of vākyas

Different types of vākyas emerged over a period of time, to code different types of numerical values in connection with various astronomical computations. Among them we find a large number of them representing longitudes. For example a group of 248 specifying the true longitudes of the Moon (known as Candravākyas) [Kunhan Raja 1984] pertaining to 9 anomalistic months and a set of 2075 called Samudravākyas, Mandala-vākyas or Kujādi-pañcagraha-vākyas for the five planets (570 for Mars, 528 for Mercury, 231 for Jupiter, 195 for Venus and 551 for Saturn) can be cited. These vākyas present longitudes in terms of sign  $(r\bar{a}\dot{s}i)^1$  and its subdivisions i.e., degrees and arc-minutes. We find a set of 37 vākyas for computing the motion of the Sun [Sastri and Sarma 1962, p. 15] which provide the longitude in degrees of arc corresponding to 10 day intervals. Besides these, there are vākyas that provide correctional values (such as equation of center) in arc-minutes [Sarma KV 1954, pp. 7-13].

Some of the  $v\bar{a}kyas$  represent a certain duration of time. The units of time used in such  $v\bar{a}kyas$  are days,  $n\bar{a}d\bar{i}s$  or  $vin\bar{a}d\bar{i}s$ .<sup>2</sup> For example:  $m\bar{a}sav\bar{a}kyas$  [Sastri and Sarma 1962, p. 251] provide the duration of each of the twelve solar months in days. The rising time of zodiacal signs is specified using *vinādīs* [Sastri and Sarma 1962, p. 77]. In some instances we find both the day and its subdivisions being represented by *vākyas*. The *sankramavākyas* which are used in the computation of the moment of transits of the Sun into twelve zodiacal signs (which is going to be discussed in section 3 belonging to this category.

Some of the *vākyas* such as *śodhyavākyas*, *maņdala-vākyas* and *parivṛttivākyas* merely stand for a particular number which are used in the computations of the true longitudes of planets [Sastri and Sarma 1962, pp. 51-55].

## 2.2 Decoding vākyas

All  $v\bar{a}kyas$  are decoded using the *kaṭapayādi* notation of numerals [Sastri and Sarma 1962, p. 36].<sup>3</sup>

The decoded numbers may represent the units of arc length such as minutes, degrees and zodiacal signs (rāśis) or the units of time like days, nādīs and vinādīs. Context quickly determines the absolute value of the grouping of the digits. As indicated in the previous section, though there are different types of vākyas a majority of them represent the longitudes of the planets. The  $v\bar{a}kyas$ specifying the longitudes more or less have a specific pattern in the sense that they represent three separate sets of numbers namely the signs  $(r\bar{a}\dot{s}is)$ , degrees and minutes. Each of them can at most a two digit number and hence these vākyas have three to six syllables. By convention, while decoding the  $v\bar{a}kyas$  the numbers should be noted down from right to left. A conventional/popular adage ankānām vāmato gatih means that "the digits go towards left". Thus the first two syllables represent minutes, the next two represent degrees, and whatever is left with represents zodiacal sign. Here, it may also be added that it is the completed zodiacal signs that are specified in vākya. So when a planet is in the first sign no number will be mentioned. When it is in the 12th sign, then  $v\bar{a}kya$ will specify the number 'eleven'. For example: the first of the candravākyas namely girnah śreyah

(1203) corresponds to the Moon's position being 03 minutes and 12 degrees. Here, since the Moon is in the first zodiacal sign, the number of the sign is not mentioned. Further computations are needed for calculating the exact position of the planet.

The  $v\bar{a}kyas$  that provide information such as day and time would have five syllables. In that case the first two syllables stand for  $vin\bar{a}d\bar{i}s$ , the next two for  $n\bar{a}d\bar{i}s$  and the remaining one for day. This will be illustrated with example while explaining the *sankramavākyas* (in section 3.3).

### 2.3 History of vākyas

The earliest *vākyas* that are extant today are the *candravākyas* of Vararuci (4th century AD) [Pingree 1994, p. 558]. The *candravākyas* are also known as *Vararuci-vākyas* [Kunhan Raja 1984, p. 56]. These *vākyas* provide the longitudes of the Moon. There are 248 *candra-vākyas* corresponding to 9 anomalistic cycles of the Moon.

Another important text in which the *vākyas* appear is the *Grahacāranibandhana* of Haridatta (c. 683 AD). Haridatta clearly says that he is going to state *vākyas* (*atha vakṣyāmi vākyāni...*) [Sarma KV 1954, p. 6, verse no. 35] before listing the literal expressions of numbers.

The Vākyakaraņa, (c. 13th century AD) presents an abundant number of elegant vākyas to specify the longitudes of the planets. This became much popular in south India especially among almanac-makers. The Veņvāroha of Mādhava (c. 1350 AD) [Sarma KV 1956], Sphuṭacandrāpti of Mādhava (c. 1350 AD) [Sarma KV 1956], Sphuṭacandrāpti of Mādhava (c. 1350 AD) [Sarma KV 1973] and Dṛggaṇita of Parameśvara (c. 1430 AD) [Sarma KV 1963] are prominent among the texts which present vākyas.

#### 2.4 Earlier studies on vākyas

Scholars like Van der Waerden [Waerden 1955, pp. 230-233], G. J. Toomer [Toomer 1964, p. 13] and K. Chandrahari [Chandra 2001] have already thrown light on the vākya system of computation. Van der Waerden initiated an enquiry into the basis of calculation of the tables used by astronomers from Tamil speaking region of India. He claimed that the solar table was derived from Āryabhata [Waerden 1955, pp. 230-233]. A mathematical analysis of Tamil solar table was presented by G. J. Toomer [Toomer 1964] with the computations of the equation of centre and mean longitudes corresponding to the days given in the table. The values given in the solar table (shown by G. J. Toomer [Toomer 1964, p. 13]) exactly match with the values of sankramavākyas of the Vākyakaraņa. But neither the name of the sańkramavākyas nor that of the Vākyakarana does appear in his article. In another article, he points that the *vākya* system comes originally from a work written in Sanskrit, Parahita [Toomer 2 1964, p. 255]. Clearly, Van der Waerden and Toomer were not aware of the text Vākyakaraņa. Now, it is clear that 'Tamil' astronomical calculations were based on 'Vākyakaraņa', which is itself based on the works of the Āryabhata school, and the parahita system of Haridatta prevalent in Kerala.

## 3. THE VĀKYAKARAŅA (C. 1300 AD)

The Vākyakaraņa is an anonymous work generally believed to have been composed around 1300 AD. It has an erudite commentary called *Laghuprakāśikā* by Sundararāja (c. 1500 AD) [Sastri and Sarma 1962], an eminent astronomer of Tamilnadu who is supposed to have been a contemporary of Nīlakaṇṭha Somayāj (1444-1544 AD).<sup>4</sup>

From the citations appearing in the commentary it is clear that there exists another commentary of the *Vākyakaraņa* called *Vāsanāprakāśikā*<sup>5</sup> of Sundararāja.

There is no agreement among scholars regarding the authorship of the *Vākyakaraņa*. While some consider it as an anonymous work [Sastri and Sarma 1962], there are other who have

opined that Vararuci could be the author [Helaine 1997, p. 955], [Sarma PD 2004, pp. 141-142]. Prof. Pingree [Pingree 1994, p. 559] states that "The Vākyakaraņa...., is sometimes attributed to Vararuci". Possibly the readings found in some of the manuscripts<sup>6</sup> have led him to make this remark. Even if one were to accept Vararuci as the author of the Vākyakarana, this Vararuci cannot be identified with the other Vararuci, the author of candravākyas who is supposed to have lived around the 4th century AD. This is because one of the *vākvas* describe the rise and decline of the Chola kingdom found in the Vākyakaraņa. Since the period of Chola rule in Tamilnadu is fairly well known, this is considered as clear evidence that this text must have been composed during the 13th century AD [Sastri and Sarma 1962, pp. viii-ix]. In fact, K.V. Sarma states in an article that the Vākyakarana is apocryphically (falsely) ascribed to Vararuci, and that the author hailed from Karisaila or Kanci in Tamilnadu, as he himself states in his work [Sarma, 1997, p.995].

#### 4. Sańkramavākyas of the Vākyakaraņa

The word *saikrama* or *saikramana* is used in Indian astronomy to refer to the transit of the Sun into a  $r\bar{a}\dot{s}i$  (zodiacal sign) from another. A  $r\bar{a}\dot{s}i$  is one-twelfth part of the ecliptic. The division of the ecliptic into different  $r\bar{a}\dot{s}i$  starts from a fixed point along the ecliptic. That fixed point is taken to be the star  $a\dot{s}vini$  (Aries) instead of the vernal equinox. The time of entry of the true Sun into the twelve zodiacal signs is given by the following verses.

śrīrguņamitrā | bhūrvidhipakṣā | strratiśūrā | bhogavarāte ||

bhāvacaroriķ/ tenavaśatvam/ lokajabhtiķ/ sthūlahayo'yam //

angadhigāraḥ/ stambhitanābhiḥ/ nityaśaśīśo/ yāgamayo'yam //

tāvurupūrvam/ sankramavākyam/ tatkramayojyam/pādavaśena ||

VK 4-A

The first three verses provide twelve  $v\bar{a}kyas$ . They help to find the time of the transits (of the Sun); hence known as *sankramavākyas*. The fourth verse states how to use those  $v\bar{a}kyas$ . The literal meaning of the above verses is given below:

Wealth is a friend of virtues; A land supported by law; A very skillful women; A better [source of] enjoyement for you. The [actual] enemy resides in thought; Being controlled by that; Fear arising out of the world; This is quite a huge horse. One who has crooked parts; One whose centre is fixed; Eternal lord of the Moon; This is full of sacrifice – are the *sańkramavākyas* beginning with [that of] Taurus which have to be added [to the *dhruva*] in their order according tothe quarter [they belong to].

VK 4-A

Sundararāja explains the structure of the *sańkramavākyas* (see section C in the appendix) by pointing that the verses given above are constructed in the *pańkti* meter. Further, he informs us the following characteristics of the *pankti*. The pańkti is a variety that comes under the supratisthā meter. It contains five syllables in each quarter. A bhagana<sup>7</sup> and two guru syllables constitute a pańkti. It is a uniform meter (samavrtta) having the same pattern in all the quarters.<sup>8</sup> In order to get the time of the transit of the Sun into twelve zodiacal signs, first we have to decode the  $v\bar{a}kyas$ given above using the katapayādi system of numeration. The five syllables of each quarter give the week day, *nādis* and *vinādis*<sup>9</sup> sequentially but in the reverse order. The day, nādis and vinādis correspond to the transit of the Sun into twelve signs beginning from Taurus. The numerical values provided by the *sankramavākyas* are the following:

2;55;32, 6;19;44, 2;56;22, 6;24;34, 2;26;44, 4;54;06, 6;48;13, 1;18;37, 2;39;30, 4;06;46, 5;55;10 and 1;15;31.

#### 4.1. Importance of sankramavākyas

By definition, a solar month is the time interval between the transit of the Sun from one

 $r\bar{a}si$  (zodiac sign) to another. Though  $r\bar{a}si$  are equally spaced segments along the ecliptic, this time interval is not regular because of the variation in the rate of motion of the Sun. As the transit can happen at any moment during the day (0–24h) the actual moment of the beginning of a solar month can be any time during the day or night. However, certain rules exist for the purpose of observing rituals either on the previous day or the successive day depending upon whether the transit were to happen during the night or day at a given location. When there was a need to have an easy way to know the moment of the transits of the Sun, the *sankramavākyas* came as a great aid in this regard.

## 4.2. Can the *saṅkramavākyas* be considered as numerical tables?

A general feature that could define numerical tables is that they consist of a few rows and columns. Typically the first column would present a list of values of an independent variable against which one or more columns would list the values of dependent variable. Keeping this in mind and noting that in the case of the *sankramavākyas*, since it seems to be merely a list of numbers corresponding to the value of the dependent variable, a doubt has been encouraged as to, whether these numbers are to be treated as a part of numerical tables or should they be merely categorized to be versified list of numbers.

In the case of *saikramavākyas* we do not have lines and columns. But it is important to note that here exists a functional relationship between the argument and result. Since the values of the independent variable are the multiples of degrees (true longitude of the Sun during the transit) they are not mentioned explicitly in the verses. The order/rank of the  $v\bar{a}kyas$  itself tells us the zodiacal sign to which the Sun enters. The fourth verse that starts with  $t\bar{a}vurup\bar{u}rvam$  states that the given set of entries (i.e., *saikramavākyas*) corresponds to the zodiacal signs starting from Taurus.

The second column in Table 1 displays the *sankramavākyas* which are given by the verses. The third column shows the literal meaning of the  $v\bar{a}kyas$ . Actually user is interested in the mathematical/astronomical information provided by the  $v\bar{a}kyas$  along with their sequence. The values represented by them as well as their order are also given in last two columns. The order of the gives the longitude covered by the Sun. The mathematical/astronomical information (provided by the sequence as well as by the  $v\bar{a}kyas$ ) are given in the third and fourth column respectively.

No.	Vākya	Literal meaning	Longitude crossed	<b>Transit on</b> $2^d 55^n 32^v$	
01.	śrīrguṇamitrā	Wealth is a friend of virtues	30°		
02.	bhūrvidhipakṣā	A land supported by law	60°	$6^{d}19^{n}44^{v}$	
03.	strratiśūrā	A very skillful woman	90°	$2^{d}56^{n}22^{v}$	
04.	bhogavarāte	A better [source of] enjoyment for you.	120°	$6^{d}24^{n}34^{v}$	
05.	bhāvacaroriķ	The [actual] enemy resides in thought	150°	$2^{d}26^{n}44^{v}$	
06.	tenavaśatvam	Being controlled by that	180°	$4^{d}54^{n}06^{v}$	
07.	lokajabhtiḥ	Fear arising out of the world	210°	$6^{d}48^{n}13^{v}$	
08.	sthūlahayo'yam	This is quite a huge horse	240°	$1^{d}18^{n}37^{v}$	
09.	angadhigārah	One who has crooked parts	270°	$2^{d}39^{n}30^{v}$	
10.	stambhitanābhih	One whose centre is fixed	300°	$4^{d}06^{n}46^{v}$	
11.	nityaśaśīśo	Eternal lord of the Moon	330°	$5^{d}55^{n}10^{v}$	
12.	yāgamayo'yam	This is full of sacrifice	360°	$1^{d}15^{n}31^{v}$	

**Table 1:** *Sańkramavākyas*, their meanings as per the commonly employed classical Sanskrit and their decoded numerical values

Looking at these points, we consider the *sankramavākyas* to be a versified numerical table.

## 4.3. Finding the time of transits using the sankramavākyas

The *sankramavākyas* are added to the time of the first transit of the year to provide the successive transits. That additive value (the time of first transit in weekday, *nādis* and *vinādis*) which is fixed for a year is termed *dhruva* (a fixed [initial value]). The word *dhruva* literally means a constant. Here, the additive is the same for all twelve *vākyas*, and hence called so. In fact, this number remains fixed for a year. The number of days elapsed till the beginning of the present year (*ahargaṇa*) is used to compute the fixed [initial value]. The *Vākyakaraṇa* provides the following relation<sup>10</sup> to get *ahargaṇa* using *D*, the number of passed years (of *Kaliyuga*) [Sastri and Sarma 1962, p. 3].

Ahargana,

$$D_{y} = y \left(365 + \frac{1}{4}\right) + \frac{5y - 1237}{576}.$$
 (1)

The above relation<sup>11</sup> (1) gives an integer of days and part of the day (in  $n\bar{a}dis$  and  $vin\bar{a}dis$ ). Normally, *ahargaṇa* is an integer of civil days. But here, fraction of the day is also taken into account. The integer represents the number of passed days and the fraction tells us when the new year begins on the coming day.<sup>12</sup> The remainder obtained by dividing the number of days ( $D_y$ ) by seven indicates the weekday that precedes the day of the transit. This remainder and the fractional part together constitute the fixed [initial value] (*dhruva*). Thus the value of the fixed [initial value] indicates the day and moment of Sun's entry into the first zodiacal sign, *Meşa*.

It is this quantity that is computed first and added to all the *sańkramavākyas* for getting the time of the Sun's successive transits. If a user wants to know the time of the transit into  $(i + 1)^{th}$  sign then  $i^{th} v\bar{a}kya$  should be added to the value of the first transit or the fixed [initial value] (where i=1 to 12). The user has to keep in mind that the transit obtained using  $i^{th} v\bar{a}kya$  takes place in  $(4i + 1)^{th}$  week of the given year. The first digit indicates the previous weekday and the other two couples of digits give the moment of transit in  $n\bar{a}dis$  and  $vin\bar{a}dis$ .

### An example

Let us calculate the transits of the Sun for the 5112th year of *Kali* era (2011 AD).

*The number of elapsed years,* y = 5111

Using (1), we get

the number of days elapsed,  $D_y = 1866834;57^n;06^v$ 

The fixed [initial value],  $Dhruva = D_y$ modulo 7

$$=4^{d}; 57^{n}; 06^{v}.$$

Here, the number 4 indicates the weekday passed before the first transit. Monday happens to be the fourth day counting from Friday,<sup>13</sup> thus we come to know that the first transit occurs on Tuesday. The moment of the transit is given by the numbers 57 and 06. It means that the transit occurs at 57 *nādis* and 06 *vinādis* after sunrise. Table 2 displays the time of successive transits obtained by adding the fixed [initial value] to the *sankramavākyas*.

For example, to know the time of the transit into the 9th zodiacal sign, we have added 8th  $v\bar{a}kya$  (*i*=8) to the fixed [initial value]. Then we get the time of the transit as  $6^{d}15^{n}42^{v}$ . This indicates that the transit takes place when 6 days and 15  $n\bar{a}dis$  and 42  $vin\bar{a}dis$  are passed. (Also, one should know that 32 weeks have passed since the beginning of the year.)

Sundararāja prescribes [Sastri and Sarma 1962, p. 11] two further corrections to the above

**Table 2:** The time of the Sun's transits in 5112th year of*Kali* era (2011-2012 AD)

Transit to	Value from <i>vākya</i>	Dhruva + vākya
1. Meșa		4 <sup><i>d</i></sup> 57 <sup><i>n</i></sup> 06 <sup><i>v</i></sup>
2. Vṛṣabha	$2^{d}55^{n}32^{v}$	$0^{d}52^{n}38^{v}$
3. Mithuna	$6^d 19^n 44^v$	$4^{d}16^{n}50^{v}$
04. Karka	$2^{d}56^{n}22^{v}$	$0^{d}53^{n}28^{v}$
05. Simha	$6^{d}24^{n}34^{v}$	$4^{d}23^{n}40^{v}$
06. Kanyā	$2^{d}26^{n}44^{v}$	$0^{d}23^{n}50^{v}$
07. Tulā	$4^{d}54^{n}06^{v}$	$2^d 53^n 12^v$
08. Vṛścika	$6^{d}48^{n}13^{v}$	$4^{d}45^{n}19^{v}$
09. Dhanus	$1^{d}18^{n}37^{v}$	$6^d 15^n 42^v$
10. Makara	$2^{d}39^{n}30^{v}$	$0^{d}36^{n}36^{v}$
11.Kumbha	$4^{d}06^{n}46^{v}$	$2^{d}03^{n}52^{v}$
12. Mīna	$5^{d}55^{n}10^{v}$	$3^{d}49^{n}16^{v}$
01. Meṣa	$1^d 15^n 31^v$	$6^d 12^n 37^v$

values in order to obtain the precise time of transit.<sup>14</sup> One of them depends on the longitude and the other one on the latitude (of the place of the observer). For the passage explaining the need of these corrections see section B in the appendix.

## 4.4. How the numbers represented by the *sańkramavākyas* are obtained?

The *saṅkramavākyas* provide the values which are ready to use. Now, let us see how the *saṅkramavākyas* (i.e., the numerical values presented by them) are obtained. Here, the main step behind the curtain is to compute the duration between the first and other transits. The number of days taken by the Sun to cross the zodiacal signs is computed by adopting the inverse process of computation (i.e., computing the number of days corresponding to a given longitude) which is explained below. It involves the computation of the equation of centre.

# 4.4.1. Computation of the duration between transits

In order to construct *sankramavākyas* the duration between the first transit and the transit to

the given zodiacal sign should be known. The procedure for the computation of that (the exact duration between the first transit and the transit in question) involves the following steps (the commentary based on which the following steps are explored is given in section D of the appendix):

### Step 1:

Finding the mean time  $(t_i)$  taken by the Sun to cross multiples of 30 degrees along the ecliptic using:<sup>15</sup>

$$t_{i} = \frac{No.of \ civil \ days \ in \ a \ Mah\overline{a}yuga}{No.of \ solar \ months \ in \ a \ Mah\overline{a}yuga} \times i \ (i = 1 \ to \ 12)$$
$$= \frac{157, 79, 17, 500}{43, 20, 000 \times 12} \times i$$
$$= 30.4382238i. \tag{2}$$

## Step 2:

Find the equation of center  $(E_i)$ corresponding to the longitude  $(\lambda_i)$  of the Sun. Here,  $\lambda_i = i \times 30$  (i = 1 to 12).  $E_i$  is given by

$$E_i = R\sin(\lambda_a - \lambda_i) \times \frac{r_0}{R}.$$
 (3)

where  $\lambda_a$  is the longitude of the apogee of the Sun.

 $\frac{r_0}{R}$  is the ratio of the circumference of Sun's epicycle to that of the deferent circle. By inserting their values as given by Bhāskara I [Shukla 1960], the equation (3) can be rewritten as

$$E_i = 57.3\sin(78 - \lambda_i) \times \frac{13.5}{360}.$$
 (4)

Equation (4) gives the magnitude of the correction in degrees which is the difference between the mean and the true longitudes of the Sun.

## Step 3:

Here, we find the time  $(\Delta_{t_i})$  taken by the Sun to cover the above difference of longitudes. That is obtained using the true rate of motion of the Sun at  $\lambda_i$ .

$$\Delta_{t_i} = \frac{E'}{d_t}.$$
(5)

where  $d_i$  is the true daily motion of the Sun in degrees.<sup>16</sup> The answer will be in terms of days. If the equation of center is positive then the obtained duration should be subtracted from  $t_i$ otherwise added.<sup>17</sup>

$$t'_{i'} = t_i \pm \Delta_{t_i}. \tag{6}$$

The obtained quantity,  $t'_{i'}$ , is the time required for the Sun to reach  $\lambda_i$ .

### Step 4:

Here,  $t'_{i'}$  gives the time required for  $i^{th}$ 

transit from the beginning of the mean year. Actually we need the time elapsed since the beginning of true year. The true year begins before the mean year by  $2^d$ ; $08^n$ ; $51^v$ ;15. Hence, this difference is added to  $t'_{i'}$  to get the actual time difference between the first transit and the desired transit  $(D_i)$ .

Now, we have

$$D_i = t'_i + 2^d; 08^n; 51^v; 15.$$
<sup>(7)</sup>

Having found the durations between the transits, it has to be divided by seven to get remainder. The day of the transit is given by the remainder.<sup>18</sup> If  $D_i$  (i = 1 to 12) represents the number of days (including the fractional part of it) taken by the Sun to cover the multiples of degree segments of the ecliptic, then the  $v\bar{a}kya$  is given by

$$V_i = D_i \mod 7. \tag{8}$$

It is quite unlikely that  $D_i$  will be an integer, and usually has a fractional part. The fractional part of a day obtained here is converted into  $n\bar{a}dis$ and  $vin\bar{a}dis$ . And finally, by using the relation shown in (8) we get the values of the *sankramavākyas*.

The procedure given above is illustrated below using some examples.

## Illustration

1. First, let us compute the time taken by the Sun to reach the second zodiacal sign.<sup>19</sup> Hence we have i = 1 (the number of elapsed signs) and  $\lambda_i = 30^\circ$ . Now, using (2), we get

$$t_1 = \frac{157, 79, 17, 500}{43, 20,000 \times 12} \times 1$$

= 30.43822338.

Using equation (3), we get

$$E_1 = R\sin(48) \times \left(\frac{13.5}{360}\right)$$

$$= 1.596832444^{\circ}$$

By dividing by the Sun's true daily motion (which is calculated for the 30<sup>th</sup> day of the year i.e., when the Sun is likely to cross the first sign), we get

$$\Delta_{t_1} = \frac{1.596832444}{0.960841027} = 1.661911179.$$

Now, from (6), we get

 $t'_1 = 30.43822338 - 1.661911179$ 

$$= 28.776312201 = 28;46;34;43.$$

Now, using (7), we get

$$D_1 = 30;55;25;58.$$

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By using (8), we obtain

$$V_1 = 02;55;25;58$$

2. The time taken by the Sun to transit two zodiacal signs is obtained as follows:

$$\lambda_2 = 60^{\circ}$$

$$t_2 = 60.87644676$$

Using equation (3), we get

$$E_2 = R\sin(18) \times \left(\frac{13.5}{360}\right)$$

 $= 0.664000267^{\circ}$ 

By dividing by the Sun's true daily motion (which is calculated for the  $60^{th}$  day of the year i.e., when the Sun is likely to cross the second sign), we get

$$E'_2 = \frac{0.664000267}{0.950524254} = 0.698562151$$

Now, from (6), we get

 $t'_{2} = 60.87644676 - 0.698562151$ 

= 60.177884609 = 60;10;40;23.

Now, using (7), we get

$$D_{2} = 62;19;31;38.$$

By using (8), we obtain

 $V_2 = 06; 19; 31; 38.$ 

Similarly, we get the time of other transits also. The values of we get are approximately equal to those given by the *sankramavākyas*.<sup>20</sup>

#### **5.** CONCLUSION

Normally, the computations of the true longitudes of planets and the time of the transits involve the operations of huge numbers like the number of revolutions of the planet and the number of civil days in an aeon and so on. Also, trigonometric functions should be employed as we have seen above. But the use of the  $v\bar{a}kya$  method of computation allows to circumvent the laborious procedures. The  $v\bar{a}kya$  'tables' greatly simplify the computations by replacing trigonometric functions and reducing the multiplications and divisions of big numbers.

#### 6. ACKNOWLEDGMENT

I am grateful to Dr. Agathe Keller, Prof. Clemency Montelle and Prof. Kerine Chemla for their precious suggestions in refining the article. I express my sincere gratitude to Prof. Ramasubramanian, Prof. M. S. Sriram, Prof. Balachandra Rao for their guidance on the subject. My special thanks go to Kerala University Oriental Manuscript Library, Tiruvanantapuram, Adyar Library, Chennai and Government Oriental Manuscript Library, Chennai. I am thankful to the project HTN of CNRS and Universite de Paris-7 under the aegis of which this article has been prepared.

## Appendix

#### Sankramavākyas of the Vākyakaraņa with the Laghuprakāśikā<sup>21</sup>

#### A. Dhruva and sankramavākyas

अथ सौरवर्षसङ्क्रमभूतश्रीर्गुणादिधुवज्ञानायाह (?ज्ञानमाह) - श्रीर्गुणादिना (?श्रीर्गुणेत्यादिना) दृश्यत इत्यन्तेन। .....सेनाभक्तावशेषितम्॥ VK 1.3b श्रीर्गुणादिधुवं विद्याद् भान्तं वाक्यैस्तु दृश्यते। VK 1.4a

पूर्वानीतशुक्रवासरादित्योदयकाले कलियुगसङ्क्रमात्, शुक्रवारोदयमारभ्य दिनगणे प्रवृत्ते सप्तहृतावशिष्टं शुक्रवारादि गणयित्वा वर्तमानवर्षसङ्क्रमणज्ञानावगतिः। सेनाभक्तावशेषितदिनादिकं मेषरविसङ्क्रमणभूतम्। वृषभादिरविसङ्क्रमणसाध्य(?साधन)भूतघ्रवं प्रतिसंवत्सरं वाक्यनिबन्धं कुर्यात्।

Now, this is stated for the knowledge of the fixed [initial value] (*dhruva*) of the transits of the solar year, by means of [the verse] which starts with *śrīrguṇa* and ends with *drśyate*.

The remainder of the division by seven  $(sen\bar{a})$  has to be known as the fixed [intitial value] for  $\dot{s}r\bar{i}rguna$  etc. Indeed, the ending of the signs (zodiac) is observed by means of .

(VK 1.3b- VK 1.4a)

Because the [first] transit of *Kaliyuga* [happened] at the time of sunrise on Friday, which is calculated earlier, when the group of the days (*dinagana*) is commenced from the morning of Friday, the transit in the present year is known by counting the remaining [days], that obtained from the division [of days] by seven, from Friday. The remainder of the division by seven in days etc. is [the day] of the Sun's transit into Aries. The fixed [initial value] which is the means in [the knowledge of] the transits of the Sun into Taurus etc. should be constructed in  $v\bar{a}kya$  every year.

#### B. Correction due to longitude and ascensional difference

अत्र चरार्धदेशान्तरदोर्विवरकलासमानीतघटिकादिशोधनयोजनद्वारा स्फुटसङ्क्रमणावगतिः। तत्र देशान्तरज्ञानाय श्लोकौ -

लङ्कातः खरनगरं सितोरुगेहं पाणाटं<sup>22</sup> मिसितपुरी तथातपर्णी। उत्तुङ्गं सितवरनामधेयदेशो लक्ष्मीवत्पुरमपि वात्स्यगुल्मसंज्ञम्॥ विख्याता वरनगरी तथा ह्यवन्ती। स्थानेशो सुदितमनास्तथा च मेरुः॥ अध्वाख्यः करणविधिस्तु मध्यमाना-मेतेषु प्रतिवसतां न विद्यते तु॥

इति। एवं लङ्कातः खरनगर-सितोरुगेह-पाणाट-मिसितपुर्यातपर्णी- उत्तुङ्गसितवरनामधेय-लक्ष्मीवत्पर-वात्स्यगुल्म-वरनगर्यवन्ती-स्थानेश्वरमदितमनो-

(महाभास्करीयम् II 1-2)

महामेर्ववगाहिरेखासमपूर्वापारेखासमवस्थायिनां द्रष्टणां देशान्तरसम्भवः। न तु रेखास्थानाम्। लङ्का-यवकोटि-सिद्धपुर-रोमकावगाहि- समपूर्वापररेखा-समतिरश्चीनरेखानिवासिनां द्रष्टणामेव स्थितद्रष्टदृष्ट्यनुरूपं लङकाक्षितिजमण्डलरूपोन्मण्डल-चरसम्भवः. तत्र स्वदेशक्षितिजमण्डलान्तरविवरसम्भवात्। पूर्वापररेखास्थानाम्। न तु पूर्वापरदक्षिणोत्तररेखासम्पाताधिष्ठितद्रष्टुः देशान्तरचरौ न स्तः। दोर्विवरन्त मध्यमस्फुटार्कभुक्तज्योतिश्चक्रप्रदेशविशेषत्वेन सर्वत्रनिवासिनामस्त्येव। एषां गणनप्रकारम् उपरिष्टादाचार्यः स्वयमेव वक्ष्यति। तच्च तत्रोक्तधनर्णवैपरीत्येन अत्र सङ्क्रमे संस्कर्तव्यम्। तथा सति स्फटवर्षादिसङक्रमसिद्धिः। एतज्ञावश्यं ज्ञातव्यं यत्र चरार्धदेशान्तरयोर्महत्ता, तत्र महतो भेदस्य विद्यमानत्वात्। वर्षादिसङ्क्रमध्रुवेण श्रीर्गुणादिवाक्यैश्च भान्तं राश्यन्तं द्रष्टव्यम्। भशब्दो राशिवचनः।

राशिक्षेत्रग्रहर्क्षभानि भवनं चैकार्थसम्प्रत्ययाः। (वृहज्जातकम् १-४) इत्याचार्यवचनात्।

Here, obtaining the true transit is by means of either addition or subtraction of the *ghațikās* etc. calculated from the minutes of half the ascensional difference, longitudinal difference and the equation of time (*deśāntara, carārdha* and *dorvivara*) Among these, to know the the longitudinal difference, two verses are [given] as –

From<sup>23</sup> Lankā (towards the north, we have the following places on the prime meridian): *Kharanagara, Sitorugeha, Pāņāṭa, Misitapurī, Taparņī,* the lofty mountain called *Sitavara,* the wealthy town called *Vātsyagulma,* the well known *Varanagarī, Avantī, Sthaneśa, and then Meru,* which is inhabited by happy people. For those who reside in these places; the correction for the longitude does not exist.<sup>24</sup>

(Mahābhāskarīya, Ch. II, 1-2)

Like this, there exists longitudinal difference for the observers who reside on the line which is towards the east or west of the line (standard meridian) that passes through Lankā, Kharanagara, Sitorugeha, Pāņāma, Misitapurī, taparņī, the lofty [mountain] named Sitavara, Lakśmvatpura, Vātsyagulma, Varanagara, Avantī, Sthaneśvara, Muditamanā and the great Meru; and not for those residing on the line.

The ascensional difference does exist only for the observers those who reside on the line that is inclined to the east-west line passing through Lanka, Yavakoti, Siddhapura and Romaka; because of the existance of the difference between the horizon of one's place and the six o'clock circle (unmandala) that is the horizon of Lankā corresponding to the sight of the observer staying there. Indeed [it is] not for those who reside on the east-west line (equator). There are no longitudinal difference and ascensional difference for the observer located on the intersection of east-west [line] and south-north lines. Indeed, the Rsine difference dorvivara being the difference of the portions of the ecliptic that are covered by the mean and true Sun, certainly exists (or applicable) for all the residents (observers), there exists [the application of]. The master himself will say their computational method later on. And that should be applied herein the transit according to the contrariety of its additivity and negativity stated there. Being so, the true transit at the beginning of the year is obtained. Also, necessarily this has to be found where there is a large [amount of] ascensional difference and longitudinal difference because of the existence of a great difference. By means of the fixed [initial value] which is the first transit of the year and by means of the beginning with *śrīrguna* etc., [the Sun's transit at] the end-point of the rāśis (bhāntam) should be noted. The word *bha* stands for  $r\bar{a}si$ , since there is a statement of the master.<sup>25</sup>

"The words *rāśi*, *kṣetra*, *gṛha*, *ṛkṣa*, *bha* and *bhavana* mean the same object."

#### (Brhajjātaka Ch. 1, verse 4)

#### C. Structure of the sankramavākyas

अथ तानि वाक्यानि श्रीर्गुणमित्रेत्यादीनि सुप्रतिष्ठायां छन्दसि समवृत्तैः प्रदर्शितैः पङ्क्तिनामभिश्चतुर्भिः श्लोकैः श्रीर्गुणमित्रेत्यादिभिः पादवशेनेत्यन्तैराह - श्रीर्गुणमित्र इति। श्रीर्गुणमित्रा भूर्विधपक्षा स्त्रीरतिशूरा भोगवराते॥ भावचरोरिः तेनवशत्वं लोकजभीतिः स्थूलहयोयम्॥ अङ्गधिगारः स्तम्भितनाभिः नित्यशशीशो यागमयोयम्॥ ताबुरुपूर्वं सङ्क्रमवाक्यं तत्क्रमयोज्यं पादवशेन॥ ४ - А॥ एकैकः पादः पञ्चाक्षरः। तथा च वृत्तरत्नाकराचार्यवाक्यं - "सुप्रतिष्ठायां भगौदिति पङ्क्तिः" इति। तस्य श्रीनाथव्याख्यानवाक्यं - 'भगणगुरुगुरुगुरुग्रान्य्यः प्रतिप्रादं चेत् पङ्क्तिमि वृत्ते स्वति इति। तानि वाक्यानि श्रीर्गुणमित्र इत्यादीनि पङ्क्तिवृत्तस्य पादरूपाणि पञ्चाक्षरमितानि

विनाड्यादीनि दिनान्तानि सङ्क्रमद्वयावगाहीनि ज्योतिश्चक्रद्वादशांशरूपसौरभाग-रविभोगप्रमाणदिक्चक्रप्रमाणप्रमितिजनितभास्करभूमिभ्रमणभूतभगणविवरलक्षणसावनदिननि करसप्तावतक्षणावशिष्टानि द्वादशसङ्क्रमसम्बन्धीनि। ताबुरुपूर्वं सङ्क्रमवाक्यं तत्क्रमयोज्यं पादवशेन इति। तत्तन्मासोक्तसङ्क्रमवाक्यं वृषभादितो वर्षादिसङ्क्रमे युक्त्वा चरार्धादि च वैपरीत्येन संस्कृत्य स्फुटरविसङ्क्रमस्यावगतिः। Now [the author] states those which start as  $\hat{srrgunamitra}$  in 'supratistha' metre; with four verses that appear with uniform metres called pankti starting with  $\hat{srrgunamitra}$  and ending with  $p\bar{a}davasena$  –

śrīrguņamitrā bhūrvidhipakṣā |strratiśūrā| bhogavarāte ||

bhāvacarorih| tenavaśatvam| lokajabhtih| sthūlahayo'yam ||

angadhigārah stambhitanābhih |nityaśaśīśo| yāgamayo'yam ||

tāvurupūrvam sankramavākyam tatkramayojyam pādavašena ||

VK 4-A

are the *sankramavākyas* beginning with [that of] Taurus which have to be added in their order [to the *dhruva*] according to the quarter [they belong to].

(VK 4-A)

Each and every quarter has five syllables. Also, the saying of the master in Vrttaratnākara is so "In supratisthā [category] it is the pańkti when there are one bha and two gurus." The statement in its commentary by Srinātha is "In each quarter if there are a bha group, a guru and a guru at the end, then it is a metre called pankti". Those which begin as śrīrgunamitrā form the quarter of the *pankti* metre, limited to five syllables, start with *vinādis* [and] end with days, immersed between two transits; the remainder of the division-by seven-of the group of the civil days which is characterised by the duration (vivara) of the revolutions which is the motion of the Sun around the Earth that is generated through [the rule of three with] the knowledge of the Sun's longitude in the form of twelve segments of the ecliptic, the rate of motion of the Sun and the measure of the equator. The time corresponding to the longitude is measured on the equator are related to twelve transits. The sankramavākyas starting with Taurus should be added in their order depending on [their] quarter. Starting from Taurus (Vrsabha), by adding the corresponding sankramavākya which are stated for each month to the first transit of the year and by applying the corrections like ascensional difference and etc. reversely, the knowledge of the true transit [is obtained].

#### D. Construction of the sankramavākyas

#### अत्र वासना

श्रीर्गुणादिवाक्येषु यावतिथं वाक्यं जिज्ञासितं तावतिथं वाक्यं ज्ञात्वा तत्पूर्वराश्यन्तमध्यमं च ज्ञात्वा तदन्तरालभूतमासिकसौरमध्यमभोगेन स्फुटमध्यमाहर्गणान्तरज्ञानेन उक्तत्रैराशिकसिद्धदिनादि। सप्तावतक्षणावशिष्टदिनादिकं सङ्क्रमवाक्त्वेन विज्ञेयम्। पुनरप्यत्र बहु वक्तव्यमस्ति। ग्रन्थविस्तरभयान्नोच्यते। अतः परं गणितप्रक्रियाप्रकाशनमात्रमेव क्रियते। वासना त्वस्मदुक्तसुन्दरराजीयवाक्यकरणवासनाप्रकाशिकायां द्रष्टव्या ॥ ४-А॥

#### Here is the exposition (vāsanā)

Among the vākyas that begin with śrīrguna, having known [the rank of] that  $v\bar{a}kva$  which is desired to be known. and having known the mean number [of days] corresponding to the end of the preceding rāśi, by means of the Sun's average monthly longitude which is the difference between those two [ $r\bar{a}\dot{s}is$ ]; and by finding the difference of true and mean number of days (ahargana); (we get) the days etc. that are calculated through the rule of three which has been [already] stated. The remainder of the division [of the ahargana] by seven in days etc. should be known as a sankramavākya. Again much more has to be stated here. It is not said because of the fear of expatiation of the text. Hereafter, the exposure of the calculational procedure only will be done. The exposition should be seen in the Vākyakarana-vāsanā-prakāsikā of Sundararāja which has been stated by us.

#### **END NOTES**

- 1. A  $r\bar{a}si$  (zodiacal sign) is one-twelfth part of the ecliptic. There are twelve  $r\bar{a}sis$ ; each comprising 30 degrees.
- 2. A day is divided into 60 *nādis*, and a *nādi* into 60 *vinādis*.

#### 3. Katapayādi notation:

The following table shows the numerals corresponding to the letters of Sanskrit [Plofker 2009, pp. 75-76]:

#### Vowels

 $a \ \overline{a} \ i \ \overline{\iota} \ u \ \overline{u} \ r \ e \ ai \ o \ au \ am \ ah \ \mathbf{0}$ 

#### **Consonants**

1	2	3	4	5	6	7	8	9	0
ka	kha	ga	gha	'na	ca	cha	ja	jha	ña
ţa	ţha	фа	ḍha	ņa	ta	tha	da	dha	na
ра	pha	ba	bha	ma					
ya	ra	la	va	śa	<u>ș</u> a	sa	ha	ļa	

4. The following statement appears in the *Laghuprakāśikā* [Sastri and Sarma 1962, p. 119]:

nilakņmhāryeņa... asmadanugrahārthe sundaraārjapraśnottarākhye granthe pratipāditam

In order to bless us, it is explained in the text called "The answers to the questions of Sundararāja" by  $N\bar{l}akantha...$ 

 ... sundararārjīya-vāsanāprakāśikāyām drastavyā (...should be seen in the Vāsanā-prakāśikā of Sundararāja) [Sastri and Sarma 1962, p. 13].

- 6. *vararucinā gaņanakarmoktam* [Sastri and Sarma 1962, p. 7].
- 7. One *guru* followed by two *laghu* syllables in order constitute a *bhagaṇa*. The syllable which is either long or preceded by a conjunct syllable is called *guru*. A *laghu* syllable is that which is short but not followed by a conjunct one.
- 8. There are three types of meters; (i) uniform: the one whose quarters follow uniform pattern (ii) non-uniform: that which has its quarters with different styles and (iii) half-uniform: that whose two quarters appear with same pattern.
- 9. A day is divided 60 *nādis* and a *nādi* into 60 *vinādis*.
- 10. The following verse gives the procedure to compute *ahargana:*

kalyabdo mātulaguņah varcavāmsena samyutah |

punarabdānmānaguņāt sālapriyavivarjitāt ||VK 2||

tatsamāptairdinairyuktam śukravārādikam dinam |

- Here, the term is introduced in (1) to deduct an amount equal to due to the equation of center. This deduction is done in order to provide the exact number of days (*sphuțāhargaṇa*) passed at the junction of true solar year (see the commentary of Sundararāja [Sastri and Sarma 1962, p. 8])
- 12. By definition, a solar year begins when the Sun crosses a fixed point ( $A \pm vin \bar{i}$  star in Indian tradition) of the ecliptic. That moment need not be at the time of the beginning of a day (i.e. sunrise).
- 13. The weekdays are counted from Friday, because the Kaliyuga began on Friday. The Laghuprakāśikā of Sundararāja says that "pūrvānitaśukravārādityo-dayakāle kaliyugasankramāt śukravārodayamārabhya dinagaņe pravrtte..." [Sastri and Sarma 1962, p. 10]. See section A in the appendix for translation.
- 14. *atra carārdhadeśāntaradorvivara...* [Sastri and Sarma 1962, p. 11].
- 15. A *Mahāyuga* 43,20,000 years. Here, we have used the parameters of *Laghubhāskarīya* as given by Sundararāja [Sastri and Sarma 1962, p. 6].
- 16. True daily motion of a planet is the difference in its true longitude in 24 hrs. It is calculated by taking the difference of its two (true) longitudes calculated for two successive days. Here in our computation the days are chosen such that when the Sun is likely to enter the zodiac signs (i.e., 30<sup>th</sup> & 31<sup>st</sup> days for second zodiac sign, 60<sup>th</sup> & 61<sup>st</sup> for the third etc. ).

- 17. Since we are employing the inverse process of computing the number of days from the longitude, the correction has to be applied in the opposite way.
- 18. See section D in the appendix for the text in Sanskrit and its translation.
- 19. Here, we have used the parameters of *Laghubhāskarīya* as given by Sundararāja [Sastri and Sarma 1962, p. 6].
- 20. Here, the computations are done using computer.
- 21. This portion of the and its commentary is taken from [Sastri and Sarma 1962, p. 10-15].
- The reading in Ms. B is taken instead of the edited version. The reading in *Mahābhāskarīya* [Shukla 1960, p. 47] is *pāņāṭaḥ*.
- 23. The *Mahābhāskarīya* [Shukla 1960, p. 47] gives *Muditajanaḥ* instead of the name *Muditamanāḥ*.
- 24. Translation of these verses are taken as it is from [Shukla 1960, p. 47].
- 25. The word *bha* can refer to 27 constellations also. Hence this clarification is given by the commentator.
- 26. The time corresponding to the longitude is measured on the equator.

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