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THE *KARAṆAKESARĪ* TABLES FOR COMPUTING ECLIPSE PHENOMENA

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Abstract

The *Karaṇakesarī* is a work which includes a set of tables and an accompanying text for computing the circumstances, details, and astrological aspects of lunar and solar eclipses. The tables have an epoch of 1681AD and the text reveals that they were composed by a certain Bhāskara, son of Rāma of the Kavīndrakula. We present here a preliminary survey of the contents and format of the tables, examining some aspects of the details of presentation, the use of paratexting, and the ways in which tables and text interact.

Key words: Astronomical tables, Eclipses, Elongation tables, Indian astral sciences, Lords of the Eclipse, Tabular layout

1. INTRODUCTION

Beginning in about the twelfth century, a distinct new way of presenting numerical data appeared to rise in popularity in the Indian astral sciences. This was the format of numerical tables of a sort which used spatial arrangement and alignment, and over the centuries it became an integral part of Indian astronomy. Reliant in many ways on inspiration from Greco-Arabic features,[1] astronomers working in the Sanskrit astral sciences also developed their own innovations for data-storage and retrieval related to this format. Dozens of such texts were produced between the late fifteenth and late eighteenth centuries and tables of this sort form a significant part of the corpus of Sanskrit astral science in the second millennium.

One such work is the *Karaṇakesarī* of Bhāskara (fl. 1681), a short explanatory text accompanying a set of astronomical tables for computing the circumstances and details of lunar and solar eclipses from an epoch of 1681 AD. The tables contain tabulated data to establish nodal elongation, apparent lunar and solar diameters, measures of the shadow cone, half-durations, ‘deflection’, and parallax, as well as other relevant astronomical parameters and their astrological correlations. The short accompanying text is divided into two ‘chapters’ (*adhikāras*): a lunar eclipse chapter (*candraparva*) which contains thirteen verses, and a solar eclipse chapter (*sūryaparva*) which contains seven verses. Together the tables and text provided their users with a set of precomputed numerical data relating to all the features relevant to eclipse phenomena and instructions on how to use them. In principle, this format allowed its users to enter the tables with some fundamental details (such as date, length of daylight, etc.) and simply ‘look-up’ the various aspects they were interested in determining.

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The title of this table text is intriguing. *Karaṇakesarī* is a compound formed from the word *kesarin* that means, among other things, a lion, and *karaṇa* which usually refers to the more traditional astronomical handbook whose contents are made up of versified rules and parameters. Invoking the word *karaṇa* for a table text work, rather than the more typical *koṣṭhaka* or *sāraṇī* may reveal something of the status that the author Bhāskara wanted to convey and the audience he hoped might use it. More instances of the uses of the terms *koṣṭhaka*, *sāraṇī*, and indeed *karaṇa* need to be collected before we can better appreciate the scope of these terms and the ways in which they could have been understood by the original actors.

Tables and their accompanying texts, such as the *Karaṇakesarī*, offer much potential for gaining insight into the scope and practice of astronomy during the period in which they were compiled. They also directly and indirectly testify to the goals and priorities of those that compiled them and the ways in which these compilers responded to the potentialities of this format and how they addressed the many technical challenges associated with computing and displaying large quantities of data. Tabular arrangements of these sorts also offer a glimpse into the interface between content and use, and often attest to a preoccupation on the part of their creators with enhancing, and in many cases simplifying, the ways in which users interact with the data. This is particularly pertinent in the case of eclipse reckoning when essential questions such as the visibility of an eclipse, and its duration and magnitude are dependent on many different components and several intermediary steps of calculation. We will explore these details through a careful study of selected features of the *Karaṇakesarī*.

2. The Tables and Text and their Contents

2.1. Description of the Primary Sources

This study of the *Karaṇakesarī* is based on a single manuscript copy of a set of tables[2] and a single manuscript copy of the accompanying text.[3] Tables under this name have also been identified in at least a dozen other sources which are in collections around the world.[4] The number of copies along with the fact that the latest of those that are dated were copied in the mid-nineteenth century, suggests that these tables were somewhat popular for a significant period of time after their original introduction.

2.2. Contents of the Tables

The *Karaṇakesarī* contains 27 distinct tables[5]. The majority of these have titles, and although there are still some challenges in determining the nature and function of the numerical content of each table, they are provisionally identified as set out in Fig. 2. The script of these tables, their titles, and marginalia is Devanāgarī and the language is Sanskrit[6]. This copy contains many ruled tables of varying sizes. Each has a title of varying length and occasionally additional explanatory text. Commonly, there are several tables per folio. Occasionally pages are only partially filled, and some tables extend over several pages. Sometimes rows are labelled, but this practice is not consistently applied. A typical page from the tables can be seen in Fig. 1.

These contents directly attest to the details practitioners were concerned with when modeling eclipses and their application more broadly. In this particular case, we see that practitioners were interested in answering questions such as timing, magnitude, and duration, but also a central part of eclipse reckoning was addressing other concerns as well, such as the astrological signification of the eclipse. Here, the key questions are: when will the next eclipse occur (date, day, and time of the

Fig. 1. f. 4v from the *Karaṇakesarī**

Folio Number	Table Number	Content
f.3	1	Elongation between mean Sun and Lunar Node 130 130-year periods
f.3v	2	Elongation between mean Sun and Lunar Node 130 1-year periods
f.4	3	Elongation between mean Sun and Lunar Node 1-27 *avadhi*s
f.4	4	Digits of the obscuration
f.4	5	Apparent solar diameter
f.4v	6	Apparent lunar diameter and diameter of cone of Earth's shadow
f. 4v	7	Corrections with reference to the sun
f. 4v	8	Duration of totality of a lunar eclipse
f. 4v	9	Half-duration of a lunar eclipse
f. 4v	10	True longitudes and daily progresses of the sun on first day of 1-27 *avadhi*s, and differences
f. 4v	11	Differences of table 9 for 1 to 21 digits
f. 5	12	Thirds(?) of duration of the eclipse
f. 5	13	Corrections to the obscuration(?)
f. 5-5v	14	Latitudinal deflection (*akṣavalana*) and the differences
f. 5v	15	Lords of the eclipse possibilities (*parveśa*)
f. 6	16	Deflection (*ayanavalana*)
f. 6v	17	Corrected deflection of the Sun
f. 6v	18	Corrected deflection of the Moon
f. 7	19	Half-lengths of daylight
f. 7v-8	20	Rising-times (*lagna*) of the zodiacal signs
ff. 8v-9v	21	Rising-times of sixtieth parts(?) of the zodiacal signs
f. 9v	22	Parallax in longitude
f. 10	23	Multipliers for correcting the longitudinal parallax
f. 10v	24	Parallax in latitude
f. 11	25	Half-duration of a solar eclipse
f. 11	26	The *vikālas*? of the planets in horoscopy
f. 11v	27	Attributes of the 28 constellations (*nakṣatras*)

Fig. 2. The contents of the *Karaṇakesarī* tables indicating folio and table numbers

*From a facsimile from the David E. Pingree Collection at the Brown University's John Hay Rare Books and Manuscripts Collection.

impact, mideclipse, and release), what will the magnitude be (partial, total), and how long the eclipse will last (duration). However, as these tables reveal, there are other features that these tables address, whose primary purpose is answering questions of an astrological nature. These include the deflection (*valana*), which is defined as the angle of impact of the eclipse and the lords of the eclipse (*parveśa*), a scheme in which certain gods or sages were assigned to eclipse possibilities. These aspects provide no technical information useful from a strictly astronomical point of view (although they rely on it for the underlying organization of their schemes), but they are crucial for 'interpreting' the ominous effects of the eclipse.

The order of topics covered by the tables loosely reflects the order of computation. This is important as almost all features of eclipse reckoning are cumulative. Lunar eclipses are treated first, and solar eclipses and the corrections due to parallax follow. It is typical to treat lunar eclipses prior to solar eclipses as the majority of computations for lunar and solar eclipses are essentially the same, but solar eclipses have the additional requirement of the application of parallax (an effect not relevant to lunar eclipses), hence are treated second. First, an eclipse possibility needs to be established which is done by examining the relative positions, or elongation, of the sun and the lunar node. The apparent diameters of the sun, the moon, and the earth's shadow are required next and this information, combined with the eclipse limits establish whether or not an eclipse will occur. Next we are given information on the duration of the lunar eclipses. Tables 14–18 concern the angle of impact of the eclipse which is a parameter that imparts astrological significance and is necessary for the graphical rendering of an eclipse.[6]

Then follow tables which give the half length of daylight (presumably to establish whether or not the eclipse will be visible) and the rising times (to tell the time during the night). Tables 22-24 give longitudinal and latitudinal parallax, and, presumably with parallax factored in, the last table gives the half-duration of a solar eclipse. Two tables then give astrological information useful for horoscopes, and the various attributes to be associated with the 28 lunar constellations (an animal, a 'world' (of Gods, of men, of demons), and one of three 'bands' in the sky (high, middle, low)). These are presumably for astrological purposes also. These tables then seem to support the fact that one of the main motives for predicting and modeling eclipses and accounting for their features was ultimately for astrological reasons.

The only topics which are left out here that are generally included as part of the Indian eclipse reckoning treatment are the colours of eclipses, and the graphical representation of the eclipse. This latter aspect, however, is covered in quite some detail in the accompanying text, as its subject matter is not readily amenable to a tabular format, being diagrammatic.

2.3. Contents of the Text

The text which is intended to accompany the tables is only twenty verses long and is an extremely condensed account of eclipse reckoning. The verses of text are arranged into two chapters, numbered 1–13 and 1–7 respectively, and are followed by a colophon. The colophon gives both the epoch of the tables (1681 AD) as well as the date in which this manuscript was copied (1762 AD). While the text does in some instances give a guide on how to use the tables, two verses give details on the author of the text, Bhāskara, son of Rāma,[8] another gives an aphoristic saying, and several are dedicated to expounding the rules for the graphical construction of an eclipse, something that (for obvious reasons) do not feature in the tables. So in this sense, the text functions both as a set of instructions for using the tables, but also as a supplement to the tables for those aspects of eclipse computation that the tables do not cover. An edition of the text and a thorough technical analysis is in preparation.[9]

3. Tables and Format

3.1. Tabular Layout

There are many different types of tables in the *Karaṇakesarī*, displaying a variety of organizing principles. However, several general observations about these tables can be made. Firstly, the tables are all single argument tables. That is, all of the tables represent a function with a single variable as their argument. However, in terms of layout, these single argument functions can be organised as a single-entry table (see, for instance, 3.1.1) or a double-entry table (see for instance 3.1.3).[10] Tables in Sanskrit source usually have a horizontal orientation; that is, the argument runs horizontally across the page and the entries are placed vertically underneath. This is most probably a result of the paper dimensions—Indian manuscripts tend to be wider than they are long; a feature which may predispose the scribe towards this horizontal table orientation. Some tables are to be used as a collection (see, for instance, the elongation tables in Fig. 4), some on their own, and some span over several pages.

Chapter	Verse Number	Topic(s)
1	1	Salutation to Kṛṣṇa
	2	Nodal Elongation
	3	Lunar Latitude and Diameters of sun, moon, and shadow
	4	Obscuration
	5	Beginning and End of Eclipses; Zenith Distance; *Akṣvalana*
	6	*Ayanavalana*
	7	Arcs
	8	The Depression
	9	Precession; the Half-length of Daylight
	10-13	Diagrammatic Representation of an Eclipse
2	1	General Saying
	2	Oblique Ascensions
	3-6	Parallax
	7	Conclusion: Reverence, Location and Family Details

Fig. 3. The contents of the text

Therefore, the tables of the *Karaṇakesarī* exhibit many sorts of layouts, each specific to the content and phenomena they contain. This suggests that to a certain extent, the compiler of the table (or perhaps the scribe) was mindful of tailoring the format of the table to the specific properties of the content. A few have been selected and described in more detail below.

3.1.1. A Single Entry Table: Elongation

F. 3r (see Fig. 4) contains a single table which occupies the entire page; this table gives the elongation[11] of the mean sun and the moon's node in 130-year intervals (for the technical details, see section 4). It is a single-argument (single entry) table which in oriented horizontally; the argument runs from left to right horizontally across the page and the corresponding entries are placed underneath also spanning left to right. Double horizontal lines demarcate the top and bottom of the table from the margins, and single lines separate argument from entry and successive rows of arguments and entries. It seems the table has been preruled, which is suggested by the surplus boxes at the end. The final columns remain empty, and there are three empty entries at the end of the table; from these, all but the final one have an argument though, despite the corresponding entry box being empty.

The argument runs from 1 to 132, and the corresponding entries, given in zodiacal signs, degrees, minutes, and seconds, are oriented vertically; seconds sit directly underneath minutes, which sit directly underneath degrees, underneath zodiacal signs. This vertical orientation is not a

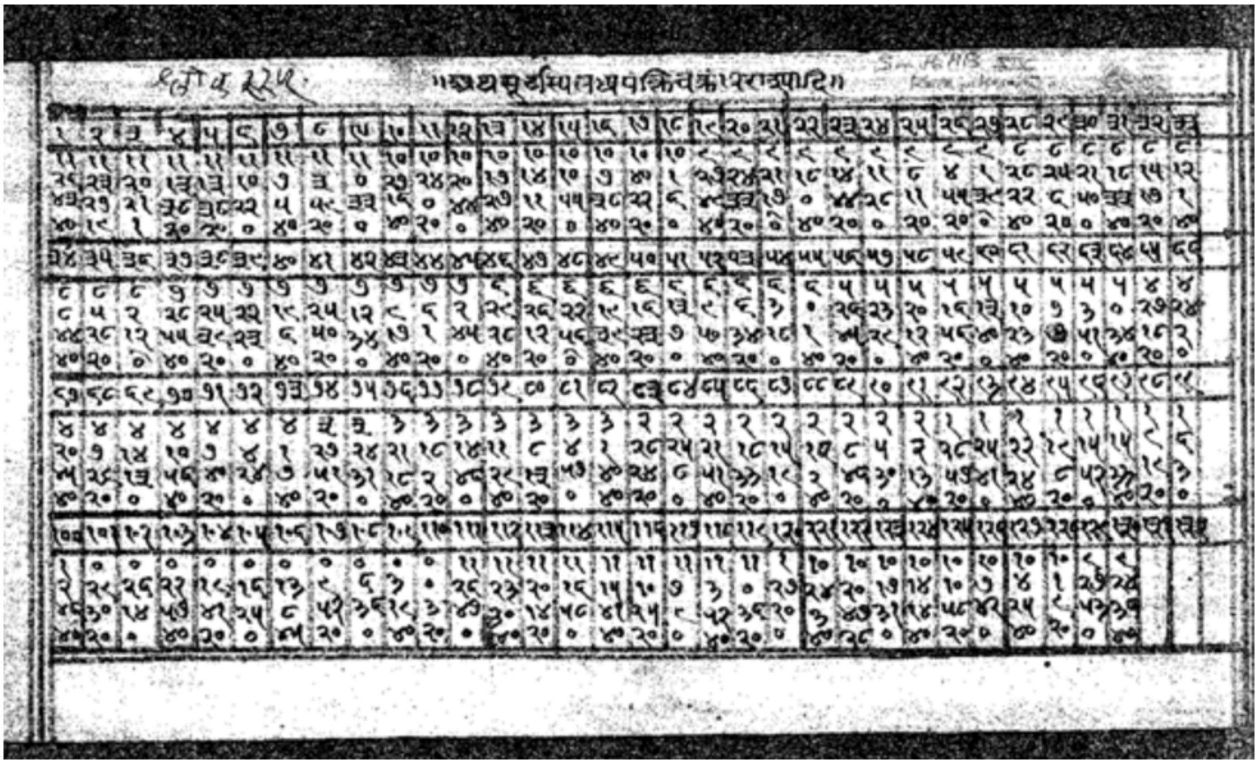

Fig.4. f.3r: Table for nodal-solar elongation over 130-year periods

typical way to express numbers, however it has its advantages. By a simple horizontal sweep one can gauge changes in the 'minutes' place, the 'seconds' place, and the like, as they are all aligned on the same horizontal line. This may have been advantageous for the purposes of checking the correctness of the entries. The first proper row contains entries for arguments 1 to 33, the second 34 to 66, and 3rd 67 to 99, and the last to 132. There appears to be no correlation between an entry in one row, and that directly below it.

There is a single title for this table which is justified in the middle. Neither the argument row nor the entry row are labeled, although the title reveals the contents of both, by specific use of a compounded preposition (for details see section 6). The table entries appear to have been checked (albeit imperfectly). For instance, the entry for argument 4 and 5 has been repeated (entry 4 is wrong). This has been noted by a thin line crossing out the numbers. There do exist other clear errors which have not been corrected: for instance the '19' in the second entry, the '45' in the 106th entry, and so on.

3.1.2. Single Entry Tables of Various Shapes and Sizes

In contrast, another page (f. 4v) from the work contains six tables (see Fig. 1). Here, each table has its own title, and some tables have individual headings labeling the rows. Three tables are included in the same ruled grid, but there have been several scribal errors with ruling to delimit each of these three tables, which again suggests (imperfect) pre-ruling. Explanatory text has been crammed in the spaces on the page. Several tables on this page have more than one entry for each argument.

Furthermore, two separate tables are closely related-table 9 for half-durations and table 11 of differences; the latter table is a table giving the differences between entries in the former.

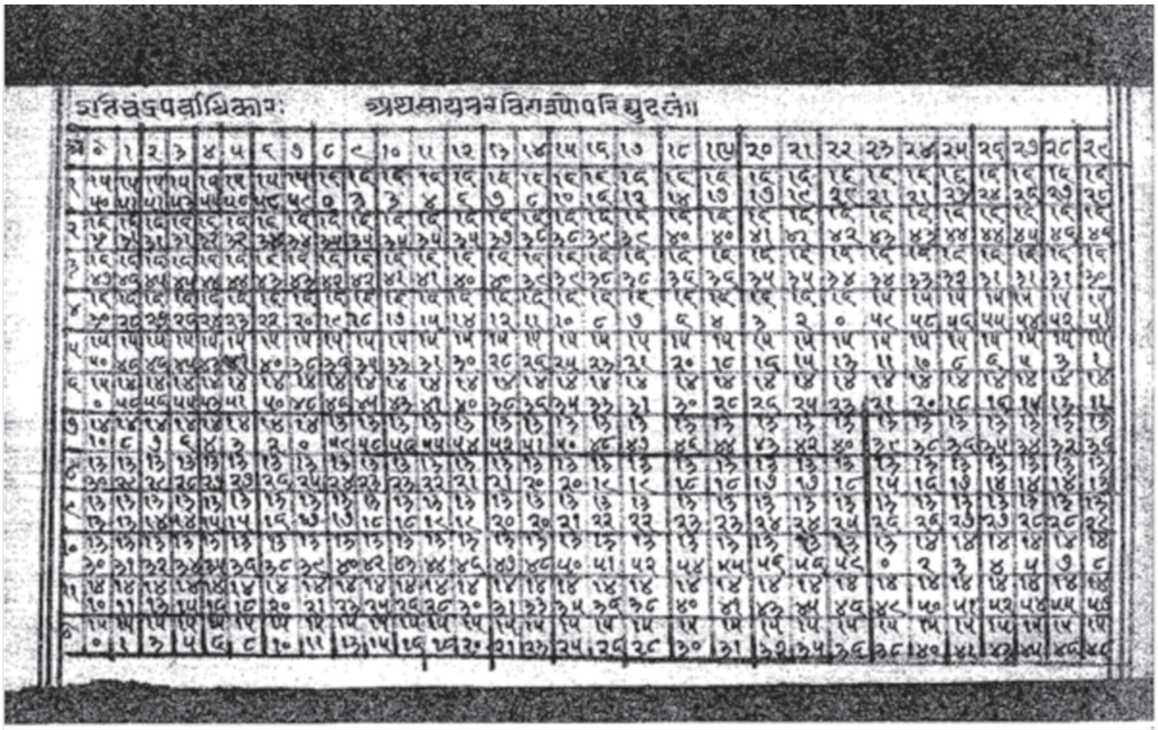

Fig. 5. f. 7r showing vertical-horizontal layout

However, these tables are not contiguous as one might expect, but rather are separated by another table (table 10: True longitudes). This seems to be an oversight on the part of the scribe, or perhaps in fact table 11 is merely an after-thought on their part, given the desire to fill the page. This ‘differences’ table provides no new information, it simply would assist in interpolating between non-tabulated values of the table of half-durations.

3.1.3. A Double Entry Table: Half-Length of Daylight

A strict horizontal orientation is not always used, as the table on f. 7r reveals (see Fig. 5). This page presents a table of the half-lengths of daylight for the position of the sun on the ecliptic. 360 entries are required, and rather than arrange this in several rows (as, for instance, has been done in the elongation tables, see Fig.4), one on top on the other, we see here a clever combination of the vertical and horizontal orientation for economy of space. Each of the 12 zodiacal signs form the vertical arrangement, and for each of these the 30 degrees have been arranged horizontally. One then searches for the desired entry by first moving vertically to the appropriate zodiacal sign, and then horizontally to the desired degree within this zodiacal sign. This is still a single argument table, however it shows a certain increased sophistication in layout.

3.1.4. Tabular Symmetries: Oblique Ascensions

One table has used various symmetry considerations to economise the layout. F. 8v is a table of rising times for each zodiacal sign, divided into sixty parts (see Fig. 6).

Rising times are a particular astronomical feature that due to the relative position of the zodiacal circle with respect to the equator are symmetrical for certain pairs of signs: the values associated with the first zodiacal sign are identical with the twelfth, the second with the eleventh, the third with the tenth, and so on. This has been taken

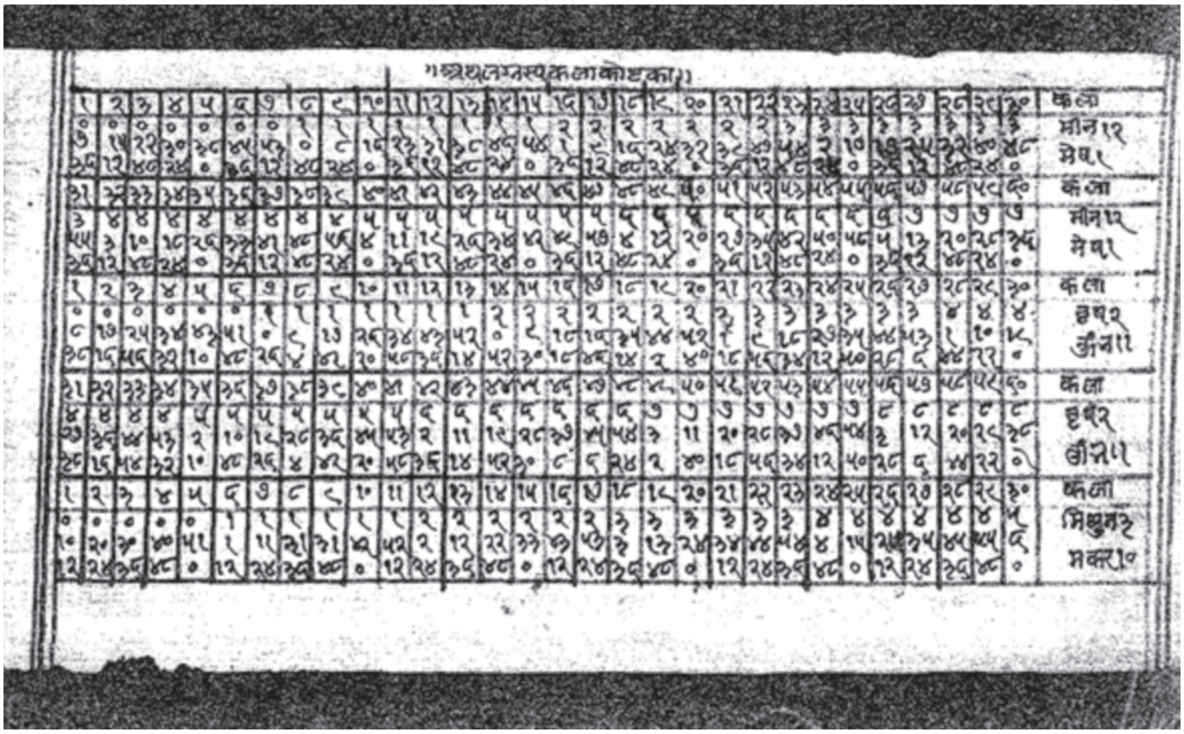

Fig. 6. f.8v showing tabular symmetry

into account when organising the table. Rather than including a separate section for each of the twelve zodiacal signs, six sections suffice, one for each appropriate pair. This is clearly indicated in the right hand side cells where each pair of signs have been named and numbered.

3.2. Tabular Paratext

As we have seen, the tables presented in the *Karaṇakesarī* are surrounded by accompanying text on the same page, as an interface between the users and the data. Such appendages, or paratext[12], are crucial for the general intelligibility of the technical content. In this context, they include titles, column and row headings, inter-tabular commentary, corrections, and accompanying text, which itself contains paratextual elements such as invocatory verses, authorship, dating, and scribal acknowledgement. All of these elements contribute essential and enhancing details to the tabular data. We focus here on the impact of the titles and the ways in which they function in the *Karaṇakesarī* as well as the ways in which columns have been labelled.

3.2.1. Titles of Tables

The titles of the tables in the *Karaṇakesarī* have a multi-purpose function, for they not only convey the content of the table, but they may also do any of the following as well: specify the astronomical context, identify the rows and the columns, identify the relevant units, or mention dependencies with other works, for instance. In this way titles function as the initial synopsis of the content of the table as well as a source of explanation and contextualisation. By its location, it is conspicuous; it is therefore the user's point of entry into the table.

We consider some examples below. We draw them from a single manuscript copy which was all that was available to us. We have made no attempt to emend these titles even though the

Sanskrit is irregular or incorrect. In some cases this means we have left obvious errors in the Sanskrit but in other cases discrepancies may reveal the influences of the author's vernacular. Our translations attempt to extract sense from difficult phrases using context.

The simplest sort of title clearly gives what the table computes and the units in which it is computed. For instance,

> *atha lagnasya kalākoṣṭhakā*
> Here, the tabular entries of minutes of rising times
> *Karaṇakesarī* (f. 8v)

However, there is no standardisation of approach when composing a title for a table in the *Karaṇakesarī*. To highlight this lack of standardisation, consider a trio of titles which head three separate tables that tabulate the same astronomical amount, only at different intervals (the first at 130-year intervals, the second at 1-year intervals, and the third at 14-day (*avadhi*) intervals; these are the tables of elongation between the mean sun and the lunar node, as discussed in section 4). The titles for each of these tables are formulated quite differently. Each of the titles, with a translation, is as follows:

> *atha suryasya labdhapaṃkticakraṃ 12 rāśyādi*
> Here, the cycle of the collection of the quotient (relating to the elongation between the lunar node and) of the sun; (expressed as) 12 zodiacal signs and their subdivisions.
> f. 3r
>
> *atha śrīkaraṇakeśarigraṃthe sūryasya śeṣapaṃkticakraṃ 12 | 30*
> Here, in the book of the *Karaṇakesarī*, the cycle of the collection of the remainder (relating to the elongation between the lunar node and) of the sun; (expressed as) 12 (zodiacal signs) 30 (degrees and their sub-divisions).
> f. 3v
>
> *atha karaṇakesarigrathokte siddhāntarahasye sūryedvoḥ parvanayanārthe caṃdrasya koṣṭakāvadhyopari* ||
> Here, in the *Siddhāntarahasya* quoted in the book of the *Karaṇakesarī*, for the sake of the computation of an eclipse of the sun and the moon, a table of the moon (tabulated) with *avadhi*s above.
> f. 4r

The first table deals with the elongation between the mean sun and the lunar node in 130-year increments. Note the information that the title leaves out; it does not mention elongation, nor the lunar node, nor the fact that the increments are in 130-year periods. This is only implicit from the construction of the table and the data contained therein.

The title of the second table is a little more descriptive, but not much more. In addition to the information given as in the first title, it gives the title of the work that the table appears in. One important distinction between these two tables is the reference in the first to *labdham* (quotient) and in the second to *śeṣa* (remainder). This is an indirect reference to the process referred to in the verse which accompanies this table in the text (*Karaṇakesarī* verse 2; see section 4.2), in which the relevant look-up entry is determined by subtracting the epoch year from the current year, dividing by 130, and considering the resulting quotient (the number associated with the complete number of 130-year cycles elapsed since the epoch) and the remainder (the number of single years left over).

The third title is strikingly different. Firstly this title refers to a source called the *Siddhāntarahasya*. This is in fact a reference to a work of that name written by Gaṇeśa in the early sixteenth century, better known by the name, the *Grahalāghava*. This suggests a parameter dependency. Secondly, the title reaffirms the astronomical context, citing this is for the sake of computing an eclipse of the sun and the moon. What is interesting is the use of the proposition *upari* in this context. Here it has been used in a compound as a post-positive, that is, it modifies the noun which precedes it. *Upari* conveys the sense of 'over', 'above', 'upon'. Here, in the tabular context it refers to the argument of tabulation. In this sense the argument is 'above' the entries. This is used in several other instances throughout the table with the same meaning. Notably critical information is missing from this table, including the fact that it appears that the true motion of the sun has been used here to compute the elongation, the fact that the changing daily velocity (at each 14-day interval) has been included as another row, as well as a third row of 'corrections'.

Another title reveals ways in which tables with multiple rows were specified. For instance, on f. 4v (see Fig. 1) there appears a table to compute the apparent lunar diameter and of the shadow of the earth with respect to the length of a *tithi*. The title reads:

> *atha tither mānaghaṭyopari caṃ drabiṃbaṃ tathābhūbhāṃ gulādi* || Here, the (apparent) diameter of the moon with the measure of the *ghaṭika*s (in one day) above, and then the (shadow) of the earth (measured) in *aṅgula*s and so on.
> f. 4v

Here, we see the two particles *atha* and *tathā* which indicate this table has two distinct rows of data tabulated; *atha* gives the first tabulated row of entries (apparent diameter of the moon); *tathā* gives the second row (the diameter of the shadow of the earth). *Upari* used here has been compounded in a distinctive way and is to be understood in the context of tables as referring to the argument of tabulation, i.e., that (row of data) which is 'above' the others. This is used frequently throughout the tables of the *Karaṇakesarī* as well as the text. It is also used in other astronomical tables, for instance, Nāgadatta's *Brahmatulyasāraṇī* [Pingree, 1968].

Therefore, many of the titles in the *Karaṇakesarī* seem more like instructions for how to undertake the astronomical procedure with the data the table provides. Often these titles can be quite long and there is no standard practice to which all titles conform.

3.2.2. Labelling the Rows

An important part of working with tables is knowing what the tables contain. This is in part done by the titles, however another way in which this can be achieved is via labelling of the rows. Labelling of this sort has been done rather inconsistently in the *Karaṇakesarī*. Some arguments and entries are clearly labelled. For instance, the table of the lords of the eclipses possibilities (see Fig. 8) labels the argument '*rāśayaḥ* ' (Numbers) and the entries '*Parveśāḥ*' ((Lords). Some columns are not labelled, despite the fact that spaces are left (see Fig. 7).

As the previous section showed, sometimes the rows are identified in the title paratext explicitly, using the word *upari* to distinguish between the two spatially.

4. Case Study: Elongation Tables (Ff. 3–4)

4.1. Will an Eclipse be Possible? Computing the Elongation.

Lunar and Solar eclipses are a result of a precise alignment of the three heavenly bodies:

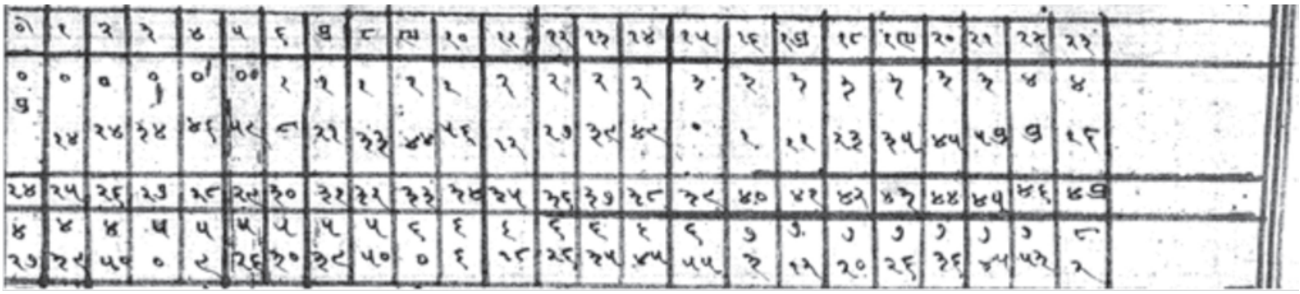

Fig. 7. f.6v showing no row headings despite spaces left.

the earth, the moon, and the sun. In the case of a lunar eclipse, the sun, the earth, and the moon are aligned, and in that relative order; the earth passes between the sun and the moon, thus depriving the moon of the sun's rays, and creates a shadowing effect. In the case of a solar eclipse, the sun, the moon, and the earth are aligned, and in that order; the moon passes between the sun and the earth, hence depriving the observer on earth of the sun's light. As the sun and the moon travel on orbits which are inclined with respect to one another, alignment can only occur when these orbits intersect (or close to it). These points are called (lunar) nodes. The first and most critical piece of information one needs when determining an eclipse possibility is therefore the relative positions of the sun and the moon with respect to each other and with respect to the earth and the associated nodal elongation. This is purpose of the first three tables of the *Karaṇakesarī*.

The tables are based on the basic period relation which invokes the ratio of the revolutions of the Moon's node over a specific time interval. According to one dominant school in the Indian astronomical tradition, the *Brahmapakṣa* school.[13], the Moon's node makes -232,311,168[14] revolutions in 4,320,000,000 years,[15] which gives a mean yearly motion:

$$\frac{-232{,}311{,}168}{4{,}320{,}000{,}000} = -19;21,33,21,\ldots/\,year$$

There are three tables that collectively can be used to determine the nodal elongation on the beginning of any 14-day period (referred to as an *avadhi*). Two of them are based on this mean parameter. The third is based on variable solar velocity.

The first computes the elongation in zodiacal signs and so forth in increments of 130 years. This table has 130 entries. The second computes the elongation in zodiacal signs and so forth in increments of 1 year. This table has 130 entries also. The third, and final table, computes the elongation in 27 *avadhi* or 14-day periods. A quick calculation reveals that these tables in fact cover 130 × 130 or 16,900 years, making them extend to the year 18,581AD! The first table begins at the beginning of the zodiac. The second table begins at 6,29;24,36, which is the epoch elongation for these tables, that is, the elongation in 1681. This allows us to establish the mean elongation for the year we are looking for. Successive entries in this table differ too by a constant amount: the mean yearly nodal motion of -19;21 as derived above.

The third table allows us to compute the elongation where we are in any given year. It does this by dividing up the year into 27 *avadhis* or 14-day periods. This is particularly useful for eclipses as conjunction and opposition occur roughly 2 weeks apart. These entries do not have a constant difference, but rather differences which vary sinusoidally. Another feature which is distinctly different from the other two tables is the structure. Under each of the 27 entries are three rows of distinct data. The first gives the elongation, the second, labelled *gatayaḥ* (lit. daily motions), gives the true velocity of the sun presumably for the beginning of each *avadhi*—with a minimum around 60,5 and maximum 64,59, and the third, which is unlabelled, has various symbols underneath–1's and 2's and markings to indicate positive and negative–which don't appear to be in any recognisable pattern or relation. It is conceivable that this row was intended to be some sort of correction to the solar velocity to account for its variation on a daily basis.

4.2. Using the Tables: Text and Table Interact

There is a verse contained in the text which instructs the user how to compute the elongation via these three tables. One verse suffices (1, 2):

> *śako rāmaviṣṇupadāṅgendu 1603 hīno*
> *viyadrāmacandrair hṛto 130 labdhaśeṣau*
> *yutau śeṣakarṇaṃ hy avadhyanvitaṃ tat-*
> *sapātendutātkāliko bāhubhāgaiḥ ||2||*
>
> The śaka year diminished by 1603 (and) divided by 130. (The tabular entries corresponding to the amounts given by) the quotient and the remainder are added (together). The 'result' is increased (by the tabular amount corresponding to the number of elapsed) *avadhis*. That (resulting sum is) 'the elongation' at that time.[16]

The procedure described can be explained as follows. Take the current (śaka) year and subtract 1603 from it. This gives the number of years elapsed since the beginning of the epoch of the tables. Divide by the resulting difference by 130 to extract the multiples of 130-year periods. The remainder will be the number of single years. With the first amount (the 'quotient', *labdha*), enter the table to find the elongation after the required number of 130-year periods. With the second (the remainder, *śeṣa*), enter the second group of tables to determine the elongation after the required number of single years. The two retrieved tabular entries are then to be added together. This amount represents the nodal-solar elongation at the beginning of the current year. The third table then incorporates the change in elongation since the beginning of the current year, presumably up to the nearest 14-day interval (as opposition or conjunction occur 14 days apart), although the text is not explicit about this.

What is notable about this passage is its brevity, although this is characteristic for Sanskrit mathematical works which are composed in verse. Despite this, much information has been conveyed, and more still can be directly inferred from context. The epoch of the tables (*śaka* 1603) is made explicit, and the existence of three tables can be inferred from the operations described. The cumulative effect of the trio of tables is made clear. However, any reference to the tables themselves and the corresponding manual manipulations required to extract the correct parameters is absent. The author has assumed that these details and the related look-up tasks are clear from the context.

5. Case Study: Aligned Table, Versified Data, Algorithm. Determining the 'Lord of the Eclipse'

Aligned tables offer a distinct way of presenting data which alter the ways in which the users are compelled to interact with technical content. To explore this point in this context we will consider an aligned table from the *Karaṇakesarī*, and some expositions of the same content in a non-aligned format. This will allow us to explore some of the impacts of these presentation-styles on the user of the table, as well as the author.

One table contained in the *Karaṇakesarī* whose purpose is primarily astrological is the so-called 'table of lords of the eclipse'17 (see Fig. 8). The lords of eclipses (*parveśa*), refer to a traditional practice in which each eclipse possibility (*parvan*) is assigned a particular deity which in turn imbues the possible eclipse with portentous significance. Traditionally there are

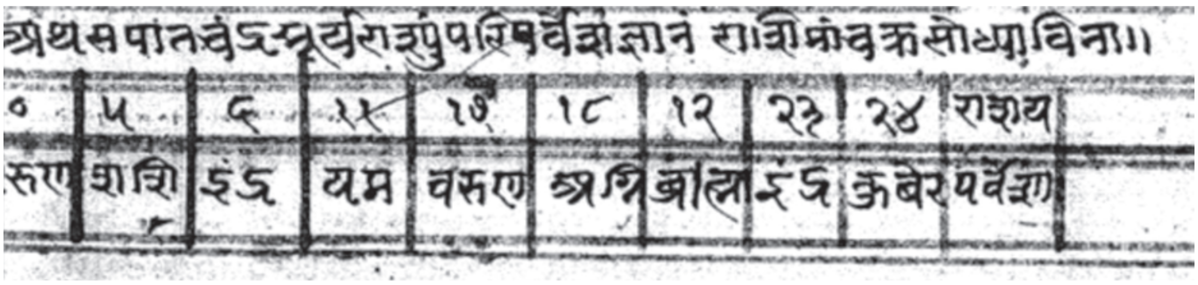

Fig. 8. F.5v Table of the Lords of the Eclipse

The table can be transcribed and translated as follows:

0	5	6	11	17	18	12	23	24	Numbers
Varuṇa	Śaśi	Indra	Yama	Varuṇa	Agni	Brāhma	Indra	Kubera	Lords of the Eclipse

seven such lords and these associations indicate a different effect, such as the prosperity of cattle, good crops, drought, famine, destruction of crops, trouble for the king, and so forth.

Basic lunar eclipse intervals entail that eclipse possibilities occur almost every six months but sometimes five. Therefore in the above scheme 'lords' have been associated to potential eclipses for every possibility which fall under multiples of six,[18] but also $6k - 1$ intervals (i.e., 5, 11, 17, and 23), to cover the 5-month interval possibility. The seven lords invoked here are (in no particular order) Varuṇa, Śaśi, Indra, Yama, Agni, Brāhma, and Kubera, and the exact pattern (if there exists one) in which each lord has been associated to a particular eclipse possibility is unclear.

One of the earliest instances of this scheme was enunciated by Varāhamihira in the early fifth century, who lists the lords in a prescribed order and alludes to the rule to assign them, as follows:

> *ṣaṇmāsottaravṛddhyā parveśāḥ saptadevatāḥ kramaśaḥ* |
>
> *brahmaśaśīndukuverā varuṇāgniyamāś ca vijñeyāḥ* ||
>
> The seven deities, in order, Brahma, Śaśi, Indra, Kubera, Varuṇa, Agni, and Yama are to be known as the lords of the Parvans by the increase of the difference of six months.
>
> Varāhamihira *Bṛhat-Saṃhitā* V,[19]

There are several important contrasts with this exposition and the one in the *Karaṇakesarī*. First of all Varāhamihira's account is in a versifed form; the *Karaṇakesarī* in aligned tabular form. Varāhamihira explicitly gives a rule ('in- crease of the difference of six months'), whereas in the table these increments are implicit. Varāhamihira's rule is the more basic (6 month intervals only), the *Karaṇakesarī* is slightly more nuanced (combinations of five and six month intervals). From the point of view of the user, the aligned table has precomputed both the argument and the entry for them; in this way the user is relieved from engaging with the rule. They simply need to 'look up' the information set out for them, and then, need only concern themselves with the relevant entry, and none of the others. However, the user who is following Varāhamihira's scheme must reconstruct the scheme for themselves. They must individually articulate the six- month intervals, and reciting the seven deities given in verse in order, assign each one to the eclipse possibilities. Because the content is encoded in verse, they are compelled to begin with the circumstances for the first eclipse possibility and then cycle through them (at least) until they reach the one desired. In this way they must negotiate their way through the whole scheme for a single entry, something they can avoid when they consult the table. The difference is perhaps negligible for this particular example, but may entail a huge reduction in computation in more complex schemes.

To highlight this contrast even more, consider a mathematical algorithm given to compute the eclipse possibility and its corresponding lord. Take for instance, the exposition given by Āmarāja, a twelfth century commentator on Brahmagupta's *Khaṇḍakhādyaka* (665 AD). Āmarāja cites a rule from an earlier commentary by the tenth-century commentator Bhaṭṭopala on the same work:

> *dinavṛndāt khaśara 50 ghnāt ṣaṇ-navaravi 1296 bhir vibhājitād āptam |*
> *rāmarttukhendu 1063 sahitaṃ kṣiped dyuvṛ nde bhajet khadhṛ ti- 180 bhi-stat||*
> *labdhaḥ kamalajapūrvaḥ parvagaṇ aḥ saptabhājite śeṣaḥ |*
> *gatagamye tithyū 15 ne khaguṇ o 30 ne candrasūryaparva syāt ||*

> One should add the quotient, (produced) from the number of days multiplied by 50 and divided by 1296, increased by 1063, to the number of days. One should divide this by 180. The result is the quantity of the *parvan* beginning with Brahma. When what has passed or what is to come is divided by 7, then the remainder if it is less than 15 or less than 30, is the parvan of the Moon or Sun (respectively).
>
> *Khaṇḍakhādyaka*, p. 145 ll. 17–20

To summarise, the scheme works as follows. Given the number of days since the epoch (a), the eclipse possibility (EP) is computed as follows:

$$\text{The current EP + a remainder} = \frac{50a + 1063}{1296}$$

where the exact numbers in the numerator and denominator are a consequence of the foundational parameters of Brahmagupta's astronomical system. Again, the versified format has some consequences for the user. First, the user must compute the eclipse possibility themselves, using the algorithm given. Second, they must recall the the order of the lords (here prompted by the cue 'beginning with Brahma'). Presumably they must recall to mind another verse which lists them in versified form, thus having to cycle through the whole order, rather than 'look-up' the relevant one, which they can do in the aligned table format. This is indicative of the verse format in general. For given the predominantly oral context in which this verse is to be recalled, users must recite the whole verse from the beginning to extract the particular datum they require.

The implications of these contrasts in format are somewhat profound. Among the many differences, perhaps the most important is the level of interaction required from the user with the various formats. Astronomical or astrological data presented in precomputed aligned tables minimises the engagement the user must have in the computation themselves. In fact, they can avoid any knowledge of the way in which these numbers were derived, as well as any theoretical or cosmological suppositions, and need only concern themselves with the information pertaining to the relevant entry at their convenience. This is not entirely the case for formats which describe the system in verse. Here, the user is required to recompute the scheme, and where appropriate, reconstruct it in its entirety themselves (or up until the entry they require at the very least) simply for a single entry. In this way, these formats are demanding on the user. Aligned tables on the other hand, require in this sense far less effort on the part of the user.

6. Concluding Remarks

Sources reveal that Sanskrit scholars commonly included tabulated data within their works, however listing this data in a versified format seems to have been standard practice and persisted for over a millennium. Extant sources reveal that the practice of presenting data with recourse to spatial alignment, that is, in rows and columns, didn't become common until the early second millennium, and was a practice that seems to have predominated in North and Western India [4, p. 41–46]. By the time the *Karaṇakesarī* appeared, this format was a popular medium and well developed in the Indian tradition. Our survey of selected aspects of the tables and accompanying text of the *Karaṇakesarī* reveal some of the features of this mode of presentation.

Indeed, the intellectual circumstances in India had some appreciable effects on the storage and delivery of information and the significance of tabular alignments for this is only now being examined. The themes explored here in the context of the *Karaṇakesarī* is just a beginning. Further studies will help us to understand better how the ambient intellectual circumstances influenced the arrangement and presentation of data in the Indian tradition. Detailing the aspects and features of these sorts of tables and determining the impact they had on the astral sciences are very much in their preliminary stages. As research begins to probe and account for the aspects of this genre of presentation, we will be increasingly in a position to understand the manner in which this medium affected both the data being presented and the practitioners using it, in nuanced and direct ways, and the appreciable epistemological effects that these works had on broader scientific activity.

NOTES

1. It has been argued that the prominence of the mathematical table in Sanskrit astronomy was linked to Islamic influences ([Plofker (2009), pp. 274–277], [Pingree (1981), pp. 41–46]) particularly through the popularity of the *Zīj* compositions, works which contained mathematical tables and accompanying explanatory texts.
2. See [Pingree (1981), p. 328] and [Pingree (1968), p. 70]. Poleman 4946 (Smith Indic MB) XIV ff. 3–11 which has 2ff duplicated (f. 7v-9v) in Poleman 4946 (Smith India MB) XXVII. Paleographic similarities suggest that these were copied by the same scribe.
3. This is from the Baroda Central Library, Baroda 11268 which covers 2 folia.
4. See the relevant entry in [Pingree (1981), p. 328] for a comprehensive list.
5. Poleman 4946 (Smith India MB) XXVII 2ff duplicates the material on f. 7v-9v of Poleman 4946 (Smith Indic MB) XIV.
6. In some marginal notes there are occasional irregularities in the language, suggesting the influence of a vernacular. For instance, the header material on f. 5r includes: *sapātacaṃdratuṃjapadeho yato dhanakījaite mokṣabāṇathāye ṛ ṇ akījaite spārśabāṇ-athā i jo yugma pade sapātaho yato mad- hyaśaramāṃ dhanakījaite sparśabāṇathā i ane ṛṇa kījai to mokṣathā i*
7. For further details of the *valana* or angle of inclination, see [Montelle (2011)].
8. We learn that he is from the *Kavīndra* family (*kula*), he is a '*moḍ ha*' (Brahmana group), and a member of the *Aupamanyava* gotra.
9. *Karaṇakesarī of Bhāskara: a 17th century table text for computing eclipses* by C. Montelle and K. Plofker, forthcoming.
10. This latter layout is not to be confused with a double-argument table which tabulates a function with two independent variables.
11. The elongation refers to the relative distance between these two objects. When this is small enough, an eclipse is possible.
12. [Genette (1987)] first introduced this term in a broader literary context to refer to the elements which accompany a text, such as title, preface, illustrations, all of which are crucial elements of the work. This term is aptly suited in this context, as it naturally captures important elements in the 'tabular' environment
13. For details on the main *pakṣa*s and their various parameters, see [Pingree (2008)].
14. The negative sign indicates that this motion is retrograde.
15. This is a traditional chronological division of time.
16. The last compound of this verse *bāhubhāgaiḥ* belongs with the next verse.

17. The full title of this table given by our manuscript is *prathamapātacaṃ drasūryarāśyupariparveśajn˜ānaṃ rāśimāṃ cakrasodhyāvinā* ||

18. There is an obvious scribal error, with the entry for '12' being out of place. Whether or not Brāhma should also be moved as well it is difficult to establish, as correlations that are made for astrological significance are not always logically reconstructable.

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