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MAKARANDA SĀRIṆĪ AND ALLIED *SAURAPAKṢA* TABLES – A STUDY

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Compilers of annual calendrical–cum-astronomical almanacs (*Pañcāṅgas*) depend on traditional astronomical tables called differently as *sāriṇī*, *padakas*, *vākyas* and *koṣṭhakas*. There are a large number of such tables belonging to different schools (*pakṣas*) like *Saura*, *Ārya*, *Brāhma* and *Gaṇeśa*. Among the *Saurapakṣa* tables *Makaranda sāriṇī* (*MKS*) is the prominent and the most popular one. It is composed, by Makaranda, son of Ānanda at Kāśī (Vāraṇāsī, Benares) in 1478 AD. In the present paper we discuss some features of not only the *Makaranda Sāriṇī* but also of the lesser and locally used *Tyāgarti* manuscript and the *Pratibhāgī padakas*, all belonging to the *Saurapakṣa*. A comparison of parameters in these tables among themselves as also with those of another *pakṣa* is attempted. Procedures for eclipses and lunar parallax are essayed with examples.

Key words: *Makaranda sāriṇī*, *Padakas*, Indian astronomical tables, *Saurapakṣa*, *Sūryasiddhānta*, *Pratibhāgī padakas*, *Tyāgarti*.

1. INTRODUCTION

The *Makaranda sāriṇī* (*MKS*) is a popular Sanskrit text containing a large number of calendrical and astronomical tables based on the popular *siddhāntic* treatise *Sūryasiddhānta* (*SS*). These tables are worked out with immense effort by Makaranda, son of Ānanda at Kāśī. At the commencement of the text this fact is mentioned following the author's salutations to lord Gaṇeśa and goddess Sarsvati, the deities of learning and knowledge[1]:

śrī gaṇeśāyanamaḥ śrī sarasvatyainamaḥ|

atha makaranda sāriṇī likhyate ||

śrī sūryasiddhāntamatena samyag viśvopakārāya guruprasādāt |

tithyādi patraṃ vitanotikāśyāmānan-dakando makaranda nāmā ||

– *MKS, Śl.1*

"Prostrations to *Śrī Gaṇeśa* and *Śrī Sarasvatī*.

Now Ānanda's son by name Makaranda, brings forth at Kāśī by the blessings of the preceptor (*guru*), folios of *tithi* etc., based on the *Sūryasiddhānta* school of thought, properly for the benefit of the world".

The major tables in *MKS* are for (i) the ending moments of *tithi*, and *yoga*, (ii) the mean longitudes of the Sun, the Moon and the five *tārāgrahas* viz, Kuja (Mars), Budha (Mercury), Guru (Jupiter), Śukra (Venus) and Śani (Saturn), (iii) the *mandaphala* (equation of the centre) of each of the heavenly bodies, (iv) the (equation of the conjunction) of the five planets, (v) the moments of solar ingress (*saṅkarmaṇa*) into the *rāśis* (zodiacal signs) and *nakṣatras* (the twenty-seven asterisms), (vi) the Sun's declination (*krānti*), (vii) the latitude (*śara*, *vikṣepa*) of the

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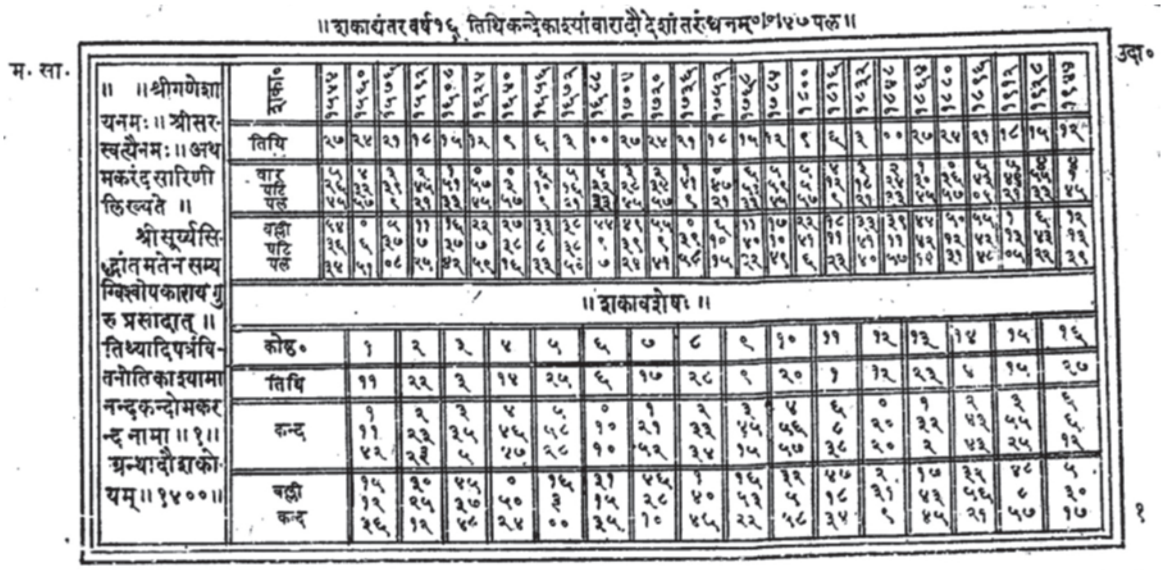

॥शाकाद्यंतरवर्ष१६ तिथिकन्देकाद्यांवारादौ देशांतरंधनम्०।१४७पल॥

म॰ सा॰ — उदा॰

॥ ॥श्रीगणेशाय नमः॥ श्रीसरस्वत्यैनमः॥ अथ मकरंदसारिणी लिख्यते ॥ श्रीसूर्य्यसिद्धांतमतेन सम्यग्विश्वोपकाराय गुरुप्रसादात्॥ तिथ्यादिपत्रंवितनोति काश्यामानन्दकन्दो मकरन्द नामा॥१॥ ग्रन्थादौ शाकोयम्॥१४००॥

शाकाः	१५४४	१५६०	१५७६	१५९२	१६०८	१६२४	१६४०	१६५६	१६७२	१६८८	१७०४	१७२०	१७३६	१७५२	१७६८	१७८४	१८००	१८१६	१८३२	१८४८	१८६४	१८८०	१८९६	१९१२	१९२८	१९४४
तिथि	२७	२४	२१	१८	१५	१२	९	६	३	००	२७	२४	२१	१८	१५	१२	९	६	३	००	२७	२४	२१	१८	१५	१२
वार घटि पल	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
वल्ली घटि पल	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

॥शाकावशेषः॥

कोष्ठ॰	१	२	३	४	५	६	७	८	९	१०	११	१२	१३	१४	१५	१६
तिथि	११	२२	३	१४	२५	६	१७	२८	९	२०	१	१२	२३	४	१५	२७
कन्द	१ ११ ४२	२ २२ २३	३ ३५ ५	४ ४६ ४७	५ ५८ २८	० १० १०	१ २१ ५२	२ ३३ ३४	३ ४५ १५	४ ५६ ५७	६ ८ ३८	० २० २०	१ ३२ २	२ ४३ ४३	३ ५५ २५	६ ६ १२
वल्ली कन्द	१५ १२ २६	३० २५ १२	४५ ३७ ५८	० ५० २४	१६ ३ ००	३१ १५ ३५	४६ २८ १०	१ ४० ४५	१६ ५३ २२	३२ ५ ५८	४७ १८ ३४	२ ३१ ९	१७ ४३ ४५	३२ ५६ २१	४८ ८ ५७	५ ३० १७

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Fig. 2.1: *Thithikanda* for *śaka* (16 yrs.int.) and *śakāvaśeṣa* (see Tables 2.1, 2.2), a folio from *MKS*

Moon and (viii) angular diameters (*bimba*) of the Sun, the Moon and the earth's shadow-cone (*bhūcchāyā*, *bhūrbhā*) for computing lunar and solar eclipses.

David Pingree has provided a detailed description of *MKS* with his learned critical comments, in his extremely useful and exhaustive two catalogues: *Sanskrit Astronomical Tables in the United States* (SATIUS)[2] and *Sanskrit Astronomical Tables in England* (SATE)[3].

In the chapter " *Tithyādi sādhanādhikāra*" (obtaining *tithi* etc.), under the *tithikanda*, the ending moments of *tithis* (one thirtieth of lunar month) at the beginning of solar years are given for intervals of 16 years starting with the *gata* (elapsed) year of the *Śālivāhana śaka* 1544 (i.e. 1622-23 AD). This table is followed by the *tithi* parameter for each year of the interval. Similar pattern is followed for *nakṣatra* and *yoga*.

Makaranda has made quite a few innovations in the procedures for planetary positions and eclipses. In order to elucidate the procedures of *MKS*, the famous commentator Viśvanātha Daivajña composed the very useful commentary with a large number of examples in *Śā. śaka* 1540 (1618 AD). Prior to that Divākara had composed the explanatory commentary,. In *śaka* 1688 (1766 AD) Gokulanātha Daivajña wrote the *Upapatti* (derivations and rationales) for *MKS*. For further elucidation of the text Daivajña Nārayaṇa Śarmā, published his *Makaranda prakāśa*[4] in *śaka* 1831 (1909 AD). All this shows how popular *MKS* is among the *Pañcāṅga* makers, especially the followers of the *Saurapakṣa*.

2. Obtaining *tithi kandas* (*Tithi Kandānayanam*)

The word *kanda* in Sanskrit literally means "root" (of a tree or plant) and the word *vallī* means creeper. For finding the *tithi* details at the beginning of a given *Śālinvāhana śaka* year, *MKS* gives the *kanda* and *vallī* comprising the weekday number (*vāra saṅkhya*) and the time in *daṇḍa* and *palas* (also called *ghaṭī* and *vighaṭī* or *nāḍī* and *vināḍī*).

In Table 2.1 that follows, the first topmost row contains the *Śā. Śa* years with an interval of 16 years, starting with 1544 (1622 A D) and continuing with 1560, 1576, …. upto 1944 (2022 A D). Incidentally, while the text says that the commencing year is *śaka* 1400 (1478 AD), the table for *tithikanda*, in the published version of

MKS, starts from 1544(1622 AD). This may be because the published work is based on Viśvanātha's manuscript composed during the first quarter of 17th century. In the second row *tithi* numbers 27, 24, 21, …. are given. The third row contains the *vāra* (weekday number) *ghaṭī* and *palas*. The fourth and the last row has *vallī* followed again by *ghaṭī* and *palas*.

Note: 1 *ghaṭī* = 1 *nāḍī* = 1 *daṇḍa* = 60 *palas* = 24 minutes
1 *vighaṭī* = 1 *vināḍī* = 1 *pala* = 24 seconds

Table 2.1: *Tithikanda* for *śaka* years (16 yrs. interval)

Śaka	1544	1560	…..	1880	…..	1944
Tithi	27	24	…..	24	…..	12
Vāra	5	4	…..	0	…..	4
Ghaṭī	26	32		36		1
Pala	45	57		57		45
Vallī	54	0	…..	50	…..	12
Ghaṭī	36	5				
Pala	34	51		31		39

Table 2.2: *Śakāvaśeṣa tithikanda*

Koṣṭhaka	1	2	3	….	11	….	16
Tithi	11	22	3	….	1	….	27
Kanda	1	2	3		6		6
	11	23	35	….	8	….	6
	42	23	5		38		12
Vallī Kanda	15	30	45		47		5
	12	25	37	….	18	….	30
	36	12	48		34		17

In Table 2.2, the *tithi* and the corresponding *vārādikanda* and *vallī* are given for each year of the sixteen years interval used in Table 2.1. *Śakāvaśeṣa* means the remainder when the *śaka* interval is divided by 16. We have to take note of the following points:

(i) In Table 2.1, the *tithyādi* for successive *śaka* years with 16 years interval from the epoch is obtained by subtracting the following *tithyādi* from the preceding one:

Tithi	*Vāra*	*Ghaṭi*	*Pala*
3	0	53	48

Example 2.1

Śaka	*Tithi*	*Vāra*	*Ghaṭi*	*Pala*
1544	27	5	26	45
Subtract	3	0	53	48
1560	24	4	32	57

Note: While adding or subtracting, a cycle of 30 *tithis* (one lunar month), *vāra* cycle of 7 weekdays, each weekday of 60 *ghaṭīs* and each *ghaṭī* of 60 *palas* are used.

(ii) The *vallī* under the *tithyādi* for any tabulated *śaka* year is obtained by adding 5|30|17 (*vallī, gh., palas*) to the corresponding previous entry (of 16 years interval).

Example 2.2

Śaka	*Vallī*	*Ghaṭi*	*Palas*
1544	54	36	34
Add	5	30	17
1560	0	06	51

Note: For *vallī* a cycle of length 60 is used. Therefore, if addition of corresponding *vallī* exceeds 60, then the nearest multiple of 60 must be removed.

3. Obtaining *Nakṣatrakandas*

The *nakṣatrādi* (i.e. *nakṣatra, vāra, ghaṭī, palas*) for each tabulated *śaka* entry of 24 years interval is obtained by adding 23 *nak.*, 2 *dina*, 12 *gh.* 35 *palas* to the previous entry. Here, *dina* means a day to be added to the weekday number.

Table 3.1: *Nakṣatrakanda* for *śaka* years (24 yrs. interval)

Śaka	1592	1616	1640	….	1976
Nakṣatra	16	12	8	….	6
Vāra	2	4	6		2
Ghaṭī	34	47	59	….	56

contd...

Pala	49	24	59		8
Vallī	10	19	27		24
Ghaṭī	40	3	25	….	42
Pala	33	9	45		9

Table 3.2: *Śakāvaśeṣa nakṣatrakanda* for each year of *śaka* (24 yrs. interval)

Koṣṭhaka	1	2	3	….	24
Nakṣatra	10	20	3	….	23
Vāra	1	2	3		2
Ghaṭī	18	36	54	….	12
Pala	3	6	10		35
Vallī	15	30	46		8
Ghaṭī	26	52	19	….	22
Pala	27	54	21		36

Table 3.2 gives the *nakṣatra* and *vallī* for each of the years of the twenty-four years interval used in Table 3.1.

Example 3.1

Śaka	*Nak.*	*Vāra*	*Ghaṭi*	*Pala*
1592	16	2	34	49
Add	23	2	12	35
1616	12	4	47	24

Note: For *nakṣatra* a cycle of 27 *nakṣatras* is used. The zodiac of 360° is divided into 27 *nakṣatras* of 13°20′ angular range each.

The *vallī* under the *nakṣatrādi* for any tabulated *śaka* year is obtained by adding 8\|22\|36 to the corresponding *vallī*, *gh*. and *palas* of the previous entry (24 years earlier).

Example 3.2

Śaka	*Vallī*	*Ghaṭi*	*Pala*
1592	10	40	33
Add	08	22	36
1616	19	03	09

4. Obtaining *yogādikanda*

The tables of *yogākanda*, for 24 years interval (Table 4.1) and for each of 24 years (Table 4.2) are obtained similarly.

Table 4.1: *Yogākanda* for *śaka* years (24 yrs. Interval)

Śaka	1520	1544	1568	….	1904
Yoga	1	24	20	….	18
Vāra	2	5	00		3
Ghaṭī	59	11	24	….	20
Pala	16	51	26		36
Vallī	45	53	2		59
Ghaṭī	34	56	19	….	35
Pala	2	38	14		38

Table 4.2: *Śakāvaśeṣa yogakanda*

Koṣṭhaka	1	2	3	….	24
Yoga	10	20	3	….	23
Vāra	1	2	3		2
Ghaṭī	17	35	53	….	12
Pala	53	46	38		35
Vallī	15	30	46		8
Ghaṭī	26	52	18	….	22
Pala	4	8	12		36

5. *Bījas* (corrections) to Civil Days and Mean Daily Motions

It is truly a noteworthy practice among the ancient and medieval Indian astronomers that they always insisted that there should be concordance between the observed and the computed results. They called it "*dṛgganitaikya*". Right from the *Vāśiṣṭha siddhānta* upto the remarkable Kerala contributions of the late medieval period the updation of parameters and procedures in classical Indian astronomy has been strongly recommended and periodically effected also. For example, the famous Kerala astronomer Parameśvara, (1362-1455 AD) insists:

> *kālāntare tu saṃskāraś cintyatāṃ gaṇakottamaiḥ* |
>
> – In course of time may corrections (in parameters) be thought over by the best among mathematicians.

The *Vāśiṣṭha siddhānta* declares:

> *yasmin pakṣe yatrakāle dṛgganitaikyaṃ dṛsyate tena pakṣena kuryāt tithyādi sādhanam* |
>
> – That *pakṣa* (school of thought) which yields results (by computations) tallying

with observations during any period, from that *pakṣa* the (calendrical and astronomical) results like *tithi* etc. must be obtained for that period.

Nīlakaṇṭha Somayāji (1444–1545 AD), the crown jewel of Kerala astronomers, in a lengthy passage in his *Jyotirmīmāmsā*, admonishes a certain commentator who laments that on account of our ancient *siddhāntas* going wrong , the observances, religious rites and their expected merits are all going haywire:

hā dhik! saṇkaṭe mahati patitāḥ smaḥ

– "Alas, we are befallen into a great crisis!".

Nīlakaṇṭha further recommends[5]:

…pancasiddhāntāstāvat kvacitkāle pramāṇameva ityavagantavyam |

… ye punaranyathā prāktana siddhāntasya bhede sati yantraiḥ

parīkṣya grahāṇām bhagaṇādi saṅkhyām jñātvā abhinava siddhāntaḥ praṇeya ityarthāt |

– It must be known that the five *siddhāntas* had been indeed correct during some period… When earlier *siddhāntas* despite corrections, show discord, the revolutions etc. of the heavenly bodies must be known based on (actual) observations of eclipses etc. and a new *siddhānta* (astronomical treatise) must be composed!

The author of the *Makaranda sāriṇī* has incorporated many changes to yield better results (during his time). For example, mean motion of the Sun is tabulated under *Ravi vāṭikāpatram*. There are 59 columns, serially numbered from 1 to 59. Each column gives the Sun's mean motion for the number of days, represented at the top of the column, multiplied by 10. For example, in the column headed by 1 (i.e. for one day) the numbers moving downwards, in successive sexagesimal subunits, are 9|51|21|41|44|02|05.

Dividing this sequence by 10 we get 0|59|08|10|10|24|12|30

i.e., $0°59'08''10'''10^{iv}24^{v}12^{vi}30^{vii}$ which corresponds to 0°.9856026705264996 (*SDM*) correct to 16 decimal places.

(i) Now, the length of the (*nirayaṇa*, sidereal) solar year apparently adopted by *MKS* comes to

Solar year = 360 / *SDM* = 365.258750575109 days.

According to the *Sūryasiddhānta*,

$$\text{Solar year} = \frac{\textit{Civil days in M.Y}}{\textit{Sun's revns. in Mahāyuga}} =$$

$$\frac{1,57,79,17,828}{43,20,000} = 365.2587564814815 \text{ days.}$$

∴ *Bīja* to the solar year = $-3.5438235 \times 10^{-4}$ *Ghaṭī* = -0.51031 sec

(ii) A *Mahāyuga* (*M.Y.*) is defined as the period of 43,20,000 solar years. At the revised rate of the Sun's mean daily motion, the number of civil days (*sāvanadinas*) according to *MKS* comes to

$$\text{Civil days in } M.Y. = \frac{43,20,000 \times 360}{SDM} =$$

$$1,57,79,17,802.48447 \approx 1,57,79,17,802 \text{ days}$$

Now, according to the *Sūryasiddhānta* (*SS*), civil days in *M.Y.* = 1,57,79,17,828.

Bīja in civil days in *M.Y.*

= 1,57,79,17,802 – 1,57,79,17,828 = -26 days.

Similarly, we can work out the *bhagaṇas* (revolutions) of the other bodies also based on their mean daily motions given under the respective *vāṭikā* tables in *MKS*. These results are provided in Table 5.1.

(i) In Table 5.1, under 'Revised revns', the figures are given correct to 4 decimal places;

(ii) in the last column, under '*Bīja*', the figures are given to the nearest integer; and

Table 5.1: *Bījas* to revolutions of bodies

Body	Mean daily motion °	′	″	‴	iv	v	vi	vii	Revised revns.	*SS* revns.	*Bīja*
Candra	13	10	34	52	03	49	08	0	5,77,53,335.0879	5,77,53,336	–1
Mandocca	0	06	40	58	30	41	28	0	4,88,198.9998	4,88,203	–4
Rāhu	-0	03	10	44	43	51	0	31	2,32,238.5688	2,32,238	+0.57
Kuja	0	31	26	28	11	08	56	30	22,96,831.8929	22,96,832	0
Budha *śīgh.*	4	05	32	21	29	09	48	30	1,79,37,075.7218	1,79,37,060	+16
Guru	0	04	59	08	48	56	31	30	3,64,212.0116	3,64,220	–8
Śukra *śīgh.*	1	36	07	43	01	47	58	48	70,22,363.9911	70,22,376	–12
Śani	0	02	00	23	28	54	40	42	1,46,580.0052	1,46,568	+12

(iii) in the first column, under 'Body', *Mandocca* refers to the Moon's apogee.

(iv) Pingree in his SATIUS[6] provides the mean daily motions.

The extension *śīgh.* following Budha and Śukra is '*śīghrocca*' in short. This word means the 'apex of conjunction' of the inferior planets, Mercury and Venus. In classical Indian texts, while the mean Sun is taken as the *śīghrocca* for the superior planets, two different points are taken as *śīghrocca* for Budha and Śukra in the epicyclic theory. However, Nīlakaṇṭha Somayāji maintains, in his *Tantrasaṅgraha* (1500 AD) that the mean Sun is the common *śīghrocca* for all the planets. In that case, 'anomaly of conjunction', *śīghrakendra* = (mean planet – mean Sun), the mean planet's elongation from the mean Sun. Of course, some texts define *śīghrakendra* as (*śīghrocca* – mean planet) in which case the resulting correction will have the opposite sign.

6. Constants for determining *tithis*

For determining true values of *tithi*, *nakṣatra* and *yoga*, *MKS* gives separate tables for each of them, in intervals of 6 as 0,6,12,….,48. In the first row (*koṣṭhaka*) at the top of daily *vallīs*, successive numbers from 0 to 59 are given.

A *vallī* has three numbers; the topmost one is called *mastaṅka* ('head number') and the middle one *saralāṅka*. The last number is called *adhiṣṭhāṅka*. In a *vallī*, subtracting the earlier written *siddhāṅka* from the *saralāṅka* (i.e. the middle number of the *vallī*), the resulting number is the *guṇaka* (multiplier) for obtaining the *tithi*. If the number below the *vallī* is greater than 30, then 1 added to the *saralāṅka* is the *guṇaka* (multiplier).

We have, 1 solar year exceeding a lunar year by $11^{ti} | 1^{dina}\, 11^{gh.} 41.7^{pa.}$. Therefore, 16 solar years exceed 16 lunar years by

$$16 \times (11^{ti} | 1^{dina}\, 11^{gh.} 41.7^{pa.})$$

$$= 176^{ti} | 19^{dina}\, 07^{gh.} 07.2^{pa.}$$

$$= 26^{ti} | 5^{dina}\, 07^{gh.} 07.2^{pa.}$$

(from 176^{ti}, subtracting 150 *tithis*, being 5 complete lunar months and removing multiples of 7 from 19 *dinas*).

Example 6.1: For *śaka* 1891, we have from Table 2.1 of *tithikanda* and *vallī* (for the *śaka* years of 16 years interval):

For *śaka* 1880 : $24^{ti}\, 0^{di}\, 36^{gh}\, 57^{pa} | 50^{va}\, 12^{gh}\, 31^{pa}$

śeṣa varṣa 11 : $1^{ti}\, 6^{di}\, 08^{gh}\, 38^{pa} | 47^{va}\, 18^{gh}\, 34^{pa}$

Adding : $25^{ti}\, 6^{di}\, 45^{gh}\, 35^{pa} | 37^{va}\, 31^{gh}\, 05^{pa}$

Since $25^{ti} > 15^{ti}$ the *tithi* is 25-15=10 of the *kṛṣṇa pakṣa*.

Table 6.1: *Tithisaurabha* — *Tithi* corrections for *mastāṅkas* and *saralāṅkas*

	0	1	…	23	…	37	…	46	…	59
0	24	27		40		09		00		22
	57	50		32		22		00		04
6	25	28		40		09		00		22
	15	08		21		10		00		21
…	…	…		…		…		…		…
…	…	…		…		…		…		…
30	26	29		39		08		00		23
	24	16		34		26		04		30
36	26	29		39		08		00		23
	42	33		22		15		05		47
…	…	…		…		…		…		…
…	…	…		…		…		…		…
54	27	30	…	38		07	…	00	…	24
	33	24	…	44		42	…	08	…	39

Note: In Table 6.1, the topmost row (*koṣṭhaka*) consists of *mastāṅka* (*vallī*) successively from 0 to 59; (ii) the first column has *saralāṅka* (*gh.*) from 0 to 54 at intervals of 6 *gh.*; and (iii) corrections to the *tithis* are listed in *ghaṭīs* and *palas* against the *mastāṅka* and *saralāṅka* mentioned in (i) and (ii). Here, *mastāṅka* = 37, *saralāṅka* =31 and *adhiṣṭhāṅka* = 5.

Now, the *saralāṅka* lies between the *sthirāṅkas* (constants) 30 and 36. From Table 6.1 ("*Tithisaurabha*") in the vertical column under *mastāṅka* 37, in the rows against *saralāṅkas* 30 and 36 respectively we have 8|26 and 8|15 *ghaṭīs*. The difference between these numbers, *phalāntara* = (8|15) – (8|26) = –0|11 *ghaṭīs*. The difference between the given *saralāṅka* 31 and the earlier tabulated *saralāṅka* 30 is (31-30) = 1. Therefore, proportionately, for this difference, the correction

$$= \frac{-(0\,|\,11)\times 1}{6} \approx -0\,|\,2\ gh.$$

Combining this to the *phala* 8|26 (corresponding to *saralāṅka* 30), we get

spaṣṭaphala = (8|26) – (0|2) = 8|24 *gh.*

For the beginning of the *śaka* solar year 1891, we have

Mean *tithyādi*: $10^{ti}\,|\,6^{di}\,47^{gh.}10^{pa.}$ in the *kṛṣṇa pakṣa*.

Add *spaṣṭaphala*: $8^{gh.}\,24^{pa.}$

True *tithyādi* : $10^{ti}\,|\,6^{di}\,55^{gh.}34^{pa.}$

This means that the new solar year *śaka* 1891 commenced (with solar ingress into *Meṣarāśi*) on the 10th *tithi* (i.e. *Daśamī*) of the dark fortnight, the 6th *dina* (Friday) at 55*gh*, 34 *palas* (after the mean sunrise).

Note: (1) The *dinas* 1 to 7 (or 0) of the week represent respectively Sunday to Saturday. Hence *dina* 6 is a Friday. (2) Similarly, true *nakṣatrādi* and *yogādi* can be obtained from the respective tables.

7. *Pratibhāgī padakāni*

The *Pratibhāgī* (*PRB*) tables[12] are very popular among the *pañcāṅga* makers in Karnataka and Andhra regions. Most possibly the name of the text comes from the fact that the relevant tables are computed for each degree (*prati bhāga*).

Āryabhaṭa I (b.476 AD) and the now popular *Sūryasiddhānta* provide *R*sine differences (*R* = 3438') to get *R*sine for every 3°45′. Some texts (handbooks) provide brief tables for the *manda* and *śīghra* equations for the respective anomalies at even higher interval (step-) lengths. For example, Gaṇeśa Daivajña in his *Grahalāgavam* (1520 AD)[8] tabulates the *manda* and *śīghra* equations of the planets at intervals of 15°. Another popular handbook, *Karaṇakutūhalam*[9] of Bhāskara II (b. 1114 AD) gives the *jyākhaṇḍas* (blocks of *R*sine values) for every 10°. In such cases intermediate values are obtained by interpolation. While generally linear interpolation is expected to be used, it is truly noteworthy that as early as in the seventh century the great Indian astronomer Brahmagupta (c.628 AD) provides the 'second order' interpolation to obtain more accurate values for the equations of the centre and of 'conjunction' in his *Khaṇḍakhādyaka*[10,11].

Now, the *pratibhāgī* in contrast to the *siddhānta* and *karaṇa* texts, provides tables for *each degree*. In the photocopy with us, no mention of either the author or of the period of the composition is mentioned. A critical edition based on the available manuscripts in due course might throw light on these details. The mean positions of the heavenly bodies have to be worked out using the *Kali ahargaṇa*, the elapsed number of civil days for the given date from the beginning of the *Kaliyuga* (the mean midnight between 17th and 18th, February 3102 BC). Therefore the *Pratibhāgī* text has no need to mention or use a later epoch.

The popularity of *PRB* in parts of Karnataka and Andhra regions is very clear from the fact that a good number of manuscripts of the main text as also its commentaries are listed in the Catalogue of O.R.I., Mysore.

The important tables in *PRB* are on (1) the mean motions of the Sun, the Moon, apogee (*mandocca*) and the ascending node (*Rāhu*) of the Moon and the five planets; (2) the *mandaphala* (equation of the centre) of the bodies and (3) the *śīghraphala* (equation of conjunction) of each planet; (4) the Sun's declination (*krānti*) and lastly (4) Moon's latitude (*vikṣepa*, *śara*).

The tables of mean motions of the bodies for each day from 1 to 9 days, every 10 days from 10 to 90 days, every 100 (*nūru* in Kannada) days from 1 to 9 hundreds, every 1000 (*sāvira* in Kannada) from 1 to 9 thousands, from 10 to 90 thousands, 1 to 9 lacs (hundred thousand, *lakṣa* in Sanskrit and Kannada) and finally for 10 and 20 *lacs* (i.e. one and two million) days.

7.1 Mean motion, revolutions and sidereal periods in *PRB*

From the mean motion of the Sun for two million days given in *PRB*, we have 5475 $^{\text{Rev.}}$ 6^{S} 25° 18' 33" 02"' (the superscript *S* stands for 'signs' i.e. *rāśis* of the zodiac). This gives us the Sun's mean daily motion, *SDM* = 0°.985602617263794. From *SDM*, we obtain the length of the *nirayaṇa* (sidereal) solar year = 365.2587703139661 days and *sāvanadinas* (civil days) in a *Mahāyuga* (of 432×10^4 years) as 1,57,79,17,888 days.

The number of civil days in a *M.Y.* according to *SS* is 1,57,79,17,828 so that the *bīja* (correction) for civil days is +60.

Remark: The present authors, in an effort to update the *pañcāṅga* elements, recommend adoption of 1,57,79,07,487 as the *sāvanadinas* (civil days) for a *M.Y.*

We list the mean daily motions, revolutions (*bhagaṇas*) and the sidereal periods of the bodies according to *PRB* in Table 7.1

Note: In Table 7.1, (i) the mean daily motions are given correct to 15 decimal precision (on computer), (ii) the revolutions in a *Mahāyuga* (of 432×10^4 solar years) are given to the nearest integer and (iii) the sidereal periods are correct to 4 or 5 decimal places.

Remark: While the proposed number of civil days in *M.Y.* is 1,57,79,07,487 (see earlier remark), the

Table 7.1: Daily motion, revns. and sidereal periods in *PRB*

Body	**Mean daily motion**	**Revns. in *M.Y.***	**Sid. period (days)**
Moon	13°.17635250091553	577533340	27.32167
Moon's *Mandocca*	0°.1113829091191292	488203	3232.0937
Rāhu	0°.0529848113656044	232238	6794.4
Kuja	0°.5240193605422974	2296832	686.9975
Buddha *śīgh*	4°.092318058013916	17937061	87.9697
Guru	0°.08309634029865265	364220	4332.32076
Śukra *śīgh*	1°.60214638710022	7022376	224.69857
Śani	0°.03343930840492249	146568	10765.7729

suggested figure for a *kalpa* (432×10^7 years), to yield a more accurate value, is 15,77,90,74,87,027.

As mentioned in the earlier remark, the present authors have proposed revision of *bhagaṇas* (revolutions) in a *kalpa* (432×10^7 years), sidereal periods of the bodies as shown in Table 7.2. in comparison with *Sūryasiddhānta*.

8. *Tyāgarti* manuscript (*TYGMS*)

We procured recently a copy of a manuscript, called *Grahagaṇita padakāni*, from a private collection. The manuscript belongs to a small place called Tyāgarti[12] (also Tāgarti) of Sagar taluk in Shimoga district of Karnataka. The latitude (*akṣa*) of the place is given in terms of *akṣabhā* (*palabhā*). This value coincides closely with the known modern value of the latitude of Tyāgarti.

TYGMS explicitly mentiones that it is based on the *Sūryasiddhānta*. Even like the *Pratibhāgī*, *TYGMS* does not need and does not mention a contemporary epoch. Both of them need the *Kali ahargaṇa KA* for a given date. *KA* represents the number of civil days elapsed since the beginning of the *Kaliyuga* viz, the mean midnight between 17th and 18th of February 3102 BC.

This *KA* accumulated to more than ten *lakhs* (one million) days around 365 BC. For example, as on August 1, 2011, *KA* = 18,67,309, more than 1.8 million days. Therefore both *PRB* and *TYGMS* manuscripts provide the mean motion tables even for a *lakh*, ten *lakhs* (a million) and a *crore* (ten million) days for the sake of accuracy. These data help us to obtain the sidereal period and the *bhagaṇas* (revolutions in a *Mahāyuga*) of a heavenly body.

Table 7.2: Proposed *bhagaṇas* in a *kalpa*

Body	***Bhagaṇas* (Revolutions)**		**Sid. Period(days)**
	Sūryasiddhānta	**Proposed**	**Proposed**
Sun	4,32,00,00,000	4,32,00,00,000	365.256362738
Moon	57,75,33,36,000	57,75,29,85,910	27.32166
Moon's *Mandocca*	48,82,03,000	48,81,25,074	3232.589
Rāhu	23,22,38,000	23,22,68,618	6793.46
Kuja	2,29,68,32,000	2,29,68,76,453	686.9797
Budha *śīgh*	17,93,70,60,000	17,93,70,33,867	87.96926
Guru	36,42,20,000	36,41,95,066	4332.589
Śukra *śīgh*	7,02,23,76,000	7,02,22,60,402	224.7008
Śani	14,65,68,000	14,66,56,219	10759.23

TYGMS contains 32 folios of tables for astronomical computations. One or two folios are missing in between. For example, the folio for the mean motion of Saturn (*Śani madhya padakāni*) is missing in the bundle of folios.

Interestingly, the manuscript is in *Nāgarī* script with numerals completely in Kannada script. Even many Kannada words, by the way of instructions or descriptions, are in the *Nāgarī* script. Folio 31 (back) mentions "*akṣaliptāḥ* 842|17" i.e. the latitude in arcminutes is 842|17. This means the local latitude $\phi = 842'17'' = 14°02'17''$. Further, folio 32 mentions "*laṅkodaya viṣuvacchāyā ṅgula* 3". This means that the equinoctial shadow (called *akṣabhā* or *palabhā*) is 3 *aṅgulas* (with the gnomon of length 12 *aṅgulas*). This gives:

Latitude, $\varphi = \tan^{-1}\left(3/12\right) = \tan^{-1}(0.25) = 14°02'10''.48.$

Folio 11 (front) mentions "*kalivarṣa* 4813". Now, *kali* year 4813 corresponds to 1712 AD. In the same folio the *mandoccas* (apogees) and the *pātas* (nodes) of the planets are given.

Although for obtaining the mean positions contemporary epoch is not needed, the author of *TYGMS* perhaps desired updation of the apogee and nodes of the planets. However, the rates of motion of these special points as given in the *Sūryasiddhānta* are unrealistic from the point of view of our modern known results.

In addition to giving the *Kali* year as 4813 (1712 AD), *TYGMS* mentions the *nirayaṇa* mean position of the Sun as $11^{Ra}\ 10°\ 08'\ 03''$ which gives the date as March 22 of the year 1712 AD with *Ayanāmśa* (amount of equinoctial precession) as about 18°. From this data the *TYGMS* can be dated as **March 22, 1712 C.E.**, three centuries old.

8.1 Solar year, civil days, revolutions etc. in *TYGMS*

TYGMS gives the Sun's mean motion for 1 *crore* (10^7) days as $10^{Ra}\ 06°\ 33'\ 20''$ (along with 27377 revolutions as can be calculated). From this we get (i) Sun's mean daily motion, *SDM* = 0°.9852676868.Therefore, in a *Mahāyuga* of 43,20,000 solar years, the number of civil days (*sāvanadinas*):

$$\frac{4320000 \times 360°}{SDM} = 1,57,79,17,792.$$

The corresponding value according to *SS* is 1,57,79,17,828. Therefore, *Bīja* (correction) of civil days is –36 and

(ii) the length of the *nirayaṇa* solar year = 360°/ *SDM* = 365.2587563 days.

Based on the mean motions of the bodies for ten million days in *TYGMS*, we have worked out *bhagaṇas* (revolutions) and hence the *Bījas* as shown in Table 8.1

Table 8.1: Mean daily motions, revns. and *bījas* in *TYGMS*

Body	Mean motion for 1 crore days					Revolutions. in *M.Y.*		*Bījas*
	Revn.	*Ra*	D	M	S	*TYGMS*	*SS*	
Moon	366009	09	11	27	08	5,77,53,332	5,77,53,336	–4
Moon's *Mandocca*	3093	11	19	06	20	4,88,202	4,88,203	–1
Rāhu	1471	09	18	08	0	2,32,237	2,32,238	–1
Kuja	14556	01	03	46	40	22,96,832	22,96,832	0
Budha *śīgh*	113675	06	0	26	30	1,79,37,059	1,79,37,060	–1
Guru	2308	02	23	25	20	3,64,219	3,64,220	–1
Śukra *śīgh*	44504	0	23	56	0	70,22,375	70,22,376	–1

Note: In Table 8.1, (i) the mean motions are given for one crore (10 million) days in terms of revolutions, *rāśis* (signs), degrees (*amśa*), minutes(*kalās*) and seconds (*vikalās*), (ii) revolutions in a *Mahāyuga* are to the nearest integer, (iii) the last column gives the *bījas* (correction) to the revolutions given in the *Sūryasiddhānta* and (iv) details of Śani do not appear in the table since the related folio is missing in *TYGMS*.

9. *Mandaphalas* and *Śīghraphalas* in *PRB, TYGMS* and *MKS*

In finding the true longitudes of the Sun and the Moon we need apply only the major correction, *mandaphala* (equation of the centre). But, in the case of the five planets, besides the *mandaphala*, the other major equation to be applied is *śīghraphala*.

9.1 *Mandaphala* in the *saura* tables

The *mandaphala* (equation of the centre) of a heavenly body is given by the classical expression:

$$\sin(MP) = \frac{p}{R}\sin(MK)$$

where *MP* is the required *mandaphala*, *MK* is the *mandakendra* (anomaly from the apogee), *p* is the *manda paridhi*, the periphery of the related epicycle, *R*=360°, the periphery of the deferent circle. The *mandakendra MK* is defined as

MK = (*Mandocca* – Mean planet) where *mandocca* is the mean apogee.

Āryabhaṭa I (b. 476 AD) takes the peripheries of the Sun and Moon as constants at 13°.5 and 31°.5 respectively and those for the five planets are variable ones. On the otherhand, the *Sūryasiddhānta* and the tables under consideration here adopt variable peripheries for all the seven bodies. Table 9.1 lists the limits of these *paridhis* (peripheries) according to *SS*.

Table 9.1: *Manda paridhis* according to *SS*

Body	***Manda Paridhi***	
	(*MK* = 0°, 180°)	**(*MK*=90°, 270°)**
Sun	14°	13°40'
Moon	32°	31°40'
Kuja	75°	72°
Budha	30°	28°
Guru	33°	32°
Śukra	12°	11°
Śani	49°	48°

The *manda paridhi* is maximum at the end of an even quadrant (i.e. for *MK* = 0°, 180°) and minimum at the end of an odd quadrant (i.e. for *MK* = 90°, 270°).

If the peripheries at the ends of *even* and *odd* quadrants are denoted respectively by p_e and p_o, then the variable periphery for *mandakendra MK* is given by

$$p = p_e - (p_e - p_o) \times |\sin(MK)| \quad \ldots(9.2)$$

where $|\sin(MK)|$ means the numerical or absolute value of sin(*MK*).

Thus, according to *SS*, the *mandaphala MP* is given by (9.1) using (9.2). The values of *MP* of the Sun as per the three tables, for *MK* at intervals of 10°, are compared with the actual ones, obtained from (9.1) and (9.2) in Table 9.2.

In Table 9.2, we have compared the *mandaphala* values for the Sun whose *mandaparidhi* varies from 13°40' to 14°. For *MK* = 90°, *MP* = 130' 31" = 2°10' 31" according to *TYGMS*. We notice that all the three tables for the Sun give *MP* in *kalās* and *vikalas* (arcminutes, arcseconds). The values differ by a maximum of 5 arcseconds.

According to the Indian classical texts, the greatest *MP*, among the seven heavenly bodies, is for Kuja (Mars) whose *mandaparidhi* varies from 72° to 75°. For *MK* = 90°, the *mandaparidhi*,

Table 9.2: *Mandaphala* of the Sun

MK	***Mandaphala* (Equation of the centre)**							
	TYGMS		***PRB***		***MAKARANDA***		**Formula (9.1)**	
	ka	***vik***	***ka***	***vik***	***ka***	***vik***	***ka***	***vik***
10°	23	07	23	07	23	07	23	07
20°	45	19	45	19	45	22	45	21
30°	66	03	66	03	66	02	66	03
40°	84	36	84	35	84	42	84	37
50°	100	31	100	33	100	36	100	33
60°	113	25	113	21	113	25	113	24
70°	122	47	122	47	122	51	122	50
80°	128	31	128	32	128	35	128	36
90°	130	31	130	31	130	32	130	31

Table 9.3: *Mandaphala* of Kuja

MK	***Mandaphala* (Equation of centre)**					
	TYGMS	***PRB***		***MAKARANDA***	**Formula (9.1)**	
	kalās	***kalās***	***vik***	***kalās***	***kalās***	***vik***
10°	123	123	31	111	123	31
20°	242	241	31	219	241	48
30°	352	351	31	320	351	32
40°	449	449	23	414	449	48
50°	534	533	47	498	533	58
60°	602	601	57	570	601	49
70°	651	651	15	627	651	36
80°	681	681	29	667	681	59
90°	692	692	03	689	692	13

$p = p_o = 72°$ so that the corresponding *mandaphala* $MP = 72°/2\pi = 11°27'33'' = 687'33''$. To examine how the *mandaphala* values for a planet according to the *saurapakṣa* tables under consideration compare with one another these are shown in Table 9.3.

We notice in Table 9.3 that (i) *MKS* and *TYGMS* give the *mandaphala* of Kuja only in *kalās*, to the nearest arcminute while *PBR* provides the same both in *kalās* and *vikalās*. In fact this is the case with other four planets also.

Note: According to *MKS*, the *mandaphalas* of the five planets differ from those of the other texts.

For example, for *MK*=40°in Table 9.3 the *mandaphala* values according to *TYGMS* and *MKS* are respectively 449 and 414 *kalās*. The main reason for this is that, in *SS* the true position of a star planet is obtained by applying successively four corrections. Among these the *manda* correction is applied twice in between the two *śīghra* corrections. On the other hand, *MKS* simplifies the procedure by reducing only to three corrections. Here the *manda samskāra* is applied only once between the two *śīghra samskāras*. In the process *Makaranda* has consolidated the two *manda* corrections of *SS* into a single equation in *MKS*[16]. This makes the *mandaphala* value of *MKS* differ from those of *SS* and the other related tables.

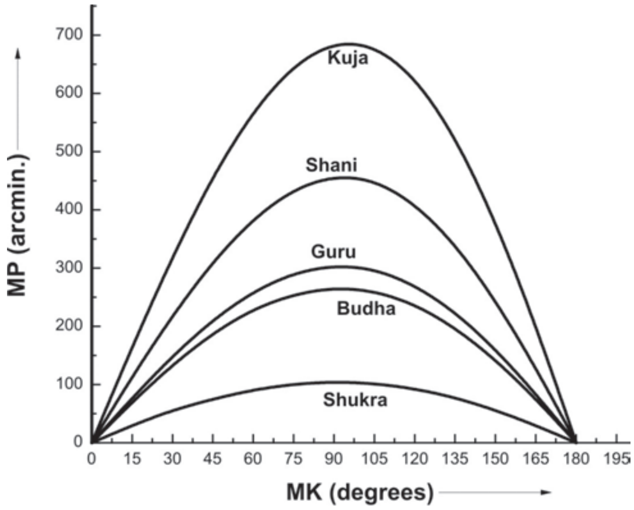

Fig. 9.1: Variation of *MP* of the planets against *MK*.

In Fig. 9.1, the variation of the *mandaphala* (*MP*) with the *mandakendra* (*MK*, the anomaly from the apogee) is shown graphically for the five planets. The behaviour of the graphs is sinusoidal with *MP* = 0° for *MK* = 0°, 180° and reaching the maximum at *MK* = 90°

9.2 *Śīghraphala* in *PRB, TYGMS* and *MKS*

As pointed out earlier, in obtaining the true planets we apply two major equations which are referred to as the *manda-samskāra* and the *śīghra-samskāra*. While the former corresponds to the equation of the centre, the latter to the transformation from the heliocentric to the geocentric frame of reference for the five *tārāgrahas* (star-planets).

The classical procedure for *śīghraphala* is based on the expression:

$$\sin(SP) = \frac{p}{SKR}\left[R\sin(SK)\right] \quad \ldots(9.3)$$

where *SP* is the required *śīghraphala*, *p* is the *śīghraparidhi*, the periphery of the *śīghra* epicycle, *R* = 3438' and *SKR* is the *śīghrakarṇa*, the *śīghra* hypotenuse given by

$$SKR^2 = \left(Sphuṭakoṭi\right)^2 + \left(Doḥphala\right)^2 \ldots(9.4)$$

Let $r = \frac{p}{360^o}$ then

$$Doḥphala = r\left[R\sin(SK)\right] \quad \ldots(9.5)$$

$$Koṭiphala = r\left[R\cos(SK)\right] \quad \ldots(9.6)$$

$$\begin{aligned} Sphuṭakoṭi &= R + r[R\cos(SK)] \\ &= R[1 + r\cos(SK)] \end{aligned} \quad \ldots(9.7)$$

The *śīghrakarṇa SKR* is given by

$SKR^2 = \left(Doḥphala\right)^2 + \left(Sphuṭakoṭi\right)^2$ using (9.5) and (9.7)

$$= R^2\left[\left\{r\sin\left(SK\right)\right\}^2 + \left\{1 + r\cos\left(SK\right)\right\}^2\right]$$

$$= R^2\left[r^2 + 2r\cos\left(SK\right) + 1\right]$$

$$\therefore SKR = R\sqrt{r^2 + 2r\cos\left(SK\right) + 1} \quad \ldots(9.8)$$

Substituting (9.8) in (9.3), we get

$$\sin(SP) = \frac{r(R\sin SK)}{SKR}$$

$$= \frac{r(R\sin SK)}{\sqrt{r^2 + 2r\cos(SK) + 1}} \quad \ldots(9.9)$$

so that the *śīghraphala*,

$$SP = \sin^{-1}\left[\frac{r(R\sin SK)}{\sqrt{r^2 + 2r\cos(SK) + 1}}\right] \quad \ldots(9.10)$$

Example 9.1: Find the *śīghra* correction for Śani (Saturn) given the following:

Śani's *śīghrakendra*, *SK* = 62°.0406 and Śani's corrected *śīghraparidhi*, *p* = 39°.88328

We have

(i) $Doḥphala = \frac{39°.88328}{360} \times 3438' \times \sin(62°.0406) = 336'.4284$

(ii) $Koṭiphala = \frac{39°.88328}{360} \times 3438' \times \cos(62°.0406) = 178'.5765$

(iii) $Sphuṭakoṭi = 3438' + 178'.5765 = 3616'.5765$

(iv) $Śīghrakarṇa = \sqrt{(336'.4284)^2 + (3616'.5765)^2} = 3632'.1907$

(v) $R\sin(SP) = \frac{3438' \times 336'.4284}{3632'.1907} = 318'.44166$

$\therefore$ $Śīghraphala,\ SP = \sin^{-1}\left[\frac{318'.44166}{3438'}\right] = 5°18'53''.$

The *śīghraphala* is additive or subtractive according as the *śīghrakendra SK* is less than or greater than 180°.

In the above example, since $SK = 62°.0406 < 180°$, $SP > 0$ i.e. $SP = +5°18'53''$.

It should be noted that in the case of the *śīghra* correction also, as for the *mandaphala*, the *śīghraparidhi* (periphery) *p* is a variable given by

$$p = p_e - (p_e - p_0) \times |\sin(SK)| \quad \ldots(9.11)$$

The peripheries *p*, for different planets, at the ends of *even* and *odd* quadrants according to the *Sūryasiddhānta* are given in Table 9.4

Table 9.4: *Śīghraparidhi* of planets

Planet	*Śīghraparidhi*	
	SK = 0°, 180°	*SK* = 90°, 270°
Kuja	235°	232°
Budha	133°	132°
Guru	70°	72°
Śukra	262°	260°
Śani	39°	40°

The *śīghraparidhis* for Kuja, Budha and Śukra are greater at the end of the *even* quadrents (*SK* = 0°, 180°) than at the *odd* quadrants (*SK* = 90°, 270°). But it is the other way for Guru and Śani.

A sample folio from *PRB*, displaying Śani's *śīghraphalas* for *SK* = 36° to 84° is shown in Fig. 9.2. The numerals are in Kannada script. From this folio of *PRB* an extract of the *śīghraphala* values for *SK* = 42° to 44° are reproduced in Table 9.5.

Among the five *tārāgrahas*, Śukra (Venus) has the maximum *śīghraparidhi* and hence we choose to tabulate its values according to the

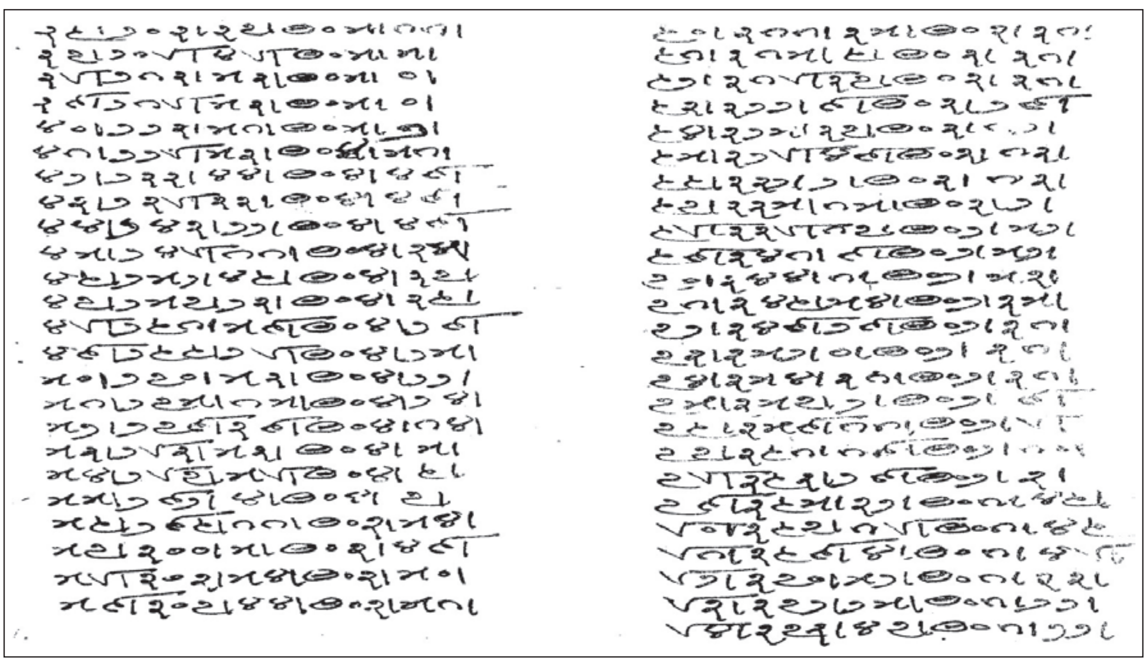

Fig. 9.2: *śīghrapadaka* of Śani, a folio from *Pratibhāgī* ms

Table 9.5: A sample of *śīghraphalas* of Śani according to *PRB*

SK	*Śīghraphala*	**Difference**
42°	233' 44''	+ 04' 49''
43°	238' 33''	+ 04' 49''
44°	243' 22''	+ 04' 49''

different *sāriṇīs* and *padakas*, at intervals of 15° for *SK* = 0° to 180° in Table 9.6

In Table 9.6 the *Śīghraphalas* of Śukra according to the three astronomical tables, *MKS*, *PRB* and *TYGMS* are compared with the corresponding values according to the popular *Karaṇa* text *Grahalāghavam* (*GL*)[5] and those obtained from formula (9.10)

The three texts of tables are all based on the *Sūryasiddhānta* and hence their *śīghraphala* results are close to those obtained from formula 9.10 based to *SS*.

In the first column the *śīghrakendra* (*SK*), the 'anomaly of conjunction' is taken from 0° to 180° at intervals of 15°. *GL* has given the *śīghrankas* for every 15° of *SK*. To get the actual *śīghraphala* in degrees, we have to divide the *śīghrāṅka* (col.2) by 10. For example, the for *śīghrāṅka* for *SK* = 15° is 63. By dividing 63 by 10 we get 6.3 i.e. 6°18' as shown in col. 3. Thus, the *śīghrāṅka* in col. 2 are divided by 10 and expressed as degrees and arcminutes (*aṃśa* and *kalā*) in col. 3.

While *MKS* gives the *śīghraphala* values in degrees and minutes (col. 4), *PBR* gives them in *kalās* and *vikalās* (col. 5) and *TYGMS* only in *kalās* (col. 6). However, for the sake of immediate comparison the values from all the five sources are expressed in degrees etc. We notice that the three texts of *sāriṇīs* (or *padakas*) are loyal to the basic text *SS* on which these are based, and their *śīghraphala* values are much closer to the formula-based last column. But, *Grahalāghavam*, on which the *Gaṇeśapakṣa* is based has different set of parameters and completely dispenses with the all important trigonometric ratio *sine* by adopting a very good algebraic approximation.

A folio from *TYGMS* giving Śukra's *śīghraphala* (from *Karka*) is shown in Fig 9.3.

Table 9.6: *Śīghraphala* of Śukra

SK	*Grahalāghavam*		*MKS*	*PBR*	*TYGMS*	**Formula**
	Śīghrāṅka	*śī. phala*				**(9.10)**
0°	0°	0°	0°	0°	0°	0°
15°	63	6°18'	6°18'	6°18'17"	6°18'	6°18'16"
30°	126	12°36'	12°33'	12°32'19"	12°33'	12°33'14"
45°	186	18°36'	18°43'	18°42'21"	18°42'	18°42'13"
60°	246	24°36'	24°44'	24°43'32"	24°44'	24°41'47"
75°	302	30°12'	30°28'	30°27'32"	30°28'	30°27'01"
90°	354	35°24'	35°52'	35°51'32"	35°52'	35°50'16"
105°	402	40°12'	40°39'	40°39'06"	40°39'	40°38'19"
120°	440	44°0'	44°27'	44°27'30"	44°28'	44°26'16"
135°	461	46°6'	46°23'	46°23'05"	46°23'	46°21'23"
150°	443	44°18'	44°16'	44°16'37"	44°17'	44°14'56"
165°	326	32°36'	32°12'	32°14'13"	32°14'	32°12'36"
180°	0	0°	0°	0°	0°	0°

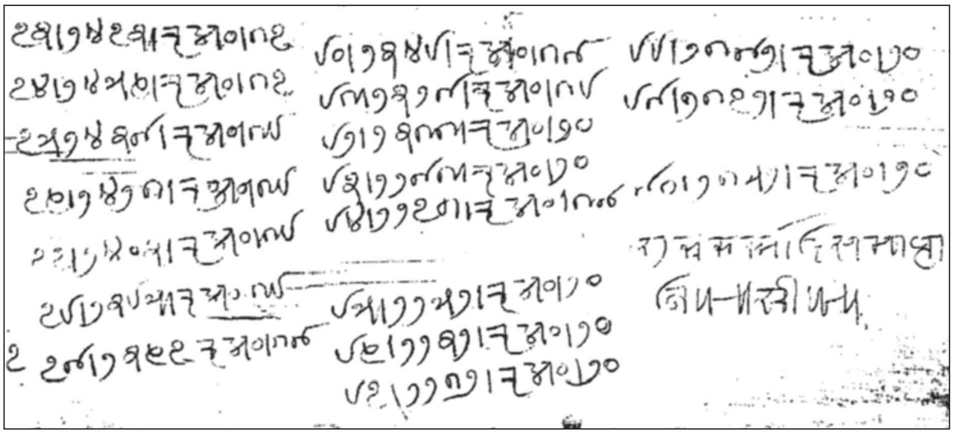

Fig. 9.3: Śukra's (*Karkādi*) *śīghraphala*, a folio from *TYGMS*

Table 9.7: A sample of *śīghraphalas* of Śukra (*Karkādi*) according to *TYGMS*

SK	*Śīghraphala*	Difference
80°	2348'	- 19'
81°	2329'	- 18'
82°	2311'	- 20'

9.3 Maximum *śīghraphala* and critical *śīghrakendra*

The *mandaphala* of a body attains its maximum for the argument, *mandakendra* = 90° as can be seen from equation (9.1)

However, surprisingly *MKS* differs from the other two texts and also from the basic source *SS* in as far as the *mandaphalas* of the planets attain their *maxima* not at *MK* = 90° but over a range beyond 90°. However, for the Sun and the Moon, *MKS* is in line with *PRB* and *TYGMS* .

The behaviour of the *śīghraphala* (*SP*) variation is truly interesting. Here also the *sine* term of the argument occurs, even as in the case of the *mandaphala*, as a factor in the numerator. But, unlike the other case, the expression has *sine* and *cosine* terms, under square-root in the denominator. This structure of the expression for *SP* causes it to have different *critical values* for the *śīghrakendra* (*SK*). Of course the maximal values of *SP* are different for the different planets though these bodies share the common ground value 0 at *SK* = 0° and 180° i.e. when a mean planet is in *conjunction* or *opposition* with the mean Sun. Table 9.6 gives the *critical* values of *SK* and the corresponding maximal *śīghraphalas* for the different planets.

Table 9.6: Maximum *śīghraphala* and critical *SK*

Planet	Critical SK	Maximum SP
Kuja	130°.8	40°16′26″
Budha	111°.7	21°31′19″
Guru	101°.2	11°31′50″
Śukra	136°.7	46°22′55″
Śani	96°.2	06°22′42″

Since the classical tables give *SP* for each degree, we can trace the critical *SK* to the nearest degree and the corresponding *SP*. These results are shown in Table 9.7

(i) From Table 9.7 we observe that *PRB* tables for *SP* is unique among the three texts in giving the *SP* of each planet in *vikalās* (arcseconds) also. While *MKS* lists the *SP* in degrees and arcminutes (*aṃśa* and *kalā*), *TYGMS* provides the values only in *kalās* and *PRB* gives in *kalās* and *vikalās*. In Table 9.7 we have expressed the values of *SP* in degrees etc. for easy comparison. (ii) Since *MKS* does not give *SP* in *vikalās*, the critical *SK* values are shown to lie within a range of 2° to even 5° (as for Śani). However, in the case of *TYGMS*, though here also

Table 9.7: Maximum *SP* in *Sāriṇīs*

Planet	*Makaranda Sāriṇī*		*Pratibhāgī ms.*		*Tyāgarti ms.*	
	Cr. *SK*	Max. *SP*	Cr. *SK*	Max. *SP*	Cr. *SK*	Max. *SP*
Kuja	130°–132°	40°16'	131°	40°17'13"	131°	40°17'
Budha	109°–113°	21°31'	112°	21°32'14"	112°	21°31'
Guru	100°–103°	11°31'	101°	11°31'36"	101°	11°32'
Śukra	136°–138°	46°24'	135°	46°23'05"	135°–138°	46°23'
Śani	94°–99°	6°22'	98°	6°22'42"	97°	6°23'

vikalās are not given for *SP*, it is possible to locate the critical *SK* correct to a degree for each planet. But in the case of Śukra, the critical *SK* lies between 135° and 138° since the corresponding *SP* is given the same, 46°23' (= 2783 *kalās*). (iii) Unlike *MKS* and *PRB*, the *Tyagarti ms.* lists the *SP* against *SK* in two parts: 0° to 90° *Mṛgādi* (from the beginning of Capricorn) and 0° to 90° *Karkādi* (from the beginning of Cancer). Because of this arrangement, if we need *SP* for *SK* > 90° (< 180°), say of the form 90° + θ (where θ is acute), then to get the related *SP* we have to look for the same in the second part (*Karkādi*) tables against the argument (90° – θ).

Thus, for example, in the tables of *śīghraphala* for Śani, to get *SP* for *SK* = 98° = 90° + 8° (i.e. θ = 8°) we have to look for the argument 90° – θ i.e. 90° – 8° = 72° in the second part of the *śīghra* tables.

In Fig. 9.4, the variation of *śīghraphala* (*SP*) the with the *śīghra* anomaly (*SK*) is shown graphically for the five planets. The graphs, with *SP* = 0° for *SK* = 0° and 180°, reach the maxima not at *SK* = 90° but at different critical points for different planets as given in Table 9.6. Both *SK* and *SP* are in degrees.

10. Eclipse computations

An important phenomenon to which two separate chapters are devoted in the *siddhantic* texts is eclipse (*grahaṇa, uparāga*). In fact, the benchmark for the validation of the parameters and procedures was the observation of lunar and

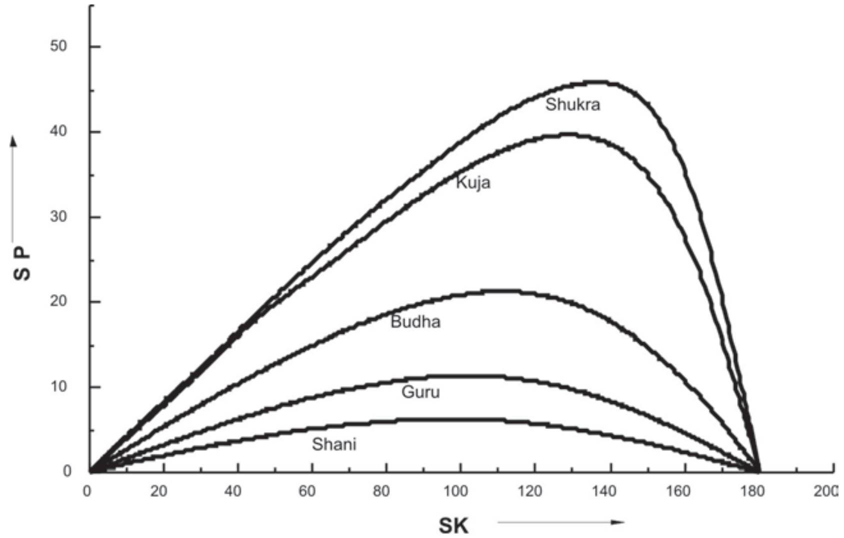

Fig. 9.4: Variation of *SP* with *SK* for planets

solar eclipses and planetary conjunctions, especially the lunar occultations of stars and planets.[11,12] Nīlakaṇṭha Somayāji (1500 AD) rightly remarks how his *paramaguru* (grand-preceptor) Parameśvara composed his text *Samadṛggaṇita* based on fifty-five years' astute observation of eclipses and planetary conjunctions (*nirīkṣya grahaṇa grahayogādiṣu*).

Viśvanātha Daivajña in his *Udāharaṇa*[1] commentary on *MKS* provides an example each for lunar and solar eclipses.

Example 10.1: Lunar eclipse of *Śaka* 1534, lunar month *Vaiśākha śuddha* (bright fortnight) 15 (fullmoon day, *paurṇimā*) 54 | 40 *gh*. *Anurādhā nakṣatra* with *gataiṣyayoga* (sum of the elapsed and to be covered durations) 58 | 36 *gh*. The given traditional date corresponds to **May 15, 1612 AD.** The instant of fullmoon is taken approximately as 54 | 40 *gh*.

Viśvanātha gives the longitudes of the Sun, Moon and Rāhu (Moon's node) as follows:

Sun: 1^R 06°30'37", Moon: 7^R06°34'35" and Rāhu: 1^R14°18'11".

10.1 Angular diameters of the Moon and the earth's shadow cone

Interestingly, *MKS* gives the angular diameters (*bimba*) of the Moon and the earth's shadow cone (*bhūccāyā, bhūbhā*) as determined by the total duration of the running *nakṣatra* (of the Moon). The image of the related folio is in Fig.10.1.

An eatract of Fig. 10.1 is given in Table 10.1.

Note : In Table 10.1 the word "*pāta*" refers to the shadow and not the Moon's node.

1 *Aṅgula* (*Aṅg*.) = 60 *pratyaṅgula* (*pra*.)

Table 10.1: *Candra bimba* and *Bhūcchāyā bimba*

Duration of *nakṣatra* in *Ghaṭī*		**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**
Candra	*Aṅg*	11	11	11	10	10	10	10	10	10	09	09
bimba	*pra*.	34	22	10	59	48	37	27	17	07	58	48
Pāta	*Aṅg*	29	28	28	27	27	26	25	25	24	24	24
bimba	*pra*.	34	54	16	38	02	27	53	20	49	48	48

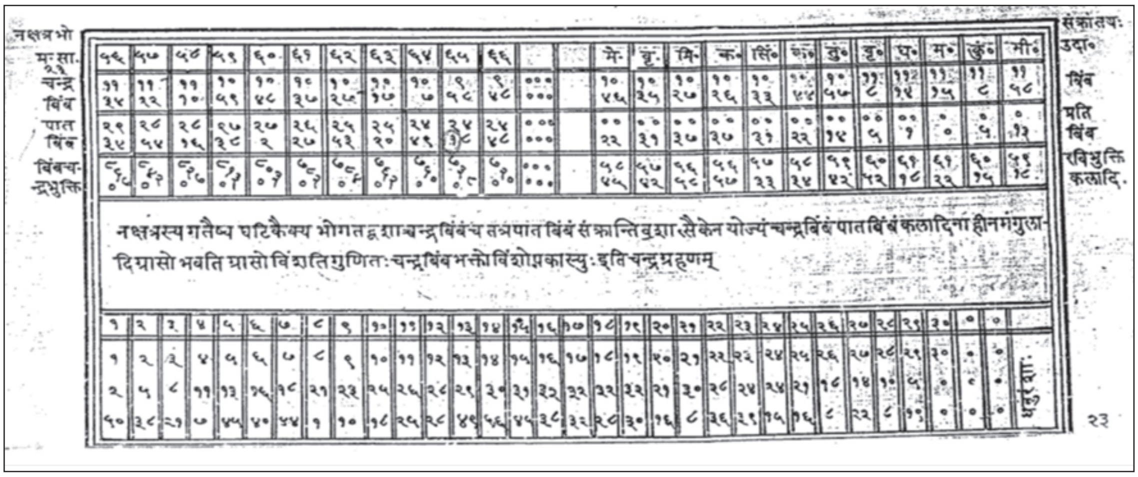

Fig.10.1: Tables of *bimbas* and *dhanu*, folio from *Makaranda sāriṇī*

For the given example 10.1 we have to find the angular diameters of the Moon and the earth's shadow cone using Table10.1. The duration of the running *Anurādhā nakṣatra* is given as 58 | 36 g*h*. This value lies between 58 and 59 *ghaṭīs* for which the corresponding values of the Moon's angular diameters are respectively 11 | 10 and 10 | 59 *aṅgulas*. Now, by the rule of three (*trairāśi*, *anupāta*) we obtain the Moon's angular diameter as 11 | 3.4 *aṅgulas*.

Similarly the diameter of the shadow cone (*bhūccāyā bimba*) is calculated. In the above Table 10.1, under 58 and 59 *ghaṭīs* against *pāta bimba*, we have 28 | 16 and 27 | 38 *aṅgulas*. Therefore, for the argument 58 | 36 g*h*. in between, proportionately we get 27 |53.2 *aṅgulas* as the *mean* diameter of the earth's shadow. This needs to be corrected to get the *true* (*spaṣṭa*) diameter.

In the same folio of *MKS*, corrections to the mean diameter of the shadow cone are given (in *aṅgulas* and *pratyaṅgulas*) for the Sun's ingressions to different *rāśis* (*Meṣa* etc). In the example under consideration, the true *nirayaṇa* Sun is 1^R 06°30'37" i.e. *Vṛṣabha rāśi* 06°30'37". Now, under *Vṛṣabha* the correction given is 0 | 31 *aṅg*. and that under the next *rāśi Mithuna* is 0 | 37 *aṅg*. The difference between them is + 0 | 06 *aṅg*. Therefore, for the balance 06°30'37" we get

$$\frac{06°30'37"}{30°} \times (0 \mid 06)\, aṅg. = 0 \mid 1.3\, aṅg.$$

Adding this to the value 0 | 31 *aṅg*. corresponding to the beginning of *Vṛṣabha*, we get the correction = 0 | 31 + 0 | 1.3 = 0 | 32.3 *aṅg*. Adding this correction to the mean diameter 27 | 53.2 *aṅg*. obtained earlier, we get the *true* diameter:

spaṣṭābhūbhā = 27 | 53.2 + 0|32.3 = 28 | 25.5 ≈ 28 | 26 *aṅgulas*.

Already we have the Moon's diameter, *Candrabimba* = 11| 3.4 *aṅg*. The sum of the semi-diameters of the Moon and the shadow cone,

$$Mānaikya\ khaṇḍa = \frac{1}{2}(11 \mid 3.4 + 28 \mid 26) \approx 19 \mid 45\, aṅg.$$

(ii) Moon's latitude (*Candra śara*)

Moon's nodal distance, *Virāhucandra*,

VRCH = Moon's longitude – Rāhu's longitude
= 7^R06°34' 35" – 1^R14°18'11"
= 5^R22°16' 24" = 172°16' 24".

Bhuja of *VRCH* = 180°–172°16'24" = 7°43'36". Now the table for the Moon's *śara* in *MKS* gives the latitude for the values of the argument (*bhuja* of *VRCH*) from 1° to 90°. From Table 10.2, under the argument values 7° and 8°, we have *śara* given respectively as 32 | 52 and 37 | 32 in *kalās* (arcminutes) with a difference of 4 | 40 *kalās* (Table 10.2). By proportions, for the balance of 43'36" between 7° and 8° we get the increment in *śara* as 3 | 23.4 *kalās*. Adding this increment to the *śara* 32 | 52 *kalās* (for 7°), we get

Candra śara = 32 | 52 + 3|23.4 = 36 |15.4 *kalās* ≈ 12 | 5.1 *aṅg*.

Note : *Śara* is positive or negative according as *VRCH* is less or greater than 180°.

Table 10.2: A sample of *Chandra śara* in *MKS*

VRCH	*śara*
1°	4' 43"
2°	9' 25"
…..	……
7°	32' 52"
8°	37' 32''
…..	…..
90°	270' 0"

(iii) *Grāsa* and *sthiti*

By definition, *grāsa* = *Manaikya khaṇḍa* – | *śara* |

noting that if *śara* is negative, then its numerical value is considered.

In the example, *grāsa* = 19 | 45 – 12 | 5.1 ≈ 7 | 40 *aṅg*.

MKS gives the following Table10.3 for *sthiti* (half-interval) as a function of *grāsa* (amount of obscurity):

Table 10.3: *Sthiti* (Half-duration) for lunar eclipse

Grāsa (*aṅg.*)	1	2	3	4	5	6	7	8	9	10	11
Sthiti Gh.	1	2	2	2	3	3	3	3	3	4	4
Pa.	29	4	30	50	7	22	35	46	56	4	11
Grāsa (*aṅg.*)	12	13	14	15	16	17	18	19	20	21	22
Sthiti Gh.	4	4	4	4	4	4	4	4	4	4	4
Pa.	18	23	28	31	34	36	37	37	38	38	39

In the example under consideration, *grāsa* = 7 | 40 *aṅg*. In Table 10.3 for half- duration (*sthiti*), we find that below the entries 7 and 8 *aṅgulas* of *grāsa* we have the corresponding *sthiti* values respectively as 3 | 35 *gh*. and 3 |46 *gh*., the difference between them being 0 | 11*gh*. For 1 *aṅg*. of *grāsa*. Therefore, for the balance of 0 |40 *aṅg*., the corresponding increment in *sthiti* is 0 |07 *gh*. Adding this to 3 |35 *gh*., we get *sthiti* = 3 | 35 + 0 |07 = 3 |42 *gh*.

Commentator Viśvanātha stops the example at this stage recommending the further procedure to be continued as per the relevant *karaṇa* (handbook). However, respectively subtracting *sthiti* from and adding the same to the instant of the *fullmoon* we get the *sparśa* (beginning) and the *mokṣa* (end) of the lunar eclipse. Thus we have:

Sparśakāla: 54 | 40 – 03 | 42 = 50 | 58 *gh*. and *Mokṣakāla* = 54 |40 + 03|42 = 58|22 *gh*.

Remark: Computations of eclipse according to the **Improved *Siddhāntic* Procedure (ISP)**[14,15], developed by the present authors, give the following circumstances:

Moon's diameter = 31'.72 = 10.573 *aṅg*. and

shadow's diameter = 86'.756 = 28.9187 *aṅg*.

Moon's latitude (*śara*) = +0°38'.72.

Summary of the eclipse

Beginning (*sparśa*): 1^h 50^m a.m. (IST)

Middle (*madhya*): 3^h 15^m a.m. (IST)

End (*mokṣa*): 4^h 40^m a.m. (IST)

Half-duration: 1^h 25^m (correct to a minute).

According to Viśvanāthā's *udāharaṇa* on *MKS*, the half-duration (*sthiti*) is 3 | 42 *gh*.

i.e. $1^h\ 28^m 48^s$. There is a difference of $3^m 48^s$ in the half duration. This is due to the approximate values taken in the traditional tables for the related parameters.

Example 10.2: We now consider the example for a solar eclipse given by Viśvanātha in his *udāharaṇa* commentary on *MKS*: *Śaka* 1532, lunar month *Mārgasīrṣa kṛṣṇa* (dark fortnight) 30, Wednesday, 11 | 59 *gh*. This traditional date corresponds to **December 15, 1610 AD** (Gregorian). The parameters of the participating bodies at the instant of the new moon are as follows:

True *nirayaṇa* Sun, $S = 8^R 05°26'20''$

Lagna (ascendant), $L = 11^R 02°05'34''$

(i) *Sūryabimba*: From the related table Viśvanātha obtains the Sun's angular diameter,

Sūryabimba = 11 | 24 *aṅgulas*.

(ii) *Lambana*: Subtracting 3 *rāśis* (*tribhā*) from *Lagna*, we get:

Tribhonalagna: $8^R 02°05'34'' \equiv TBL$

$$\therefore\ S - TBL : 8^R 05^o 26' 20'' - 8^R 02^o 05' 34'' = 3^o 20' 46''$$

From the table for *lambana* (the longitude component of the lunar parallax), Viśvanātha's obtains *lambana* = 0 | 14 *gh*. corresponding to $S - TBL = 3°20'46''$.

(iii) *Krānti* (declination)

Next, the *krānti* of the Sun is determined. For the given year, *śaka* 1532 (1610 AD) the accumulated amount of precession, *ayanāmśa* = 16°39'54". Adding this to the (sidereal) Sun we get the *sāyana* (tropical) Sun. Thus, we have

Sāyana Sun = 8^R05°26'20" + 16°39'54" = 8^R22°06'14".

Bhujāmśa of *sāyana* Sun = 8^R22°06'14" – 6^R = 82°06'14".

Dividing the *bhujāmśa* by 6, the quotient is 13 and the remainder 4°06'14". Now, from the table for *krānti* (declination), under entries 13 and 14 in the top row against the *koṣṭhaka* readings are respectively 3 | 54 | 26 and 3 | 58 | 36 in *ghaṭīs*. Now, by the rule of proportions, the *krānti* for the above obtained *bhujāmśa* of the *sāyana* Sun comes out as 3 | 57 | 20 *gh*. Multiplying this result by 6, we get *krānti* degrees (*bhāgāḥ*) as 23°44'. Since *sāyana* Sun > 6 *rāśis*, declination δ is negative i.e. δ = – 23°44'. Note that classical Indian astronomers always took the Sun's maximum declination as 24°.

A folio from *TYGMS* giving Sun's *krānti* is shown in fig. 10.2.

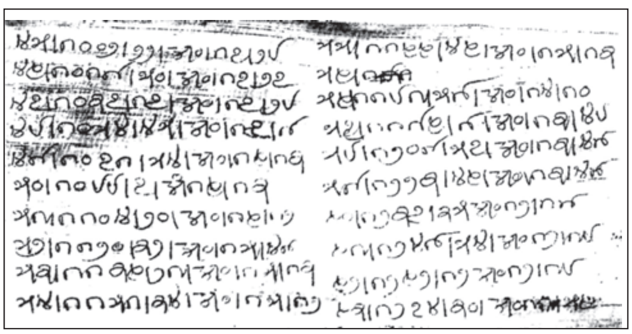

Fig. 10.2: *Krānti* (declination) table of the Sun, a folio from *TYGMS*

Table 10.4: A sample of Sun's *krānti* according to *TYGMS*

λ	*Krānti* = δ	Difference
45°	1002' 22"	17' 28"
50°	1088' 7"	16' 13"
55°	1199' 46"	15' 13"
60°	1237' 35"	12' 19"

Example: Suppose Sun's tropical longitude λ = 45°.

According to Table 10.4, δ = 1002' 22". Putting λ = 45° and taking ε = 24° (the traditional value) in the expression

$$\delta = \sin^{-1}(\sin\varepsilon \sin\lambda)$$

we get δ = 1002' 52" 55"'.02. We see that the value of δ by *TYGMS* is close to the actual value with in an error of 30".

Remark: We have the expression for the declination δ of the Sun:

$$\sin\delta = \sin\varepsilon \sin\lambda$$

Now, taking ε = 24° and the tropical longitude of the Sun,

λ = 8^R22°06'14" i.e. 262°06'14". we get δ = -23°45'30". However, with the better value ε = 23°.5, δ = –23°15'50".

Viśvanātha determines the Sun's *krānti* by another method. Now, *sāyana*

Sun = 8^R22°06'14". Subtracting this from one revolution (*bhagaṇa*) i.e. 12^R, we have

12^R – 8^R22°06'14" = 3^R07°53'46" i.e. 97°53'46". Dividing this by 6, we get the quotient (*labdhi*) 16 and the remainder 1°53'46". By the rule of proportions Viśvanātha obtains the Sun's declination as 23°44', in its numerical value, the same as the one obtained earlier. Further, he refines this value to get δ = –23°44'58".

(iv) *Candraśara* (Moon's latitude): We have the *bhuja* of *virāhucandra* = 7°43'46". Although Viśvanātha has not given explicitly Rāhu's longitude, he seems to have taken it as 2^R13°10'06". In that case we have *virāhucandra*, *VRCH* = 8^R05°26'20" – 2^R13°10'06" = 5^R22°16'14". *Bhuja* of *VRCH* = 6^R – 5^R22°16'14" = 7°43'46".

From the *śara* table, for *bhuja* 7°43'46", the Moon's latitude (*śara*) comes out as 36'16". Since *VRCH* < 6^R, the *śara* is positive. Dividing this *śara* in *kalās* (arcminutes) by 3 we get *śara* ≈ 12 | 05 *aṅgulas*. Viśvanātha stops his example here and expects the readers to continue working as per the of the

Remark: It is interesting that Viśvanātha Daivajña works out the same example in his *udāharaṇa* commentary on Gaṇeśa Daivajña *Grahalāghavam* (epoch: March 19, 1520 AD). We summarize the result for comparison. For the given date, *cakra* = 8, *varṣagaṇa* = 90 and *ahargaṇa* = 1005. Here *cakra* is a cycle of 4016 days, close to 11 sidereal solar years.

At the instant of new moon i.e. at 13 | 04 *gh*. after sunrise, we have

True Sun = True Moon = $8^R05°26'.4$ and *Rāhu* = $2^R11°41'.3$;

Natāmśa = δ – φ = – 49°04'52" where δ = – 23°38'10" the declination of *vitribhalagna* and φ = 25°26'42", the latitude of Vāraṇāsī (Kāśī). From this, the *lambana* = 0 | 11 *gh*. so that the apparent conjunction of the Sun and the Moon, *spaṣṭa darśānta* is at 12 | 53 *gh*. after sunrise. The mean half duration (*sthiti*) is 2 | 44 *gh*. Finally, the beginning (*sparśa*), the middle (*madhya*) and the end (*mokṣa*) timings are respectively 9 | 03 *gh*., 13 | 04 *gh*. and 16 | 44 *gh*. after the local sunrise at (Kāśī).

Conclusion

In the present paper we have discussed the different aspects of Indian astronomy and calendrical system like (i) planets' true positions involving *manda* and *śīghra* equations, (ii) *tithi* and *nakṣatra*, (iii) eclipses involving *krānti* (declination) and *śara* (latitude) using various tables of the *saura pakṣa* like *Makaranda sāriṇī*, *Pratibhāgī* and *Tyāgarti* manuscripts.

These tables are based on the popular Sanskrit treatise, *Sūryasiddhānta*. We find that these tables yield close values. Interestingly *MKS* simplifies the procedure for a true planet by reducing the steps of successive corrections from four (as in *SS*) to only three by composing separate tables of *mandaphala* by consolidating the two conventional ways of applying the *manda* equation twice. The traditional Hindus were required to perform their daily rituals and observances by declaring the daily *tithi* and *nakṣatra* etc. This purpose was adequately served by using the *sāriṇī* (tables) rather than using the main metrical texts of *siddhāntas* and *karaṇas*.

The very fact that the traditional priestly class had the practice of declaring the daily calendrical details at the time for thousands of years implies that they had simple algorithmic procedures without using the texts every time.

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