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BOOK REVIEW

Sita Sundar Ram — ***Bījapallava of Kṛṣṇa Daivajña: Algebra in Sixteen Century India – A Critical Study***, The Kuppuswamy Sastri Research Institute, Chennai-600004, First Edition, 2012; Pages xxiv +287, price Rs.400/-.

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In this era of science and technology, it is relevant to bring to light the contributions made by ancient and medieval texts and commentaries written in the traditional Sanskrit language. The KSRI (Kuppyswami Sastri Research Institute, founded in 1945) has been doing pioneering work in promoting Sanskrit learning and research on ancient as well as modern lines. Editions of early Sanskrit texts on Indian exact sciences have been brought out by it from time to time including the famous *Vākyakaraṇa* (with Sundararāja's commentary). Recently it has brought out the text and translation of Śaṅkara Varman's *Sadratnamāla* which is almost the last noteworthy work of the Late Āryabhaṭa School of South India.

The book under review is the published version of Mrs. Sita Sundar Ram's thesis which was awarded Ph.D. degree by the University of Madras, Chennai. The thesis was prepared under the supervision of Dr. V Kameswari (the Director of KSRI) and was submitted in 2008. Dr. Sita's qualifications as B.Sc. (Mathematics) and M.A. (Sanskrit) were suitable for the involved research work.

The main body of the book consists of seven chapters which are as follows:

1. A Brief Survey of Indian Mathematics (pp.1-17).
2. Six Mathematical Operations (18-38).
3. *Kuṭṭaka* — Linear Equations of First Degree (39-70)
4. *Varga-prakṛti* — *Cakravāla* (Indeterminate Equations of the Second Degree – Cyclic Method) (71-118)
5. *Ekavarṇa Samikaraṇa* – *Madhyamakaraṇa* (119-146).
6. *Anekavarṇa* – *Samīkaraṇa* – *Madhyamakaraṇa* – *Bhāvita* (147-215).
7. *Kṛṣṇa's* Erudition : An appraisal (216-231).

Then follow an equal number of appendices. Of these, the first six are short and on miscellaneous topics. Appendix VII (pp. 248-268) is entitled *Sūtras* (text) of *Bījagaṇita*. It consists of the Sanskrit text (in *devanāgari*) of 219 numbered items which have various rules etc. of Bhāskara II's Algebra along with its introduction and epilogue/colophon (size of the items vary from few words to four lines). In the end there is the Selected Bibliography and a General Index. In the beginning of the book there are Foreword (by Takao Hayashi), Appreciation (by MS Rangachari), Preface (by V. Kameswari) and the Author's Note (pp. xi-xvi). The table on some Illustrious Indian Mathematicians (p. xxiv) is not listed or mentioned in the Contents.

Sita was registered for Ph.D. in 2001 and the thesis was submitted seven years later. The long intervening period for study and research may lead one to expect that the resulting doctoral would be quite elaborate and comprehensive. The source material consulted by her (as reflected in the bibliography at the end) does not lead so. The commentary (about 1600 AD or earlier) of Kṛṣṇa (son of Ballāla) on the *Bījagaṇita* (Algebra) of Bhāskara II (12th century AD) has been very popular among scholars both ancient and modern.

It is usually called *Bījapallava*, but often it is given other names such as *Bijāṅkura*, *Navāṅkura* and *Kalpalatāvatāra*. Three printed editions of the commentary are known. These are as follows:

i) Poona edition (1930) which used about half of a dozen manuscripts (oldest being dated 1704 AD).

ii) Tanjore edition (1958) (=*BP* of the present book). It is said to use a transcribed copy of the local M. Sarfoji's Saraswathi Mahal Library ms. D 11523 (which is the earliest reported ms. dated Kāśi, Śaka 1523 = 1601 AD).

iii) Jammu edition (1982) which follows Poona ed.

By now about four score manuscripts of Kṛṣṇa's *Bījapallava* commentary are known (see D. Pingree's famous *Census of the Exact Sciences in Sanskrit* or *CESS*, Series As, Vol. II, 1971, pp. 53-54). And a serious researcher is expected to consult at least a few fresh manuscripts. Sita says (p. xiii) that Kṛṣṇa "has carefully read more than one manuscript of *Bījagaṇita*". But for her own study of *Bījapallava*, she lists only one manuscript that was already used for Tanjore edition (=BP).

Let us take an example of crucial reading. The Poona (p.39) and Jammu (p.48) editions both have the correct reading

… *ācāryeṇa pāṭyāmuktam* etc. etc.

This refers to Bhāskara's *Pāṭi* (=*Līlāvatī*) and the end part of the passage shows that Kṛṣṇa wrote a commentary on it. The result is a new (5th) work to Sita's list of Kṛṣṇa's four works (pp.13-14). But the Tanjore edition (*BP* p.56) spoils the matter by having the reading '*pādyām*' instead of '*pāṭyām*'!

It is surprising to find that Sita (p.10) cites a poor (and uncritical) list of six commentaries of *Bījagaṇita* while a bigger, better and more consolidated list of about a dozen commentaries is available in a local book (see the famous *New Catalogues Catalogorum* or NCC. Vol. XIV, Univ. of Madras, 2001, under *Bījagaṇita* , pp 11-13).

Chapter II of the book covers the six fundamental operations namely addition, subtraction, multiplication, division, squaring and extraction of square-root. Both positive and negative numbers are considered. Of course, Indians were aware of the fact that square root of a negative number does not exist (*navidyate*) among real numbers.

Kṛṣṇa's commentary has good exposition of N/*d* when *d* tends to zero ($N \neq 0$) and illustrates the equivalence of a/0 and b/0 ($a \neq b$) by using an astronomical example (p.29). But care is needed in giving credit for stating rules and results. The rule of ^{n}Cr (combinations of *n* things taking *r* at a time), here credited (p.3) to Mahāvīra (9th cent. AD), already appears (in equivalent form) a century earlier in Śrīdhara's *Pāṭigaṇita* (rule 72). Bhāskara II's sign reversal rule (p.25)

$$0 - (\pm a) = \mp a$$

is already found a century earlier in a work of Śrīpati (see R C Gupta's quoted paper p.153).

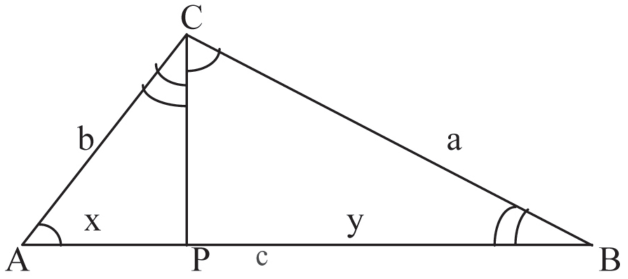
Chapter IV deals elaborately with the indeterminate equation (called *varga prakṛti)*

$$Nx^2 + 1 = y^2, \text{ N being non-square.}$$

Solution of this by using the cyclic method (*cakravāla*) is regarded as "the crowning glory of Indian mathematics" (p. xii). The French mathematician Pierre de Fermat sent the above equation with N = 61 as a challenge to Bernard Frenicle de Bessay in February 1657 and then to William Brouncker and John Wallis. But, as pointed out in the book (p.93), the same equation had been solved in India more than five centuries earlier!

According to Wallis's *A Treatise of Algebra Both Historical and Practical* (London, 1685, p. 363) Fermat also challenged mathematicians of Europe to solve the equation for N=109, 149 and 433. Brouncker solved the three cases and also N=151 and 313 cases. The last case solution is fully mentioned by Sita (p.93). The German historian of mathematics Herman Hankel has praised the Indian cyclic method by saying that "it is certainly the finest thing achieved in the theory of numbers before Lagrange" (p.118). Dr. Sita's expositions and quotations in this chapter IV are quite good and inspiring. She could also point out some lapses on the part of Indian scholars in this context (p.107).

The present book seems to skip over the very short Indian proof of the Pythagoras theorem essentially found in Bhāskara II. This is given by Kṛṣṇa in equivalent more general way (*BP*, p.197). Briefly it is as follows. In a right angled triangle ABC, the perpendicular CP on the hypotenuse AB (= c) makes x and y (see Fig.) as segments.



From similar triangles APC and ACB, we have AP/AC = AC/AB

or, $x / b = b / c$. i.e. $x = b^2 / c$.

From similar triangles BPC and BCA, we have BP/BC = BC/AB

or, $y / a = a / c$. i.e. $y = a^2 / c$

By adding these, we easily get

$x + y = (a^2 + b^2) / c$

and the theorem follows because $x + y = c$

The *BP* (pp.198-199) also outlines the traditional proof (*rūdhi-gaṇita*) of the same theorem based on assembly and dissection of figures. This proof appeared earlier in Bhāskara I's commentary (7th century A.D.) on the *Āryabhaṭīya* and also in Bhāskara II's commentary on his own Algebra.

As far as the algebraic notation is concerned, India was in the second stage of development during the ancient and medieval times (the three stages are rhetoric, syncopation, and symbolic). Indians used the first letter of the technical words usually. Kṛṣṇa gives an interesting explanation (p.31) as to why *bhā* (from *bhāvita*, 'multiplied') was chosen to denote the product of any two unknown. Thus we have (p.32)

याकाभा for या × का (or $x \times y$)

and कानीभा for का × नी (or $y \times z$)

(*yā* comes from *yāvat-tāvat*, *kā* from *kālaka*, *nī* from *nīlaka*)

However, small circle placed between two unknowns has been also used to denote their product i.e. $a \cdot b$ is used for $a \times b$ (see Poona edition p.148; Tanjore edition p.199). Interestingly, in next few years, this small circle (o) was replaced simply by a dot (.) as we find in the works of Munīśvara and Kamalākara, and even the dot was often omitted (as we do in modern times when we write *ab*).

The last chapter in the present book is quite interesting. It presents lot of information useful for historians, literary persons and general readers.

The printing and get-up of the book are good. The beautiful jacket is quite attractive. On the whole we have a fine book for Indologists as well as for Historians of Science.