Precise Determination of the Ascendant in the *Lagnaprakaraṇa* - II

Aditya Kolachana[a,*], **K. Mahesh**[b], **K. Ramasubramanian**[b]

[a]Indian Institute of Technology Madras.
[b]Indian Institute of Technology Bombay.

(Received 04 February 2020)

Abstract

Authored by the celebrated mathematician-astronomer Mādhava, the *Lagnaprakaraṇa* is an important astronomical text dedicated to the determination of the *udayalagna* or the ascendant, and is notable for its technical brilliancy and multiple approaches to a given problem. In continuation of our previous papers on this text, here we discuss some more methods for precisely determining the *udayalagna* as described in the second chapter of the *Lagnaprakaraṇa*.

Key words: Ascendant, *Dṛkkṣepajyā*, *Dṛkkṣepakoṭikā*, *Kālalagna*, *Lagna*, *Lagnaprakaraṇa*, Mādhava, *Rāśikūṭalagna*, *Rāśikūṭaprabhā*, *Udayalagna*.

1 Introduction

In our previous paper[1] we discussed the first method described by Mādhava for precisely determining the ascendant in the *Lagnaprakaraṇa*. There, the procedure involved first determining the *dṛkkṣepalagna* or the nonagesimal, and then determining the *udayalagna* therefrom. In this paper, we discuss three other methods described by Mādhava for precisely determining this quantity. These methods, described in the second chapter of the *Lagnaprakaraṇa*, involve determining the ascendant from (i) the *rāśikūṭalagna*, (ii) the *unmaṇḍalalagna*, and (iii) the *kālalagna*. It may be noted that this paper is to be read in conjunction with our earlier paper as various physical and mathematical quantities described therein are employed here as well. Thus, readers are directed to our previous paper for all references to verses 31–38 in the current paper.

In the following discussion, it may be reiterated that we employ the symbols λ, α, δ, and z to respectively refer to the longitude, right ascension, declination, and zenith distance of a celestial body. The *kālalagna*, the latitude of the observer, and the obliquity of the ecliptic are denoted by the symbols α_e, ϕ, and ϵ respectively. It may also be mentioned that all the figures in this paper depict the celestial sphere for an observer having a northerly latitude ϕ. In these figures, N, S, E, and W denote the cardinal directions north, south, east, and west, while P and K denote the poles of the celestial equator and the ecliptic respectively.

2 Determining the ascendant from the *rāśikūṭalagna*

In this section, we discuss the first of the three methods to be discussed in this paper for determining the *udayalagna*, as outlined in five verses (39–43) of the second chapter of the *Lagnaprakaraṇa*. The procedure involves determining the *udayalagna* from the *rāśikūṭalagna*, the *dṛkkṣepajyā* and other quantities. To this end, verses

DOI: 10.16943/ijhs/2020/v55i1/152339

***Corresponding author: aditya.kolachana@gmail.com**

[1]See [9]. Also see [8], [10], and [7] for our discussion on some of the foundational astronomical parameters described by Mādhava in the first chapter of the *Lagnaprakaraṇa*.

39–41 describe an alternate[2] method of determining the *dṛkkṣepajyā*, while verses 42 and 43 prescribe two relations for calculating the *udayalagna* from the *rāśikūṭalagna*, using the *dṛkkṣepajyā* and other quantities. Readers may refer to our previous paper for further details on the *rāśikūṭalagna*.

2.1 Alternate expression for the *dṛkkṣepajyā*

यद्वा कालविलग्नतः पुनरितः दोःकोटिजातौ गुणौ
नीत्वा बाहुगुणादपक्रमगुणं तत्कोटिमप्यानयेत् ।
तस्मात्कोटिगुणात्परापमहतात् दोःक्रान्तिकोट्या हृता
क्रान्तिर्मध्यविलग्नबाहुजनिता तस्याश्च कोटिं नयेत् ॥३९॥
पलगुणमध्यक्रान्त्योः अन्योन्यकोटिकाभिहतयोः[3] ।
विस्तृतिदलसंहृतयोः[4] कर्किमृगाद्योः
क्रमेण वियुतियुती ॥४०॥
कालभुजापमकोट्या हत्वा त्रिज्याहृतोऽत्र दृक्क्षेपः ।
याम्यः स च विज्ञेयो भेदे तु क्रान्तिजेऽधिके सौम्यः[5] ॥४१॥

yadvā kālavilagnataḥ punaritaḥ
doḥkoṭijātau guṇau
nītvā bāhuguṇādapakramaguṇaṃ
tatkoṭimapyānayet |
tasmātkoṭiguṇātparāpamahatāt
doḥkrāntikoṭyā hṛtā
krāntirmadhyavilagnabāhujanitā
tasyāśca koṭiṃ nayet ||39||
palaguṇamadhyakrāntyoḥ
anyonyakoṭikābhihatayoḥ |
vistṛtidalasaṃhṛtayoḥ karkimṛgādyoḥ
krameṇa viyutiyutī ||40||
kālabhujāpamakoṭyā
hatvā trijyāhṛto'tra dṛkkṣepaḥ |
yāmyaḥ sa ca vijñeyo
bhede tu krāntije'dhike saumyaḥ ||41||

Or, again having computed the Rsine and Rcosine from this *kālalagna*, one should compute the Rsine of the declination from the Rsine of the *bāhu* [of the *kālalagna*] and also the Rcosine of that [declination] (i.e. *doḥkrāntikoṭi* or *kālabhujāpamakoṭi*). That [former] Rcosine multiplied by [the Rsine of] the maximum declination (*parāpama*) is divided by the *doḥkrāntikoṭi*. That [resulting quantity] is [the Rsine of] the declination (*krānti*) obtained from the longitude of the meridian ecliptic point (*madhyavilagna*). One should also compute the Rcosine of that [declination].
The [Rsine of this] declination and the Rsine of the latitude multiplied by the Rcosines of each other and divided by the semi-diameter (*vistṛtidala*) would be subtracted or added [depending on *kālalagna*'s position] in the six signs from Cancer (*karki*) or Capricorn (*mṛga*) respectively.
Having multiplied [that sum or difference] by the *kālabhujāpamakoṭi*, and divided by the radius (*trijyā*), [one obtains the] *dṛkkṣepa*[*jyā*] here. That is to be known to be southern [generally]. It is northern if the Rsine of the declination [of the meridian ecliptic point] is greater [than the Rsine of the latitude] when they differ [in their directions].

The verses above give another relation to determine the *dṛkkṣepajyā*. To this end, the verse instructs to first compute the Rsine and Rcosine of the *kālalagna*, and then to also compute the Rsine and Rcosine of the so called declination from the *kālalagna*, which corresponds to the arc $ER = \mu$ in Figure 1.[6]

Having computed these quantities, as a precursor to determining the *dṛkkṣepajyā*, the verse gives the following relation to obtain the Rsine of the declination of the meridian ecliptic point, known as the *madhyakrāntijyā*:

$$\textit{madhyakrāntijyā} = \frac{\textit{kālalagna-koṭiguṇa} \times \textit{parāpamajyā}}{\textit{doḥkrāntikoṭi}}$$

or,

$$R \sin \delta_m = \frac{R \cos \alpha_e \times R \sin \epsilon}{R \cos \mu}. \qquad (1)$$

From this we are asked to determine $R \cos \delta_m$, and to use these quantities to determine the *dṛkkṣepajyā* as follows:

$$\textit{dṛkkṣepa} = \Big(\textit{kālabhujāpamakoṭi} \times \big[\textit{palaguṇa} \times \textit{madhyakrāntikoṭi} \pm \textit{palakoṭi} \times \textit{madhyakrāntiguṇa}\big] \div \textit{vistṛtidala}\Big) \div \ \textit{trijyā}$$

[2]See our previous paper for the first relation given in the *Lagnaprakaraṇa* for this quantity.

[3]Manuscripts read कोटिकाहतयोः. However, this does not fit any known metre. We have emended the text to satisfy the metrical constraints of the *udgīti* metre, without any change in meaning.

[4]Manuscripts read विस्तृतिदलसंहतयोः. We have emended this likely scribal error as the mathematical procedure requires division by R and not multiplication.

[5]Manuscripts read सौम्यम्. The scribal error is evident as *saumya* is used as an adjective to the masculine noun *dṛkkṣepa* in the verse.

[6]We have shown one method of determining $\sin \mu$ in our discussion of verse 36. Another method is shown below.

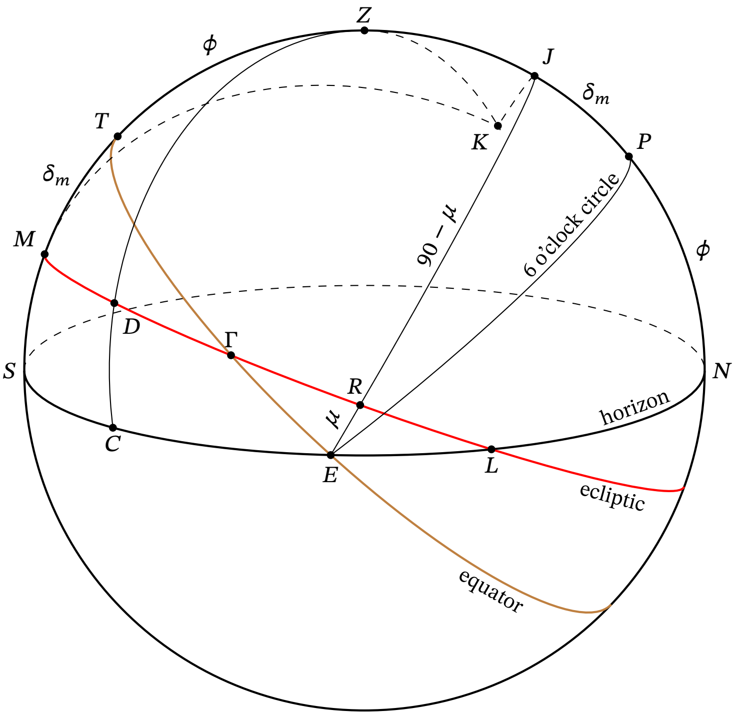


Figure 1 Visualising the *dṛkkṣepa*.

or,

$$R\sin z_d = \frac{R\cos\mu \times \frac{R\sin\phi \times R\cos\delta_m \pm R\cos\phi\times R\sin\delta_m}{R}}{R} \qquad (2)$$

$$= \frac{R\cos\mu \times R\sin z_m}{R}, \qquad (3)$$

where z_m is the zenith distance of the *madhyalagna*, discussed in our previous paper.

Obtaining the *madhyakrāntijyā*

The expression for the *madhyakrāntijyā* given in (1) can be easily verified from the spherical triangle ΓER in Figure 1. Noting that the point R in this triangle is the *rāśikūṭalagna*, we have

$$\Gamma\hat{R}E = 90, \qquad \text{and} \qquad \Gamma R = \lambda_m + 90.$$

Also, by definition

$$\Gamma E = \alpha_e, \qquad E\hat{\Gamma}R = \epsilon, \qquad \text{and} \qquad ER = \mu.$$

Applying the sine rule in this triangle, we obtain

$$\sin\mu = \sin\alpha_e \sin\epsilon, \qquad (4)$$

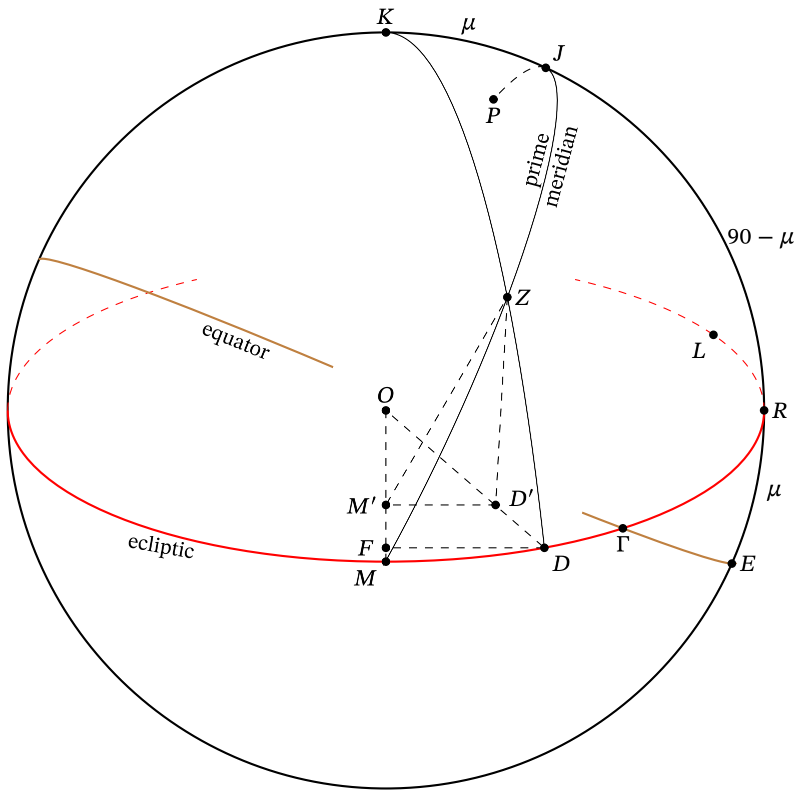
$$\text{or,} \qquad R\sin\mu = \frac{R\sin\alpha_e \times R\sin\epsilon}{R}, \qquad (5)$$

where α_e and ϵ are both known. Comparing (4) with the expression for the sine of the declination of the Sun given by $\sin\delta = \sin\lambda\sin\epsilon$, one can view the above result as the sine of the 'declination' derived from the sine of the *kālalagna* instead of the sine of the longitude of the Sun. Therefore, the expression $\sin\alpha_e\sin\epsilon$ is referred to as the 'sine of the declination derived from the *bāhu* (of the *kālalagna*)'[7] in verse 39. Having determined $R\sin\mu$, the verse prescribes to also determine the cosine $R\cos\mu$, which is easily done using basic trigonometry. This cosine is referred to as the *doḥkrāntikoṭi* in verse 40 and as the *kālabhujāpamakoṭi* in verse 41, which can be translated as 'the cosine of the declination derived from the *doḥ* or *bhujā* of the *kālalagna*'.

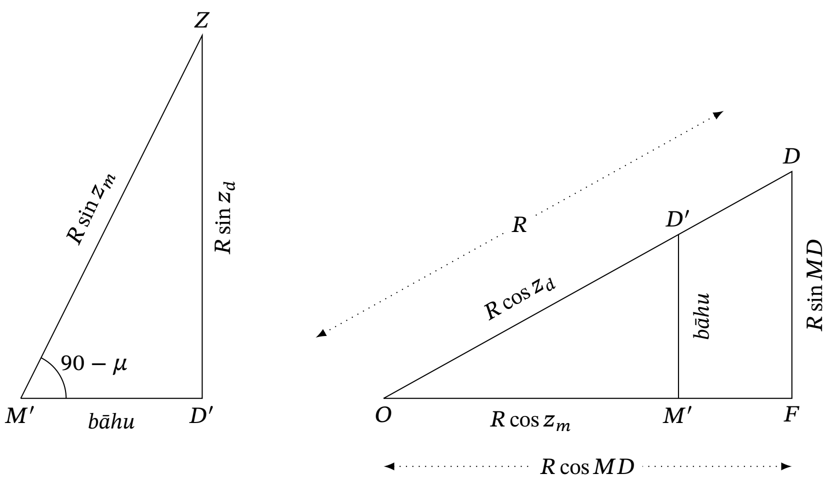
Now, applying the cosine rule in the spherical triangle ΓER, we have

$$\cos\alpha_e = \cos(\lambda_m + 90)\cos\mu,$$

[7]It may be noted that a similar usage has been encountered in verse 36 (see our previous paper), where the term *koṭikrānti* refers to the 'declination' derived from the cosine of the longitude of the *madhyalagna*.



(a) Visualising the *dṛkkṣepa* from the point of view of the ecliptic plane.



(b) Planar triangles used to determine the *dṛkkṣepajyā* and the *dṛkkṣepalagna*.

Figure 2 Determining the *dṛkkṣepajyā* and the *dṛkkṣepalagna*.

or,

$$\sin\lambda_m = \frac{-\cos\alpha_e}{\cos\mu}.$$

Now, since $\sin\delta_m = \sin\lambda_m \sin\epsilon$, we have

$$\sin\delta_m = \frac{-\cos\alpha_e \times \sin\epsilon}{\cos\mu},$$

which differs from (1) only with regards to the sign. As the author discusses the application of the sign later in the second half of verse 40, while giving the relation for the *dṛkkṣepajyā*, we too discuss this towards the end of the next section.

Determining the dṛkkṣepajyā

The given relation for the *dṛkkṣepajyā* can be readily understood from the right-angled triangle $ZM'D'$ in Figure 2b,[8] where it can be seen that

$$R\sin z_d = R\sin z_m \times \cos\mu,$$

which is the same as (3). The expression for $R\sin z_m$ given in the verse as

$$R\sin z_m = \frac{R\sin\phi \times R\cos\delta_m \pm R\cos\phi \times R\sin\delta_m}{R},$$

is the same as the one given in verse 35, discussed in our earlier paper. As explained in our discussion of the *madhyajyā* there, the terms in the above expression are added or subtracted depending upon whether the equator and the zenith are on the same or opposite sides of the ecliptic. Here, an equivalent rule is given, which states that the terms of the above expression have to be subtracted or added depending upon whether the *kālalagna* is in the six signs starting from Cancer or Capricorn respectively. That is, the second term is to be subtracted from the first when the *kālalagna* is in the range of 90 degrees to 270 degrees, and added when it is in the range of 270 degrees to 90 degrees. This rule can be understood purely from physical considerations, or through mathematical analysis.

Physically, the *madhyalagna* has southern declination when the *kālalagna* is in the range of 270 degrees to 90 degrees, and northern declination otherwise. An instance of the former case is depicted in Figure 3a, where the *kālalagna* (ΓE) is in the first quadrant, and the *madhyalagna* (M) has southern declination. An instance of the latter case is depicted in Figure 3b, where the *kālalagna* is in the third quadrant, and the *madhyalagna* has northern declination. In the former case, it is clearly evident that

$$\sin z_m = \sin(\phi + \delta_m) = \sin\phi\cos\delta_m + \cos\phi\sin\delta_m,$$

while in the latter case, clearly

$$\sin z_m = \sin(\phi - \delta_m) = \sin\phi\cos\delta_m - \cos\phi\sin\delta_m.$$

Approaching the problem purely from a mathematical viewpoint, we observe that the first term ($\sin\phi\cos\delta_m$) in the above expressions is always positive,[9] while the second term ($\cos\phi\sin\delta_m$) can be positive or negative depending upon the sign of the declination.[10] From (1), we see that $\sin\delta_m$ takes the sign of the term $\cos\alpha_e$,[11] which being a cosine function is negative in the range 90 degrees to 270 degrees (Cancer etc.), and positive otherwise (Capricorn etc.). Therefore, the absolute magnitude of the term $\cos\phi\sin\delta_m$ is to be subtracted from or added to the term $\sin\phi\cos\delta_m$ depending upon whether the *kālalagna* lies in those quadrants where the cosine function is negative or positive. This is exactly the rule stated in the verse.

For an observer in the northern hemisphere, the *dṛkkṣepajyā* will generally be seen in the southern hemisphere as shown in Figure 4a. However, for observers at lower latitudes ($\phi < \epsilon$), occasionally the *dṛkkṣepajyā* may appear in the northern hemisphere when the declination of the *madhyalagna* is northwards, and also greater than the latitude of the observer, as shown in Figure 4b. These indeed are the observations that are made in the second half of verse 41.

2.2 Determining the *udayalagna*

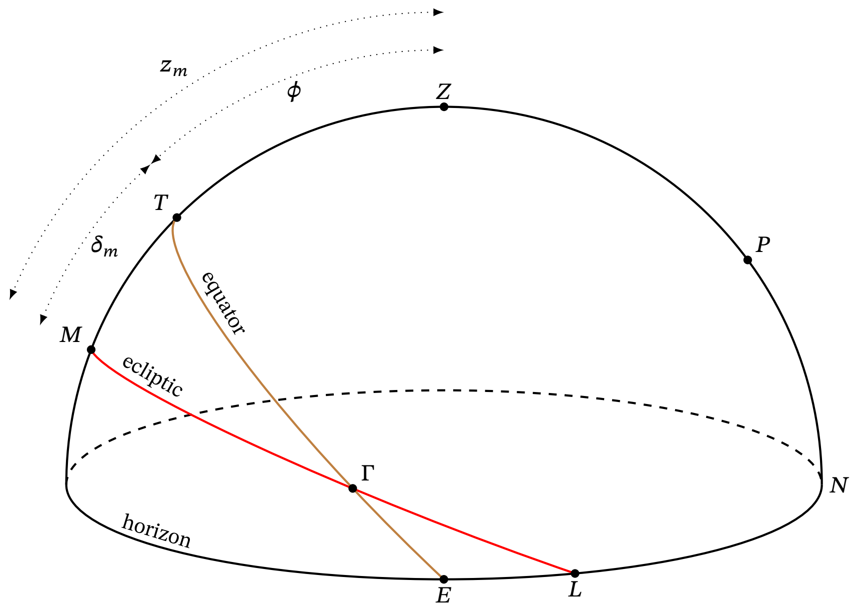
The following two verses give two relations for obtaining the *udayalagna* from the *rāśikūṭalagna*, employing the *dṛkkṣepajyā* and some other quantities derived earlier.

[8]The planar triangles in this figure are the same as those shown in Figure 2a, which depicts the celestial sphere from the point of view of the ecliptic plane. For further details on this figure, see our discussion on verse 36 in our previous paper.

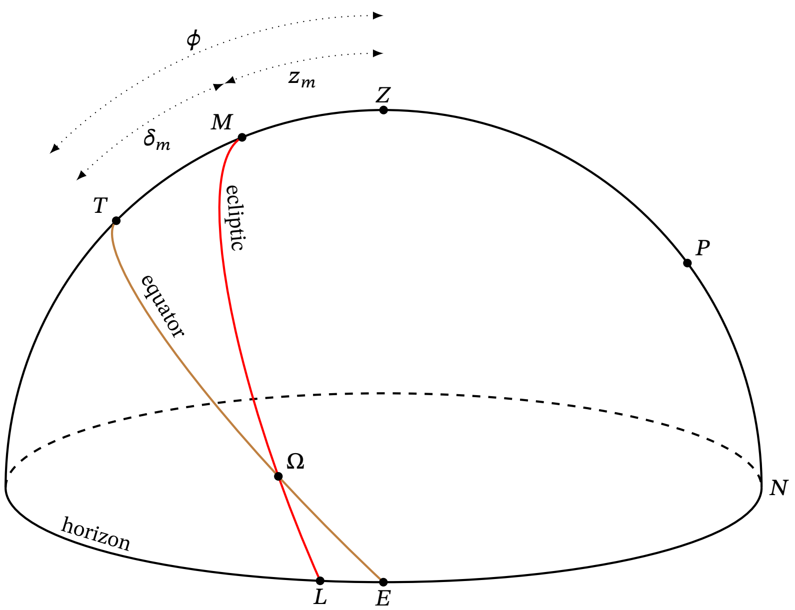
[9]As $0 < \phi < 90$, the term $\sin\phi$ is always positive. Also, as $-\epsilon < \delta_m < \epsilon$, the term $\cos\delta_m$ is also always positive.

[10]Here again, since $0 < \phi < 90$, the term $\cos\phi$ is always positive.

[11]In (1), the term $\sin\epsilon$ is always positive. Also, from (4), we can deduce that $-\epsilon < \mu < \epsilon$. Therefore, the term $\cos\mu$ too is always positive.

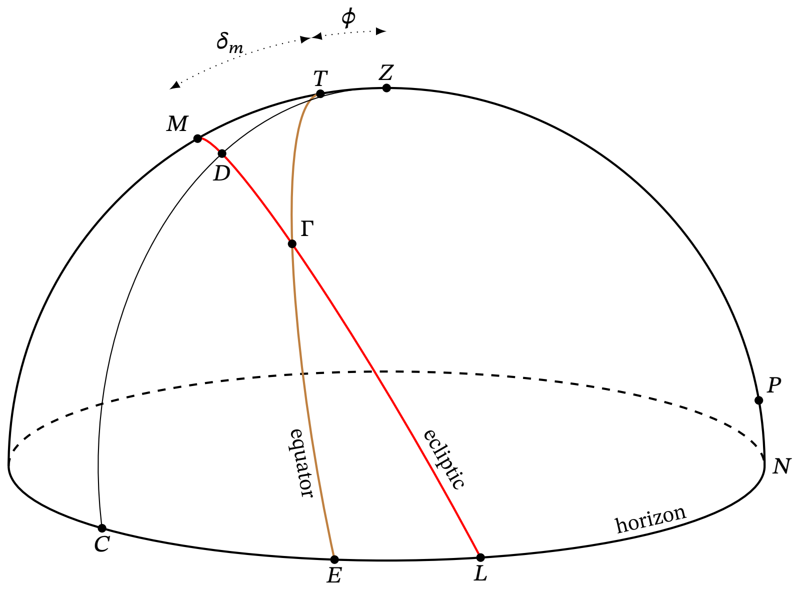


(a) Same direction.

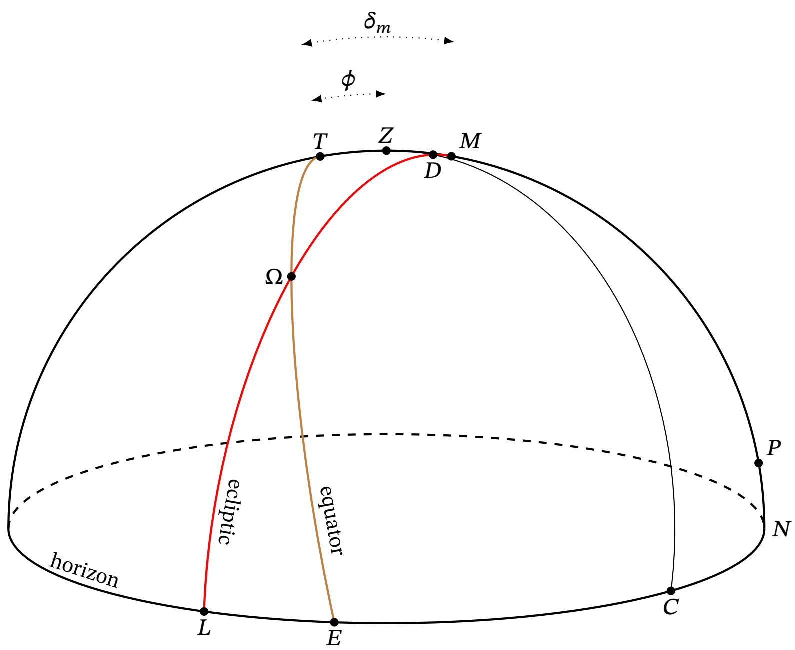


(b) Opposite directions.

Figure 3 The direction of the equator and the zenith with respect to the ecliptic for determining the *madhyajyā*.



(a) Southern hemisphere.



(b) Northern hemisphere.

Figure 4 Direction of the *dṛkkṣepa*.

2.2.1 Method 1

दृक्क्षेपाहतकालबाह्वपमतो दृक्क्षेपकोट्या हृतं
विष्कम्भार्धसमाहतं भुजभवद्युज्याहृतं चापितम् ।
स्वर्णं कालविलग्नके कृतकलाप्राणान्तरे च क्रमात्
दृक्क्षेपापमदिग्भिदैक्यवशतः प्राग्लग्नसंसिद्धये ॥४२॥

dṛkkṣepāhatakālabāhvapamato
dṛkkṣepakoṭyā hṛtaṃ
viṣkambhārdhasamāhataṃ bhujabhava-
dyujyāhṛtaṃ cāpitam |
svarṇaṃ kālavilagnake kṛtakalā-
prāṇāntare ca kramāt
dṛkkṣepāpamadigbhidaikyavaśataḥ
prāglagnasaṃsiddhaye ||42||

[The result] from the division of [the Rsine of] the declination calculated from the *kālalagna* (*kālabāhu-apama*)—which is multiplied by the *dṛkkṣepa*[*jyā*]—by the Rcosine of the *dṛkkṣepa* (*dṛkkṣepakoṭi*), is multiplied by the semi-diameter (*viṣkambhārdha*) and divided by the *bhujabhavadyujyā*. The arc of this [result] is applied positively or negatively to the *kālalagna* which is corrected by the difference in [own] longitude and right ascension (*kalāprāṇāntara*), depending on the difference or sameness in direction of the *dṛkkṣepa* and the declination [from the *kālalagna*] (i.e. *kālabāhu-apama*), for obtaining the orient ecliptic point (*prāglagna*).

This verse gives the following relation for determining the orient ecliptic point or the *udayalagna*, in terms of the *dṛkkṣepajyā* and the 'declination' (μ) derived from the *kālalagna*:

$$\begin{aligned} \textit{udayalagna} &= \textit{kālalagna} \pm \textit{nija-prāṇakalāntara} \\ &\pm \textit{cāpa}\left(\frac{\textit{kālabāhu-apamajyā} \times \textit{dṛkkṣepajyā}}{\textit{dṛkkṣepakoṭi}} \times \frac{\textit{viṣkambhārdha}}{\textit{bhujabhava-dyujyā}}\right) \end{aligned}$$

or,

$$\lambda_l = \alpha_e \pm |\lambda_r - \alpha_e| \pm R\sin^{-1}\left(\frac{R\sin\mu \times R\sin z_d}{R\cos z_d} \times \frac{R}{R\cos\mu}\right). \quad (6)$$

As the procedure described in the verse involves converting the *kālalagna* into the longitude of the *rāśikūṭalagna*, the term *kalāprāṇāntara* in the verse is to be understood as the *nija-prāṇakalāntara* discussed in verse 31.

The term *kālabāhu-apamajyā*[12] is to be interpreted as the sine of the 'declination' (μ) derived using the *kālalagna*, and is equivalent to (5). The term *bhujabhava-dyujyā* in the verse is to be understood as the day-radius corresponding to the declination (μ) derived from the *bhuja* of the *kālalagna*, and is therefore equal to $R\cos\mu$.[13] Finally, the *dṛkkṣepakoṭi* is nothing but the cosine of the zenith distance of the *dṛkkṣepalagna*, or $R\cos z_d$.

With the terms understood in this manner, the above relation can be easily derived as follows. Figures 5a and 5b depict two instances when μ and z_d are on the opposite sides, and the same side of the ecliptic respectively. From Figure 5a it is evident that the longitude (λ_l) of the *udayalagna* (L) is given by

$$\lambda_l = \lambda_r + RL,$$

where λ_r is the longitude of the *rāśikūṭalagna* (R). However, from verse 31, we already know that

$$\lambda_r = \alpha_e \pm |\lambda_r - \alpha_e|.$$

The length of the arc RL can be determined as follows. In the spherical triangle REL, by definition $ER = \mu$. Also, $R\hat{L}E = 90 - z_d$, is the angle between the ecliptic and the horizon.[14] As the eastern cardinal point is the pole for the prime meridian, the angle $R\hat{E}L$ is equivalent to the length of the arc $NJ = NP + PJ$. By definition, $NP = \phi$, while we have already shown $PJ = \delta_m$.[15] Therefore,

$$R\hat{E}L = \phi + \delta_m = z_m,$$

which is the zenith distance of the *madhyalagna*. Applying the sine rule in the spherical triangle REL, we have

$$\sin RL = \frac{\sin\mu \sin z_m}{\cos z_d}. \quad (7)$$

However, from the triangle $ZM'D'$ in Figure 2b, we have

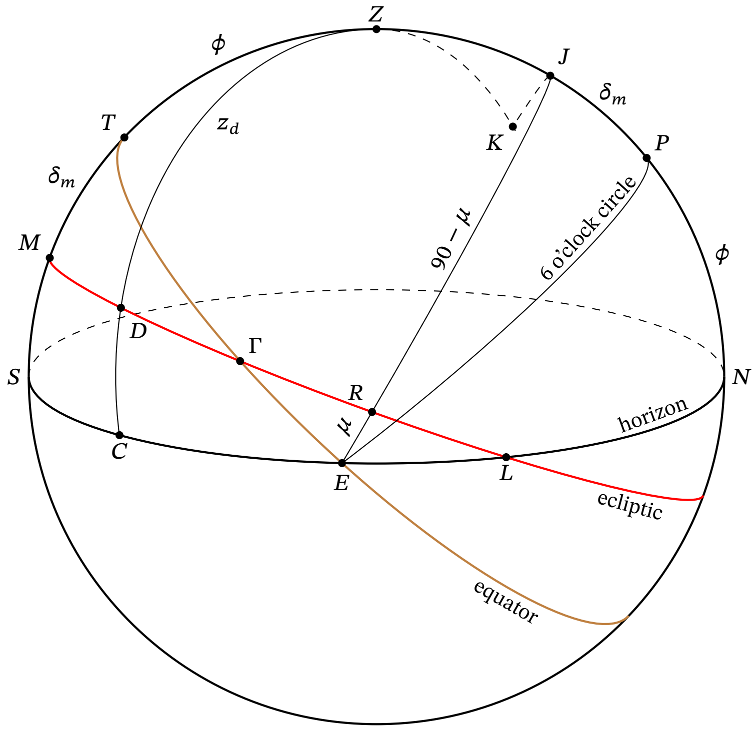
$$\sin z_m = \frac{\sin z_d}{\cos\mu}. \quad (8)$$

[12] It can be derived as कालबाहुना गणिता अपमज्या।

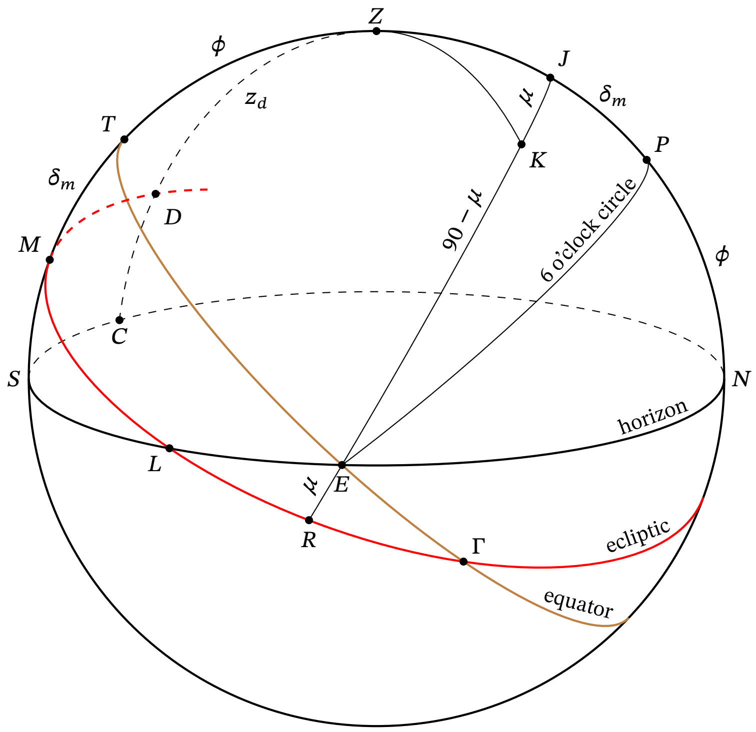
[13] Recalling that the radius of the diurnal circle of the Sun is equal to $R\cos\delta$, when its declination is δ, it is worth noting the similarity of conceptions as well as terminologies coined to describe these parameters.

[14] See (17) in our previous paper.

[15] See our discussion of verse 36.



(a) *Dṛkkṣepajyā* and *kālabāhu-apamajyā* on opposite sides of the ecliptic.



(b) *Dṛkkṣepajyā* and *kālabāhu-apamajyā* on the same side of the ecliptic.

Figure 5 Directions of *dṛkkṣepajyā* and *kālabāhu-apamajyā* in determining the *udayalagna*.

Upon solving for the arc *RL* using the above relation, we obtain the longitude of the *udayalagna* as

$$\lambda_l = \alpha_e \pm |\lambda_r - \alpha_e| + R\sin^{-1}\left(\frac{R\sin\mu \times R\sin z_d}{R\cos z_d} \times \frac{R}{R\cos\mu}\right).$$

When μ and z_d are on the same side of the ecliptic as shown in Figure 5b, then it is evident that the arc *RL* has to be subtracted from the longitude (λ_r) of *R* to obtain the longitude of *L*. Thus the verse states that the arc *RL* is to be added to λ_r when μ and z_d are on opposite sides of the ecliptic, and subtracted when they are on the same side of it. Thus, in general, we have

$$\lambda_l = \alpha_e \pm |\lambda_r - \alpha_e| \pm R\sin^{-1}\left(\frac{R\sin\mu \times R\sin z_d}{R\cos z_d} \times \frac{R}{R\cos\mu}\right),$$

which is the same as (6).

The above expression can also be derived alternatively as follows. Upon combining the relations (13), (14), and (2) from our previous paper, we obtain

$$\lambda_l = \lambda_r \pm R\sin^{-1}\left(\frac{b\bar{a}hu \times R}{R\cos z_d}\right).$$

Further substituting for *bāhu* using (10) of our previous paper and for $\sin z_m$ therein using (8) of this paper, it can be easily seen that we again obtain (6). Comparing the above expression to (13) of the previous paper reveals that the arcs *RL* and *MD* in Figures 5a and 5b are of equal measure. That is, the separation of the *udayalagna* from the *rāśikūṭalagna* is the same as the separation of the *dṛkkṣepalagna* from the *madhyalagna*. This is also directly evident from the fact that both the arcs *MR* and *DL* in this figure are of comparable measure, equal to ninety degrees.[16]

2.2.2 Method 2

दृक्क्षेपाहतकालबाहुजगुणात् दृक्षेपकोट्या हृतात्
अन्त्यक्रान्तिसमाहतात्[17] भुजभवद्युज्याहृतं चापितम् ।
स्वर्णं कालविलग्नके कृतकलाप्राणान्तरे च क्रमात्
दिग्भेदैक्यवशात्तु गुण्यगुणयोः प्राग्लग्नसंसिद्धये ॥४३॥

dṛkkṣepāhatakālabāhujaguṇāt
dṛkkṣepakoṭyā hṛtāt
antyakrāntisamāhatāt bhujabhava-
dyujyāhṛtaṃ cāpitam |
svarṇaṃ kālavilagnake kṛtakalā-
prāṇāntare ca kramāt
digbhedaikyavaśāttu guṇyaguṇayoḥ
prāglagnasaṃsiddhaye ||43||

[The result] from the division of Rsine arising from the *kālalagna*—which is multiplied by the *dṛkkṣepa*[*jyā*]—by the Rcosine of the *dṛkkṣepa* (*dṛkkṣepakoṭi*), is multiplied by [the Rsine of] the last declination (*antyakrānti*) and divided by the *bhujabhavadyujyā*. The arc of this [result] is applied positively or negatively to the *kālalagna* which is corrected by the difference in [own] longitude and right ascension (*kalāprāṇāntara*), depending on the difference or sameness of directions of the multiplicand (*guṇya*) and the multiplier (*guṇa*), for obtaining the orient ecliptic point (*prāglagna*).

In continuation of the rule discussed in the previous verse, this verse gives the following slightly modified alternate expression for obtaining the orient ecliptic point or the *udayalagna*:

$$udayalagna = k\bar{a}lalagna \pm nija\text{-}pr\bar{a}\d{n}akal\bar{a}ntara \pm c\bar{a}pa\left(\frac{k\bar{a}lab\bar{a}hujagu\d{n}a \times d\d{r}kk\d{s}epajy\bar{a}}{d\d{r}kk\d{s}epako\d{t}i} \times \frac{antyakr\bar{a}ntijy\bar{a}}{bhujabhava\text{-}dyujy\bar{a}}\right)$$

or,

$$\lambda_l = \alpha_e \pm |\lambda_r - \alpha_e| \pm R\sin^{-1}\left(\frac{R\sin\alpha_e \times R\sin z_d}{R\cos z_d} \times \frac{R\sin\epsilon}{R\cos\mu}\right). \quad (9)$$

As in the previous verse, the term *kalāprāṇāntara* here too is to be understood as the *nija-prāṇakalāntara* of the *kālalagna*. *Kālabāhujaguṇa* is to be understood as the Rsine of the *kālalagna* or $R\sin\alpha_e$. The terms *bhujabhava-dyujyā* and *dṛkkṣepakoṭi*, explained in the previous verse, and are equal to $R\cos\mu$ and $R\cos z_d$ respectively. The term *antyakrāntijyā* refers to the expression $R\sin\epsilon$. Having understood the terms, it can be seen that (9) can be obtained by simply substituting (5) for $\sin\mu$ in (6).

The sine inverse term in the above expression, as in the case of the expression found in the previous verse, gives

[16] From (2) and (14) of the previous paper respectively.

[17] Manuscripts read अन्त्यक्रान्तिसमाहृतात्. Emended as the relation requires multiplication by $R\sin\epsilon$ and not division.

the length of the arc RL, which is to be added to or subtracted from the longitude (λ_r) of the *rāśikūṭalagna*, to obtain the longitude (λ_l) of the orient ecliptic point. The last quarter of this verse states the conditions for the positive and negative application of the arc RL to λ_r, which though appearing to be different, are equivalent to the conditions stated in the previous verse. This verse states that the arc RL has to be added or subtracted to λ_r depending upon the difference or sameness in the directions of the *guṇya* (multiplicand) and the *guṇa* (multiplier). From the first line of the verse, it is evident that the *guṇa* refers to the *dṛkkṣepajyā* ($R \sin z_d$), which is the multiplier of the quantities $R \sin \alpha_e$ and $R \sin \epsilon$. The product of these two quantities, which from (5) we know to be $R \sin \mu$, is to be considered the *guṇya* here. Therefore, the addition or subtraction of the arc RL to λ_r depends upon the difference or sameness in the directions of $R \sin \mu$ and $R \sin z_d$. This is equivalent to the rule stated in the previous verse, where we have also discussed its validity.

3 Determining the ascendant from the *unmaṇḍalalagna*

In this section we discuss the second of the three methods discussed in this paper for the determination of the *udayalagna*, as outlined in verses 44–49 of the second chapter of the *Lagnaprakaraṇa*. This method involves first determining a quantity known as the *dṛkkṣepakoṭikā* or the *rāśikūṭaprabhā*, the procedure for which is described in verses 44-47. Next, using this result, verse 48 gives the relation for the computation of the longitude of the *unmaṇḍalalagna*, or the point of intersection of the ecliptic and the six o' clock circle. Finally, verse 49 gives the method of calculating the *udayalagna* using the *unmaṇḍalalagna*.

3.1 Determining the *dṛkkṣepakoṭikā* or the *rāśikūṭaprabhā*

काललग्नस्य कोट्युत्थां क्रान्तिमक्षगुणाहताम् ।[18]
लम्बान्त्यद्युज्ययोर्घाते स्वर्णं कर्किमृगादितः ॥४४॥
कृत्वा त्रिजीवया हृत्वा तत्र लब्धा भुजा भवेत् ।
काललग्नस्य बाहूत्था क्रान्तिरेवात्र कोटिका ॥४५॥
अनयोरथ दोःकोट्योः वर्गसंयोगतः पदम् ।
राशिकूटप्रभा ज्ञेया सैव दृक्क्षेपकोटिका ॥४६॥

kālalagnasya koṭyutthāṃ
krāntimakṣaguṇāhatām |
lambāntyadyujyayorghāte
svarṇaṃ karkimṛgāditaḥ ||44||
kṛtvā trijīvayā hṛtvā
tatra labdhā bhujā bhavet |
kālalagnasya bāhūtthā
krāntirevātra koṭikā ||45||
anayoratha doḥkoṭyoḥ
vargasaṃyogataḥ padam |
rāśikūṭaprabhā jñeyā
saiva dṛkkṣepakoṭikā ||46||

Having applied the declination computed from the Rcosine of the *kālalagna*—which is multiplied by the Rsine of the latitude (*akṣaguṇa*)—to the product of the Rcosine of the latitude (*lamba*) and the last day-radius (*antyadyujyā*) positively or negatively depending on [whether the *kālalagna* is in] Cancer (*karki*) etc. or Capricorn (*mṛga*) etc., [the result] is divided by the radius (*trijīvā*). The result there would be the *bhujā*. The declination found from the *kālalagna* itself is the *koṭikā* here. Now, the square-root [taken] from the sum of the squares of these *doḥ* (i.e. *bhujā*) and *koṭi* should be known as the shadow of the pole of the ecliptic (*rāśikūṭaprabhā*). That itself is the Rcosine of the *dṛkkṣepa* (*dṛkkṣepakoṭikā*).

The above verses give a relation to determine the *dṛkkṣepakoṭikā* or the *rāśikūṭaprabhā* ($R \cos z_d$), which has also been referred to previously as *paraśaṅku* in verses 37–38, and as *dṛkkṣepakoṭi* in verses 42–43. The given relation is

$$R \cos z_d = \sqrt{bhuj\bar{a}^2 + ko\d{t}i^2} \tag{10}$$

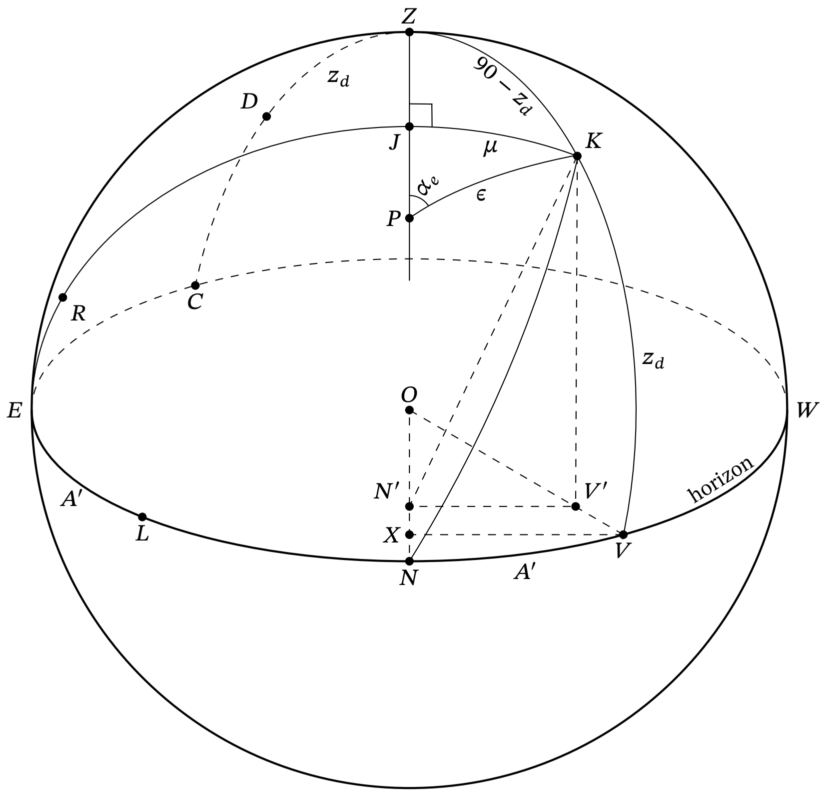
where,

$$\begin{aligned} bhuj\bar{a} &= (lambajy\bar{a} \times antyadyujy\bar{a} \ \pm ak\d{s}agu\d{n}a \times \\ &\qquad k\bar{a}lalagnasya\ ko\d{t}yutth\bar{a}\ kr\bar{a}ntijy\bar{a}) \div trij\bar{\imath}v\bar{a} \\ &= \frac{R \cos \phi \cdot R \cos \epsilon \pm R \cos \alpha_e \sin \epsilon \cdot R \sin \phi}{R}, \end{aligned} \tag{11}$$

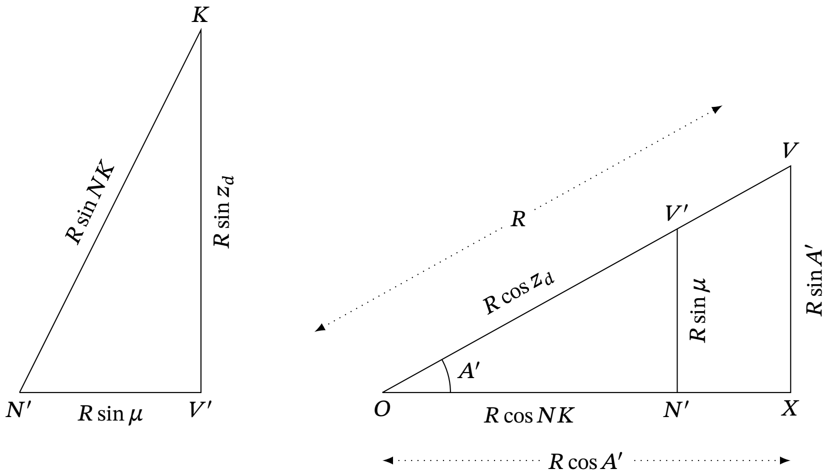
and

$$\begin{aligned} ko\d{t}i &= k\bar{a}lalagnasya\ b\bar{a}h\bar{u}tth\bar{a}\ kr\bar{a}ntijy\bar{a} \\ &= R \sin \alpha_e \sin \epsilon. \end{aligned} \tag{12}$$

[18]Manuscripts read हृताम्. Emended as the relation requires multiplication by $R \sin \phi$ and not division.



(a) Visualising the *rāśikūṭaprabhā*.



(b) The right-angled triangle having the sides *rāśikūṭaprabhā*, *bhujā* and *koṭi*.

Figure 6 Determining the *dṛkkṣepakoṭikā* or the *rāśikūṭaprabhā*.

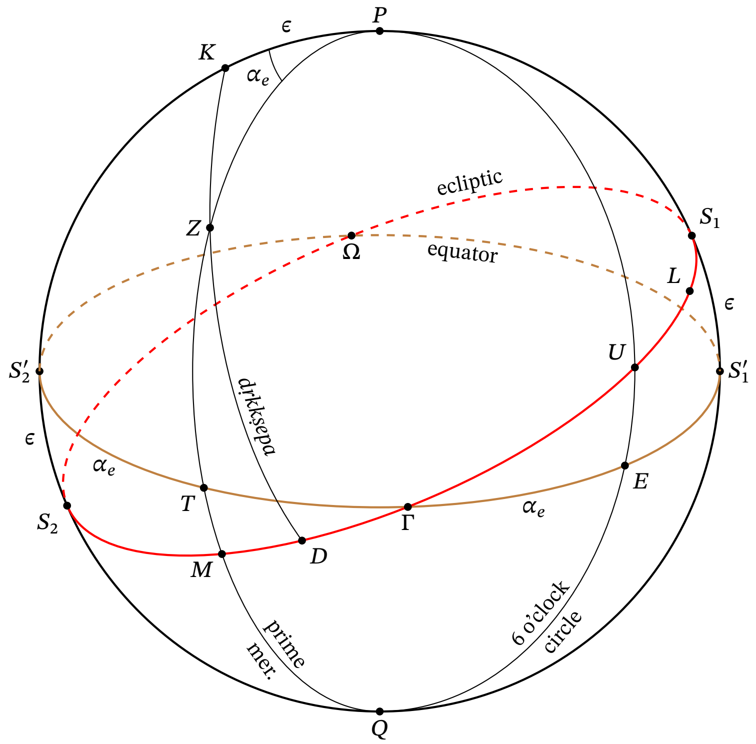


Figure 7 The hour angle of the pole of the ecliptic, and its angular separation from the celestial pole.

From (10), it is clear that the author has visualised a planar right-angled triangle inside the celestial sphere, whose hypotenuse is the desired quantity, i.e. *dṛkkṣepa-koṭikā*, and whose sides are equivalent to the expressions for *bhujā* and *koṭi*. We will show that this is the triangle $V'N'O$ in Figure 6a. This figure is the same as Figure 1, but depicts the northern hemisphere of the celestial sphere. Therefore, in this figure, the great circle arcs $NPJZ$ and $VKZDC$ correspond to the prime meridian and the *dṛkkṣepavṛtta*[19] respectively. It may be noted that the portion of the prime meridian between the points N and P in this figure is not shown so that the desired planar triangles inside the celestial sphere can be seen clearly.

In this figure, the right-angled triangle $KV'N'$, whose side KV' is the gnomon corresponding to the pole of the ecliptic (K), is perpendicular to the horizon. As the arc $KV = z_d$,[20] we have

$$KV' = R\sin z_d, \quad \text{and} \quad OV' = R\cos z_d.$$

Also, as KN' is the semi-chord corresponding to the arc KN, we have

$$KN' = R\sin NK, \quad \text{and} \quad ON' = R\cos NK.$$

In the right-angled triangle $KV'N'$, we now have

$$N'V'^2 = (R\sin NK)^2 - (R\sin z_d)^2,$$

which can be rewritten as

$$\begin{aligned} N'V'^2 &= (R\cos z_d)^2 - (R\cos NK)^2 \\ &= OV'^2 - ON'^2, \end{aligned}$$

which proves that the triangle $V'ON'$ is right-angled at N'. This triangle has the *dṛkkṣepakoṭikā* ($R\cos z_d$) as its hypotenuse.[21] Now, we will show that the sides ON' and $N'V'$ correspond to the *bhujā* and *koṭi* given in the verse.

Determining the *koṭi*

To determine the *koṭi*, consider the great circle arc $KJRE$ in Figure 6a, which is the secondary to the ecliptic from

[19] The great circle passing through the pole of the ecliptic and the *dṛkkṣepalagna*.

[20] The zenith distance of the *dṛkkṣepalagna* is given by $ZD = z_d$. As $KD = ZV = 90$, we have $KZ = 90 - z_d$, and $KV = z_d$.

[21] In the figure, it is possible to conceive of the side OV' to be the 'shadow' of the gnomon (KV') dropped from the pole of the ecliptic. This appears to be reason for calling it the *rāśikūṭaprabhā*.

its pole K, and which also passes through the east cardinal point (E). As E is the pole for the prime meridian, the arc $KJRE$ is perpendicular to the prime meridian at J. Therefore, the semi-chord corresponding to the arc KJ, whose measure is μ,[22] would be the perpendicular distance between K and the plane of the prime meridian. That is,

$$R \sin KJ = R \sin \mu.$$

As the point V' is the image of K on the horizon, and the line ON is the image of the plane of the prime meridian on the horizon, we also have

$$\begin{aligned} N'V' &= R \sin \mu \\ &= R \sin \alpha_e \sin \epsilon, \qquad \text{[using (5)]} \end{aligned}$$

which is the same as (12).

The expression for the *koṭi* can also be alternatively validated as follows. In Figure 6a, as the arcs $NE = VL = 90$, we have the arc $NV = A'$, which is the amplitude of the rising point of the ecliptic. Now, consider the similar triangles $V'ON'$ and VOX in Figure 6b, where $VX = R \sin A'$ corresponds to the Rsine of the arc NV. Applying the rule of proportionality of the sides of similar triangles, we have

$$N'V' = \frac{R \sin A' \times R \cos z_d}{R}. \tag{13}$$

Now, in the spherical triangle REL in Figure 1, where R is the *rāśikūṭalagna*, and L is the *udayalagna*, we have

$$E\hat{R}L = 90, \qquad ER = \mu, \qquad \text{and} \qquad EL = A'.$$

Also, from (17) in our previous paper, the angle between the ecliptic and the horizon $R\hat{L}E = 90 - z_d$. Applying the sine rule in this spherical triangle, we have

$$R \sin \mu = \frac{R \sin A' \times R \cos z_d}{R}. \tag{14}$$

Therefore, from (13) and (14), we have $N'V' = R \sin \mu$, which is the same as (5) and (12). By comparing (14) and (5), we can appreciate how apparently different pairs of physical quantities can give rise to the same result.

[22] The quantity μ has appeared in a number of relations, starting with verse 36, and corresponds to the 'declination' calculated from the *kālalagna*, given by the measure of the arc ER in Figure 1, as well as the expression (5). As K is the pole for the ecliptic on which the *rāśikūṭalagna* (R) lies, and as E is the pole for the prime meridian on which J lies, we have $KR = JE = 90$. Therefore, it can be seen that $KJ = ER = \mu$.

To determine the *bhuja*, we require the hour angle of the pole of the ecliptic, as well as its angular separation from the celestial pole. The method to determine these quantities is discussed next.

Determining the hour angle of the pole of the ecliptic, and also its angular separation from the celestial pole

Figure 7 depicts the celestial sphere from the point of view of the equatorial plane. In this figure, the great circle arcs $PZTMQ$ and $PUEQ$ correspond to the prime meridian and the six o'clock circle respectively. Now, consider the great circle arc $PKS_2'S_2Q$, which is a meridian passing through the pole of the ecliptic (K), and meets the ecliptic at S_2. Naturally, this arc would also be a secondary to the ecliptic, which implies that it is perpendicular to the equator as well as the ecliptic. This is only possible when it intersects the ecliptic at the solstitial point (S_2). As the solstitial point is the point of the maximum declination of the ecliptic, we have $S_2S_2' = \epsilon$. Also, as $KS_2 = PS_2' = 90$, we have $KP = S_2S_2'$. Therefore, the angular separation between the celestial pole and the pole of the ecliptic is given by

$$KP = \epsilon. \tag{15}$$

Now, the hour angle (H_k) of the pole of the ecliptic is given by the spherical angle $S_2'PT$, or the measure of the arc $S_2'T$, where T is the intersection of the prime meridian and the equator. As S_2 is the solstitial point, we have $S_2'\Gamma = S_2\Gamma = 90$. As the east cardinal point is the pole of the prime meridian, we also have $ET = 90$. Therefore, we obtain $S_2'T = \Gamma E$. However, as ΓE is nothing but the *kālalagna* (α_e),[23] we have the hour angle of the pole of the ecliptic

$$H_k = S_2'T = \alpha_e. \tag{16}$$

Determining the *bhujā*

To determine the *bhujā*, consider the spherical triangle NPK in Figure 8a. In this triangle, we have

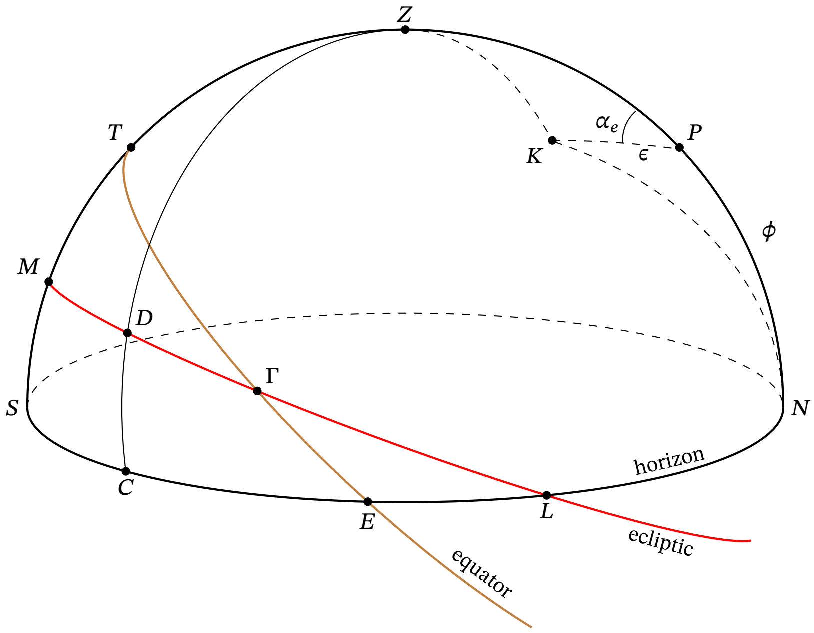
$$PN = \phi, \quad KP = \epsilon, \quad \text{and} \quad K\hat{P}N = 180 - H_k,$$

where H_k is the hour angle of the pole of the ecliptic. As $H_k = \alpha_e$ from (16), we have[24]

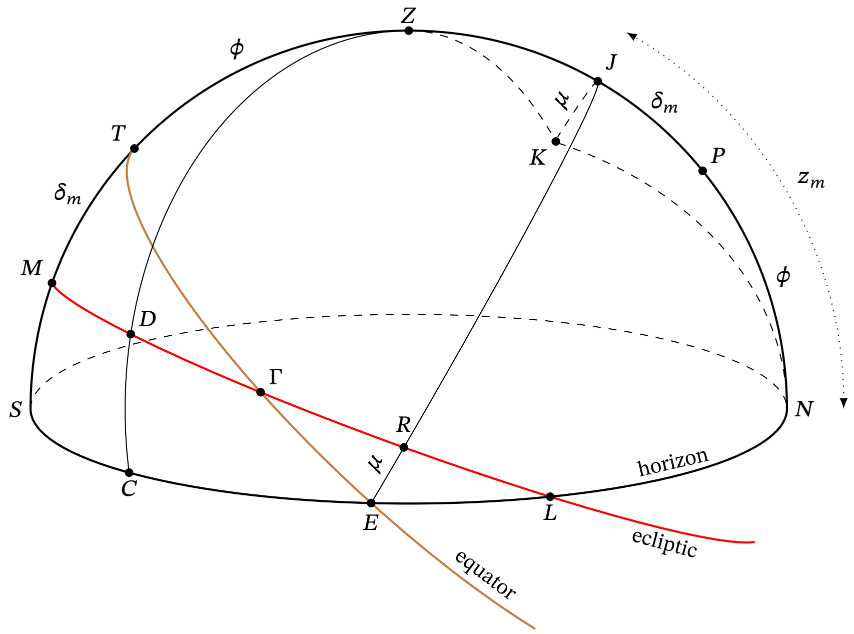
$$K\hat{P}N = 180 - \alpha_e.$$

[23] See our discussion of verse 30 in [7].

[24] In Figure 8a, the pole of the ecliptic is in the western hemisphere. Instead, when the pole of the ecliptic is in the eastern hemisphere, we



(a) Determining the *bhujā* in verses 44–46.



(b) Determining the equivalent *doḥprabhā* in verse 47.

Figure 8 Determining the *bhujā* or the *doḥprabhā* of the *rāśikūṭa*.

Applying the cosine rule of spherical trigonometry in this triangle, we have

$$\begin{aligned}\cos NK &= \cos\phi\cos\epsilon + \sin\phi\sin\epsilon\cos(180-\alpha_e)\\ &= \cos\phi\cos\epsilon - \sin\phi\sin\epsilon\cos\alpha_e.\end{aligned}$$

The function $-\cos\alpha_e$ is positive in the range 90 to 270 degrees, and negative in the range 270 to 90 degrees. Thus, the above relation can be written as

$$\cos NK = \cos\phi\cos\epsilon \pm |\sin\phi\sin\epsilon\cos\alpha_e|$$

where the second term is to be applied positively when α_e is in the range of 90 to 270 degrees (i.e., Cancer etc.), and negatively when α_e is in the range of 270 to 90 degrees (i.e., Capricorn etc.).[25] Multiplying both sides by R, the LHS of the above equation denotes the side ON' in the triangle $V'N'O$ in Figure 6a, and the RHS satisfies the magnitude and conditions for (11) stated in the verse.

In the first half of verse 46, the author refers to the *bhujā* as the *doḥ*. This same quantity is also referred to as the *doḥprabhā* in the next verse, which gives an alternative method for calculating the same.

काललग्नभुजाद्युज्या मध्यज्याकोटिकाहता ।
त्रिज्याप्ता वा भवेदत्र राशिकूटस्य दोःप्रभा ॥४७॥

kālalagnabhujādyujyā
madhyajyākoṭikāhatā |
trijyāptā vā bhavedatra
rāśikūṭasya doḥprabhā ||47||

Or, the day-radius (*dyujyā*) corresponding to the *kālalagnabhujā* is multiplied by the Rcosine of the *madhyajyā* (*madhyajyākoṭikā*) and divided by the radius (*trijyā*). The remainder would be the *doḥprabhā* of the pole of the ecliptic (*rāśikūṭa*).

This verse gives the following relation to determine the *doḥprabhā*, which is another term for the *bhujā* described in verses 44-46. The given expression is:[26]

$$\begin{aligned}do\d{h}prabh\bar{a} &= (k\bar{a}lalagnabhuj\bar{a}dyujy\bar{a}\times\\ &\qquad madhyajy\bar{a}ko\d{t}ik\bar{a})\div trijy\bar{a}\\ &= \frac{R\cos\mu\times R\cos z_m}{R}. \qquad (17)\end{aligned}$$

The term *doḥprabhā* can be understood as the *doḥ* or the *bhujā* (i.e. the lateral) in a right-angled triangle, where the hypotenuse is the *rāśikūṭaprabhā*. Therefore, the *doḥprabhā* is the same as the *bhujā* given by (11), and corresponds to the side ON' in the right-angled triangle $V'ON'$ in Figure 6a. In our discussion of verses 44–46, we have shown that $ON' = R\cos NK$, where NK is the great circle arc passing through the north cardinal point and the pole of the ecliptic. There, this quantity was determined using the spherical triangle NPK in Figure 8a. The expression given in (17) for this quantity can be derived by considering the spherical triangle NJK in Figure 8b. In this triangle, we have $NJ = z_m$,[27] $JK = \mu$, and $N\hat{J}K = 90$.[28] Applying the cosine rule in this spherical triangle, we have

$$\begin{aligned}\cos NK &= \cos\mu\cos z_m\\ \text{or,}\qquad R\cos NK &= \frac{R\cos\mu\times R\cos z_m}{R}.\end{aligned}$$

which is the same as (17).

3.2 Determining the *unmaṇḍalalagna*

काललग्ने स्वकोट्युत्थं व्यस्तं प्राणकलान्तरम् ।
कुर्यात्तदा भवेदेतत् उन्मण्डलविलग्नकम् ॥४८॥

kālalagne svakoṭyutthaṃ
vyastaṃ prāṇakalāntaram |
kuryāttadā bhavedetat
unmaṇḍalavilagnakam ||48||

To the *kālalagna*, one should apply its own

have $K\hat{P}N = H_k - 180 = \alpha_e - 180$. As $\cos(180-\alpha_e) = \cos(\alpha_e - 180)$, the result does not change in the following calculations.

[25] It may also be noted that $\sin\epsilon$ and $\sin\phi$ are always positive.

[26] It may be noted that the expression *kālalagnabhujādyujyā* in the verse is to be interpreted as काललग्नभुजासंबन्धि-द्युज्या. When the Sun's declination is δ, the corresponding day-radius or *dyujyā* is equal to $R\cos\delta$. Similarly, when the 'declination' corresponding to the *kālalagna* is μ, its corresponding day-radius is to be taken as $R\cos\mu$.

[27] We have $JM = 90$, as the *madhyalagna* is the pole of the great circle arc $KJRE$ (see our discussion of verse 31). We also have $PT = 90$, as P is the pole for any point on the equator. Therefore, we have $PJ = TM = \delta_m$. Now, $NJ = NP + PJ = \phi + \delta_m = ZT + TM = z_m$.

[28] For the values of JK and $N\hat{J}K$, see our discussion in the previous section titled 'Determining the *koṭi*'.

koṭi-prāṇakalāntara reversely. Then, this would be *lagna* on the six-o-clock circle (*unmaṇḍala-vilagna*).

This verse gives the method to determine the longitude of the point of intersection of the ecliptic and the six o' clock circle. Called the *unmaṇḍalalagna*, this point is determined by applying the *koṭi-prāṇakalāntara* to the *kālalagna* reversely:

$$\text{unmaṇḍalalagna} = \text{kālalagna} \mp \text{koṭi-prāṇakalāntara}$$

$$\text{or,} \qquad \lambda_u = \alpha_e \mp |\lambda_u - \alpha_e|, \qquad (18)$$

where λ_u is the longitude of the *unmaṇḍalalagna*, which is depicted by the point U in Figures 9a and 9b.

The six o' clock circle is the meridian which passes through the east cardinal point (E). As the *unmaṇḍalalagna* also lies on the this meridian, at the point of its intersection with the ecliptic, the right ascension corresponding to the longitude of the *unmaṇḍalalagna* is the arc ΓE, which is nothing but the *kālalagna*. Thus, applying the *prāṇakalāntara* to the longitude of the *unmaṇḍalalagna* would give the *kālalagna*. That is,

$$\alpha_e = \lambda_u \pm \text{prāṇakalāntara} = \lambda_u \pm |\lambda_u - \alpha_e|,$$

where, the *prāṇakalāntara* can be obtained by any of the techniques described in an earlier paper.[29] However, to determine the *unmaṇḍalalagna* from the *kālalagna*, the *prāṇakalāntara* has to be applied reversely. That is,

$$\lambda_u = \alpha_e \mp |\lambda_u - \alpha_e|,$$

which is the same as the relation given in the verse. It may be noted that, when considered with respected to the equatorial point E, the above *prāṇakalāntara* is called the *koṭi-prāṇakalāntara*, as E and U lie of the same meridian.[30] Applying the *nija-prāṇakalāntara* to the *kālalagna* would give the *rāśikūṭalagna*,[31] and not the *unmaṇḍalalagna*. Thus, to avoid confusion, the author clearly states in the verse that one has to apply the *koṭi-prāṇakalāntara* reversely to the *kālalagna* to obtain the *unmaṇḍalalagna*.

[29] See [8].

[30] See our discussion on verse 31.

[31] Again, see our discussion on verse 31.

3.3 Determining the *udayalagna*

भूयस्तदुन्मण्डललग्नके स्व-
दोःक्रान्तिमौर्व्याः पलताडितायाः ।
भकूटभाप्तस्य धनुश्च कुर्यात्
व्यस्तं तदा वोदयलग्नकं स्यात् ॥४९॥

bhūyastadunmaṇḍalalagnake sva-
doḥkrāntimaurvyāḥ palatāḍitāyāḥ |
bhakūṭabhāptasya dhanuśca kuryāt
vyastaṃ tadā vodayalagnakaṃ syāt ||49||

Again, to that *unmaṇḍalalagna*, one should apply positively[32] or reversely (*vyastam*) the arc corresponding to the quotient obtained from the division of the product of the Rsine of the declination corresponding to own longitude [i.e., of the *unmaṇḍalalagna*] and the latitude [of the observer], by the *bhakūṭabhā*. Then [the result] would be the *udayalagna*.

This verse gives the following relation to determine the *udayalagna* from the *unmaṇḍalalagna*:

$$\text{udayalagna} = \text{unmaṇḍalalagna} \pm \text{dhanuṣ}\left(\frac{\text{svadoḥkrāntimaurvī} \times \text{palajyā}}{\text{bhakūṭabhā}}\right)$$

$$\text{or,} \qquad \lambda_l = \lambda_u \pm R\sin^{-1}\left(\frac{R\sin\delta_u \times R\sin\phi}{R\cos z_d}\right), \qquad (19)$$

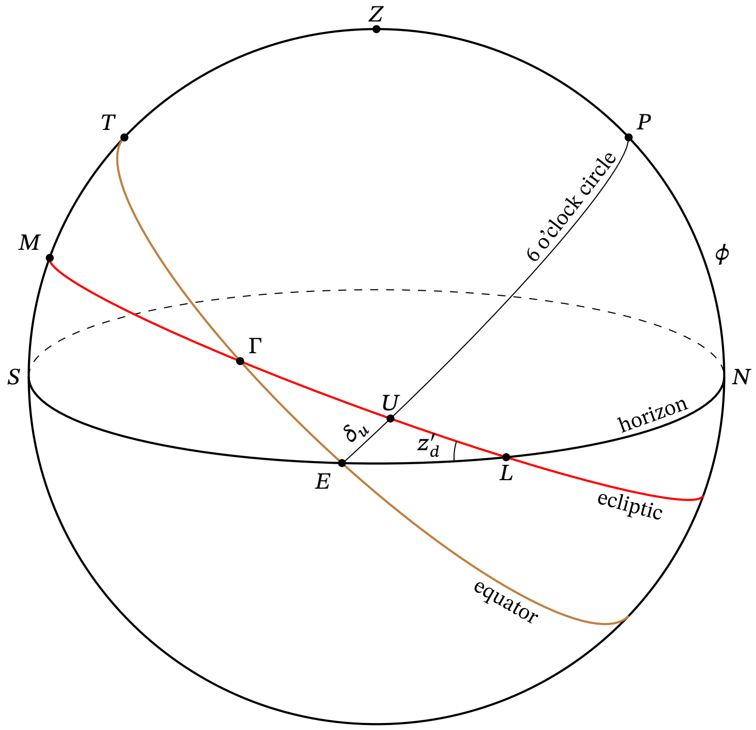
where δ_u is the declination of the *unmaṇḍalalagna*, represented by the great circle arc UE in Figures 9a and 9b. The term *bhakūṭabhā* in the above expression is nothing but the *rāśikūṭaprabhā*, or the *dṛkkṣepakoṭikā* discussed in verse 46.

The validity of the above relation can be verified from the spherical triangle UEL in Figures 9a and 9b, where we have $UE = \delta_u$, and $U\hat{E}L = \phi$. Also, from (17) in our previous paper, we have the angle between the ecliptic and the horizon $U\hat{L}E = 90 - z_d$. Applying the sine rule of spherical trigonometry, we have

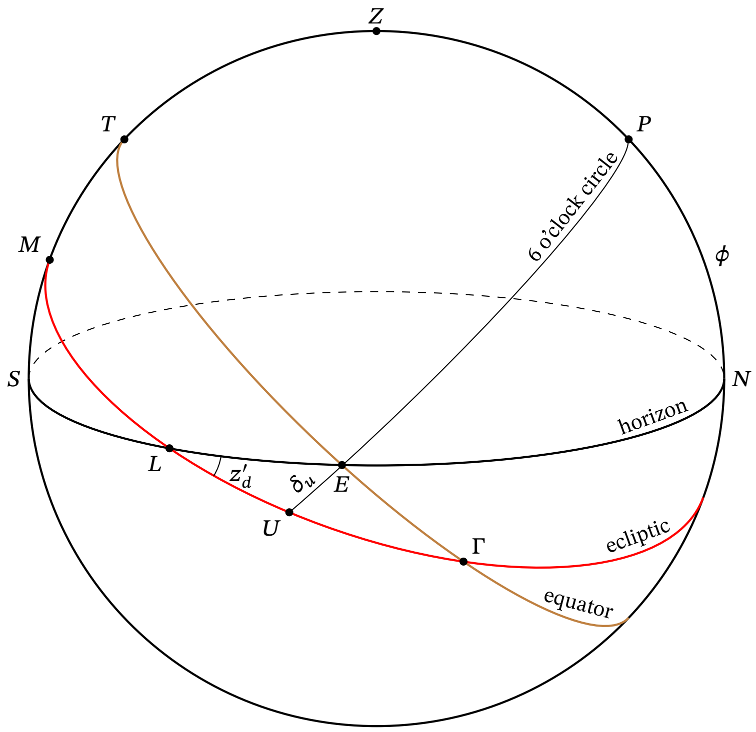
$$\sin UL = \frac{\sin\delta_u \times \sin\phi}{\cos z_d}.$$

In Figure 9a, where the *unmaṇḍalalagna* is above the horizon, we can see that adding the arc UL to the longitude of the *unmaṇḍalalagna* (ΓU) gives the *udayalagna* (ΓL). In Figure 9b, where the *unmaṇḍalalagna* is below

[32] The word *kuryāt* in the verse is interpreted here as 'apply positively'.



(a) *Unmaṇḍalalagna* above the horizon.



(b) *Unmaṇḍalalagna* below the horizon.

Figure 9 Determining the *udayalagna* from the *unmaṇḍalalagna*.

the horizon, the arc *UL* needs to be subtracted from the longitude of the *unmaṇḍalalagna* to obtain the *udayalagna*. Thus, the prescription given in the verse to add or subtract the arc *UL* to the *unmaṇḍalalagna* to obtain the *udayalagna* is found to be valid.

4 Determining the ascendant from the *kālalagna*

In this section we discuss the third method for determining of the *udayalagna* as outlined in verses 50–52 of the second chapter of the *Lagnaprakaraṇa*. Whereas verses 50 and 51 describe yet another technique for calculating the *dṛkkṣepajyā*, verse 52 makes use of this quantity as well as the *kālalagna* to present a very interesting method for the determination of the *udayalagna*. Indeed, the method described in verse 52 attests to the spatial understanding and mathematical genius of Mādhava.

4.1 Another method of obtaining the dṛkkṣepajyā

यद्वा कालविलग्नतो भुजगुणं क्रान्तिं च कोट्युद्भवां
नीत्वा कोट्यपमावलम्बकवधात् व्यासार्धभक्तं फलम् ।
अन्त्यापक्रमकोटिकाक्षवधतो व्यासार्धभक्ते फले
कर्क्येणादिवशात् क्रमादृणधनं[33] दृक्क्षेपजीवाप्तये ॥५०॥
क्षितिजाद्राशिकूटस्य सोन्नतिः परिकीर्तिता ।
तस्मादेनं वदन्त्यत्र राशिकूटनरं बुधाः ॥५१॥

yadvā kālavilagnato bhujaguṇaṃ
krāntiṃ ca koṭyudbhavāṃ
nītvā koṭyapamāvalambakavadhāt
vyāsārdhabhaktaṃ phalam |
antyāpakramakoṭikākṣavadhato
vyāsārdhabhakte phale
karkyeṇādivaśāt kramādṛṇadhanaṃ
dṛkkṣepajīvāptaye ||50||
kṣitijādrāśikūṭasya sonnatiḥ parikīrtitā |
tasmādenaṃ vadantyatra
rāśikūṭanaraṃ budhāḥ ||51||

Or, having computed the Rsine from the *kālavilagna*, and [therefrom] the declination from its Rcosine (*koṭyapama*), the product of the *koṭyapama* and the Rcosine of the latitude (*avalambaka*) divided by the semi-diameter (*vyāsārdha*) is the result. This has to be applied negatively or positively to the result of the division of the product of the Rcosine of the last declination (*antyāpakramakoṭikā*) and [the Rsine of] the latitude (*akṣa*) by the semi-diameter (*vyāsārdha*), depending on [whether the *kālalagna* lies in the six signs] Cancer (*karki*) etc., or Capricorn (*eṇa*) etc., in order to obtain the Rsine of the *dṛkkṣepa* (*dṛkkṣepajīvā*). That is stated to be the altitude of the pole of the ecliptic from the horizon. Therefore, scholars state it to be the gnomon of the pole of the ecliptic (*rāśikūṭanara*).

These verses describe yet another method to determine the *dṛkkṣepajyā*, and note that this quantity is equivalent to the gnomon (*nara*) corresponding to the pole of the ecliptic (*rāśikūṭa*). The relation prescribed in the verses is as follows:

$$\textit{dṛkkṣepajīvā} = \frac{\textit{antyāpakramakoṭikā} \times \textit{akṣajyā}}{\textit{vyāsārdha}} \pm \frac{\textit{avalambakajyā} \times \textit{koṭyapama}}{\textit{vyāsārdha}}$$

or,

$$R\sin z_d = \frac{R\cos\epsilon \times R\sin\phi}{R} \pm \frac{R\cos\phi \times R\cos\alpha_e\sin\epsilon}{R}, \tag{20}$$

where the term *koṭyapama* (i.e. *koṭi-apama*) is to be understood as the 'declination' calculated using the Rcosine of the *kālalagna*. Thus, this expression is equal to $R\cos\alpha_e\sin\epsilon$.

In our discussion of verses 44–46, we have already shown that the measure of the gnomon dropped from the pole of the ecliptic is equal to the *dṛkkṣepajyā* ($R\sin z_d$). This is evident in Figure 6a, where the gnomon KV' is equal to $R\sin z_d$. This same gnomon is also shown in Figure 10, where, by considering the spherical triangle PKZ, we can show the validity of (20).[34] In this triangle, we have $PZ = 90 - \phi$, $KP = \epsilon$, and $K\hat{P}Z = \alpha_e$.[35] Noting that $KZ = 90 - z_d$, and applying the cosine rule in this triangle, we have

$$\sin z_d = \cos\epsilon\sin\phi + \sin\epsilon\cos\phi\cos\alpha_e.$$

[33] Manuscripts read क्रमाद्धनमृणं. Emended as the reverse order is required.

[34] Nīlakaṇṭha in his *Tantrasaṅgraha* describes a similar relation to determine the *dṛkkṣepajyā*. For a detailed discussion, see [15], pp. 242–245. The following proof borrows partly from this discussion.

[35] The latter two relations from (15) and (16) respectively.

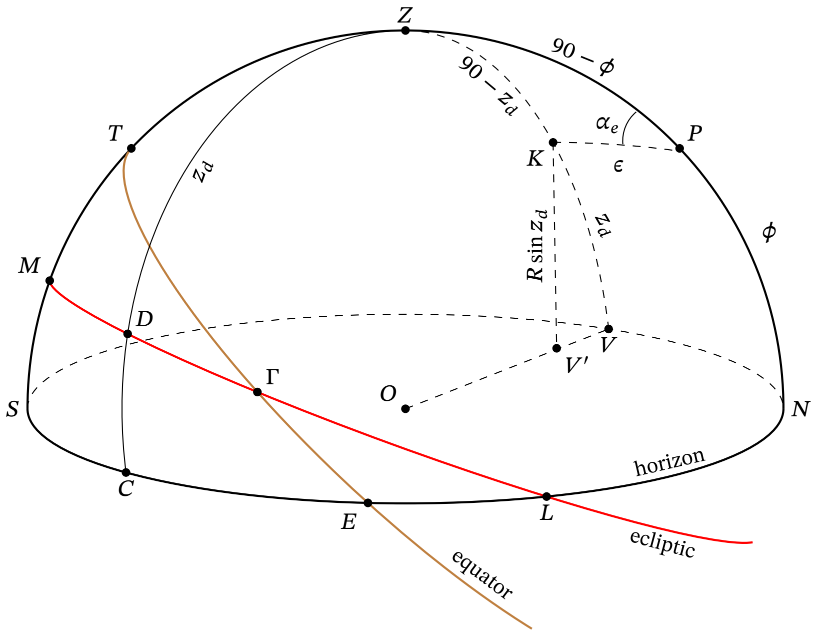


Figure 10 Another method of determining the *drkksepa*.

The function $\cos\alpha_e$ is positive in the range 270 to 90 degrees, and negative in the range 90 to 270 degrees. Thus, the above relation can be written as

$$\sin z_d = \cos\epsilon\sin\phi \pm |\sin\epsilon\cos\phi\cos\alpha_e|,$$

where the second term is to be applied positively when α_e is in the range of 270 to 90 degrees (i.e. Capricorn etc.), and negatively when α_e is in the range of 90 to 270 degrees (i.e. Cancer etc.) respectively.[36] Multiplying by R, we have the expression

$$R\sin z_d = R\cos\epsilon\sin\phi \pm R\sin\epsilon\cos\phi\cos\alpha_e,$$

which is equivalent to (20).

4.2 Determining the *udayalagna*

कृत्वा दृक्क्षेपमक्षे धनमृणममुना
काललग्नस्य दोर्ज्यां
हत्वा दृक्क्षेपकोट्या हृतमथ चरम-
क्रान्तिबाणेन हत्वा ।
हृत्वान्त्यक्रान्तिमौर्व्या फलमिह तु पुनः
चापितं काललग्ने
स्वर्णं दिग्भेदसाम्ये भुजगुणगुणयोः
प्राग्विलग्नस्य सिद्ध्यै ॥५२॥

kṛtvā dṛkkṣepamakṣe dhanamṛṇamamunā
kālalagnasya dorjyāṃ
hatvā dṛkkṣepakoṭyā hṛtamatha carama-
krāntibāṇena hatvā |
hṛtvāntyakrāntimaurvyā phalamiha tu punaḥ
cāpitaṃ kālalagne
svarṇaṃ digbhedasāmye bhujaguṇaguṇayoḥ
prāgvilagnasya siddhyai ||52||

Having applied the positive or negative [Rsine of the] *dṛkkṣepa* to the [Rsine of the] latitude (*akṣa*), and having multiplied the Rsine of the *kālalagna* by this, [the result] is divided by the Rcosine of the *dṛkkṣepa* (*dṛkkṣepakoṭi*). Now, having multiplied [the previous result] by the Rversine of the maximum declination (*caramakrāntibāṇa*) and divided by the Rsine of the last declination (*antyakrāntimaurvī*), the result is converted to arc again. [That arc] becomes additive or subtractive to the *kālalagna*, depending on the difference and similarity of the directions of the *bhujaguṇa* (i.e. Rsine of the *kālalagna*) and the semi-chord (*guṇa*) [whose arc is determined above], in order to obtain the orient ecliptic point (*prāglagna*).

The above verse gives the following relation to deter-

[36] It may also be noted that $\sin\epsilon$ and $\cos\phi$ are always positive.

mine the *udayalagna*:

$$udayalagna = k\bar{a}lalagna \pm c\bar{a}pa \left(\frac{(ak\d{s}ajy\bar{a} \pm d\d{r}kk\d{s}epa) \times k\bar{a}lalagnasya\ dorjy\bar{a}}{d\d{r}kk\d{s}epako\d{t}i} \times \frac{caramakr\bar{a}ntib\bar{a}\d{n}a}{antyakr\bar{a}ntimaurv\bar{\imath}}\right)$$

or,

$$\lambda_l = \alpha_e \pm R\sin^{-1}\left(\frac{(R\sin\phi \pm R\sin z_d) \times R\sin\alpha_e}{R\cos z_d} \times \frac{R\,\mathrm{versin}\,\epsilon}{R\sin\epsilon}\right). \quad (21)$$

The above expression reveals an impressive technique for determining the *udayalagna*, and really attests to the genius of the author of the text. This expression can be derived with the help of Figures 11 and 12. The former figure is representative of the situation when the *kālalagna* is in the first two quadrants ($0 \leq \alpha_e \leq 180$), and the latter figure is representative of the situation when the *kālalagna* is in the third and fourth quadrants ($180 \leq \alpha_e \leq 360$). Each of these cases is dealt separately below.

Kālalagna in the first two quadrants

Figure 11, which depicts the *kālalagna* in the first quadrant, is representative of the case when the *kālalagna* is in the first two quadrants. In this figure, we have drawn the great circle arc EY such that $\Gamma Y = \Gamma E$. As $\Gamma E = \alpha_e$, the longitude of the *udayalagna* is clearly

$$\lambda_l = \Gamma Y + YL = \alpha_e + YL.$$

In what follows, we show how to determine YL.

In the spherical triangle ΓEY, as $\Gamma Y = \Gamma E$, we also have $\Gamma\hat{Y}E = \Gamma\hat{E}Y$. Let these two angles be denoted by y. Also, $Y\hat{\Gamma}E = \epsilon$. Using the sine rule, we have

$$\sin YE = \frac{\sin\alpha_e \sin\epsilon}{\sin y}. \quad (22)$$

Applying the cosine rule for the side YE, we have

$$\cos YE = \cos^2\alpha_e + \sin^2\alpha_e \cos\epsilon. \quad (23)$$

Now, applying the cosine rule for the side ΓE, we have

$$\cos\alpha_e = \cos\alpha_e \cos YE + \sin\alpha_e \sin YE \cos y.$$

Using (22) and (23) in the above equation, and simplifying, we obtain[37]

$$\frac{\cos y}{\sin y} = \frac{\cos\alpha_e \times (1-\cos\epsilon)}{\sin\epsilon}. \quad (24)$$

As $\Gamma\hat{E}L = 90 + \phi$, we have $Y\hat{E}L = 90 + \phi - y$. From (17) in our previous paper, we also have $Y\hat{L}E = 90 - z_d$. Now, applying the sine rule in the spherical triangle YEL, we have

$$\sin YL = \frac{\sin YE \times \cos(\phi - y)}{\cos z_d}.$$

Substituting for $\sin YE$ using (22), and simplifying by expanding $\cos(\phi - y)$, we get

$$\sin YL = \frac{\sin\alpha_e \sin\epsilon}{\cos z_d} \times \left(\sin\phi + \cos\phi \times \frac{\cos y}{\sin y}\right).$$

Using (24) and further simplifying, we obtain

$$\sin YL = \frac{\sin\alpha_e \times (1-\cos\epsilon)}{\cos z_d \times \sin\epsilon} \times (\sin\phi + \sin\phi\cos\epsilon + \cos\phi\cos\alpha_e\sin\epsilon).$$

Employing (20) in the second term of the RHS of the above equation, we get

$$YL = \sin^{-1}\left(\frac{\sin\alpha_e \times (1-\cos\epsilon)}{\cos z_d \times \sin\epsilon} \times [\sin\phi + \sin z_d]\right).$$

Therefore, we have

$$\lambda_l = \alpha_e + R\sin^{-1}\left(\frac{(R\sin\phi + R\sin z_d) \times R\sin\alpha_e}{R\cos z_d} \times \frac{R - R\cos\epsilon}{R\sin\epsilon}\right). \quad (25)$$

It may be noted that the above result only holds when the *kālalagna* is in the first two quadrants. In this scenario, the amplitude of the *udayalagna* will always be northwards and the arc YL has to be added to the *kālalagna* to obtain the *udayalagna*. The semi-chords corresponding to the *kālalagna* (ΓY) and the arc YL lie in the plane of the ecliptic and will be perpendicular on either side to the radius of the ecliptic drawn from the point Y. Thus, the verse notes that the arc obtained above has to be added to the *kālalagna* when the semi-chords lie in opposite directions.

[37] This relation can also be directly obtained by applying the four part formula in the spherical triangle ΓEY.

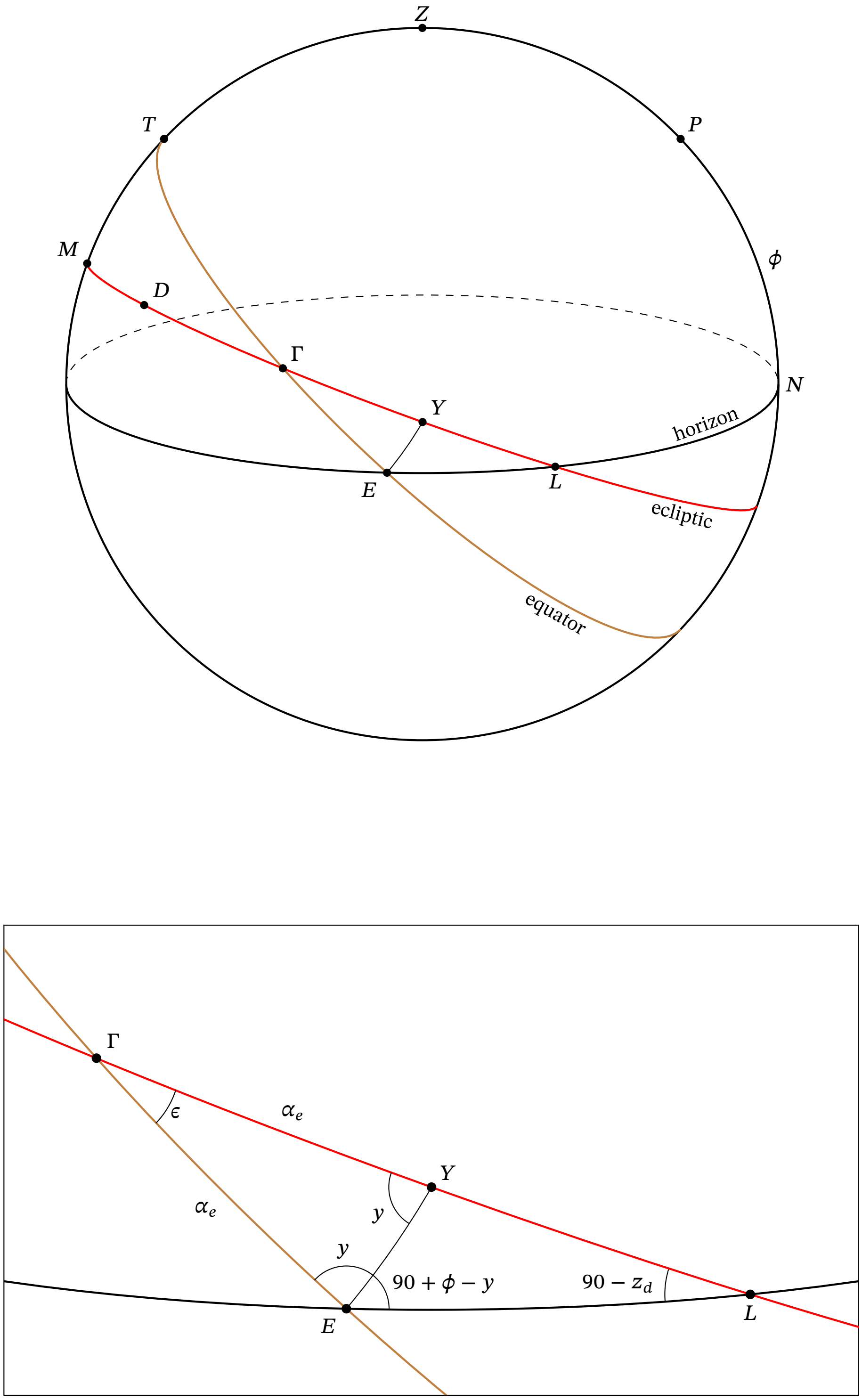


Figure 11 Determining the *udayalagna* when the *kālalagna* and the *dṛkkṣepa* are in the same direction.

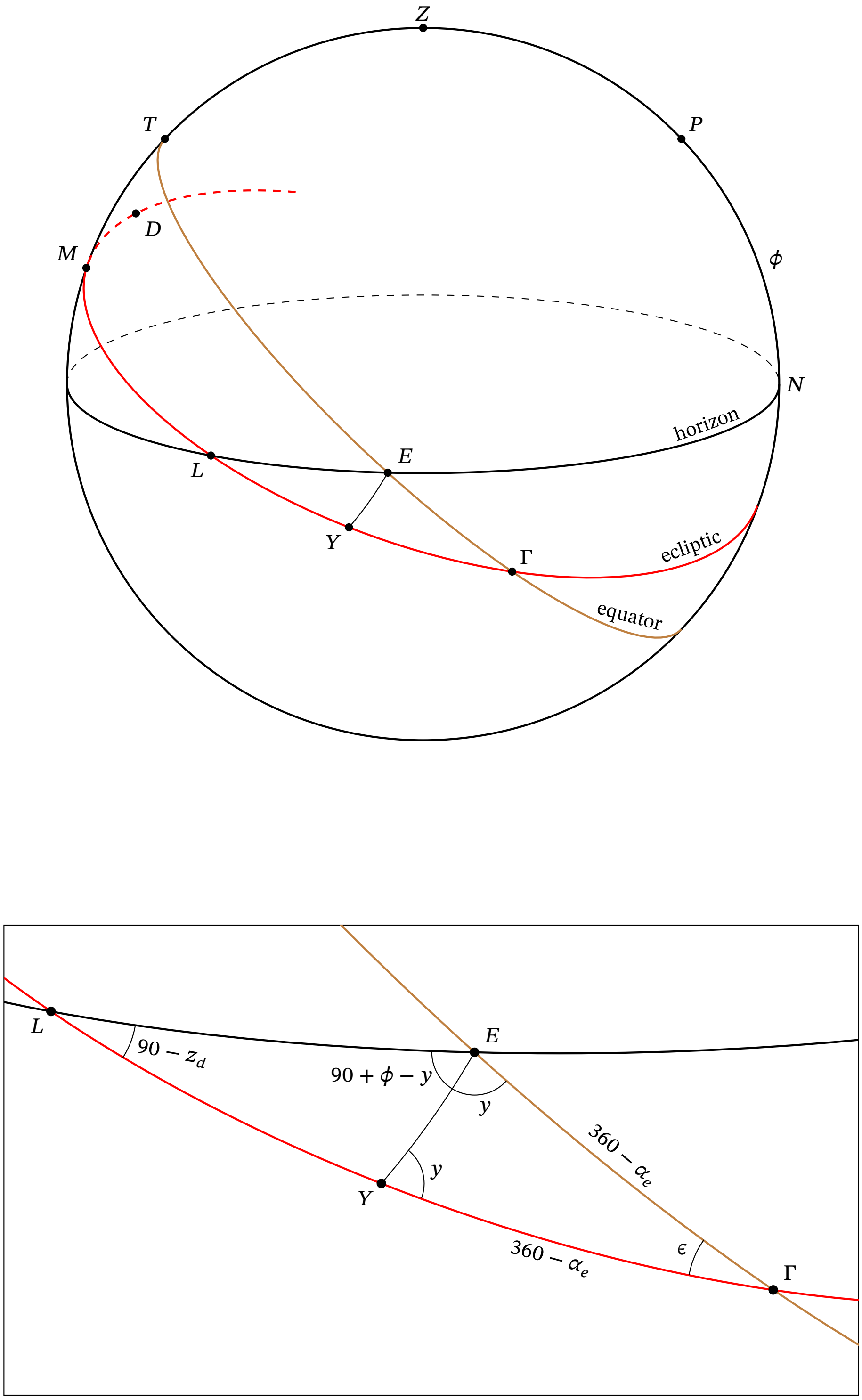


Figure 12 Determining the *udayalagna* when the *kālalagna* and the *dṛkkṣepa* are in different directions.

Kālalagna in the third and the fourth quadrants

When the *kālalagna* is in the third and fourth quadrants, the amplitude of the *udayalagna* is southwards. Figure 12 depicts one scenario when the *kālalagna* is in the fourth quadrant. In this figure, we have drawn the great circle arc EY such that $Y\Gamma = E\Gamma = 360 - \alpha_e$. Thus, we have $\Gamma Y = \Gamma E = \alpha_e$, and the longitude of the *udayalagna* is clearly

$$\lambda_l = \Gamma Y - YL = \alpha_e - YL.$$

In this case, it can be shown through a similar procedure as followed earlier that the arc

$$YL = \sin^{-1}\left(\frac{\sin\alpha_e \times (1-\cos\epsilon)}{\cos z_d \times \sin\epsilon} \times [\sin\phi + \sin z_d]\right).$$

and therefore,

$$\lambda_l = \alpha_e - R\sin^{-1} \left(\frac{(R\sin\phi + R\sin z_d)\times R\sin\alpha_e}{R\cos z_d} \times \frac{R - R\cos\epsilon}{R\sin\epsilon}\right). \quad (26)$$

In this scenario, the semi-chords corresponding to the *kālalagna* (ΓY) and the arc YL lie in the plane of the ecliptic, and will be perpendicular on the same side to the radius of the ecliptic drawn from the point Y. Thus, the verse notes that the obtained arc has to be subtracted from the *kālalagna* in this situation.

Thus, it can be seen that the two relations (25) and (26) taken together yield (21). It may however be noted that while we have the expression $R\sin\phi \pm R\sin z_d$ in (21), the corresponding expressions in (25) and (26) contain only the positive sign. We are unable to ascertain the scenario in which the negative sign may be required.

5 Conclusion

The current paper discusses three broadly different approaches for the determination of the ascendant. These amply demonstrate Mādhava's remarkable mastery in visualising various intricate projections inside the celestial sphere and his ability to expertly intuit the relationships between them. The diversity of the approaches attest to the scientific curiosity of the author, and have great pedagogical significance for attaining mastery in any science. The remarkable results discussed in the paper highlight the pressing need to preserve and study these important category of texts, and to popularise their contributions among scholars and lay people alike. To this end, in further papers we will bring out many more interesting results presented in the *Lagnaprakaraṇa*.

Acknowledgements

We would like to place on record our sincere gratitude to MHRD for the generous support extended to carry out research activities on Indian science and technology by way of initiating the Science and Heritage Initiative (SandHI) at IIT Bombay. We pay our obeisance to the late Prof. K. V. Sarma, who saved the *Lagnaprakaraṇa* for future generations by painstakingly copying the crumbling manuscripts of the text. We are also extremely grateful to the Prof. K. V. Sarma Research Foundation, Chennai, for preserving and sharing the copies of these manuscripts, which has enabled us to study this interesting and important work. Finally, we would also like to profusely thank the anonymous referee for thoroughly reviewing our paper and making constructive suggestions.

Bibliography

[1] Āryabhaṭa. *Āryabhaṭīya*. Ed., trans., and comm., with an introd., by Kripa Shankar Shukla and K. V. Sarma. New Delhi: Indian National Science Academy, 1976.

[2] Bhāskara. *Siddhāntaśiromaṇi*. With a comment. by Satyadeva Sharma. Varanasi: Chaukhamba Surabharati Prakashan, 2007.

[3] Brahmagupta. *Brāhmasphuṭasiddhānta*. Ed. and comm. by Sudhakara Dvivedi. Reprint from The Pandit. Benares: Printed at the Medical Hall Press, 1902.

[4] Śrī Kṛṣṇa Candra Dvivedī, ed. *Sūryasiddhānta*. With the commentary *Sudhāvarṣiṇī* of Sudhākara Dvivedī. Sudhākara Dvivedī Granthamālā. Varanasi: Sampurnanand Sanskrit University, 1987.

[5] Jñanarāja. *Siddhāntasundara*. Trans. and comm. by Toke Lindegaard Knudsen. Baltimore: John Hopkins University Press, 2014.

[6] Jyeṣṭhadeva. *Gaṇita-yukti-bhāṣā*. Ed. and trans. by K. V. Sarma. With a comment. by K. Ramasubramanian, M. D. Srinivas, and M. S. Sriram. 2 vols. Culture and History of Mathematics 4. New Delhi: Hindustan Book Agency, 2008.

[7] Aditya Kolachana, K. Mahesh, and K. Ramasubramanian. “Determination of *kālalagna* in the *Lagnaprakaraṇa*”. In: *Indian Journal of History of Science* 54.1 (2019), pp. 1–12.

[8] Aditya Kolachana, K. Mahesh, and K. Ramasubramanian. “Mādhava’s multi-pronged approach for obtaining the *prāṇakalāntara*”. In: *Indian Journal of History of Science* 53.1 (2018), pp. 1–15.

[9] Aditya Kolachana, K. Mahesh, and K. Ramasubramanian. “Precise determination of the ascendant in the *Lagnaprakaraṇa*-I”. In: *Indian Journal of History of Science* 54.3 (2019), pp. 304–316.

[10] Aditya Kolachana et al. “Determination of ascensional difference in the *Lagnaprakaraṇa*”. In: *Indian Journal of History of Science* 53.3 (2018), pp. 302–316.

[11] Lalla. *Śiṣyadhīvṛddhidatantra*. With the commentary of Mallikārjuna Sūri. Ed., with an introd., by Bina Chatterjee. Vol. 1. 2 vols. New Delhi: Indian National Science Academy, 1981.

[12] Mādhava. “*Lagnaprakaraṇa*”. Manuscript 414B, ff. 53–84. Kerala University Oriental Research Institute and Manuscripts Library, Thiruvananthapuram.

[13] Mādhava. “*Lagnaprakaraṇa*”. KVS Manuscript No. 37a. Prof. K. V. Sarma Research Foundation, Chennai.

[14] Mādhava. “*Lagnaprakaraṇa*”. KVS Manuscript No. 37b. Prof. K. V. Sarma Research Foundation, Chennai.

[15] Nīlakaṇṭha Somayājī. *Tantrasaṅgraha*. Trans. and comm. by K. Ramasubramanian and M. S. Sriram. Culture and History of Mathematics 6. New Delhi: Hindustan Book Agency, 2011.

[16] Putumana Somayājī. *Karaṇapaddhati*. Trans. and comm. by Venketeswara Pai et al. Culture and History of Mathematics 9. New Delhi: Hindustan Book Agency, 2017.