Retrograde motion as described in *Brahmatulya-udāharaṇam* of Viśvanātha

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Abstract

The phenomenon of retrograde motion has always been of great interest to astronomers as can be gathered from ancient texts, Indian and others. The paper discusses the phenomenon of retrograde motion as derived from the *Brahmatulya-udāharaṇam* of Viśvanātha of the 17th century. We show the rationale for the calculations following the methods already described. The use of a derivative of sine also is apparent in the procedure. The examples cited are used to verify the retrograde motion. We note that for Mars and Jupiter, the calculations agree fairly well, while for Mercury the agreement is not very good.

Key words: *Brahmatulya-udāharaṇam*, Retrograde motion, Viśvanātha.

1 Introduction

The planetary positions and movements have been described extensively and the methods of calculations of events like conjunctions are also provided in almost all Indian astronomical texts. It has been described by almost all authors as a distinct property associated with the star like planets (*tārāgraha*). This apparent motion of planets in the backward direction is referred to as *vakragati*. The cause of this apparent motion is very well known now. Earlier, the astronomers had made special efforts to explain this by invoking epicycles for each planet and relating it to the sun's position. Generally, all texts include this discussion in the chapter dealing with heliacal rising and setting of planets because the procedure for calculation is similar in both cases.

Here we discuss the procedure and examples to illustrate the accuracy of the procedure. The examples, discussed here are drawn from *Brahmatulya-udāharaṇam* of Viśvanātha Daivajña of 17th century.

2 The phenomenon as explained in the texts

All the texts provide the values of the angles required for the onset and end of retrograde motion. Here are some examples.

For instance, in the *Sūryasiddhānta* (Sarma and Subbarayappa, 1985) we have:

दूरस्थितः स्वशीघ्रोच्चात् ग्रहश्शिथिलरश्मिभिः ।
सव्येतराकृष्टतनुः भवेत् वक्रगतिस्तदा ॥ ५२॥
कृतर्त्तुचन्द्रैर्वेदेन्द्रैः शून्यत्र्येकैर्गुणाष्टिभिः ।
शररुद्रैश्चतुर्थेषु केन्द्रांशैर्भूसुतादयः ॥ ५३॥

dūrasthitaḥ svaśīghroccāt grahaśśithilaraśmibhiḥ |
savyetarākṛṣṭatanurbhavet vakragatistadā || 52 ||
kṛtarttucandrairvedendraiḥ
śūnyatryekairguṇāṣṭibhiḥ |

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śararudraiścaturtheṣu
kendrāṃśairbhūsutādayaḥ || 53 ||

Mars and the rest, when their degrees of commutation in the fourth process, are respectively 164°, 144°, 130°, 163° and 115° become retrograde and when their respective commutations are equal to the number of degrees remaining after subtracting those numbers, in each case from a whole circle, they cease retrogradation

Similarly, Lalla in his *Śiṣyadhīvṛddhida-tantra* notes (verse 3.20; 3.21):

गुणनृपतिभिर्बाणाब्ध्येकैः शराक्षिनिशाकरैः
शररसकुभिर्विश्वक्ष्माभिर्लवैश्चलकेन्द्रजैः ।
भवति नियतं वक्रारम्भः कुजादि नभःसदां
पुनरपि भवेद्वक्रत्यागश्च्युतैस्तु भमण्डलात् ॥२०॥

guṇanṛpatibhirbāṇābdhyekaiḥ śarākṣiniśākaraiḥ
śararasakubhirviśvakṣmābhirlavaiścalakendrajaiḥ |
bhavati niyataṃ vakrārambhaḥ kujādi nabhaḥsadāṃ
punarapi bhavedvakratyāgaścyutaistu bhamaṇḍalāt ||20||

रसरसाः क्रमशः शशिबाहवो
यमनिशाकरशीतमरीचयः ।
यमशरा युगपावकभूमयोऽ-
नृजुगतेर्दिवसाः कथिताः कुजात् ॥२१॥

rasarasāḥ kramaśaḥ śaśibāhavo
yamaniśākaraśītamarīcayaḥ |
yamaśarā yugapāvakabhūmayo'-
nṛjugaterdivasāḥ kathitāḥ kujāta ||

The retrograde motion of the planets beginning with Mars commences when their *śīghra-kendras* are respectively, 163°, 145°, 125°, 165° and 113°. When the *śīghra-kendras* are respectively, 360 minus each of these values, their retrograde motion ceases. It is specified that the retrograde motion of Mars, (Mercury, Jupiter, Venus and Saturn) lasts for 66, 21, 112, 52 and 132 (days) respectively (Chatterjee, 1981).

Bhāskarācārya in his *Karaṇakutūhala* states:

द्राक्केन्द्रभागैस्त्रिनृपैः शरेन्द्रैः
तत्त्वेन्दुभिः सप्तनृपैस्त्रिरुद्रैः ।
स्याद्वक्रता भूमिसुतादिकानां
अवक्रता तद्रहितैश्च भांशैः ॥५॥

drākkendrabhāgaistrinṛpaiḥ śarendraiḥ
tattavendubhiḥ saptanṛpaistrirudraiḥ |
syādvakratā bhūmisutādikānāṃ
avakratā tadrahitaiśca bhāṃśaiḥ ||5||

When the second *śīghra-kendras* (Table 1) are respectively 163°, 145°, 125°, 165° and 113°, the planets starting with Kuja attain retrogression. These values subtracted from 360° are the points of non-retrogression (direct motion) (Rao and Uma, 2008).

Figure 1 explains the phenomenon as we understand today in the heliocentric system. As the earth moves along T_1 to T_5, the corresponding positions of the planet P_1 to P_5 as seen from the earth, are also indicated by the same numerals so that their retrograde motion in the background of stars is represented by the line running along the numerals A_1 to A_5.

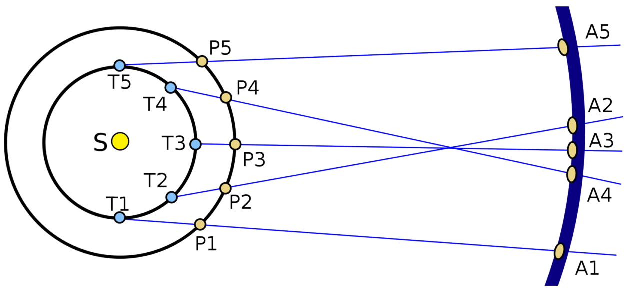

Figure 1 Retrograde motion in the heliocentric system. (courtesy: Rursus: Wikimedia)

As the earth overtakes a superior planet, owing to the difference in the speeds, the planet appears to become stationary and then trace a path in the opposite direction. This effect is temporary and the planet resumes direct motion after another halt. The points where it appears to stay still are referred to as the stationary points. The procedure describes the condition for the determination of the stationary points and also the duration of retrograde, which are decided by the distances of the planet from the earth and from the sun.

The principle based on which these values have been arrived at can be understood with the help of Figure 2 for a superior planet. The derivation shown here is based

Table 1 The longitudes of *śīghra-kendra*'s to fix stationary points (in degrees).

Planet	*Karaṇakutūhala*	*Śiṣyadhīvṛddhida-tantra*	*Sūryasiddhānta*
Mars	163	163	164
Mercury	145	145	144
Jupiter	125	125	130
Venus	167	165	163
Saturn	113	113	115

on the work of Somayājī (1971). (We have used the same notations so that the typographical errors in the original work are automatically corrected.)

2.1 The formula by Bhāskarācārya for the true or apparent motion of a planet

In the Figure 2, P is the position of the planet and the longitude is measured with respect to γ which refers to the First Point of Aries, *Aśvinī* in the *nirayaṇa* system. O_1 and O_2 denote the centres of the deferent and the eccentric circles on which the mean and the true planets are conceived to be moving. The line connecting the points O_1, O_2, A_1 and A_2 denotes the direction of *śīghrocca*. Let P_1 and P_2 represent the positions of the planet in the mean orbit and eccentric orbit on a given day. By definition, the true position P is obtained by the line joining O_1P_2. By the next day the second equation (*śīghra* anomaly) would have moved. Therefore, the corresponding shifts can be described by equal amounts P_1Q_1 and P_2Q_2 on the respective circles. By joining O_1Q_2 which cuts the mean orbit at Q we are getting the shift in *śīghra* anomaly per day. This is because A_1P_1 is the mean anomaly and A_1P is the true anomaly on the first day. On the second day they would become A_1P and A_1Q. Thus, PQ represents the true motion of the *śīghra* anomaly which is to be derived. This is called *sphuṭa-kendra-gati*. Here Bhāskara points out that Q does not represent the position of the planet itself on the second day. Because both A_1 and Q_1 would have shifted, we need to include the motion A_1 to get the true position of the planet.

Somayājī proceeds with the following constructions for deriving the value of PQ. A point L is marked on the eccentric circle such that $P_2L = P_1P =$ *śīghraphala* (second equation). Further it can be shown that O_2L is parallel to O_1P_2. ($A_1O_1P_1 = A_2O_2P_2$ and $PO_1P_1 = LO_2P_2$; therefore $A_1O_1P = A_2O_2L$ follows). P_2M is drawn perpendicular to O_2L; also, Q_2N is drawn such that it cuts O_2P_2 at J.

P_2M is the sine of arc P_2L, *śīghraphala*, θ

$$P_2M = R\sin\theta \tag{1}$$

As per the convention used by Somayājī, H is the *trijya*, which we changed to conventional R. It is easily seen that Q_2N is the sine of arc Q_2L. That is

$$Q_2N = R\sin(Q_2L) \tag{2}$$

We can also see that

$$Q_2L = Q_2P_2 + P_2L \tag{3}$$

Here Q_2P_2 is the mean motion of *śīghra* anomaly and P_2L is the *śīghraphala*.

P_1M and Q_2N are perpendicular to O_2L and therefore to O_1P_2 also. We need to find the difference Q_2J to derive PQ. For this we draw SQ perpendicular to O_1P_2. Now, from the similar triangles O_1Q_2R and O_1QS

$$\frac{QS}{Q_2J} = \frac{O_1Q}{O_1Q_2} = \frac{R'}{k} = \frac{radius\ of\ mean\ orbit}{geocentric\ radius\ vector} \tag{4}$$

QS, being small, is equated to PQ, the mean motion of *śīghra* anomaly

$$PQ = QS = Q_2\left(\frac{R'}{k}\right) \tag{5}$$

Thus, we have an expression for PQ which is needed for deciding the position of the true planet. Remembering the reason for Q not being the true position of planet, we will now implement the necessary correction shown in Figure 3. Q would have represented the true position if and only if A_1 continued to be the *śīghrocca*. However, that too has a daily motion; the exaggerated version in the figure puts it at A_1'. Consequently, the second equation corrected position Q should be corrected by the same

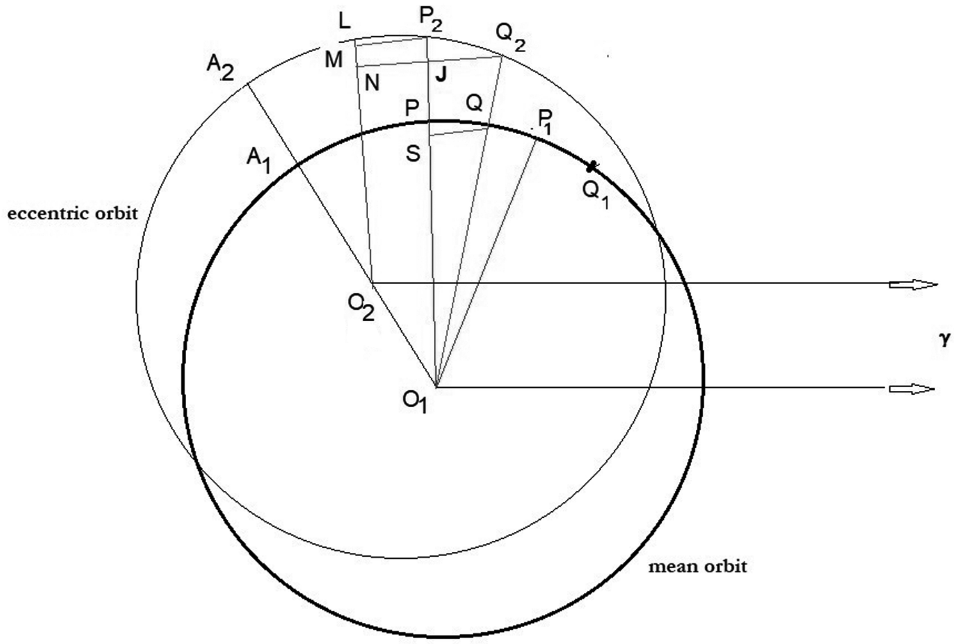

Figure 2 The formulation of the problem of retrograde motion for a superior planet, J.

amount namely by an angle equal to $A_1O_1A_1'$. Thus, we get P' as the new true position of the planet.

From the figure we see that

$$\begin{aligned} PP' &= QP' - PQ \\ &= QP' - QS \end{aligned} \tag{6}$$

QP' is the difference of the mean motion of the *śīghrocca* and QS is the incremental change in Q_2J, which is $R \sin \theta$.

Bhāskarācārya gives this expression as:

True motion of the planet PP' = Mean motion of *śīghra* anomaly – incremental change in the *śīghra-phala*

$$= U - \frac{R'}{k} \cos \theta \; \delta \upsilon \tag{7}$$

where U is the daily mean motion of the *śīghrocca* and the incremental change of *śīghra-phala* from (1), is expressed as a cosine function.

From (6) and (7) we see that

$$PQ = QS = \frac{R'}{k} \cos \theta \; \delta \upsilon \tag{8}$$

is the intended idea.

Further $\delta \; \upsilon = U – V$, where V is the mean speed of the planet. Hence the equation for true motion of the planet is given by

$$PP' = U - \frac{(U - V)}{k} R \cos \theta \tag{9}$$

The equation given above by Bhāskara is central to our discussion. If the magnitude of the second term in *RHS* is larger than the first one would have retrograde motion. If the *RHS* is zero we have stationary points.

2.2 The derivation for the rate of change of the true or apparent motion of a planet

Equation (9) above has been arrived at, by use of calculus in conventional text books of spherical astronomy and using the methods of Bhāskarācārya (Sriram, 2016). In what follows we will understand how this is arrived at. Here again we use the same notations as used by Somayājī are used in Figure 4.

Here, J is the position of the planet and the position of the sun is indicated by S; E refers to the Earth. ESJ is the angle which decides the stationary points. The distance of J from sun is represented by a_j; the earth sun distance by a and the earth- planet distance (geocentric radius vector) is represented by k. Here we draw a line EJ' parallel to SJ so that angle $m \; (= J'ES = 180 - \phi)$ represents the difference

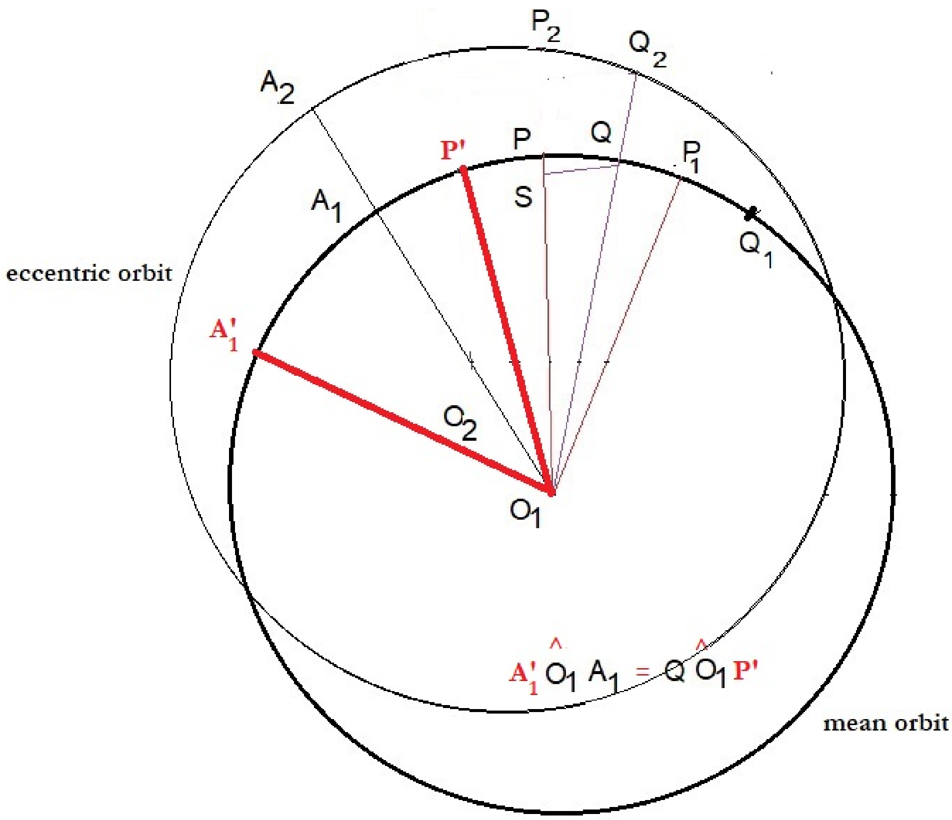

Figure 3 The true positions of the planet and *śīghrocca* on consecutive days.

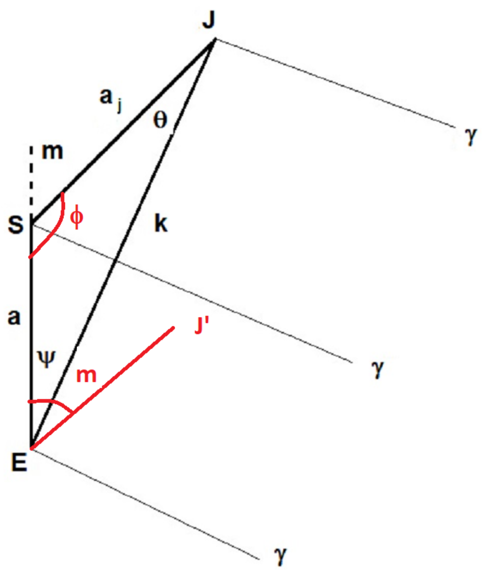

Figure 4 Schematic representation of the Sun, Earth and the planets.

between the longitudes of *śīghrocca* (γES) and the mean planet (γSJ).
The longitude of J is

$$\gamma EJ = \gamma ES - SEJ \tag{10}$$

The rate of change is given by

$$\frac{d}{dt}(\gamma EJ) = \frac{d}{dt}(\gamma ES) - \frac{d}{dt}(SEJ) \tag{11}$$

The first term is the rate of change of the longitude of *śīghrocca* or the Sun, represented by U.

From the triangle SEJ we get

$$k \cos \psi - a_j \cos m = a \tag{12}$$

$$k - a \cos \psi = a_j \cos \theta \tag{13}$$

$$k^2 = a^2 + a_j^2 + 2aa_j \cos m \tag{14}$$

$$\frac{a_j}{k} = \frac{\sin \psi}{\sin m} \tag{15}$$

Differentiating (12) we get

$$k\sin\psi\frac{d\psi}{dt}+\cos\psi\frac{dk}{dt}+a_j\sin m\frac{dm}{dt}=0 \qquad (16)$$

Differentiating (14) we get

$$2k\frac{dk}{dt}=-2aa_j\sin m\frac{dm}{dt} \qquad (17)$$

Eliminating dk/dt from (16) and (17), and applying the sine rule from (15) we get

$$\frac{d\psi}{dt}=\frac{a_j}{k}\cos\theta\frac{dm}{dt} \qquad (18)$$

Or

$$\Delta\psi=\frac{a_j}{k}\cos\theta\Delta m \qquad (19)$$

which is same as equation (8) above stated. Δm is replaced by $u-v$, since u is the mean motion of *śīghra*, v is the mean motion of the planet. The situation can be understood by the fact that the *śīghrocca* (the sun itself here) as well as the planet are in continuous motion. Therefore, the angle, which decides the stationary point for the onset and ending of retrograde motion is influenced by both these motions. The same idea has been utilized in the *Brahmatulya-udāharaṇam*.

The derivation is easily achieved by the use of calculus by calculating the rate of change of the two motions (Green, 1985 and Sriram, 2019). The same result has been arrived at by Bhāskarācārya, by a completely different approach.

2.3 The duration of retrograde motion

The time taken by the planet to move from one stationary point (before opposition) to the other, decides the duration of the retrograde motion. To obtain this, we commence our discussion by arriving at the basic condition for apparent motion of the planet as seen from the earth to be zero. This idea is represented by Figure 5 where the linear velocities (v' and u') of the planet J and the earth E are represented by JV' and EU'. These are perpendicular to the lines SJ and SE respectively.

As seen from the earth, the rate of motion or speed of J is the projection JV' along the line perpendicular to JE, which is $v'\cos\theta$. We need to include the projection of the speed of earth also along the same direction (perpendicular to JE), which is $u'\cos\Psi$. The sum of the two should be zero for the planet to be stationary.

$$u'\cos\Psi+v'\cos\theta=0$$

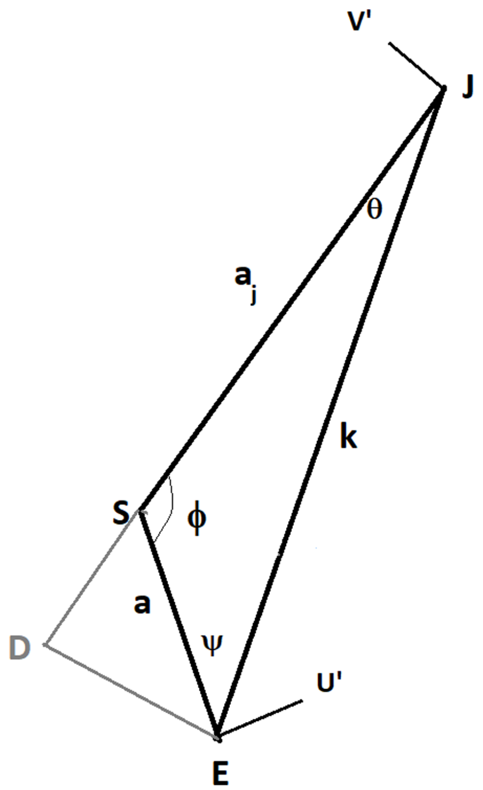

Figure 5 The linear velocities *u*' of the earth and *v*' of the planet are represented by EU' and JV'.

or

$$\frac{u'}{v'}=-\frac{\cos\theta}{\cos\psi} \qquad (20)$$

This can be further simplified. Consider SD extended along JS so that ED is perpendicular to JD.

$$\begin{aligned} a_j = JD-SD &= k\cos DJE - a\sin SED \\ &= k\cos\theta - a\cos\phi \end{aligned} \qquad (21)$$

Similarly, by drawing a perpendicular from S on to JE we can get

$$k=a_j\cos\theta+a\cos\phi \qquad (22)$$

Further, we have from the law of cosines, (14) can be written as

$$k^2=a^2+a_j^2-2aa_j\cos\phi \qquad (23)$$

Using equations (20 and (22) we get

$$\frac{u'}{v'}=\frac{-\cos\theta}{\frac{(-a_j\cos\theta+k)}{a}}=\frac{-a\cos\theta}{k-a_j\cos\theta} \qquad (24)$$

Let us consider the angular speeds of the planet and earth as v and u so that $u' = au$ and $v' = a_j v$. Then equation (24) becomes

$$\frac{au}{a_j v} = \frac{-a\cos\theta}{k - a_j\cos\theta} \tag{25}$$

Eliminating $\cos\theta$ and k in the above equation using the equations 21 to 23, we get,

$$\cos\phi = \frac{va_j^2 + a^2 u}{aa_j(u+v)} \tag{26}$$

By substituting b for a_j this essentially reduces to conventional formula given in standard texts books of spherical astronomy (see for instance Green, 1985).

$$\cos\phi = \frac{(a^2 u + b^2 v)}{ab(u+v)} \tag{27}$$

Upon application of Kepler's law $u{:}v = (a/b)^{-3/2}$, this further leads to the conventional formula

$$\cos\phi = \frac{\sqrt{a}\sqrt{b}}{a - \sqrt{a}\sqrt{b} + b} \tag{28}$$

Equation (26) has been modified to be expressed in terms of the number of revolutions n_j for planet and n_s for the sun, in a *Mahāyuga* and the ratio of the planetary radii as (Sriram, 2019)

$$\cos\phi = \frac{\left(\frac{a}{a_j}\right)^2 + \frac{n_j}{n_s}}{\left(1 + \frac{n_j}{n_s}\right)\frac{a}{a_j}} \tag{29}$$

The estimated values of the angle for the different planets are shown in Table 1.

The angle corresponds to a time t for reaching the stationary point from opposition. There are two solutions for the cosine function. The interval between the two stationary points is the duration of retrograde motion.

3 Examples from *Brahmatulya-udāharaṇam*

The examples are for the date given as *samvat* 1685 (which corresponds to 1627 CE). The *śaka* is given as 1551 and this corresponds to 1629 CE. The date is given as *Śravaṇa kṛṣṇa* (10). The *Ahargaṇa* count is given as 163063. The ambiguity about the year was resolved with the value of *Ahargaṇa* as 1629 CE. The date corresponds to August 14.

The mean position of the sun is $4^R|4°|36'|5'' = 124°|36'|15''$ which gets corrected to the true position of $4^R|3°|1'|11' = 123°|1'|11''$; the *gati* or the daily motion is calculated as $57'|28''$.

We consider the three planets Mars, Mercury and Jupiter and estimate the status of retrograde motion as provided in the text.

The mean longitude of the planet is corrected by *mandaphala* (first equation) which takes care of the eccentricity of the orbit. Further another correction called the second equation *śīghraphala* also is added to get the true positions, which essentially takes care of the fact that the five planets do not go around the earth like the moon and the sun. The other corrections are listed in the chapter called *spaṣṭādhikāra* involving 4 steps.

In what follows, we shall consider the individual planets one by one.

3.1 Jupiter

गुरोः वक्रभागाः उक्ताः १२५ गुरोः द्राक्केन्द्रभोग्यः १९८।४२।५८ विशोध्य शेषं ७३।४२।५८ । अस्य कलिकाः द्राक्केन्द्रभुक्त्या ५४।२ विभक्ताः लब्धं दिनादि ८१।५१।२२ । अधिकत्वात् एतैर्दिनैः गुरोर्वक्रः गतः ज्येष्ठशुदि ४ शनौ वक्री गुरुः ८।३८ दृष्टव्यम् । अथ मार्गसाधनम् । गुरोः द्राक्केन्द्रभागाः १९८।४२।५८ गुरोः उक्तमार्गभागाः २३५ अनयोरन्तरे जातं ३६।७७।२ । एतैरंशैर्गुरोर्मार्ग एष्यः । दिनानयनम् । अस्य कलिकाः द्राक्केन्द्रभुक्त्या विभक्ताः लब्धं दिनादि ४०।१७।२६ एतैर्दिनैर्मार्ग एष्यः ।

The calculation for Jupiter involves the stationary points situated at 125° and 235°. The estimated *śīghra-kendra* is $6^R|18°|42'|58'' = 198°|42'|58''$. Since this is more than 125°, we notice that the retrograde motion has already commenced. The daily motion of *śīghra-kendra* is $54'|2''$. The covered angle of $73°|42'|58''$ (which is $198°|42'|58'' - 125°$), therefore, corresponds to 81 days|51 *ghaṭi* |36 *vighaṭi*. This corresponds to *jyeṣṭha śukla* 4. Now we may proceed to estimate the date for the stationary point corresponding to the end of retrograde motion. This requires $36°|17'|02''$ to be covered at the dail motion of $54'|2''$. It takes 40 days 17 *ghaṭi* 26 *vighaṭi*. The corresponding *tithi* will be *Āśwayuja śukla* 5.

Table 2 shows these dates along with the dates provided by the Occult software as May 25 (3 days after new moon) and September 23 (6 days after new moon). It may be seen that the dates agree within a day.

3.2 Mars

अथ भौमस्य द्राक्केन्द्रभागाः ११२।२९।३२ उक्तवक्रभागेभ्यः एभ्यः १६३ ऊनत्वात् एष्यः वक्र इति जातम् । तद्दिनानयनम् ।

For the day of the calculation the *śīghra-kendra* of Mars is $3^R|22°|29'|34''$ which is $112°|29'|34''$. In order that the required value of 163° is to be achieved for the onset of retrograde the difference of $50°|30'|28''$ is yet to be covered. The rate of change of this angle called *śīghra-kendra gati* is $27'|8''$. Therefore, the number of days needed to achieve 163° can be calculated as 111days 41 *ghaṭi* 16 *vighaṭi*.

Adding 111 days to *Śravaṇa kṛṣṇa* 10 we get *Mārgaśira śukla* 1, which is included in the Table. We notice that the Occult software gives the date as November 23 and the *tithi* is *śukla* 8. Thus, there is a difference of 7 days.

3.3 Mercury

अथ बुधस्य द्राक्केन्द्रभागाः १८८।१२।१७ मार्गी बुधः कदा कदा भविष्यति तयालोकनार्थम् । तथा बुधस्य उक्तमार्ग भागाः २१५ उक्तमार्गभागेभ्यः द्राक्केन्द्रभागस्य ऊनत्वात् एष्यमार्गः । अस्यान्तरम् २६।४७।४३। एतैरंशैः बुधमार्गः एष्यः । दिनानयनार्थम् । अस्य कलाः द्राक्केन्द्रभुक्त्या १८५।५० ।

अनया विभक्ता लब्धं दिनादि ८।३९।५ यद्दिने बुधः स्पष्टः तद्दिना एतैर्दिनैः मार्गः एष्यः । भाद्रपदशुद्धि चतुर्थी ४ बुधे घटी ३९ प ५ समये मार्गी बुधः । तिथिपत्रे दृष्टव्यम् । इति बुधस्य वक्रमार्गो साधयित्वा ।

The calculation for Mercury also follows the same procedure. The *śīghra-kendra* is given as $6^R|8|12'|17'' = 188°|12'|17''$. The stationary point corresponding to the onset of retrograde motion is 145°, which implies that it is already over for the date of calculation. The difference of $43°|12'|17''$ indicates the angle that has already been covered. We can calculate the date of ending of the retrograde for which the angle should be 215°. The daily motion of $185'|50''$ can be used to calculate the corresponding number of days. The result of 13 days 56 *ghaṭi* and 57 *vighaṭi* can be used to find the date as *Śravaṇa śukla* 11. The calculations can be extended to find the end of retrograde motion. The other stationary point at 215°. This will be achieved after the coverage of $26°|47'|43''$ and this takes 8 days| 39 *ghaṭi* and 5 *vighaṭi*. The corresponding *tithi* is *Bhādrapada śukla* 4.

From the Table 2 we notice that there is a systematic shift of 7 and 9 days as compared to the results from Occult.

4 Discussion

The dates provided in the above example are in the lunar calendar system. The Occult software provides the phases of the moon and within a day these values can be verified. We notice from Table 2 that for Jupiter the agreement is very good while for the other two the error is large. One of the reasons is that the rate of change of *śīghrocca* is taken as on the date of calculation. It is influenced by the elliptical orbit of earth and of the planet. Similarly, the values tabulated in Table 1 for the requirements of stationary points are also subject to small variations.

The deviation of the values for Mercury is beyond any correction. This large difference is probably due to the error in fixing the position of the planet itself. The calculations provide a duration of 22 days which is in order. The true position is not given precisely as has been done for other planets. It just puts it as $4^R|52° = 172°$. Therefore, a revision based on precise position would provide a more meaningful result.

This chapter covering *udayāsta* and retrograde motion forms an integral part of almost all texts. One specific example on this is very interesting. Mercury rises in the east for a *śīghra-kendra* of 205. On the date of calculation its value is about 188 as shown above and is very close to this. Since the *gati* (daily motion) also is known as $185'|5''$ we can find the time needed which turns out to be just 5 days ahead, *Bhādrapada śukla* 1. Here the author points out - "*Udaya* recorded in *tithipatra* should be looked into". As is well known, the angle above the horizon for Mercury is fixed at 14° for calculations pertaining to visibility. However, owing to its eccentric orbit, there can be considerable variation.

5 Conclusions

In this paper we discuss the examples of retrograde motion as derived from the *Brahmatulya-udāharaṇam* of Viśvanātha. The formula for this as devised by Bhāskarācārya are provided based on the earlier works. This shows that a derivative of sine was arrived at for getting the rate of change of motion. The examples are com-

Table 2 Dates of stationary points for the year 1629 CE.

Planet	*tithis* as per calculations		equivalent dates	dates as per Occult	Remarks
Jupiter	Onset	*Jyeṣṭha śu* 6	26th May	25th May	Difference of 1 day
	End	*Bhādrapada śu* 6	23rd September	23rd September	Agreement
Mars	Onset	*Mārgaśira śu* 1	15th November	23rd November	Difference of 7 days
Mercury	Onset	*Śravaṇa śu* 11	11th August	8th August	Difference of 3 days
	End	*Bhādrapada śu* 4	23rd August	31st August	difference of 8 day

pared with the calculations from Occult software. The values for Mars and Jupiter agree approximately while for Mercury there is not a very good agreement, which is possible because of the error in fixing its position.

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Bibliography

[1] Chatterjee B. *Śiṣyadhīvṛddhida tantra of Lalla, with Commentaries of Mallikārjuna Sūri*, Indian National Science Academy, New Delhi, 1981.

[2] Green R. M. *Spherical Astronomy*, Cambridge University Press, 1985.

[3] Rao Balachandra S. and Uma S. K. *Karaṇakutūhalam* of Bhāskarācārya, Indian National Science Academy, 2008.

[4] Sarma K. V. and Subbarayappa B. V. *Indian Astronomy: a Source Book*, Nehru Centre, Bombay, 1985.

[5] Somayāji D. A. *A Critical Study of Ancient Hindu Astronomy*, Karnatak University, Dharwar, 1971.

[6] Sriram M. S. *Grahagaṇitādhyāya of Bhāskarācārya's Siddhānta śiromaṇi*, in *Bhāskara-Prabhā*, eds. K Ramasubramanian, Takao Hayashi and Clemency Montelle, Hindustan Book Agency, 2019.

[7] `https://en.wikipedia.org/wiki/Apparent_retrograde_motion#/media/File:Retrogradation.svg`